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STRUCTURAL SAFETY

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In this paper some considerations are presented concerning the problem of structural safety, a problem which has been dealt with in several publications, but for which the finding of a solution proves to be very difficult. This is mainly due to the lack of sufficient quantitative information about the variables concerned. For this reason the permissibility of the calculated stresses, forces, etc. in structures can be judged only in a manner whose accuracy is not comparable with the accuracy with which information concerning strength, for example, is obtained.

Recognition of the fact that even with the usually adopted factors of safety there remains a possibility of the occurrence of an undesirable state of the structure (failure, excessive deformation, etc.) leads to a statistical approach in which this possibility in terms of probability is adopted as a criterion of structural safety.

Since, as already stated, information on the quantities which play a part in connection with safety is lacking, it is, however, not possible to arrive at a complete solution in this way, at least not for the present. Yet it does appear possible to compare the usually adopted degrees of safety for various construction materials. In addition, the statistical safety formula derived here provides the possibility of obtaining, by graphical methods, a clear and conveniently visualised concept of the permissibility of load combinations.

1 The relation between some important factors determining structural safety

1.0 *Introduction*

In the design and construction of load-bearing structures it is not possible to make an exact prediction of the magnitude of the loads that will actually occur or of the loads at which the structure will become unserviceable. The actual loads and the actually available strength will therefore in general differ from the assumed or calculated values. On the one hand, this may result in a deficient safety of the structure or, on the other hand, it may result in wasteful use of material. In dealing with the problem of structural safety the consequences of the development of an undesirable state must be weighed against the measures aimed at limiting the risk of this occurring. As a result of specifying the magnitude of the design loads and permissible stresses, etc. a certain reserve capacity is established in the design calculations of structures, so that in general the above-mentioned uncertainties with regard to loads and strength will not render the structure unfit for service. This margin is therefore conducive to safety, but without the assignment of a quantitative significance to the degree of safety thus provided. This will be further considered later on.

In this paper the concept of safety, which cannot itself be expressed in a numerical value, will be evaluated in terms of the "probability of unserviceability". In proportion as this probability is greater, the safety will be less. In determining the permissible stresses, etc. as adopted at present, the calculus of probabilities as such is as yet seldom employed. In the present paper it will be endeavoured, with the aid of probability calculus, to determine retrospectively the chance of unserviceability which is evidently accepted for various materials by virtue of the regulations relating to them. It emerges that, in the first place, there are a number of factors involved which in the quantitative sense are not yet sufficiently known. Because of this the actual probability of unserviceability cannot be determined; but if the values of a few variables are estimated, then an approximate comparison between the various materials does become possible. A comparison of this kind is of importance in making a choice of material, if the question is posed: is the probability of unserviceability equally great for the various potentially suitable materials, or is this probability in one case much greater than in another, although the regulations have constantly been conformed to?

By collecting more data it will perhaps become possible to determine with greater accuracy the actual chance of unserviceability in the various cases. It might then become appropriate for the relevant authorities – e.g., building inspection departments – to lay down a minimum requirement as to structural safety. Such a requirement will perhaps vary for different kinds of structure, but will be the same for all potentially suitable materials. It will then also be possible to obtain more safety than the requisite minimum; with different materials this may be associated with different costs. Weighing this extra cost against the reduced risk may further affect the choice of material.

In the present paper the first question to be considered is: What safety is provided in present circumstances, and is this safety the same for different materials? Next, the safety formula which is derived is given a graphical interpretation and some conclusions are drawn therefrom.

1.1 *The factor of safety as a criterion of structural safety*

The factor of safety is often employed as a criterion of the safety of a structure. This factor can be defined in various ways. For example:

a. the "stress factor of safety", this being the factor by which the determinative *stress* (or sometimes the force, bending moment, or the like) must be multiplied in order to reach a limit that is considered to be inadmissible;

b. the "load factor of safety", this being the factor by which the determinative *external loading* must be multiplied in order to reach the undesirable state.

If there exists a linear relationship between the loading acting upon a structure and the forces and stresses acting within it, these two safety factors will be equal. For structures which are similar, are constructed of the same material and are intended for the same purpose this factor of safety does indeed constitute a criterion of structural safety. In the case of structurally different structures, or in the case of structurally comparable ones intended to serve different purposes, a higher value of the safety factor will, however, not necessarily guarantee a higher degree of safety.

For a combination of loads, one of the load components, e.g., the dead weight, may be much more accurately known than another, e.g., the live load. On the basis of this consideration certain weights are assignable to these components. If unserviceability occurs at a load or a stress S, while G represents that proportion of the load or stress which is due to dead weight and P represents the proportion due to superimposed loading, then the relationship $S \geqslant n\ (G+P)$ will no longer be valid, but instead: $S \geqslant n_g G + n_p P$ where in general $n_p > n_g$. (In these relationships the overall factor of safety is represented by n, while n_g and n_p are factors of safety associated with the actions of dead weight and of superimposed loads respectively).

Formulae of this form are already employed in various cases. The introduction of this differentiation constitutes an advance on the method in which only one factor of safety is used. In this way, by a correct choice of the factors n_g and n_p, it is possible to express the different character of the loading components.

For equal values of n_g, n_p and the ratio $m = P/G$ for two structures constructed of different materials, however, the safety will not necessarily be the same for both, because in one case the scatter in the material properties may be greater than in the other. According as this scatter is greater, the safety will be less for otherwise similar conditions. Hence it follows that the magnitude of the safety factors employed does not directly provide an indication of the degree of safety. In due course it will be seen that the assignment of different factors to different load components does indeed fit well into the system which makes use of probability calculus.

In order to be able to design with a sufficient degree of safety, certain values have been adopted for the factors of safety. On the basis of the designer's judgment and with the backing of experience, a large number of significant influences are accommodated in these commonly employed factors of safety. There is, however, no objective criterion for determining their magnitude. Since the considerations underlying particular rules and methods of design are moreover often no longer known, or no longer fully known, a transition to exceptional cases or to new construction materials and forms of construction becomes very difficult.

Summarising, it can be stated that:

- in general there exists a difference between the stress factor of safety and the load factor of safety;
- by employing different factors for the different load components it is possible to give expression to the accuracy with which they are known;

- the degree of safety associated with particular values of the stress or load factors is dependent also on the material properties, so that in general these factors do not provide a safety criterion; within certain limits they do, however, constitute a basis for comparison;
- because of the obscurity of the origins of the usual factors of safety, a transition to new cases is not objectively possible.

1.2 *The probability of unserviceability*

A more correct starting point for considerations relating to safety is the "probability of unserviceability". This concept involves only those factors which originate in the loading and in the construction materials or structural members.

Serious errors of design or construction, drastic modifications in the purpose that the structure must serve (resulting in changed loading conditions and requirements), etc. are not amenable to mathematical treatment and will therefore not be taken into consideration.

For every structure – or type of structure – it must be established at what stage it becomes unserviceable. As a result of loading this may occur either because failure develops or because the deformations become inadmissibly large. A "safe" structure is a structure for which it must be considered unlikely that the loads will attain such magnitude as to render it unserviceable. There does, however, remain the probability that this undesirable state will be attained; according as this probability is smaller, the safety will be greater.

Having regard to the above-mentioned limiting stipulations, the chance of unserviceability is considered to be due to variations in the strength and in the loads. Statistical analysis is concerned with such variable quantities. Characteristic thereof are frequency distributions from which the probability of a particular value of the variable being attained can be read. Often the actual frequency distributions are approximated by theoretical distributions; the best known frequency distribution is the so-called normal distribution or Gaussian curve. The probability with which a particular value of the variable quantity will occur depends – besides the shape of the distribution – upon the mean value and the standard deviation.

1.3 *Definitions and notation*

The following comments are presented with a view to providing a closer definition of the concepts of "strength" and "loading".

The external loads acting on a structure cause a force distribution pattern in that structure. Usually the consequences of a load are translated, through the medium of a calculation, into internal actions of forces (force, moment, stress, etc.). The magnitude of these actions is then compared with that value thereof which would give rise to a critical state (failure, yielding or the like).

In the following treatment of the subject the *internal actions of forces* will for the sake of brevity be referred to as "forces". Thus the magnitude of the action caused at a particular point of the structure by the dead weight will be termed "the force G". Similarly, "the force P" is the action caused at a point of the structure by a variable load.

Furthermore the magnitude of the force action at which a critical state develops will be called "the strength S".[1]) The strength S varies for a set of similar members. Members which are apparently identical or which differ so little from one another that the difference is not manifested in the method of analysis are said to be "similar".

The strength can be determined from the test results for a number of structural members or by calculation. In so doing it is assumed that the strength values obtained from these tests and/or calculations are representative of the strength for the cases that arise in an actual structure. In so far as tests are concerned, the material employed, the manner of its preparation and use, etc. should therefore correspond as closely as possible to reality, while a calculation should aim at the best possible approximation of the actual strength.

In certain cases the strength may change during the service life of the structure, due to corrosion, for instance, or suchlike causes. Such a change in strength will have to be expressed in the calculations for determining the structural safety. In the present paper this complication is dealt with for the material timber, whose long-term strength is lower than the strength determined by means of short-term standard tests. Here, too, it will be assumed that the frequency distribution of the strength at a particular age of the structure can serve as a basis for calculating the probability of unserviceability.[2])

The total force (or force action) Q in a structural member occurs as a result of the total external loading upon the structure. For a particular structure this loading will vary in magnitude and in location or distribution at various times. But even for similar structures the loads are not identical. Nor are the forces Q which occur in similar members of similar structures equal.

Of interest are the maximum values of Q which occur in a number of similar members in a certain period of time. Together these maximum values form a frequency distribution from which a mean value and a standard deviation are determinable (see Fig. 1). In the further treatment of the subject the term "the force" will denote the maximum force (or force action) in a structural member during the service life of the structure. In this connection it should be noted that the frequency distribution of the loading depends on the service life of the structure. The probability that a partic-

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[1]) Whether the attainment of a critical state in one member will render the entire structure unserviceable will depend on the number of members composing the structure, the manner in which they are assembled, and the material properties.

[2]) If, for example, during the service life of the structure the strength decreases linearly with time to half the original strength, then it appears – on the assumption of a normal distribution and under conditions which are otherwise similar to those of the examples to be discussed later on – that the strength at three-quarters of the service life provides a good starting point for determining the probability of failure.

ular heavy snow load will occur once in twenty years is, for example, higher than the probability that it will occur once in ten years.

Summarising, the mean values of the variables S and Q are defined as follows:

$\bar{S}$ = the mean strength of similar structural members, as can be determined by testing a sufficient number of members suitable for the purpose;

$\bar{Q}$ = the mean value of the force in a structural member. This mean value can be found as the average of the maximum loads in a series of similar members occurring once during a certain service time;

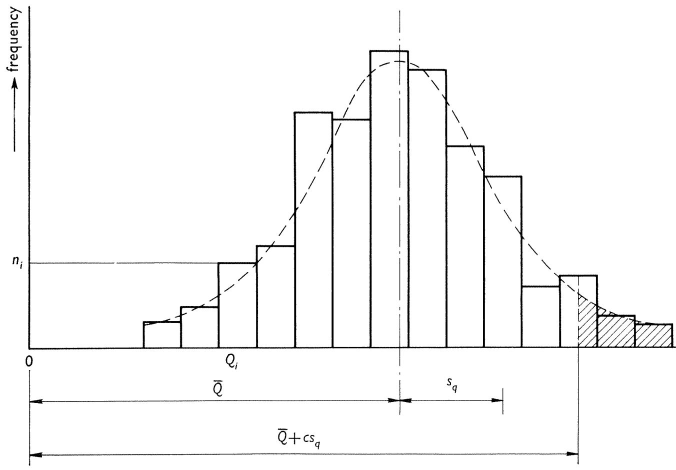

Fig. 1. Diagram representing the frequency of the maximum value of a load Q occurring on a number of similar structural members over a certain period of time. For a number n_i of these members this maximum is located in the interval Q_i. On average the highest value of Q that can be expected to occur is $\bar{Q}$; from the frequency distribution the probability of the occurrence of a maximum load in excess of $\bar{Q}+cs_q$ in the period under consideration follows from the magnitude of c.

Often the total force Q results from a number of components which can be defined in the same way as Q. Thus for example:

$\bar{G}$ = the mean value of the force in a structural member, as a result of the permanent (or dead) loading – averaged over similar structures – on the structure.

Similarly:

$\bar{P}$ = the mean value of the force in a structural member, as a result of the maximum value – averaged over similar structures – of the variable (or live) loading on the structure during the service life thereof.

