Ir. F. K. Ligtenberg

STABILITY AND PLASTIC DESIGN (2)

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In consequence of the slenderness of the structure, the collapse load of a portal frame may be lower than is indicated by the elementary methods of plastic design. In this paper it has been endeavoured to assemble the now available data in such a form as to enable this effect to be estimated. To this end, a hypothetical buckling load P_e *has been deduced from the actual collapse load* P_{cr} *and the ultimate load* P_p*, according to plastic design. This load* P_e *is determined from* $1/P_{cr} = 1/P_p + 1/P_e$.
From P_e *it is possible to deduce an effective length* l_e *which depends upon the ratio of the direct force in the bottom column to the bearing reaction of the bottom beam, upon the boundary conditions of the bottom column, and upon the shear force to be transmitted by these columns. Although there is no "exact" theoretical basis for the computation rule obtained, the collapse loads of portal frames can nevertheless be satisfactorily predicted. For the designer who bases himself on the methods of plastic design it is important to have some warning of the circumstances in which he must provide diagonal bracings or other means of ensuring the stability of a portal frame structure.*

0 Introduction

In consequence of the slenderness of the structure, the collapse load of a portal frame may be lower than is indicated by the elementary methods of plastic design. This problem of frame instability has also received a good deal of attention from investigators in many other research laboratories. In this connection the name of Baker (Cambridge), Beedle (Lehigh University) and Wood (Building Research Station) call for mention. Despite all the investigations that have been carried out, not much progress has been made beyond the point where the behaviour of a single column with known boundary conditions can be predicted. With the aid of such data it is possible to perform a complex calculation – involving the use of electronic computers – for investigating the behaviour of a simple portal frame under increasing load.

Simple procedures for approximately taking account of the effect of the occurring deformations upon the distribution of forces have, inter alia, been developed by Loof and Berkelder in the Stevin Laboratory, Delft. However, this research also has not yet made progress to the extent that practical problems can be solved in a reasonably short time.

In the practical application of plastic design it is, however, of great importance that the designer should be able to judge whether the elementary methods of plastic design provides an acceptable approximation of the actual collapse load. To meet this desire, an attempt was made to obtain, on the basis of the available data, at least some insight into the principal factors which are responsible for the fact that the collapse load calculated by means of the elementary analysis is not entirely attained.

The starting point adopted was the only simple formula that can be considered suitable for the purpose, namely:

$$\frac{1}{P_{cr}} = \frac{1}{P_p} + \frac{1}{P_e}$$

where: P_{cr} = the actual collapse load
P_p = the ultimate load according to the elementary plastic analysis
P_e = the buckling load, according to EULER, for a column with a length to be further determined

Various investigators (including W. MERCHANT [1]) have used this formula. In most cases a predetermined value was assigned to P_e (e.g., the elastic buckling load of the portal frame as a whole). In the preceding paper a method of determining P_e experimentally by means of caricature models was indicated. The theory underlying this method is somewhat disputable, however, so that it is better to achieve, as far as possible, realistic stiffness ratios in the "caricature" models.

1 Preliminary investigation

In order to find out whether something could be attained with a method of this kind, a large number of data relating to tests with portal frames having one or more storeys (constructed to scales ranging from full size to 1 : 20) have been collected from the literature. Very important information was obtained more particularly from a series of tests conducted by Low, in which numerous small models of portal frames comprising 3, 5 or 7 storeys were tested to failure. Now that P_{cr} (the observed collapse load)and P_p (the calculated ultimate load) were known, it was possible to deduce P_e from the formula:

$$\frac{1}{P_{cr}} = \frac{1}{P_p} + \frac{1}{P_e}$$

P_e was converted and expressed in an effective length l_e of the column, according to the formula:

$$P_e = \frac{\pi^2 EI}{l_e^2}$$

Low's fairly large series of tests (34 portal frames) was most suitable for obtaining a preliminary idea of the important influencing factors involved.

In Fig. 1 three lines have been drawn which give a somewhat too unfavourable estimate of the effective length for structures 3, 5 and 7 storeys in height respectively. The relevant formula is:

[1]) W. MERCHANT, "The failure load of rigid jointed frameworks as influenced by stability". Structural Engineering, 32, No. 7, 185 (July 1954).