The various formulae derived are mostly evaluated for the case $Q = G+P$.

1.4 *Application of probability calculus; the statistical safety index f_{st} as a safety criterion*

According to the foregoing definitions, unserviceability of a structural member will occur when the force in it exceeds the strength, i.e., when $Q > S$.

The critical point is reached when $Q = S$, i.e., when $V = S-Q = 0$. The frequency distributions of Q and S together determine the frequency distribution of V. Now we make use of the following expressions:

$$\bar{V} = \bar{S} - \bar{Q} \quad \text{and} \quad s_v = \sqrt{s_s^2 + s_q^2}$$

(S and Q both apply to similar structural members; they are not correlated).

For a particular form of the frequency distribution of V the probability of the occurrence of a particular value of V is again dependent on the mean value $\bar{V}$ and on the standard deviation s_v. The probability of a value of $V < 0$ is determined by the following ratio:

$$f_{st} = \frac{\bar{V}}{s_v} = \frac{\bar{S} - \bar{Q}}{\sqrt{s_s^2 + s_q^2}} = \frac{\bar{S} - \bar{G} - \bar{P}}{\sqrt{s_s^2 + s_g^2 + s_p^2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

In the further treatment of the subject the value of f_{st} thus defined will be called the *statistical safety index*.

It follows from the definition that this statistical safety index is the opposite of the coefficient of variation of V. This is represented in Fig. 2. It must be clearly stated that the value of f_{st} can permissibly be associated with a probability only if the form of the frequency distribution is known. This form is determined by the frequency distributions of the variable quantities S, G and P together, but these are not individually known. Besides, the distribution of S is not necessarily the same for all materials, so that the resultant frequency distribution of V in different cases is not necessarily the same either. Consequently, a particular value of f_{st} is not necessarily always associated with the same probability.

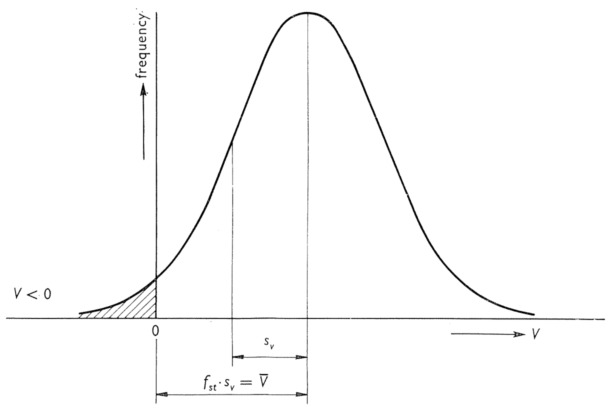

Fig. 2. Frequency distribution of $V = S - Q$. For a particular shape of the frequency distribution the magnitude of $f_{st} = \bar{V}/s_v$ is a criterion for the probability of a value $V < 0$.

So long as the frequency distributions of V for different kinds of similar structural members do not differ too greatly from one another, however, f_{st} can be regarded as a reasonable basis of comparison. These differences are not easily demonstrable, as appears, for example, from Fig. 3. Statistically it is therefore very difficult to ascertain whether any one particular theoretical distribution fits in better with a limited number of experimentally determined values than does another distribution, although the

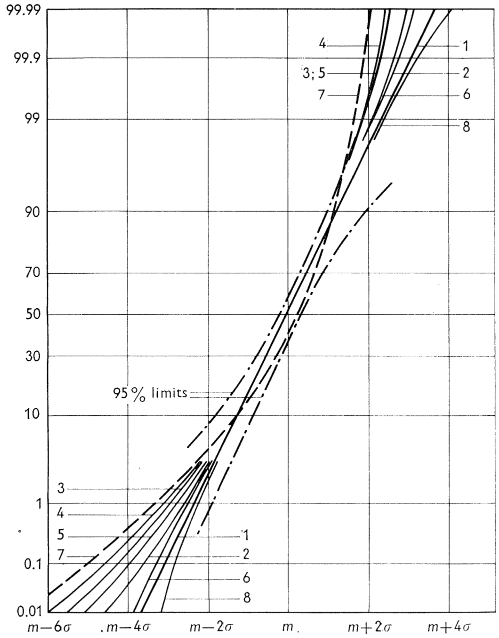

Fig. 3. Different distribution functions having the same mean value (100) and the same standard deviation (5). The 95 per cent limit is referred to the normal distribution.

1. Normal distribution.
2. Distribution of the smallest value in a sample of size $n = 10$ drawn from a normally distributed population.
3. Distribution of the smallest balue in a sample of size $n = 1000$ drawn from a normally distributed population.
4. Distribution of extreme values of the type No. II, cf. Eq. (5.35), $x_l = 0$.
5. Distribution of extreme values of the type No. II, cf. Eq. (5.28a), $x_l = 50$.
6. Distribution of extreme values of the type No. II, cf. Eq. (5.28a), $x_l = 75$.
7. Distribution of extreme values of the type No. I, cf. Eq. (5.22a).
8. Logarithmico-normal distribution.

This figure has been copied from [3].

latter may differ considerably from the former, particularly in the range of low probabilities. Since adequate data concerning the actual frequency distribution are lacking, more particularly also with regard to the loads, in the following treatment of the subject no assumption is made as to the shape of the frequency distributions of V, so that the probability of unserviceability is not quantitatively determined. It is only assumed that f_{st} may, for cases not differing too greatly from one another, be regarded as a comparative number.[3]) In this way it can be roughly ascertained what values of f_{st} provide a degree of safety which corresponds to the safety that is at present required through the medium of the existing regulations. Thus different values of f_{st} can be found for the various construction materials; to what extent these indicate differences in safety cannot be established on the basis of the foregoing. However, for new materials and forms of construction it would appear appropriate to seek a link-up with the results obtained, so that thus a more objective procedure will be applied in establishing permissible stresses and the like than has hitherto been the case.

[3]) In this paper no choice is made as to the form of the frequency distribution. In some cases, however, the normal distribution is adduced as an example. For judging the structural safety the frequency distribution of V is significant. This distribution being composed of several variable quantities will be more closely in agreement with the normal distribution than the distributions of those quantities individually. Viewed in connection with the considerable uncertainties as to the external loads that actually occur, and the schematizations and idealizations adopted in the calculations, there would, for the present, not appear to be any point in making the problem more complicated than it already is by speculating about the most likely probability distributions of loading and strength separately.

1.5 *Materials with time effects*

Before it is attempted to determine the values of f_{st} it must be noted that in certain cases there may exist an interaction between the strength of a member and the force in it. For instance, it is known that for timber or concrete a load of long duration is more dangerous than a short-term load of the same magnitude. Similarly, an alternating load may be more dangerous than a static load on a steel member. Although the susceptibility of the material to such influences is of prime importance, this property manifests itself only as a direct consequence of the forces developed in the material. The hazard due to a particular kind of loading should be expressed in the problem of safety.

This will be dealt with for timber. The starting point is provided by the experimentally determined relation between the force in a test specimen and the time that elapses until the specimen fails under the effect of that force. This period of time is longer according as the force, expressed as a percentage of the ultimate strength S_0 determined in a short-term standard test, is smaller (see Fig. 4).

Graphs of this type are to be found in various sets of regulations and manuals. In Fig. 4 the hyperbolic relationship between the force and the logarithm of the duration of load is given, as derived by WOOD [1]. A linear relationship is also presented, which – on the basis of the known test results – could equally well be adopted [2]. The objection sometimes raised with regard to the linear relationship is that with extrapolation the horizontal axis is intersected. However, in view of the actual service life of structures, this objection is of no practical significance. It is generally considered that the long-term strength can be taken as 50 to 60% of the short-term strength: a factor of 9/16 = 0.56 is often adopted, this being the value recommended by the Forest Products Laboratory at Madison. It is furthermore apparent from Fig. 4 that a specimen which is loaded to a proportion of the short-term ultimate strength S_0

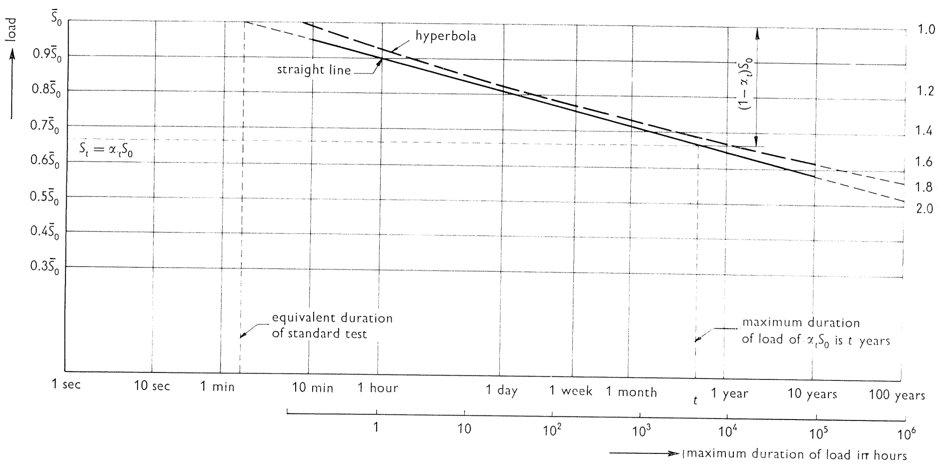

Fig. 4. Effect of duration of load.

– e.g., to $\alpha_t\ S_0$ – will fail after a certain period of time, e.g., after t years. Here this is interpreted as follows: the force $\alpha_t S_0$ has caused a strength reduction $(1-\alpha_t)S_0$ after a length of time t. Linked to this interpretation is the assumption that a force P will, during the same period t, cause a strength reduction

$$\frac{P}{\alpha_t S_0}(1-\alpha_t)S_0 = \frac{1-\alpha_t}{\alpha_t}P$$ [4])

If it is required that there must still be sufficient strength available at the time t, it will be necessary to introduce the residual strength S_t into formula (1),[5]) which then becomes:

$$f_{st} = \frac{\bar{S}_t-\bar{G}-\bar{P}}{\text{standard dev. of } (S_t-G-P)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where

$$\bar{S}_t = \bar{S}_0 - \frac{1-\alpha_g}{\alpha_g}\bar{G} - \frac{1-\alpha_p}{\alpha_g}\bar{P}$$

while α_g and α_p are the values of α from Fig. 4, associated with the total time (throughout the service life of the structure) during which the forces G and P respectively act. In calculating the standard deviation of $(S_t–G–P)$ it is now necessary to take account of the fact that S_t is dependent upon G and P.

On working out formula (2), it becomes:

$$f_{st} = \frac{\bar{S}_0-(1/\alpha_g)\bar{G}-(1/\alpha_p)\bar{P}}{\sqrt{s_{so}^2+(1/\alpha_g)^2 s_g^2+(1/\alpha_p)^2 s_p^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2a)$$

Putting $1/\alpha_g = t_g$ and $1/\alpha_p = t_p$, this becomes:

$$f_{st} = \frac{\bar{S}_0-t_g\bar{G}-t_p\bar{P}}{\sqrt{s_{so}^2+t_g^2 s_g^2+t_p^2 s_p^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

In the further treatment of the subject it will be assumed that the structure has a service life of 100 years, during which time the dead weight is always acting. The time factor for the dead weight will thus be $t_g = 1/\alpha_g = 16/9$; for practical purposes a value of $t_g = 1.8$ may, for example, be adopted. For the variable loads other values may be adopted for the time factor, depending upon the length of time during which these loads act. Values for this time factor can be read from Fig. 4.

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[4]) Whether there is, with the normally permissible stresses and loads, still any question of such a reduction in strength is doubted by some investigators; to be on the safe side, however, it is assumed that it does occur.

[5]) The requirement as to the same safety is then really too severe, however, inasmuch as part of the service life has already expired.

2 The magnitude of the determinative factors in the statistical safety formula

In any attempt to use the foregoing theory for calculating permissible stresses, etc. in the quantitative sense one comes up against the lack of sufficient data. It is one of the advantages of the statistical approach that it provides a constant reminder of this lack of knowledge, which is therefore something that can certainly not be used as an argument against the theory.

In the following, an assessment will first be made of the coefficients of variation that might be applicable to the forces G and P (see definition). Next, it will be considered what values of f_{st} would have to be taken into account in order to obtain permissible stresses comparable to those at present adopted in the regulations. In so doing it will be assumed that the regulations already embody the correct values to be adopted for the design loads, these being therefore the average maximum values throughout the service life of the structure.

2.1 *The loads*

On the assumption that, in establishing the loads to be taken into account according to the regulations, it has been endeavoured to choose such loads in conformity with the definitions given in the foregoing,[6]) the principal remaining unknowns are the dispersions (amounts of scatter) in the various load components. The coefficient of variation v is adopted as the measure of this dispersion. The dispersion in the forces due to the dead weight of materials and structures is caused by the dispersion in the specific gravity (or the bulk density) and by discrepancies between the actual dimensions and those adopted in the design. From JOHNSON's measurements on concrete slabs [3] it emerges that the coefficient of variation of the thickness is 5.7% on an average, the actual slab thickness mostly being somewhat greater than the nominal design thickness. In the following the safe value $v_g = 0.10$ will be adopted for the dead weight. Johnson has also collected data relating to the magnitude of the variable loading on floors of residential buildings. From these data an average maximum load of approx. 60 kg/m^2 (12.3 lb/ft^2) (without impact factor) and a coefficient of variation of approx. 30% could be deduced. However, the number of available data and the period of time during which they were collected are not large enough to warrant definite conclusions being drawn therefrom.

Johnson has collected Swedish and British data with regard to wind velocity. The observed maximum annual velocity in Britain (gust wind) exhibits a coefficient of variation ranging from 8 to 12% (over a 25-year period); the coefficient of variation for the average maximum wind velocity has similar values. Observations at the KNMI (Royal Netherlands Meteorological Institute) likewise present a similar dispersion in the maximum hourly averages of the annual wind velocity [4].

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[6]) Although this starting point is disputable, this is perhaps less so than is commonly supposed. After all, in defining the loads it has been explicitly stated that we are here concerned, not with the mean (or average) loading on a structure, but with the mean value of the *maximum* values that can be expected to occur during the service life of such structures.

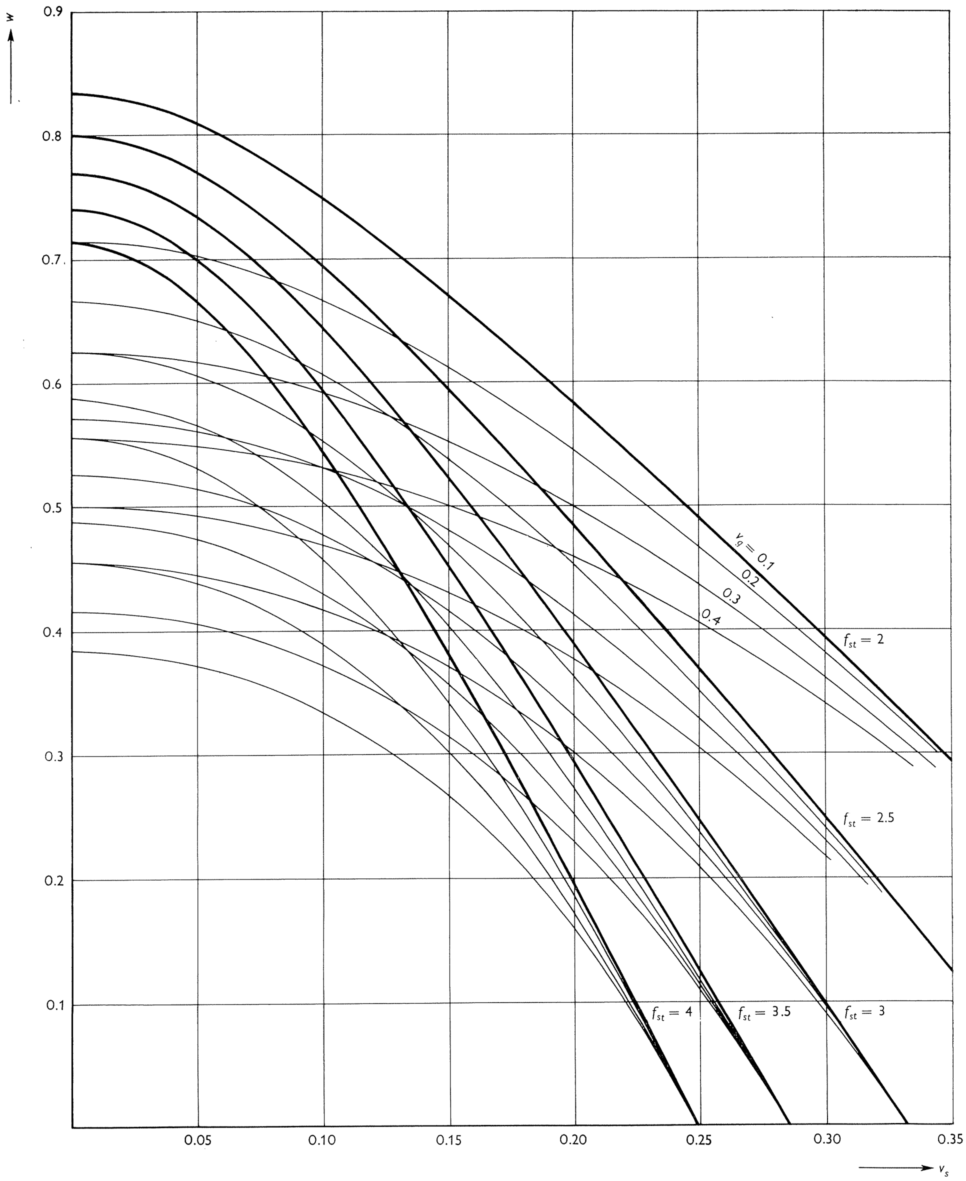

Fig. 5. The safety factor w from $\bar{G} = w\bar{S}$ for different values of f_{st}, as a function of v_s and v_g.

To what extent these data are to be regarded as representative of the variable floor loading and wind loading, respectively, is an open question; it would appear very useful to collect more data in this domain. Since it appears reasonable to suppose that the variation in the variable loading is greater than the variation in dead weight, though the magnitude itself is very doubtful, in the following treatment of the problem two values of the coefficient of variation will mostly be taken into account,

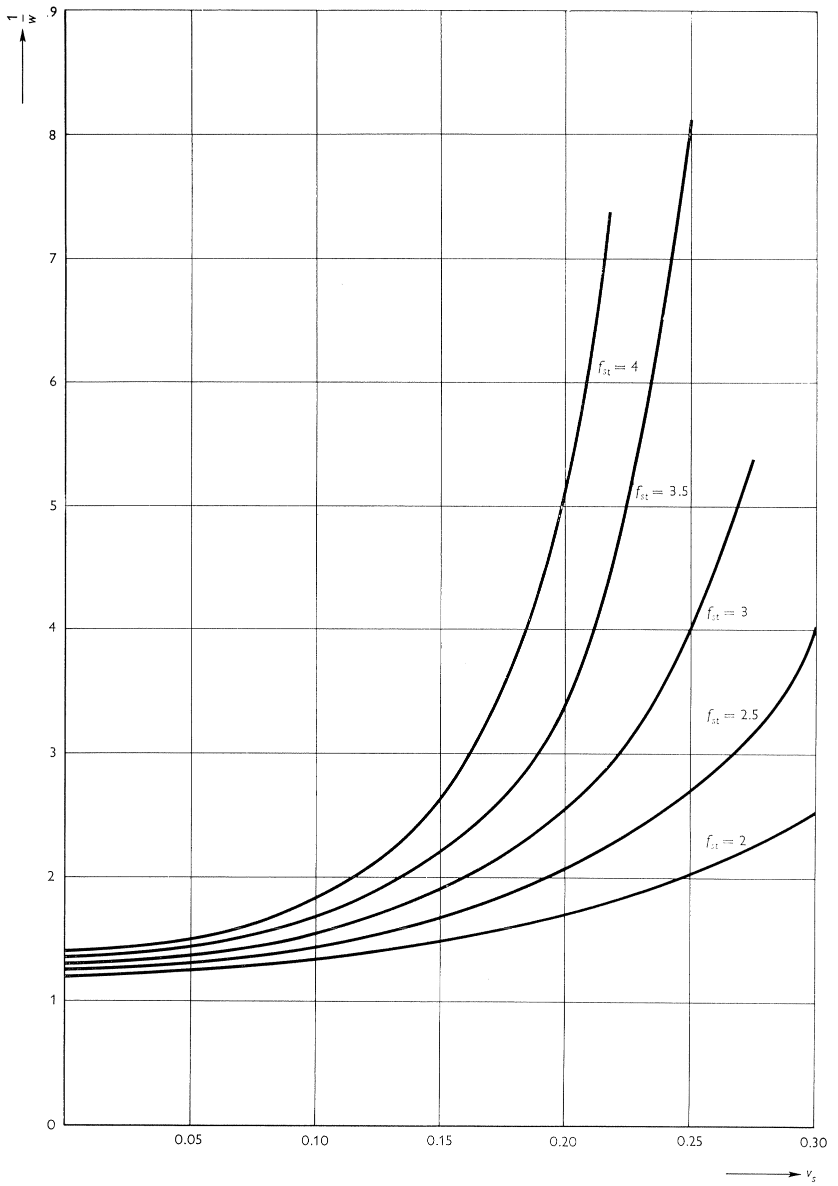

Fig. 6. The safety factor $w' = 1/w$ from $\bar{G} = \bar{S}/w'$ for different values of f_{st}, as a function of v_s. For v_g a value of $v_g = 0.10$ is chosen.

namely, $v_p = 0.20$ and $v_p = 0.30$, so that it is possible to obtain an impression of the effect of this quantity. In the case of a normal distribution of the maximum values [7]) these values adopted for the coefficient of variation signify that, for example, 16 out of 100 roofs designed for a snow load of 50 kg/m^2 will never be subjected to a snow

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[7]) In reality the frequency distribution of the loading will presumably not be entirely normal; no statement as to the actual form of the distribution is made here.

load of more than 40 and 35 kg/m^2 respectively, while on 16 other roofs the maximum snow load during their service life will exceed 60 and 65 kg/m^2 respectively.

In conformity with the foregoing, one value of the coefficient of variation for dead weight, namely, $v_g = 0.10$, will accordingly be taken into account in the further treatment of the subject, while two values will mostly be adopted for the variable loading, namely, $v_p = 0.20$ and $v_p = 0.30$.

2.2 *$\bar{G}$ and $\bar{P}$ expressed in $\bar{S}$ for different degrees of safety*

With the aid of formula (1) it is possible to express $\bar{G}$ and $\bar{P}$ in the strength $\bar{S}$. More particularly, it can be ascertained what values of $\bar{G}$ are permissible for different degrees of safety – i.e., for different values of f_{st} – in the case where $\bar{P} = 0$.

In Fig. 5 the values of w from $G = w \cdot S$ have been plotted for different values of f_{st}, v_s and v_g; those values of w could be called the 'requisite safety factor for the dead loading'. In entirely similar fashion it would also be possible to plot the safety factor for the live loading in Fig. 5, for which of course a value for v_p would have to be chosen. Since G and P are equivalent in formula (1), this safety factor would be derivable also from Fig. 5. In the following this safety factor w from Fig. 5 will be employed.

In Fig. 6 the reciprocal values $1/w$ have been plotted for the case where $v_g = 0.10$; these values are greater than 1, and can be compared with the usual coefficient by which the strength has to be divided to get the permissible stress.

From Fig. 5 the great influence of the coefficient of variation v_s is clearly apparent; low dispersion in the strength makes possible the application of large forces!

From formula (1) there also emerge combinations of $\bar{G}$ and $\bar{P}$ for different values of f_{st}, v_g, v_p and v_s. These combinations can be plotted in an orthogonal co-ordinate system with axes $\bar{G}$ and $\bar{P}$, both expressed in $\bar{S}$ (see Fig. 7). In this diagram the values on the $\bar{G}$-axis and $\bar{P}$-axis are different, this being due to the higher value assumed for the coefficient of variation v_p in the case of $\bar{P}$. In Figs. 8 and 9 a number of such curves have been plotted, calculated from formula (1), for the chosen values of v_g and v_p and for various values of v_s and f_{st}. All the combinations of values for $\bar{G}$ and $\bar{P}$ located on one curve are associated with one and the same value of f_{st} and therefore present the same risk of failure. These diagrams may also be employed for assessing the usual values of the statistical safety index for various construction materials, as will appear from the following.

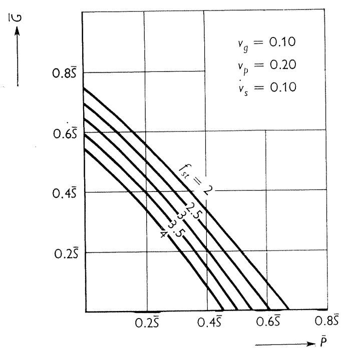


Fig. 7. $\bar{G}$ and $\bar{P}$ expressed in $\bar{S}$ for various values of f_{st}.