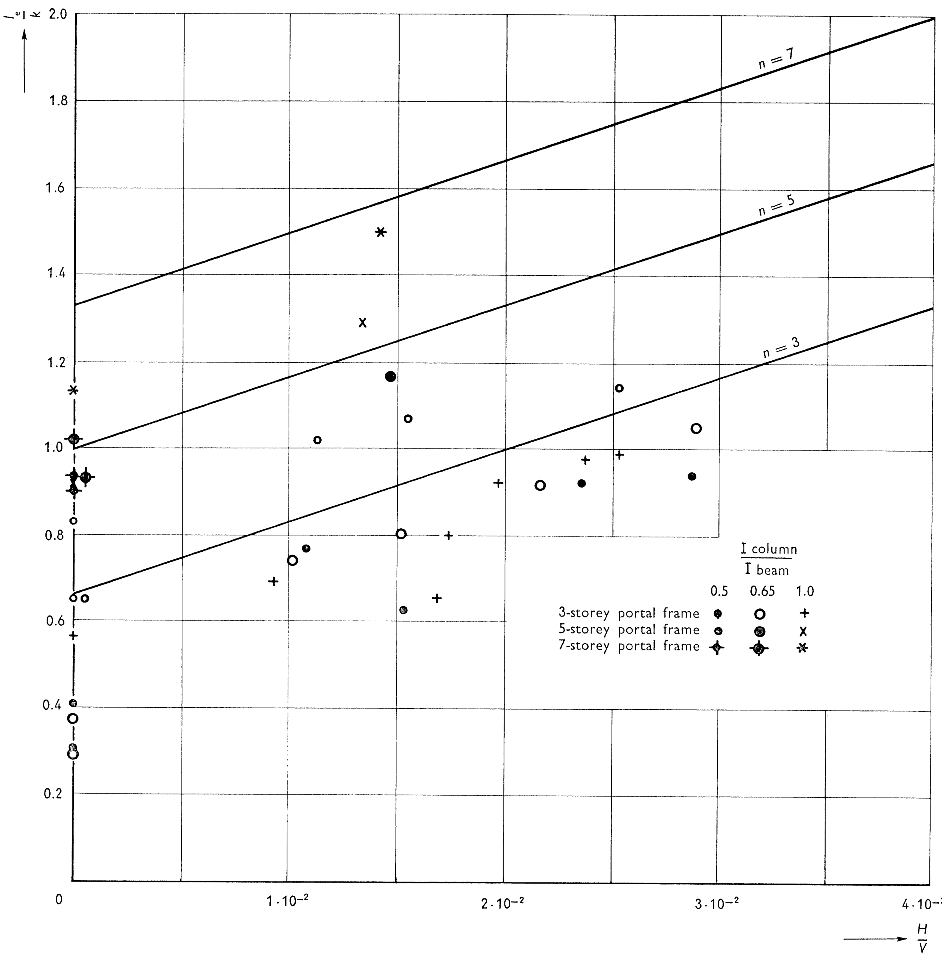


Fig. 1. Low's series of tests.

$$l_e = \left(\frac{n+1}{6} + \frac{100}{6}\frac{H}{V}\right) k$$

where: n = the ratio of the direct force (normal force) in the bottom columns to the vertical loads on the bottom beam (in Low's tests all the loads per storey were equal, so that n in the above formula is equal to the number of storeys)

H/V = the ratio of the total horizontal load to the total vertical load

k = the length of the bottom columns

This formula is very simple, but cannot be applied to all cases. Some further important factors emerge in the other portal frames investigated.

Low's columns were fixed at the base. Obviously, a pin-jointed base will produce a more unfavourable situation. Further information with regard to this is provided by the test series A and B conducted by LU and GALAMBOS.

Some of the portal frames investigated by BAKER were subjected to very large horizontal loads. In that case $100H/6V$ would become far too large. Also, it is clear that H/V – especially if the collapse mechanism calculated in the elementary analysis is a beam mechanism (Fig. 2) – can easily cause deviations from the collapse load. However, if H is so large that a combined mechanism or panel mechanism occurs, then there is no reason to anticipate a considerable influence of this kind. The same is true of portal frames such as north-light frames (saw-tooth roof frames) in which the inclined beams exert considerable horizontal forces on the columns anyway.

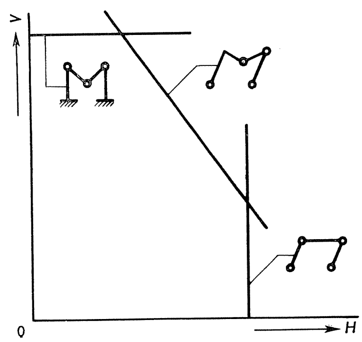


Fig. 2.

Because of these various factors the formula has been modified to:

$$l_e = \frac{n-5}{6} \cdot k + l + \frac{100}{6} \frac{H}{V} \cdot k$$

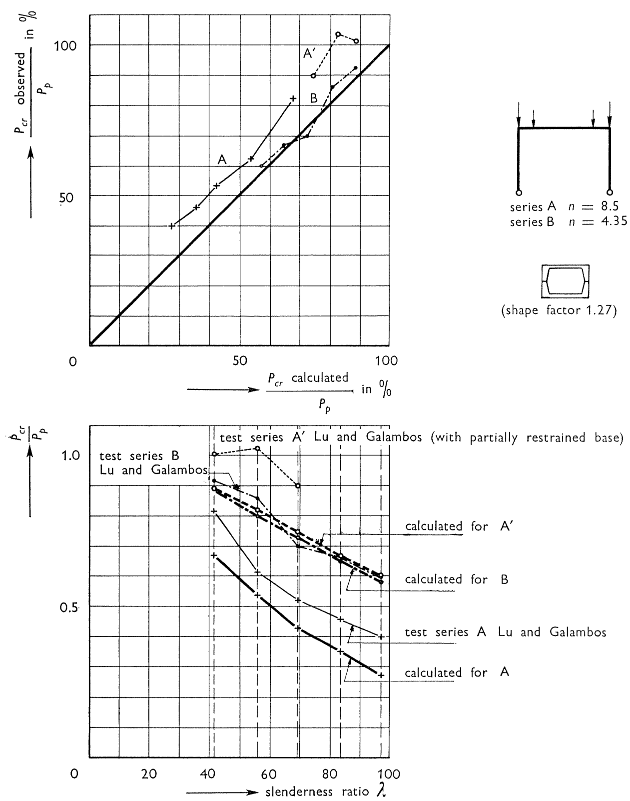
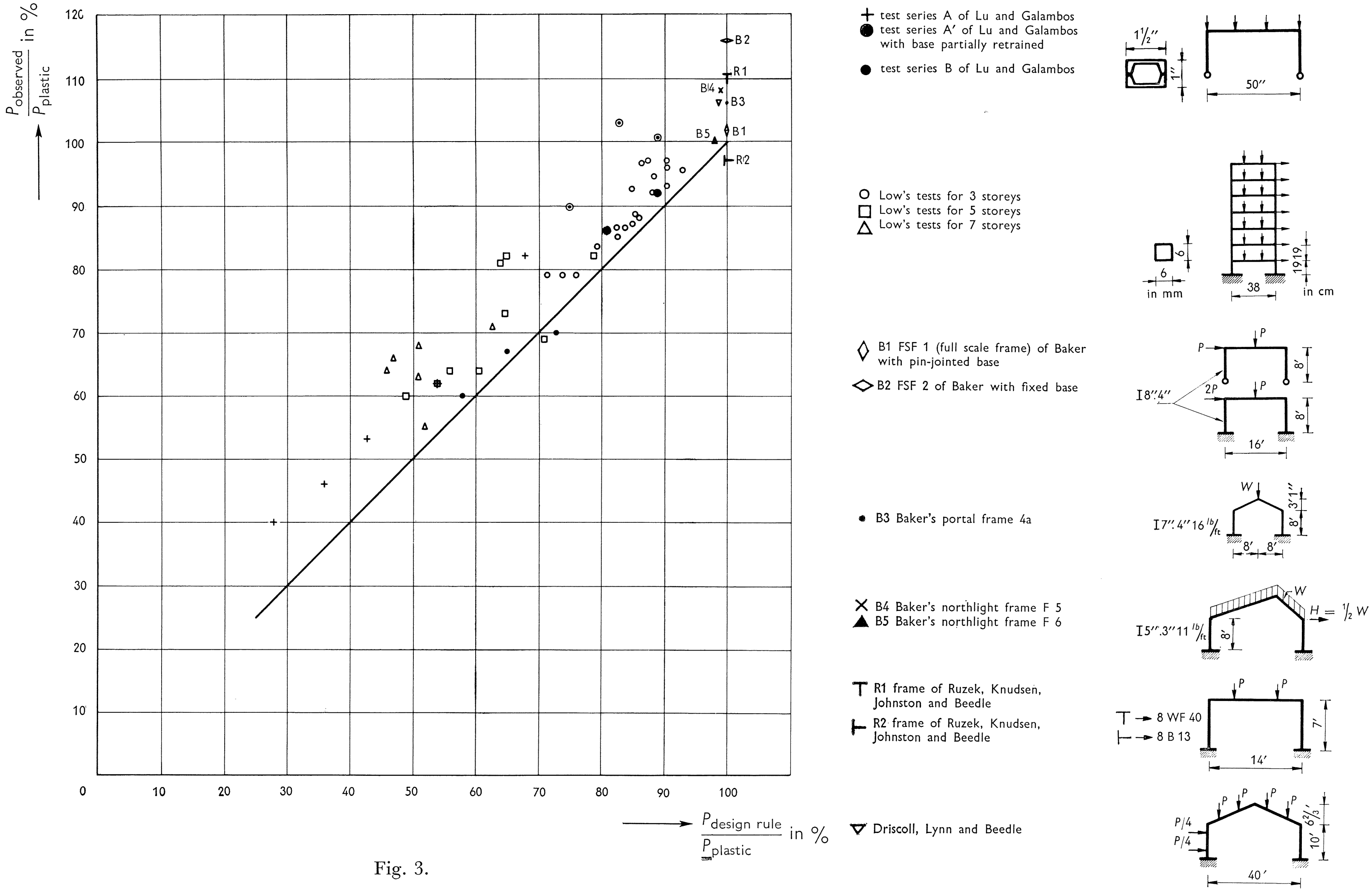


Fig. 4.