2.3 *The value of f_{st} for various materials*

2.3.1 Steel

The permissible stresses for steel are often linked to the yield point. For the widely used steel grade Fe 37 the yield point is usually taken as being 2400 kg/cm^2; the mean actual value is higher, however. For instance, from 1350 tests performed in the works laboratory of a structural steelwork firm a mean value of 2910 kg/cm^2 was found, the coefficient of variation being 11%.

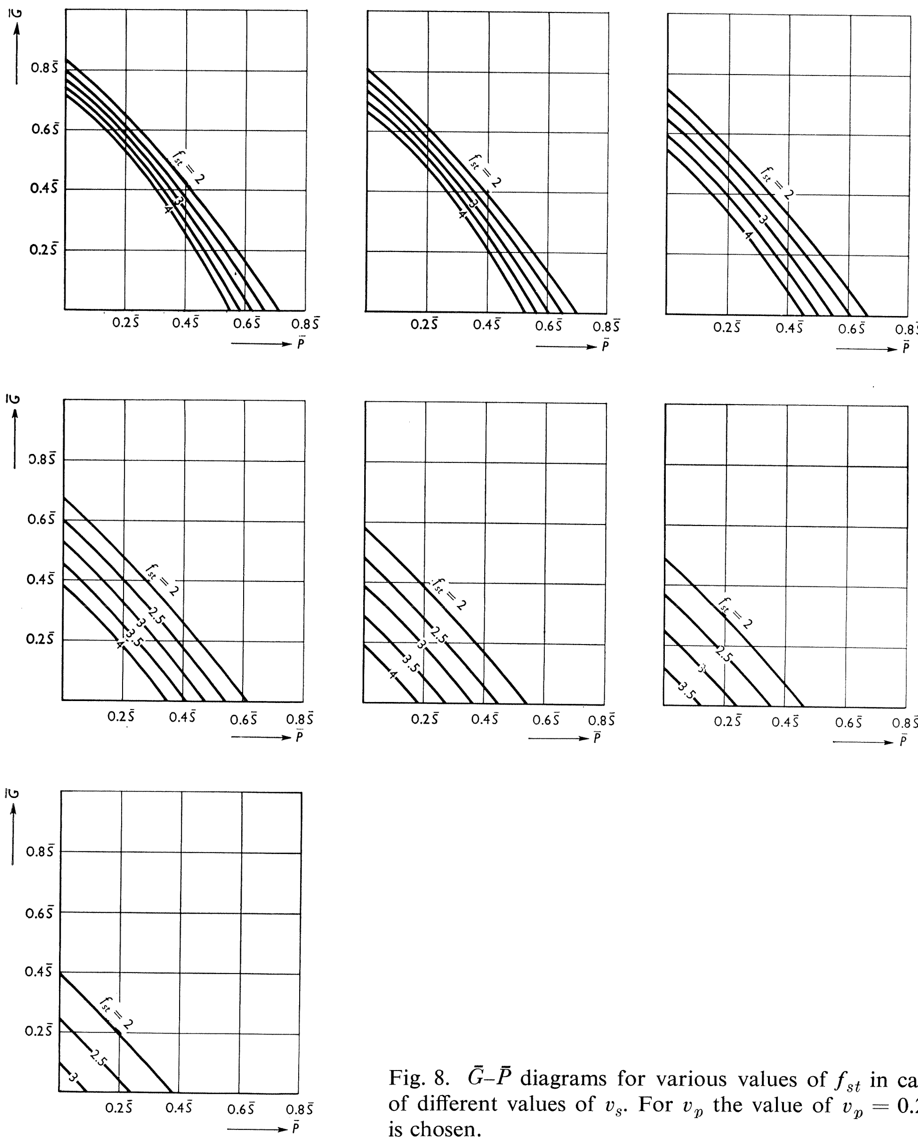


Fig. 8. $\bar{G}$–$\bar{P}$ diagrams for various values of f_{st} in case of different values of v_s. For v_p the value of $v_p = 0.20$ is chosen.

The coefficient of variation of the strength of structures is likely to be higher than that of the tensile strength of the material. In the case of rolled steel sections, for example, there are, in addition to the variations in the material properties, also the deviations from the nominal dimensions, and these deviations likewise manifest themselves in the coefficient of variation.

According to the Netherlands Standard N 1055, the permissible stress for this steel is 1400 kg/cm^2 or, under certain conditions, 1600 kg/cm^2. If the mean yield

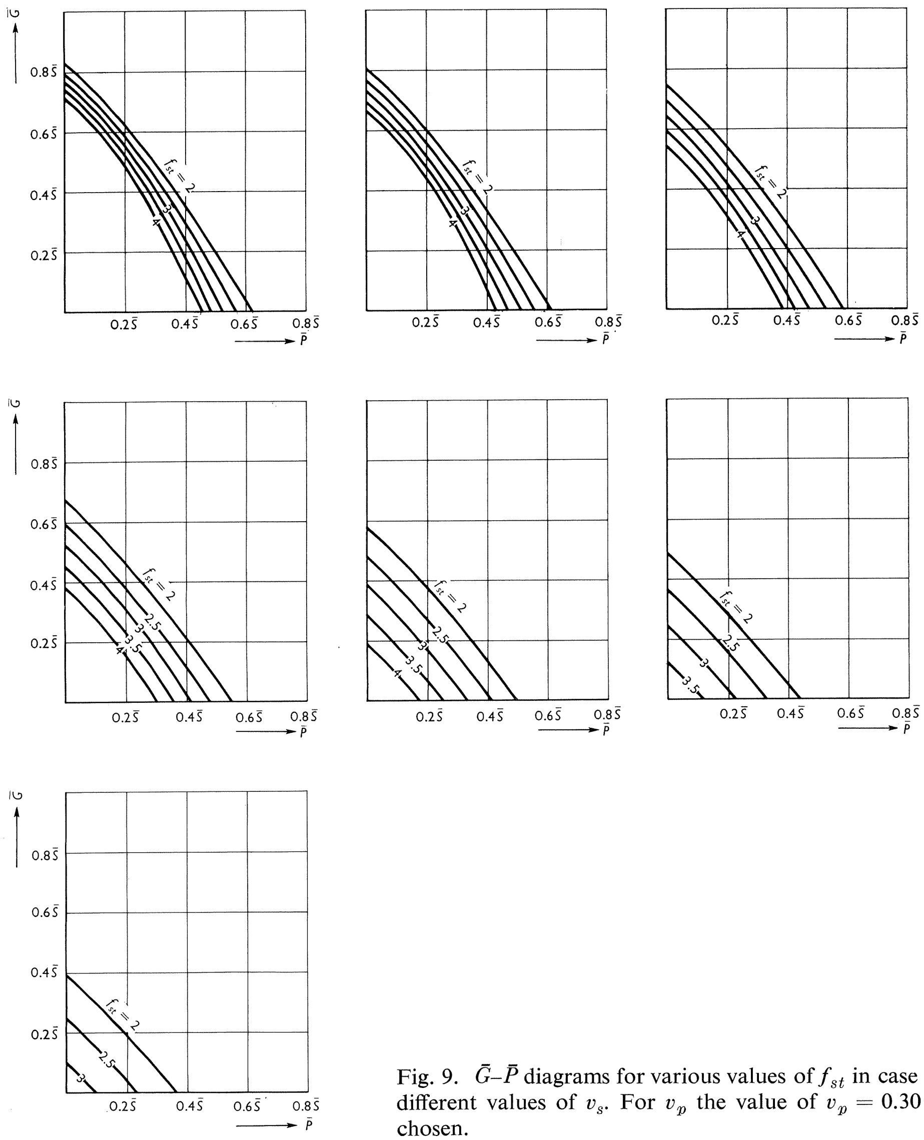


Fig. 9. $\bar{G}$–$\bar{P}$ diagrams for various values of f_{st} in case of different values of v_s. For v_p the value of $v_p = 0.30$ is chosen.

strength is situated between $\bar{S} = 2600$ kg/cm^2 and $\bar{S} = 2900$ kg/cm^2, then w will range between $w = 0.54$ and $w = 0.48$ for $\bar{\sigma}_{\text{permiss}} = 1400$ kg/cm^2 and between $w = 0.62$ and $w = 0.55$ for $\sigma_{\text{permiss}} = 1600$ kg/cm^2.

Since $\bar{\sigma}_g + \bar{\sigma}_p \leqslant \sigma_{\text{permiss}}$ or alternatively, $\bar{G} + \bar{P}$ = constant, the boundary lines in the $\bar{G}$–$\bar{P}$-diagram are linear, while the intercepts on the axes are equal. This is plotted in Fig. 10, where are also very approximately indicated the limits within which the

Fig. 10.
Values of w for the usually adopted steel stresses.

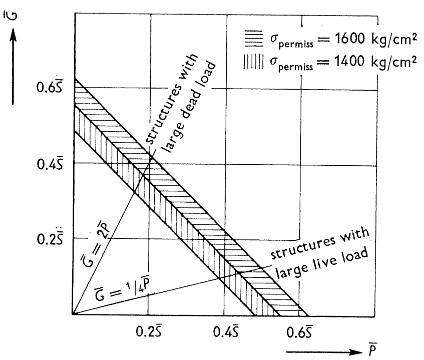
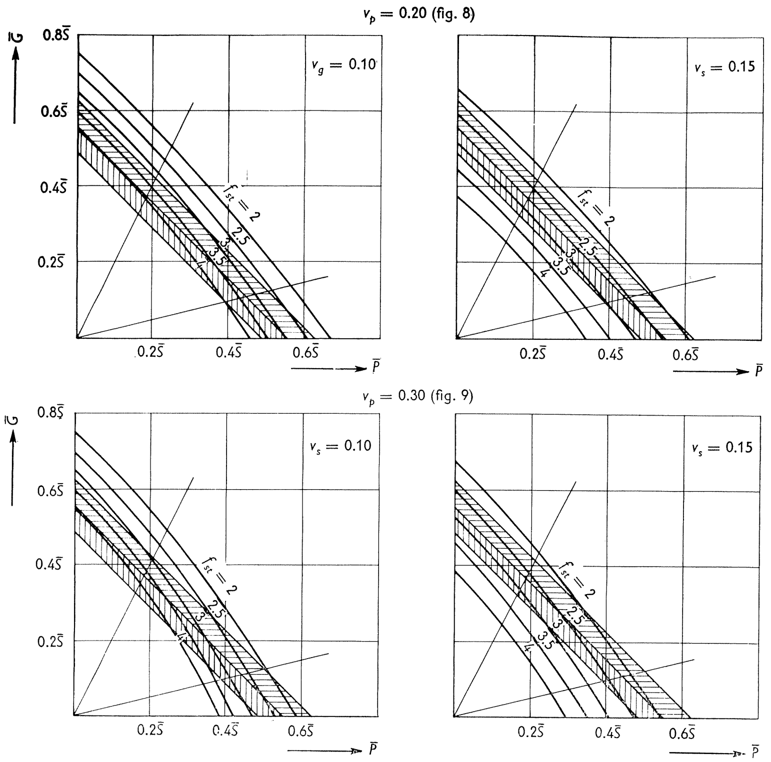


Fig. 11.
Comparison of Fig. 10 with the $\bar{G}$–$\bar{P}$ diagrams of Figs. 8 and 9.



ratio $\bar{G}/\bar{P}$ will range in normal structures. By superimposing Fig. 10 upon the graphs in Figs. 8 and 9 it is possible to read what values of f_{st} will be associated with the usual permissible stresses. The results of some cases are plotted in Fig. 11; the values obtained are given in Table 1, and they are presented in graph form in Fig. 12.