$\frac{P_{observed}}{P_{plastic}}$ in %
$\frac{P_{design\ rule}}{P_{plastic}}$ in %
0
10
20
30
40
50
60
70
80
90
100
110
120
B2
R1
B4
B3
B5
B1
R2
test series A of Lu and Galambos
test series A' of Lu and Galambos with base partially retrained
test series B of Lu and Galambos
Low's tests for 3 storeys
Low's tests for 5 storeys
Low's tests for 7 storeys
B1 FSF 1 (full scale frame) of Baker with pin-jointed base
B2 FSF 2 of Baker with fixed base
B3 Baker's portal frame 4a
B4 Baker's northlight frame F 5
B5 Baker's northlight frame F 6
R1 frame of Ruzek, Knudsen, Johnston and Beedle
R2 frame of Ruzek, Knudsen, Johnston and Beedle
Driscoll, Lynn and Beedle
1 1/2"
1"
50"
6
6
in mm
38
19 19
in cm
P
2P
8'
16'
I8".4"
W
3'1"
I7".4"16 lb/ft
H = 1/2 W
I5".3"11 lb/ft
8 WF 40
8 B 13
7'
14'
P/4
6 2/3'
10'
40'

Fig. 3.

where l denotes the effective length of the bottom column, if the bottom beam is regarded as infinitely stiff (but able to undergo displacement, if any). Hence for a fixed base we have $l = k$, for a pin-jointed base we have $l = 2k$, and for a fixed base and a beam restrained by rigid walls we have $l = 0.5k$. The term $\frac{100}{6}\frac{H}{V}\cdot k$ must not exceed the value $^2/_3k$, which is therefore the case for $H/V = 4\%$. There are not enough data available to give a better or more accurate rule for this.

The result is indicated in Fig. 3.

The collapse load represented in this fashion satisfactorily reflects the influence of various factors, even when considered in detail, e.g., compared with the test series of Lu and Galambos, in which one and the same type of portal frame with progressively increasing column lengths was tested (see Fig. 4), or with their calculations for a particular case (see Fig. 5).

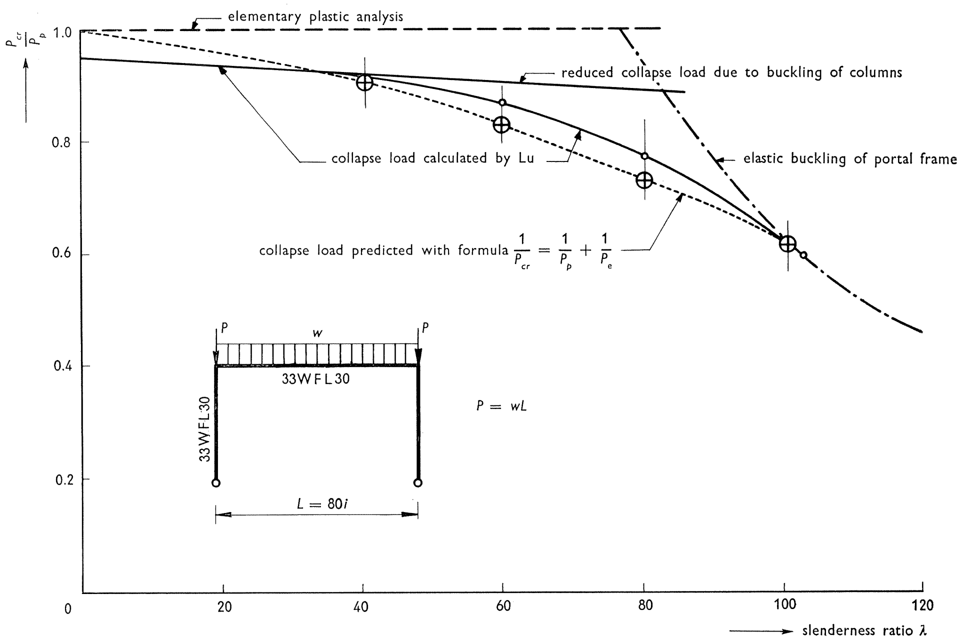


Fig. 5. Comparison with Lu's theoretical results.

2 Preliminary conclusions

The method of determining the collapse load of a portal frame, as described in the foregoing, can be expected to be suitably serviceable for practical purposes. The object is not so much to obtain very great accuracy as to provide a *warning* to the designer if a structure which he has designed should happen to have an excessively low collapse load in comparison with the collapse load determined from the elementary analysis.

In such a case the stability will have to be ensured by fairly simple means, such as, for example, the provision of local wind bracings or rigid diaphragms. The rule for calculation will have to be so extended as to enable the effect of such improvements to be ascertained and their adequacy to be judged. Unfortunately, the number of available data is still very limited. Not only are no experimental data concerning the effect of wind bracings available, but also the number of portal frame shapes investigated is very small. The most commonly employed multiple portal frame types (Fig. 6) have not yet been the subject of any investigation. Far too few data are available on portal frames in which the columns are pin-jointed at the base or in which one of the columns is stiffened in some degree. Only a limited proportion of all the portal frames investigated consisted of I-section members. Most of the smaller models were constructed of members having a rectangular or closed box-type cross-sectional shape.