Table 1. Values of f_{st} for steel

$\sigma_{permiss}$ (kg/cm²)	v_s	f_{st}: $v_p = 0.20$	f_{st}: $v_p = 0.30$
1400	0.10	4.5 to 3.5	4.5 to 2.7
	0.15	3.2 to 2.6	3.2 to 2.2
	0.20	2.5 to 2.2	2.5 to 1.8
1600	0.10	3.9 to 2.7	3.7 to 2.1
	0.15	2.8 to 2.1	2.7 to 1.7
	0.20	2.2 to 1.7	2.2 to 1.6

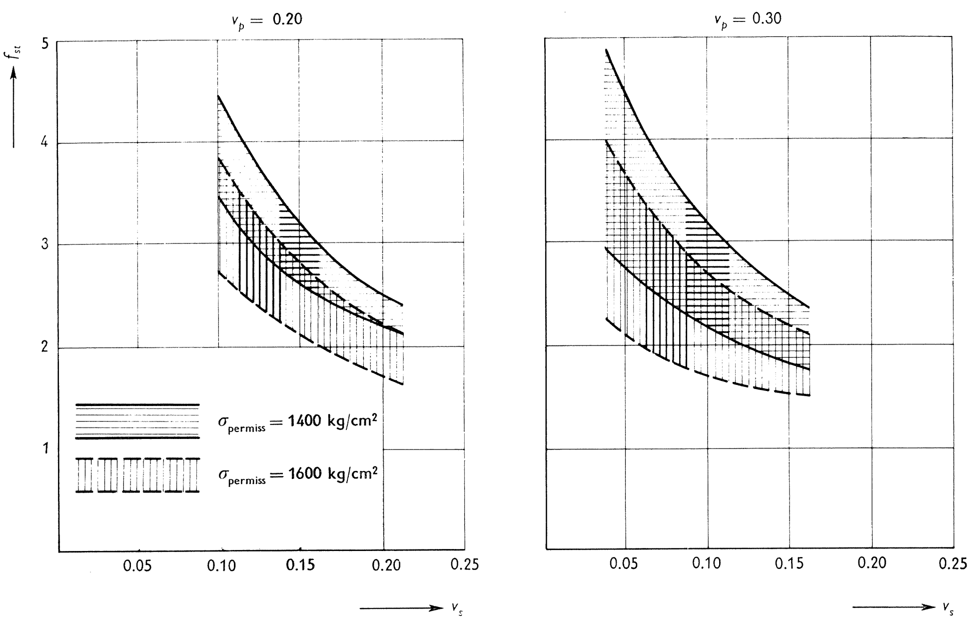


Fig. 12. Values of f_{st} for steel.

The higher permissible stress is associated with the lowest values of f_{st}. On the assumption that the permissible stress of 1400 kg/cm² is associated with a value $v_s \approx 15\%$, then f_{st} will be between 2.5 and 3.4 for $v_p = 0.20$, and between 2.1 and 3.4 for $v_p = 0.30$.

It can be assumed that, thanks to the greater amount of care that is required in the execution of the structure if the higher permissible stress ($\sigma_{\text{permiss}} = 1600$ kg/cm^2) is adopted, a smaller variation in strength will be achieved, e.g., $v_s \approx 0.12$. This results in values of f_{st} between 2.2 and 3.5 for $v_p = 0.20$, and between 1.8 and 3.4 for $v_p = 0.30$.

For structures with relatively small permanent loads the values of f_{st} are lowest; the degree of safety is therefore lowest for such structures. It is notable that with the choice that has been made here with regard to v_s the adoption of the lower permissible stress ($\sigma_{\text{permiss}} = 1400$ kg/cm^2) is still the safest course. If, in addition to a difference in the coefficient of variation, there might also be supposed to exist a difference in the mean strength $\bar{S}$ between the "ordinary" and the "carefully executed" structures, then the values of f_{st} could approach each other more closely.

From the foregoing it can provisionally be inferred that for normal steel structures the statistical index is between 1.8 and 3.5. The large difference between these lower and upper limiting values is in part due to the fact that the current regulations take no account of the difference in character between dead and live loading.

2.3.2 Reinforced concrete

The strength of reinforced concrete beams with normal percentages of reinforcement and of rectangular cross-section can be calculated from:

$$M_u = \omega_0 \sigma_e \left(1 - c\frac{\omega_0 \sigma_e}{\sigma'_u}\right) bd^2 \qquad (4$$

where:

$$\omega_0 = A/bd = \frac{\text{cross-sectional area of tensile reinforcement}}{\text{cross-sectional area of concrete}}$$

σ_e = yield point of the reinforcement
c = coefficient associated with the stress distribution in the compressive zone; for a parabolic distribution: $c = 9/16$
σ'_u = compressive strength of the concrete in the compressive zone

For the design of beams in accordance with Netherlands Standard NEN 1009 with grade QR-24 reinforcement the yield point to be adopted in this formula is $\sigma_e = 2400$ kg/cm^2. Since this is a guaranteed yield point, the mean actual value $\bar{\sigma}_e$ will be higher, e.g., 10 to 15% higher.[8]) Furthermore, a value equal to 0.6 times the cube strength must be substituted for σ'_u in the formula. This factor of 0.6 takes account of the change from cube strength to prism strength (for which the factor is approx. 0.85), as well as the combined effect of continued hardening and sustained

[8]) This percentage will depend also on the cross-sectional area of the bars and will be higher for thin bars.

loading (factor approx. 0.9) and a reduction of the mean value to a 5% lower limiting value (factor approx. 0.8).

All this means that the mean actual strength $\bar{S}$ differs from the calculated strength M_u in relation to which the permissibility of the loads applied to the structure is judged. Let $\bar{\sigma}_e$ be the mean yield point of the steel and $\bar{\sigma}'_{uk}$ the mean cube strength of the concrete; then the mean strength of the beam, including the strength reduction due to sustained loading (i.e., loading of long duration), will be:

$$\bar{S} = \omega_0 \bar{\sigma}_e \left(1 - \frac{9}{16} \cdot \frac{\omega_0 \cdot \bar{\sigma}_e}{0.9 \times 0.85 \bar{\sigma}'_{uk}}\right) bd^2$$

while the calculated strength is:

$$M_u = \omega_0 \frac{\bar{\sigma}_e}{1.1 \text{ to } 1.15} \left(1 - \frac{9}{16} \cdot \frac{\omega_0 \dfrac{\bar{\sigma}_e}{1.1 \text{ to } 1.15}}{0.9 \times 0.85 \times 0.85 \bar{\sigma}'_{uk}}\right) bd^2$$

On combining the two above expressions, we obtain:

$$\frac{\bar{S}}{M_u} = 1.11 \quad \text{to} \quad 1.17$$

According tot NEN 1009 it is necessary to allow a safety factor of 1.8 with respect to M_u, i.e.:

$$M_u \geqslant 1.8G + 1.8P, \quad \text{so that:}$$

$$\bar{S} \geqslant (2 \text{ to } 2.1)\bar{G} + (2 \text{ to } 2.1)\bar{P}$$

In the same way as has been done for steel the associated limiting lines can now be plotted; this has been done in Fig. 13.

Fig. 13. $\bar{G}$–$\bar{P}$ diagram according to NEN 1009.

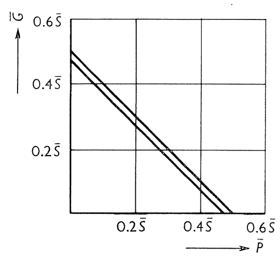
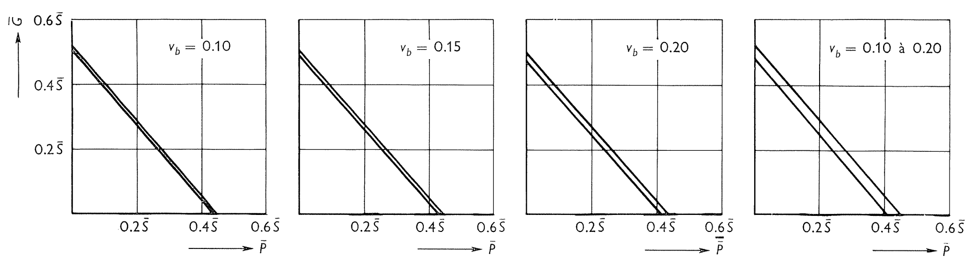


Fig. 14. $\bar{G}$–$\bar{P}$ diagrams according to C.E.B. Recommendations.



A different procedure is followed in the "Practical Recommendations" of the Comité Européen du Béton (CEB). On the one hand, it involves the use of so-called characteristic values for the strengths and loads, while, on the other, it introduces factors of safety which likewise relate to the strengths and loads.

For establishing the characteristic values, 5% probability limits are applied, which means in effect that the value substituted for the strength is the mean value minus 1.64 times the standard deviation; this applies both to steel and to concrete. Furthermore, a safety factor γ is introduced, which has the value $\gamma_s = 1.15$ for steel and which varies between $\gamma_b = 1.40$ and $\gamma_b = 1.60$ for concrete, depending on the greater or less amount of care bestowed on concrete making and workmanship.

As regards loads, the mean value is adopted for the dead weight, while the characteristic value of the live (or superimposed) load is 1.15 times the value laid down in the regulations. Finally, an enhancement factor for the loads is introduced; in general its value is 1.40.

With the aid of the previously employed formula the following expression is now obtained for the design strength S_r:

$$S_r = \omega_0 \frac{\bar{\sigma}_e(1-1.64v_{st})}{1.15}\left[1 - \frac{9}{16}\cdot\frac{\dfrac{\omega_0\bar{\sigma}_e(1-1.64v_{st})}{1.15}}{\dfrac{0.85(1-1.64v_b)\,\sigma'_{uk}}{\gamma_b}}\right]^{9)}$$

where $\bar{\sigma}'_{uk}$ is the mean cube strength. This value can again be compared with the mean actual strength $\bar{S}$, as already referred to in the foregoing. For the ratio $S_r/\bar{S}$ the following expression is obtained:

$$\frac{S_r}{\bar{S}} = \frac{1-1.64v_{st}}{1.15}\;\frac{1 - \dfrac{9}{16}\cdot\dfrac{(1-1.64v_{st})\omega_0\bar{\sigma}_e/1.15}{(1-1.64v_b)0.85\bar{\sigma}'_{uk}/\gamma_b}}{1 - \dfrac{9}{16}\cdot\dfrac{\omega_0\bar{\sigma}_e}{0.9\times 0.85\bar{\sigma}'_{uk}}}$$

This ratio has been evaluated for steel with a characteristic yield point of 2400 kg/cm^2 and a coefficient of variation $v_{st} = 0.10$, so that – since $2400 = (1-1.64v_{st})\bar{\sigma}_e$ – the mean yield point is $\bar{\sigma}_e = 2870$ kg/cm^2. Furthermore, some combinations have been chosen for concrete, namely:

$v_b = 0.1$; $\gamma_b = 1.4$,

$v_b = 0.15$; $\gamma_b = 1.5$ and

$v_b = 0.20$; $\gamma_b = 1.6$

9) In the Recommendations it is stated that the values of γ_b are based on complete loading at an age of 28 days, while attention is called to the compensation of the diminishing strength in consequence of dead load and continued hardening. For this reason no time factor is introduced with regard to the strength of the concrete.

Finally, the ratio for each of these combinations has been calculated for three concrete strengths, namely: $\bar{\sigma}'_{uk}$ = 160, 225 and 300 kg/cm^2.

According to the Recommendations it must moreover be shown that $1.4(G+1.15P) \leqslant S_r$ or, alternatively, if $S_r/\bar{S} = r$, then: $1.4G+1.61P \leqslant r \cdot S_{\text{actual}}$.

With the aid of the ratio $r = S_r/\bar{S}$ which has just been calculated it is now possible to plot the values of $\bar{G}$ and $\bar{P}$ again, as has been done in Fig. 14. These graphs were compared with those in Figs. 8 and 9, from which values of f_{st} were read, which have been plotted in Fig. 15. It appears that for $v_p = 0.20$ particularly the CEB method gives a narrow band in the diagram, this being caused by the difference in the treatment of G and P. For $v_p = 0.30$ the band in Fig. 15 is much wider because the slopes in Figs. 14 and 9 are not in good agreement with each other.

Similar reasoning is applicable to the Dutch Code of Practice for Reinforced Concrete (GBV 1962); because of the equal treatment of G and P, however, no good agreement with Figs. 8 and 9 is obtained, so that a wider band results in Fig. 15.

Comments

A few values of v_b have been chosen for the dispersion (scatter) in the *concrete* strength σ'_{uk}. Ultimately, however, we are concerned with the dispersion of the reinforced concrete *beam*, but this is something that is but little affected by the concrete quality, as is apparent from formula (4). From this same formula it also follows that a greater variation v_b hardly entails a greater variation in the strength of the beam, this being almost entirely determined by the variation in the steel strength. This would mean that in Fig. 15 only the values of f_{st} at $v_s \approx v_{st} \approx$ 0.10–0.15 are of importance.

In this connection it is to be noted that deviations in the position of the reinforcement likewise cause dispersion in the strength of the beam; this has not been taken into account.