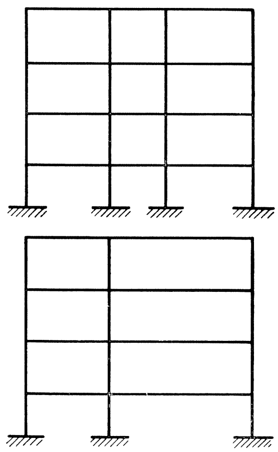


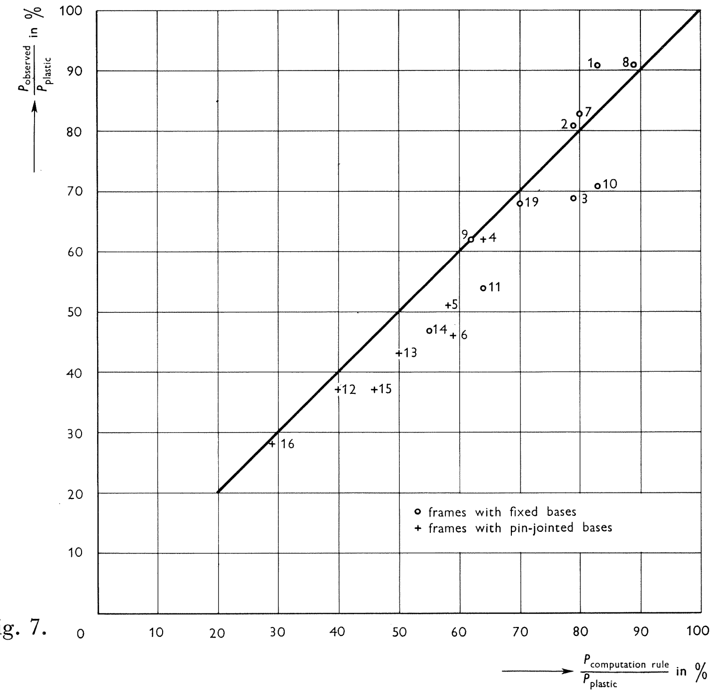
Fig. 6.

Because of these circumstances, research into the structural strength of a number of portal frames has been undertaken in collaboration with the Research Committee on Steel Structures of the Vereniging van Constructiewerkplaatsen (Netherlands Structural Steelwork Fabricators' Association). Some of these tests are reported in the following.

3 Test series

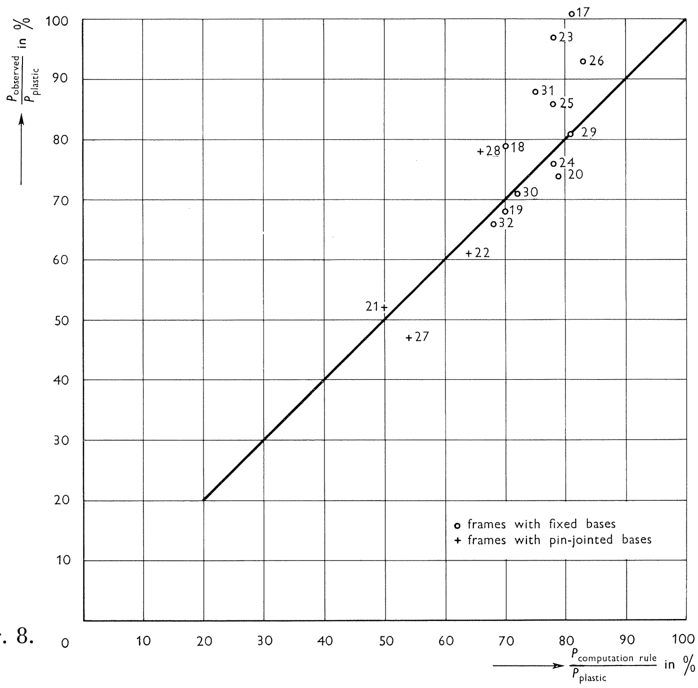
So far, a series of model tests has been carried out in which the test arrangement is comparable with that adopted by Low. Small portal frames with members of rectangular cross-section were loaded to failure. These frames can be subdivided into three groups:

1. 16 single-bay portal frames comprising 3, 5 or 7 storeys, in which the effect of the horizontal force and of the pin-jointed base was investigated in order to obtain some extension of Low's test series (see Table I). The results have been plotted in graph form in Fig. 7. The test results are in fairly good agreement with the calculation rule indicated on page 16.
2. 16 two- or three-bay portal frames, with the object of including more currently employed portal frame shapes in the investigation (see Table II). The results are plotted in the graph in Fig. 8. Although the actual collapse load is usually somewhat higher than the value obtained by means of the computation rule, especially in the cases with fixed-base columns, there is nevertheless no very good reason for establishing a different rule for portal frames of this type.



$\frac{P_{observed}}{P_{plastic}}$ in %
$\frac{P_{computation\ rule}}{P_{plastic}}$ in %
o frames with fixed bases
+ frames with pin-jointed bases

Fig. 7.



$\frac{P_{observed}}{P_{plastic}}$ in %
$\frac{P_{computation\ rule}}{P_{plastic}}$ in %
o frames with fixed bases
+ frames with pin-jointed bases

Fig. 8.

3. 4 portal frames in which one of the columns is of double construction (see Table III). The results are plotted in the graph in Fig. 9.