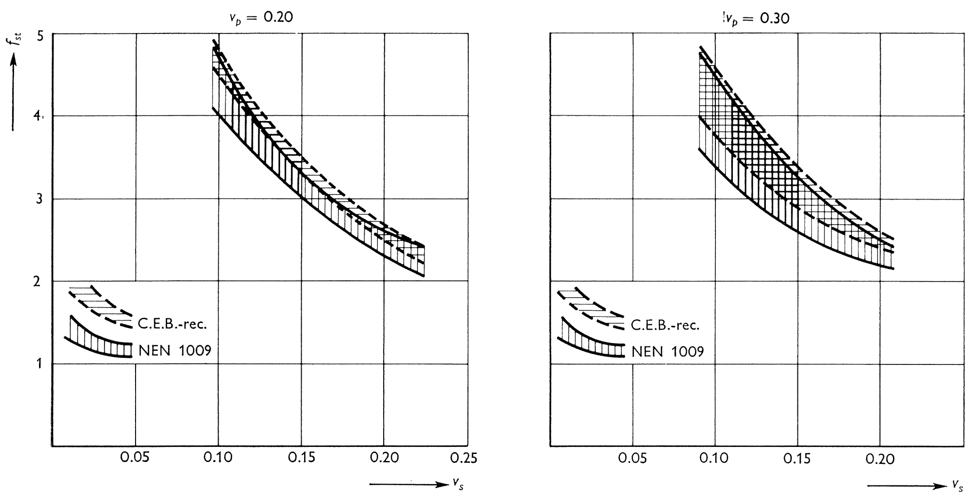


Fig. 15. Values of f_{st} for reinforced concrete.

Finally, it is pointed out that similar considerations could be established for other reinforced concrete structural members, such as columns. It would be interesting to ascertain whether, in such cases, corresponding values of f_{st} would be obtained for the various members.

2.3.3 Prestressed concrete

According to the rules applicable in Holland [10]), the following recommendation or requirement is formulated for obtaining adequate safety against failure:

$$M_u \geqslant 1.75M_g + 2.25M_p$$

For warehouses the requirements must be established as a result of due consultation; "as a rule" the safety required for such structures will not have to exceed:

$$M_u \geqslant 1.75M_g + 2.75M_p$$

In these expressions M_u is the failure moment, M_g is the moment due to dead load, and M_p is the moment due to live load.

The failure moment M_u is allowed to be determined on the basis of tests or by calculation. In such calculation the ultimate tensile strength of the prestressing steel plays a part. This steel is supplied in various grades, with tensile strength increments of 5 kg/mm^2. In the provisional directives for the testing of high-tensile steel for prestressed concrete it is specified that the tensile strength must not be lower than the stated value and must not exceed this value by more than 20 kg/mm^2. The dispersion in the steel strength will therefore be small, while the difference between the actual and the stated value will likewise not be large. The coefficient of variation of the strength of the prestressed concrete will presumably to a great extent be determined by that of the steel, but is perhaps somewhat greater because the quality of the concrete, the dimensions, etc. also play a part.

At the FIP congress in Berlin in 1958 it was proposed that the following formula be used:

$$M_u \geqslant 1.9M_g + 2.6M_p$$

Here again the failure moment M_u is determined by calculation, but the values adopted for the steel and concrete strength in the calculation are obtained from tests. Out of a series of twenty relevant test results the mean value of the ten lowest is introduced into this calculation. The actual mean strength will therefore be higher; this corresponds roughly to 1.10 times the value adopted in the calculation.

For $\bar{S} = M_u$, $\bar{G} = M_g$ and $\bar{P} = M_p$ the STUVO formula gives:

$$\bar{S} \geqslant 1.75\bar{G} + 2.25\bar{P} \text{ or (for warehouses) } \bar{S} \geqslant 1.75\bar{G} + 2.75\bar{P}$$

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[10]) Recommendations of the "Studievereniging tot Ontwikkeling van het Voorgespannen Beton" (STUVO): RVB 1952.

Adopting $\bar{S} = 1.10M_u$ and $\bar{P} = M_p$, the FIP formula gives:

$$\bar{S} \geqslant 2.10\bar{G} + 2.85\bar{P}$$

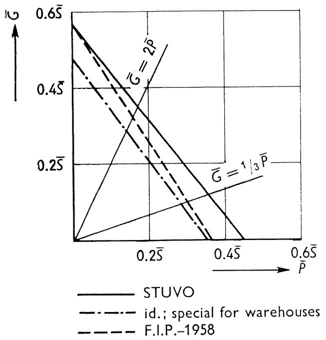


Fig. 16. $\bar{G}$–$\bar{P}$ diagram for prestressed concrete.

In Fig. 16 the straight lines associated with these requirements have again been plotted in the $\bar{G}$–$\bar{P}$-diagram. It is clearly apparent that now a difference in dispersion between G and P has indeed been taken into account. In the diagram it is indicated within what limits of the ratio of $\bar{G}$ and $\bar{P}$ the prestressed concrete structures might generally be located. The STUVO formula for normal structures is compared with gs. 8 and 9 in the manner as has been explained with reference to steel. A notable feature here is the better agreement with the theory of safety; the direction of the straight lines is almost identical with the direction in the diagrams for $v_s = 10\%$; $v_p = 20\%$ and for $v_s = 15\%$; $v_p = 30\%$. For the warehouse floors the steeper slope of the straight line in Fig. 16 is indicative of an even higher assumed value of v_p than $v_p = 30\%$ as here introduced. Finally, it should be noted that the FIP formula results in higher values of f_{st} than does the formula indicated in the Dutch rules for prestressed concrete (RVB 1962).

The values obtained for f_{st} have been stated in Table 2 and plotted in Fig. 17; the uncertainty as to the value of f_{st} which is employed is much less than in the case of steel, thanks to the load factor method of design introduced here. If the coefficient of variation v_s is taken as 0.13–0.16, then $f_{st} = 2.9$–3.4.

Table 2. Values of f_{st} for prestressed concrete

	f_{st}	
v_s	$v_p = 20\%$	$v_p = 30\%$
10%	4.2 to 4.3	4.0 to 3.5
15%	3.0 to 3.2	2.9 to 2.8
20%	2.3 to 2.4	ca. 2.3

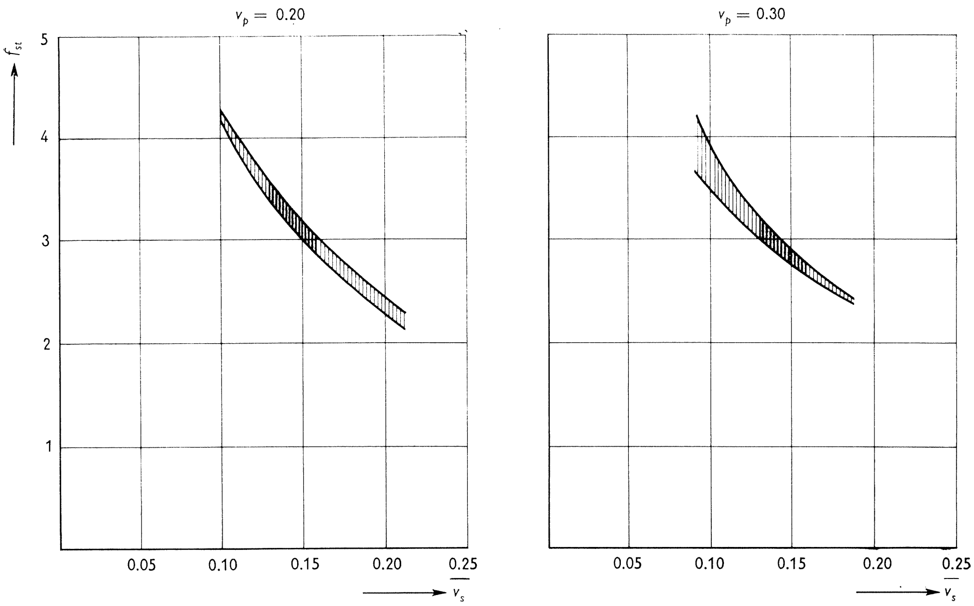


Fig. 17. Values of f_{st} for prestressed concrete.

2.3.4 Timber

For determining the permissible stresses for timber it is common practice to employ formulae in which, in addition to the mean strength, the standard deviation also occurs. In all cases the stress which is considered permissible for an unlimited length of time is calculated; for loads of shorter duration this permissible stress is allowed to be increased. In calculating the permanently permissible stress it is necessary to take account of time effects as described in 1.5. In the usual formulae this is done by introducing the reduction factor 9/16. The two formulae employed here are: [11])

CSIRO, Melbourne:

$$\bar{G} = \frac{9}{16}\,\frac{\bar{S}_0 - 2.33 s_{so}}{1.25} = \frac{9}{16}\,\frac{1 - 2.33 v_{so}}{1.25}\,\bar{S}_0$$

and

TRADA, London:

$$\bar{G} = \frac{9}{16}\,\frac{\bar{S}_0 - 1.96 s_{so}}{1.33} = \frac{9}{16}\,\frac{1 - 1.96 v_{so}}{1.33}\,\bar{S}_0$$

If the reduction factor were exactly 9/16, then, for example, the "safety factor" according to the CSIRO formula would have the following value:

$$\frac{1 - 2.33 v_{so}}{1.25}$$

[11]) For more detailed information see [5].

As pointed out in 1.5, it is not likely that this factor is indeed so precise. If the actual factor is, say, between 0.50 and 0.60, the safety factor obtained with the CSIRO formula is:

$$\frac{9/16}{0.50 \text{ to } 0.60} \times \frac{1-2.33v_{so}}{1.25}$$

These values can be compared with the values of w in Fig. 5. In Fig. 18 some curves from Fig. 5 have again been drawn and are compared with the values obtained from the CSIRO and, in similar fashion, from the TRADA formulae. The values here adopted for the actual reduction factor are 0.50, 0.55 and 0.60.[12]) In Fig. 19 is indicated at what values of f_{st} the CSIRO and TRADA formulae yield the same results as the formula presented in this paper.

As the coefficient of variation of the strength of timber is mostly between 15 and 20%, the value of f_{st} is between 2.5 and 3.2; the area in question is shown shaded in Fig. 19.[13])

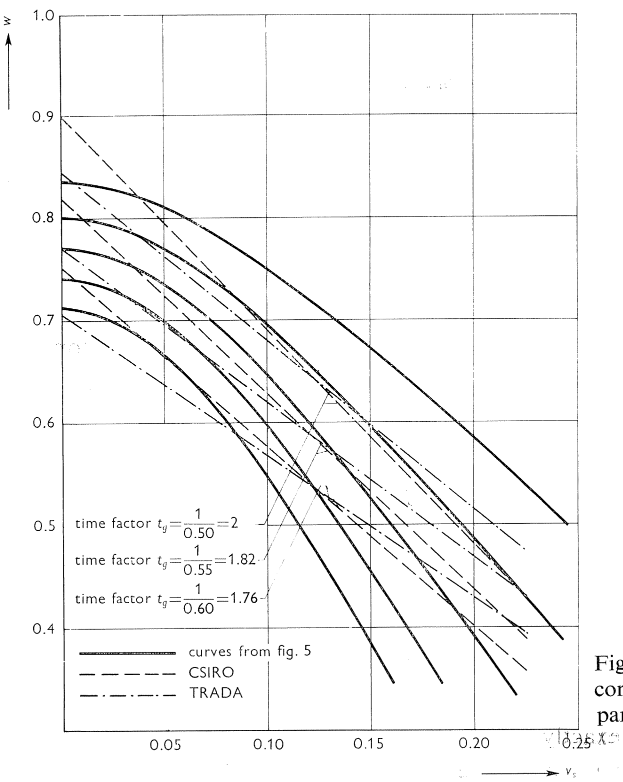


Fig. 18. $\bar{G}$–$\bar{P}$ diagrams for timber according to various regulations compared with Fig. 5.

12) The time factor for dead load will then therefore be $t_g = 1/0.50 = 2$, $t_g = 1/0.55 = 1.82$ and $t_g = 1/0.60 = 1.67$ (see 1.5).

13) The results obtained have been used in determining the permissible loads on timber connections [6].

Fig. 19. Values of f_{st} for timber.