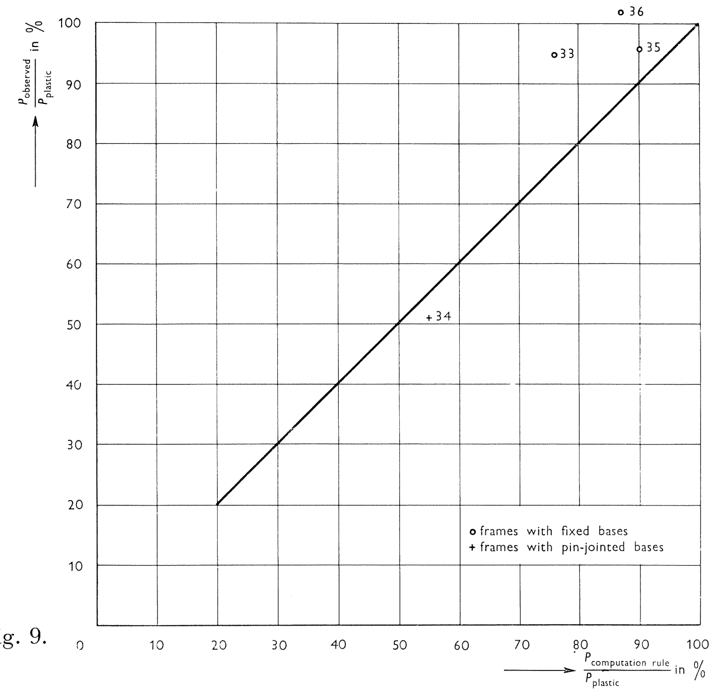


Fig. 9.

As was to be expected, a stiffening of this kind has a favourable effect upon stability. Such cases do not call for the establishment of a different calculation rule.

The investigations are continuing. In the first place, the effect of the cross-sectional shape of the members (I-section) will be studied.

4 Summary

On the basis of available experimental results concerning the collapse loads of portal frames it was possible to establish an empirical design rule which enables the effect of the slenderness and of various other factors on the collapse load to be estimated with a reasonably good approximation.

Some supplementary tests conducted by the author did not provide sufficient grounds for modifying the design rule. The effect of the cross-sectional shape of the structural members will have to be further investigated.

The importance of such a design rule to the designer is more particularly that it provides him with a simple warning if he designs a structure to which the elementary plastic design is no longer correctly applicable.

In view of the very considerable effect of the number of storeys it is advisable, in the design of high buildings, to pay a good deal attention to the prevention of a panel mechanism (e.g., by the provision of wind bracings or stiffenings).

For the dimensional design of the individual columns the normal procedure should of course be applied. In the present case only the overall stability of the framework has been investigated.

References

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2. Low, The Institution of Civil Engineers, Proceedings, July 1959, Vol. 13, Paper No. 6347.
3. LU (thesis Lehigh University, 1960).
4. RUZEK, KNUDSEN, JOHNSTON and BEEDLE, Welding Journal, Vol. 33, September 1954.
5. DRISCOLL and BEEDLE, Welding Journal, Vol. 36, June 1957.
6. BAKER, HORNE and HEYMAN, The Steel Skeleton, II.
7. LU, A survey of literature on stability of frames. Welding Research Council Bulletin, No. 81, Sept. 1962 (this publication gives 146 literature references).

TABLE I

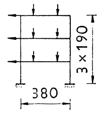
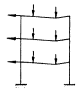
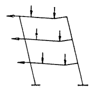
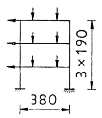
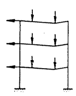
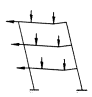
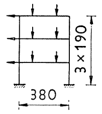
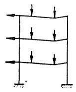
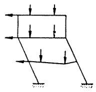
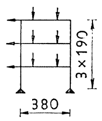
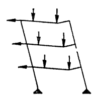
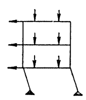
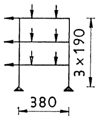
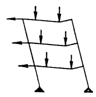
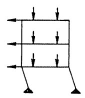
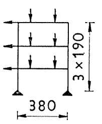
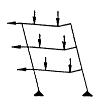
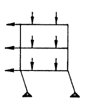
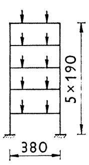
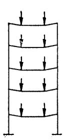
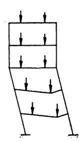
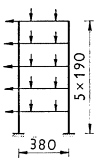
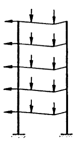
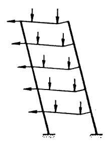
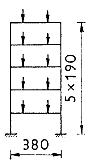
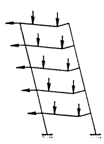
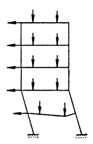
frame No.	frame shape with dimensions in mm	collapse shape according to elementary theory	actual collapse shape	beams in mm columns in mm σ_{vl} beam in kg/cm² σ_{vl} col. in kg/cm² M_p beam in kgcm M_p col. in kgcm	H/V in % P_p in kg P_w in kg P_{cr} in kg P_w/P_p in % P_{cr}/P_p in %
1				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	2 255 231 212 91 83
2				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	6 255 207 202 81 79
3				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	10 255 177 202 69 79
4				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	2 246 153 156 62 64
5				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	6 231 117 133 51 58
6				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	10 219 102 129 46 59
7				6.5 × 6.5 6.5 × 6.5 3000 3000 206 206	0 435 360 347 83 80
8				6.1 × 6.1 6.0 × 8.0 2000 2710 111 262	2 230 210 204 91 89
9				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	6 385 240 238 62 62