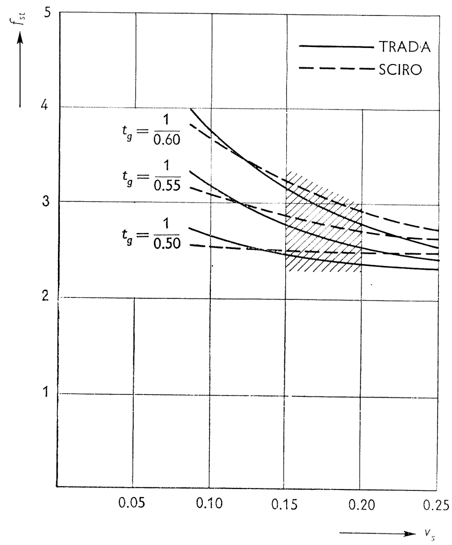
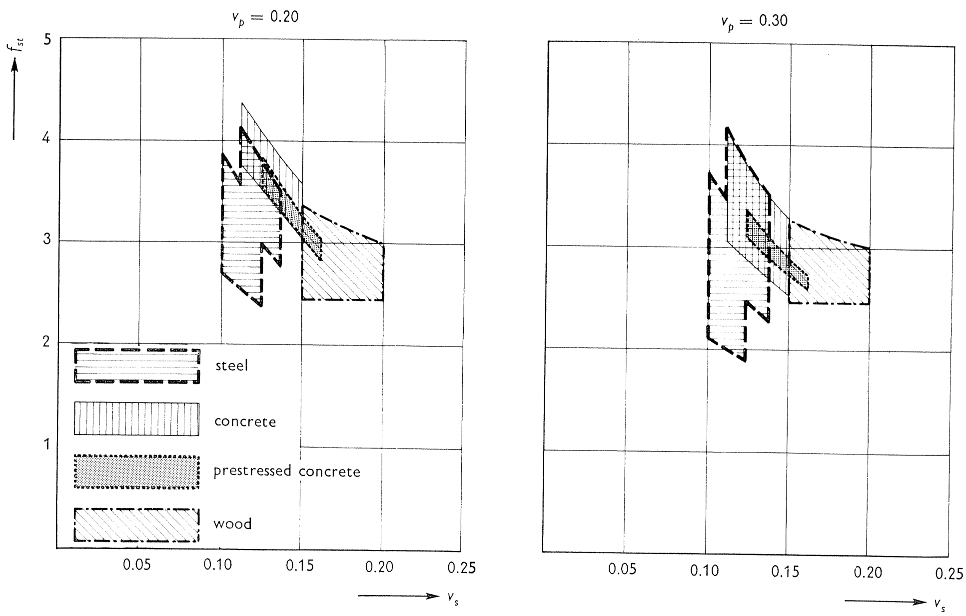


Fig. 20. Diagrams summarizing the values of f_{st} for various construction materials.



2.4 *Summary and conclusions*

In this chapter the safety formula presented in Chapter 1 has been further elaborated. For different values of the statistical safety index f_{st} it is possible to express $\bar{G}$ and $\bar{P}$ in the strength $\bar{S}$. In Fig. 5 values of the "safety factor" w can be read from $\bar{G} = w \cdot \bar{S}$ for different values of the coefficients of variation v_g and v_s and of f_{st}. After having made a choice as to probable values of v_g and v_p, it was considered what values of f_{st} are attained in the case of some construction materials. The results are summarized in Fig. 20. The great influence of the coefficient of variation v_s on the available safety is clearly manifested: adopting the currently valid permissible stresses, for each material a substantially higher degree of safety is obtained if the manufacture of the material, or the construction of the structure, is such that a low coefficient of variation v_s results. Conversely, this leads to the conclusion that a low value of v_s – obtained by careful control of the construction material – can result in higher permissible stresses, etc. while retaining the same degree of safety. Indeed, this conclusion already emerges quite clearly from Fig. 5.

The "height" of the bands in Fig. 20 is due largely to the fact that – in contrast with what has here been assumed – currently valid regulations often make no distinction between stresses caused by permanent (dead) load and those caused by variable (live) load.

Attention must be called to the region indicated for timber in Fig. 20, which region has been derived in a manner different from that adopted for other materials, namely, by comparison of standard formulae with the formula derived here. Since the standard formulae already contain the coefficient of variation v_s, the influence of v_s upon f_{st} is no longer very great; the uncertainty is now caused by the lack of certainty as to the accuracy of the time factor. The standard formulae are valid for the dead loads $\bar{G}$, and so the magnitude of v_p plays no part here; the region is the same in both cases envisaged in Fig. 20. Finally, it must be pointed out for this material that the safety theory has been applied to the residual strength S_t available after expiry of the service life; actually the value of f_{st} is therefore a little higher, and the safety greater, than is found by means of the procedure adopted here.

3 Stress regions

3.1 *Graphical interpretation of the formula for the statistical safety index*

In the foregoing, with reference to the definition formula of the statistical safety index, values of $\bar{G}$ and $\bar{P}$ were expressed in the strength $\bar{S}$. These values have already been plotted in graph form in Figs. 7, 8 and 9. In the following, these graphs will be further elaborated. The definition formula of the statistical safety index was:

$$f_{st} = \frac{\bar{S}-\bar{G}-\bar{P}}{\sqrt{s_s^2+s_g^2+s_p^2}} \quad \dots\dots\dots\dots \quad (1)$$

Since $s_s = v_s \cdot \bar{S}$, $s_g = v_g \cdot \bar{G}$ and $s_p = v_p \cdot \bar{P}$, we obtain:

$$(\bar{S}-\bar{G}-\bar{P})^2 = f_{st}^2(v_s^2\bar{S}^2+v_g^2\bar{G}^2+v_p^2\bar{P}^2) \quad \dots\dots\dots \quad (5)$$

With the aid of this formula it can be investigated what combinations of $\bar{G}$ and $\bar{P}$ are permissible in particular cases. In a graph with co-ordinate axes $\bar{G}$ and $\bar{P}$ these permissible combinations – in conjunction with particular, fixed values of the other quantities – result in a hyperbola, as indicated in Fig. 21. Only the inner branch

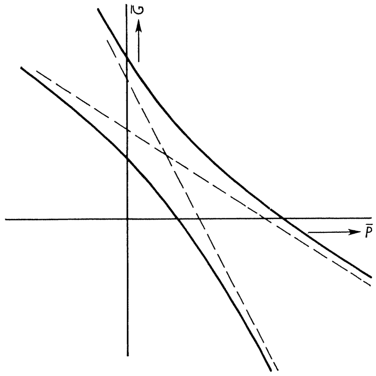
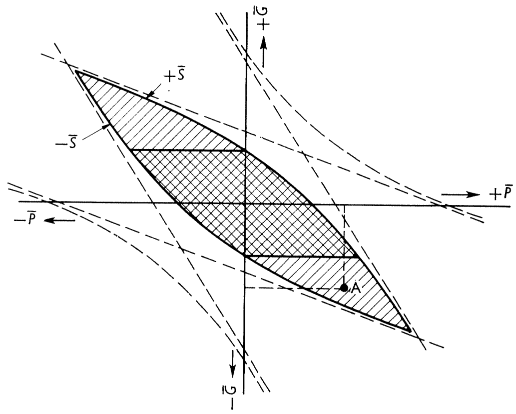


Fig. 21. Hyperbola according to formula (4).



region of permissible combinations of $\bar{G}$ and $\bar{P}$ if these are simultaneously present

ditto if P may be absent

Fig. 22. Hyperbolas associated with $+\bar{S}$ and $-\bar{S}$.

(nearest the origin) of this hyperbola is of significance to the present purpose. Points located on this curve represent combinations of $\bar{G}$ and $\bar{P}$ for which the probability of exceeding $\bar{S}$ is just so low as corresponds to the value adopted for f_{st}. On the other hand, points located on the other branch represent combinations of $\bar{G}$ and $\bar{P}$ for which the probability of *not* exceeding the strength is just equally low; these latter values of $\bar{G}$ and $\bar{P}$ will therefore very probably cause the structure to fail.

In accordance with the convention of assigning algebraic signs to stresses, etc., the strength $\bar{S}$ will likewise be given a sign. Thus, in the further treatment of the subject, a tensile strength will be considered positive, and a compressive strength will be considered negative. For a material having positive and negative strength of equal magnitude it is possible, in the same manner as discussed with reference to Fig. 21, to draw two hyperbolas associated with $+\bar{S}$ and $-\bar{S}$ respectively (see Fig. 22). The two inner branches of these hyperbolas enclose the region within which the permissible combinations of $\bar{G}$ and $\bar{P}$ are located. As the live loading P is not always present, it is necessary to reckon with G being present alone; hence the horizontal boundaries to the region between the hyperbolas, as indicated in Fig. 22.

Point A (in Fig. 22), located outside these horizontal boundaries, will, in the absence of $\bar{P}$, give a value $\bar{G}$ which in combination with $\bar{P} = 0$ is not permissible. Thus in Fig. 22 there remains the cross-hatched (heavily shaded) region within which the permissible combinations of $\bar{G}$ and $\bar{P}$ are located. Since it has been assumed that $+\bar{S}$ and $-\bar{S}$ are of equal magnitude, this is a symmetrical diagram. In certain cases, e.g., with struts subject to buckling loads, the compressive strength (negative) is lower than the tensile strength (positive), however. These smaller values of $-\bar{S}$, too, are associated with hyperbolas; in Fig. 23 it is indicated how the original total region of permissible combinations of $\bar{G}$ and $\bar{P}$ between the boundary lines $+\bar{S}$ and $-\bar{S}$ is diminished if the negative strength successively amounts to $-0.8\bar{S}$, $-0.6\bar{S}$, etc. The

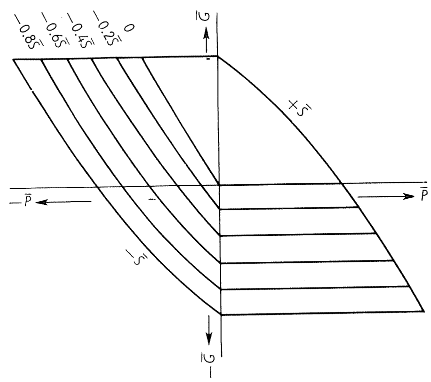


Fig. 23. Boundary lines associated with various values of the negative strength.

region within which, in the diagrams plotted in this way, permissible combinations of $\bar{G}$ and $\bar{P}$ have to be located will, in the following, be referred to as the "stress region". It should be noted that, besides stresses, $\bar{G}$ and $\bar{P}$ may also represent forces, moments, etc.

3.2 *Some characteristic properties of the curves obtained*

Before further elaborating the curves of the form represented in Fig. 23 for particular values of the respective variables, it will be appropriate to describe some general properties of these curves, whereby an insight can be gained into the influence of some of the quantities involved. In so far as that may be necessary, a value of 2.5 or 3 will be assigned to the statistical safety index f_{st}. As appears from Chapter 2, the values of f_{st} for various materials are located approximately within these limits.

a. The intersections with the axes

The intersections of the hyperbola, associated with a strength $\bar{S}$, with the $\bar{P}$-axis are determined from:

$$\bar{P} = \frac{1 \pm \sqrt{1-(1-f_{st}^2 v_s^2)(1-f_{st}^2 v_p^2)}}{1-f_{st}^2 v_p^2}\bar{S}$$

Of these the smaller value is of importance.

The intersection with the $\bar{G}$-axis is obtained by replacing v_p by v_g and $\bar{P}$ by $\bar{G}$ in the formula.

In Fig. 24 these intersections with the axes are indicated for various cases and are, for simplicity, interconnected by straight lines. Since in all cases we have chosen $v_p > v_g$, the intercept on the horizontal axis is always smaller than that on the vertical axis.