TABLE I (continued)

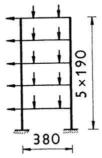
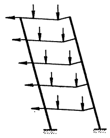
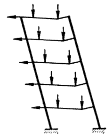
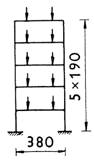
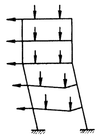
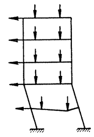
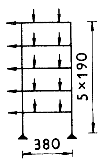
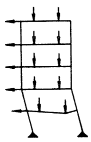
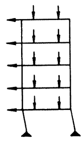
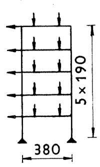
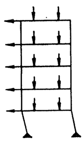
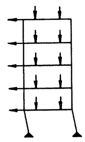
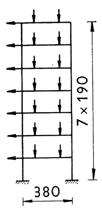
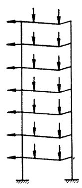
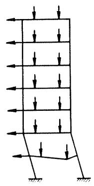
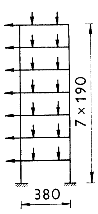
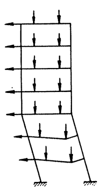
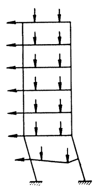
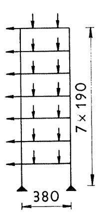
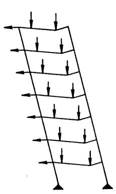
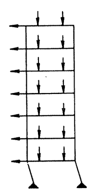
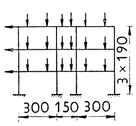
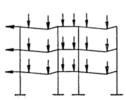
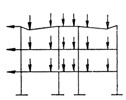
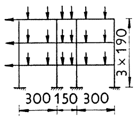
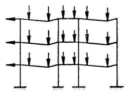
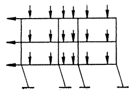
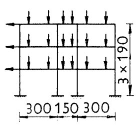
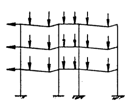
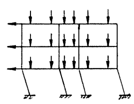
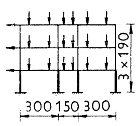
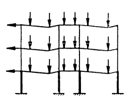
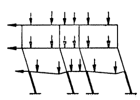
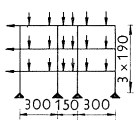
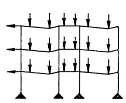
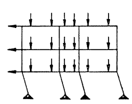
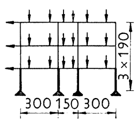
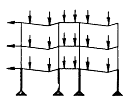
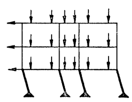
frame No.	frame shape with dimensions in mm	collapse shape according to elementary theory	actual collapse shape	beams in mm columns in mm σ_{vl} beam in kg/cm² σ_{vl} col. in kg/cm² M_p beam in kgcm M_p col. in kgcm	H/V in % P_p in kg P_w in kg P_{cr} in kg P_w/P_p in % P_{cr}/P_p in %
10				6.1 × 6.1 6.0 × 8.0 2000 2710 111 262	6 225 160 186 71 83
11				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	10 345 185 221 54 64
12				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	6 355 130 143 37 40
13				6.5 × 6.5 6.5 × 6.5 2900 2900 200 200	10 235 100 119 43 50
14				6.5 × 6.5 6.5 × 6.5 3000 3000 206 206	1 609 287 336 47 55
15				6.5 × 6.5 6.5 × 6.5 3000 3000 206 206	6 504 189 231 37 46
16				6.5 × 6.5 6.5 × 6.5 3000 3000 206 206	2 567 161 168 28 29

TABLE II

frame No.	frame shape with dimensions in mm	collapse shape according to elementary theory	actual collapse shape	beams in mm top col. in mm bott. col. in mm σ_{vl} beam in kg/cm² σ_{vl} top in kg/cm² σ_{vl} bott. in kg/cm² M_p beam in kgcm M_p top in kgcm	M_p bott. in kgcm H/V in % P_p in kg P_w in kg P_{cr} in kg P_w/P_p in % P_{cr}/P_p in %
17				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 2 840 846 680 101 81
18				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 5 840 660 584 79 70
19				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 8 840 570 584 68 70
20				6.5×6.5 6.5×6.5 6.0×8.0 3090 3090 2710 211 211	262 8 840 624 670 74 79
21				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 2 840 435 422 52 50
22				6.5×6.5 6.5×6.5 6.0×8.0 3090 3090 2710 211 211	262 2 840 510 535 61 64

TABEL II (continued)

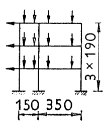
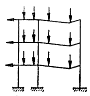
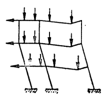
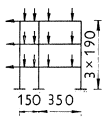
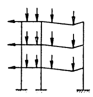
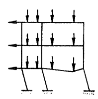
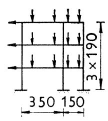
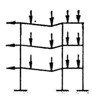
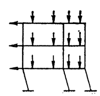
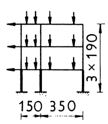
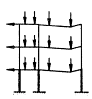
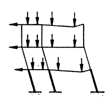
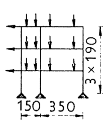
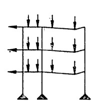
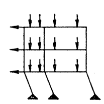
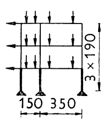
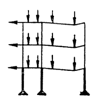
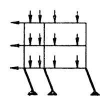
frame No.	frame shape with dimensions in mm	collapse shape according to elementary theory	actual collapse shape	beams in mm top col. in mm bott. col. in mm σ_{vl} beam in kg/cm^2 σ_{vl} top in kg/cm^2 σ_{vl} bott. in kg/cm^2 M_p beam in kgcm M_p top in kgcm	M_p bott. in kgcm H/V in % P_p in kg P_w in kg P_{cr} in kg P_w/P_p in % P_{cr}/P_p in %
23				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 4 414 402 322 97 78
24				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 10 414 315 322 76 78
25				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 10 414 357 322 86 78
26				6.5×6.5 6.5×6.5 6.0×8.0 3090 3090 2710 211 211	262 10 414 387 343 93 83
27				6.5×6.5 6.5×6.5 6.5×6.5 3090 3090 3090 211 211	211 4 414 195 220 47 54
28				6.5×6.5 6.5×6.5 6.0×8.0 3090 3090 2710 211 211	262 4 414 324 273 78 66

TABLE II (continued)

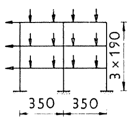
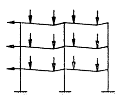
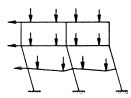
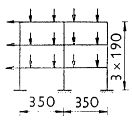
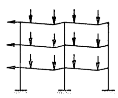
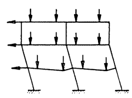
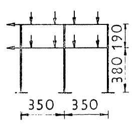
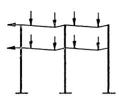
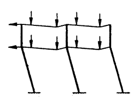
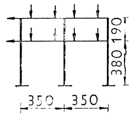
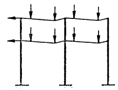
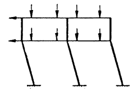
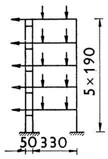
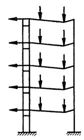
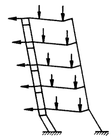
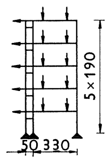
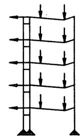
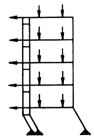
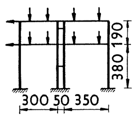
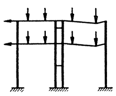
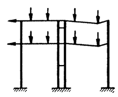
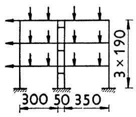
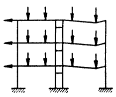
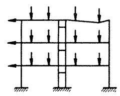
frame No.	frame shape with dimensions in mm	collapse shape according to elementary theory	actual collapse shape	beams in mm top col. in mm bott. col. in mm σ_{vl} beam in kg/cm^2 σ_{vl} top in kg/cm^2 σ_{vl} bott. in kg/cm^2 M_p beam in kgcm M_p top in kgcm	M_p bott. in kgcm H/V in % P_p in kg P_w in kg P_{cr} in kg P_w/P_p in % P_{cr}/P_p in %
29				6.5×6.5 6.5×6.5 6.5×6.5 3080 3080 3080 211 211	211 2 576 465 469 81 81
30				6.5×6.5 6.5×6.5 6.5×6.5 3080 3080 3080 211 211	211 6 576 408 412 71 72
31				6.5×6.5 6.0×8.0 6.0×8.0 3080 2710 2710 211 262	262 2 384 336 286 88 75
32				6.5×6.5 6.0×8.0 6.0×8.0 3080 2710 2710 211 262	262 6 384 256 261 66 68

TABLE III

frame No.	frame shape with dimensions in mm	collapse shape according to elementary theory	actual collapse shape	beams in mm columns in mm σ_{vl} beam in kg/cm^2 σ_{vl} col. in kg/cm^2 M_p beam in kgcm M_p col. in kgcm	H/V in % P_p in kg P_w in kg P_{cr} in kg P_w/P_p in % P_{cr}/P_p in %
33				6.5×6.5 6.5×6.5 3080 3080 211 211	10 510 485 388 95 76
34				6.5×6.5 6.5×6.5 3080 3080 211 211	6 510 260 283 51 55
35				6.5×6.5 6.0×8.0 2900 2710 200 262	10 338 336 306 96 90
36				6.5×6.5 6.5×6.5 3080 3080 211 211	10 534 543 463 102 87