For $f_{st} = 3$, which is indicative of a higher degree of safety than $f_{st} = 2.5$, the intersections with the axes are closer to the origin. Attention is called to the decreasing

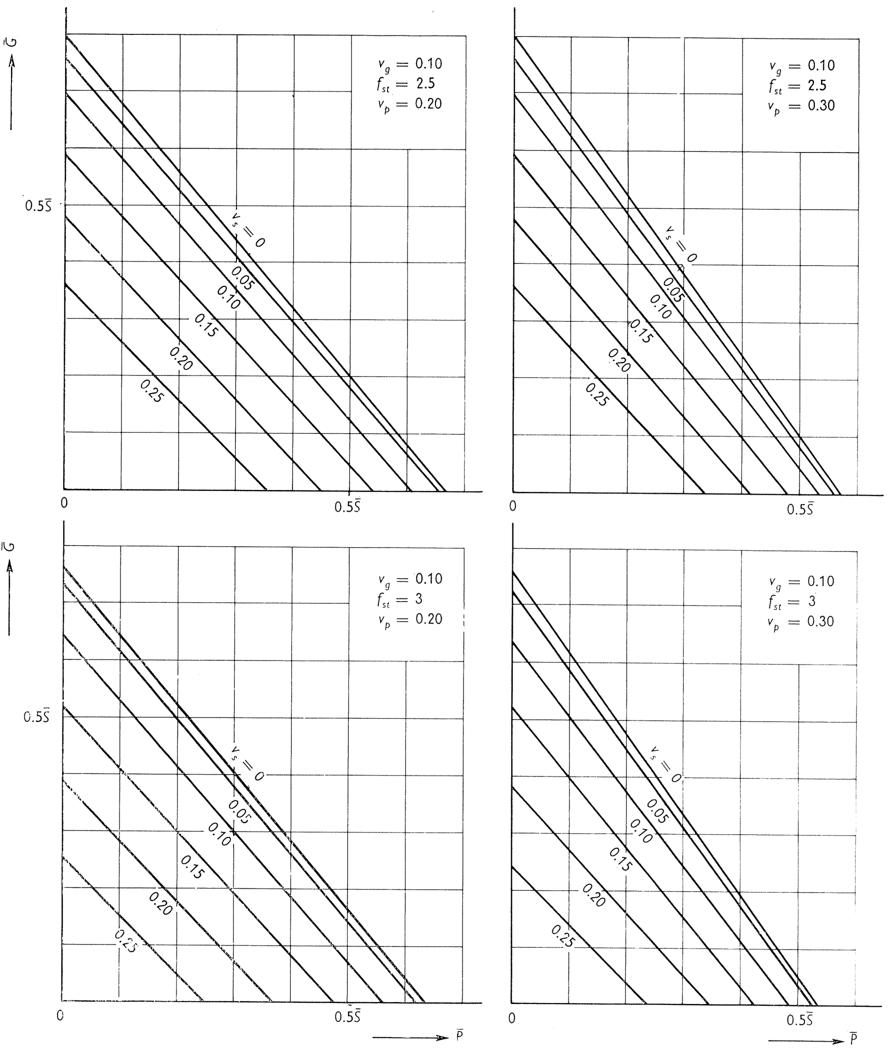


Fig. 24. The intersections of the hyperbola with the axes $+\bar{G}$ and $+\bar{P}$ for various values of the variables. For simplicity the points of intersection have been joined together by straight lines.

slope of the lines for increasing values of v_s; for the largest value chosen for v_s these lines are sloped at nearly 45° if $\bar{G}$ and $\bar{P}$ are plotted to the same scale on the axes. In practical terms this means that, for materials with little dispersion in the strength, there is reason to make a distinction between the stresses and other force actions due to dead load, on the one hand, and those due to live load, on the other, if it may be assumed that v_p differs from v_g. For materials with a large dispersion there is hardly any reason for such a distinction. The fact that for certain materials there may be other reasons (e.g., time effects) for making such distinctions has already been discussed earlier on.

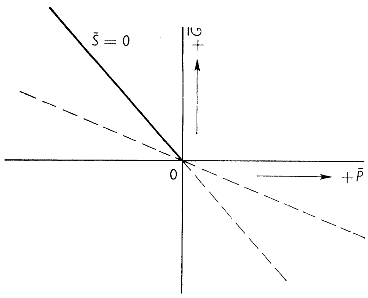


Fig. 25. Straight line through O for $\bar{S} = 0$. The portion situated below the $\bar{P}$-axis is cancelled by the truncating line which here coincides with the $\bar{P}$-axis.

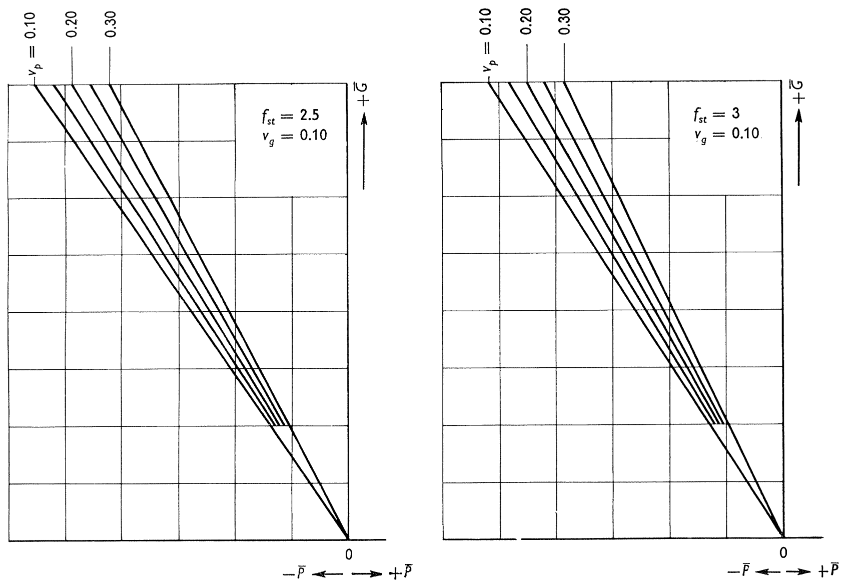


Fig. 26. The straight line for $\bar{S} = 0$ for various values of the variables f_{st} and v_p; $v_g = 0.10$.

b. The compressive strength $(-\bar{S}) = 0$

In the foregoing a distinction was made between positive (e.g., tensile) strength and negative (e.g., compressive) strength. Usually a structural component will have a positive as well as a negative strength, which strengths are not necessarily numerically equal. Consider, for example, a lattice member in which a tensile force is produced by the dead load; to resist this force a certain tensile strength $+\bar{S}$ is necessary. If a compressive force is additionally produced in that same member by a live load, then there arises the question as to how large this compressive force can permissibly be before the member in question must also be required to have a compressive strength. In the line of reasoning that has been followed, a line for $\bar{S} = 0$ denotes the combination of $\bar{G}$ and $\bar{P}$ for which it is just not yet necessary to take account of compression.

For this value $\bar{S} = 0$ the hyperbola degenerates into two straight lines through the origin (see Fig. 25). Of these lines only the portion drawn as a full line in the diagram

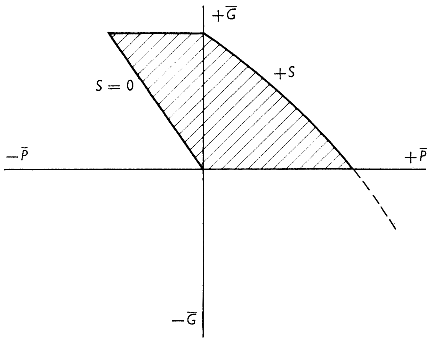


Fig. 27a. Stress region for a cord $(+\bar{S}; 0)$.

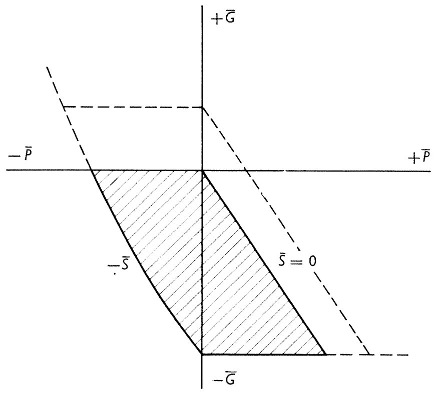


Fig. 27b. Stress region for stone with zero tensile strength $(0; -\bar{S})$. The area of the stress region increases considerably if the material can attain some tensile strength.

is of interest; the other portion is cancelled because the truncating line coincides with the $\bar{P}$-axis. The equation of this line is:

$$\bar{G} = \frac{-1-\sqrt{1-(1-f_{st}^2 v_g^2)(1-f_{st}^2 v_p^2)}}{1-f_{st}^2 v_g^2}\bar{P}$$

This line has been plotted for various values of v_p in Fig. 26. In both the graphs presented here $v_g = 0.10$; for f_{st} two values have again been chosen, namely, 2.5 and 3. The slope of the lines always exceeds 45° (even for $v_g = v_p = 0.10$). This is because the variation in the loads has been included in the consideration of the problem; the magnitude of the coefficients of variation v_g and v_p determines the slope of these straight lines. This slope is therefore independent of the construction material employed.

By way of example, in Fig. 27 the stress regions are represented for two cases where the line $\bar{S} = 0$ is significant.

Summarizing, it emerges from the foregoing that, on the basis of the formula for the statistical safety index, permissible combinations of $\bar{G}$ and $\bar{P}$ can be plotted in a graph with $\bar{G}$ and $\bar{P}$ as co-ordinate axes. These permissible combinations are located within the region enclosed by two hyperbolas and by two horizontal straight lines (Fig. 25). The diagram thus obtained is the "stress region", as first mentioned in 3.1.

The slope of the hyperbolas in the diagram increases according as the coefficient of variation v_p is greater than v_g. This effect becomes less pronounced, however, with increasing v_s and is therefore dependent on the construction material under consideration.

3.3 *Stress regions for materials with time effects*

In 1.5 attention was paid to the fact that strength and loading are not always quantities that exist independently of each other, but that there may be an interaction between them. The example of timber was elaborated, this being a material for which loads of long duration are more dangerous than loads of short duration. The line of reasoning that was followed led to the formula (3):

$$f_{st} = \frac{\bar{S}_0 - t_g\bar{G} - t_p\bar{P}}{\sqrt{s_{so}^2 + t_g^2 s_g^2 + t_p^2 s_p^2}}$$

where t_g is considered to be associated with a service life of 100 years. In formula (3) the term $t_g\bar{G}$ occupies the same place as $\bar{G}$ in formula (1); similarly, $t_p\bar{P}$ performs the same function as $\bar{P}$ in formula (1). Accordingly, for particular values of t_g and t_p it is possible to calculate stress regions in the same manner as before: here different values of $t_g\bar{G}$ and $t_p\bar{P}$ are expressed in $\bar{S}_0$.

If it is desired to read $\bar{G}$ and $\bar{P}$ directly from the graph expressed in the short-term strength $\bar{S}_0$, this means that the graph is, as it were, shortened in the ratio t_g in the vertical direction; and in the horizontal direction a shortening in the ratio t_p is

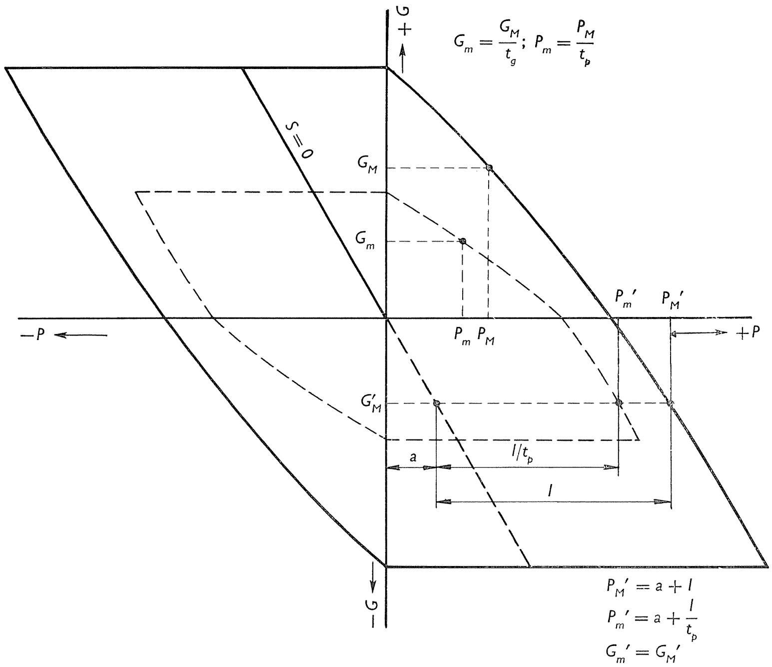
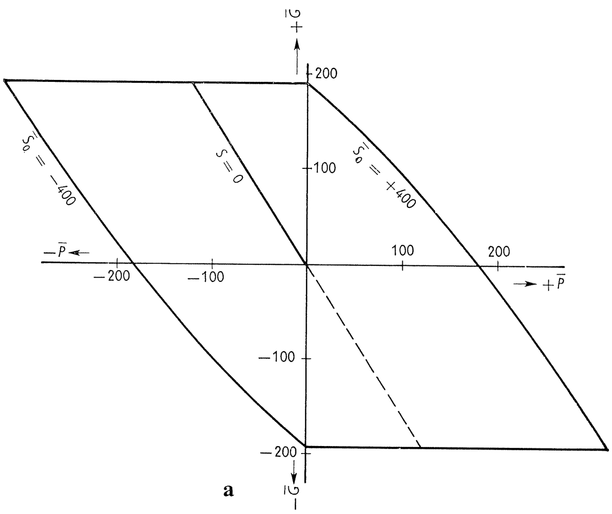


Fig. 28. Deformation of a stress region as a result of introducing time factors t_g and t_p.
——— stress region for non-time-sensitive material
– – – – – stress region for time-sensitive material

Fig. 29.
a. Non-time-sensitive material
b. Dead load
c. Dead load + snow
d. Dead load + wind



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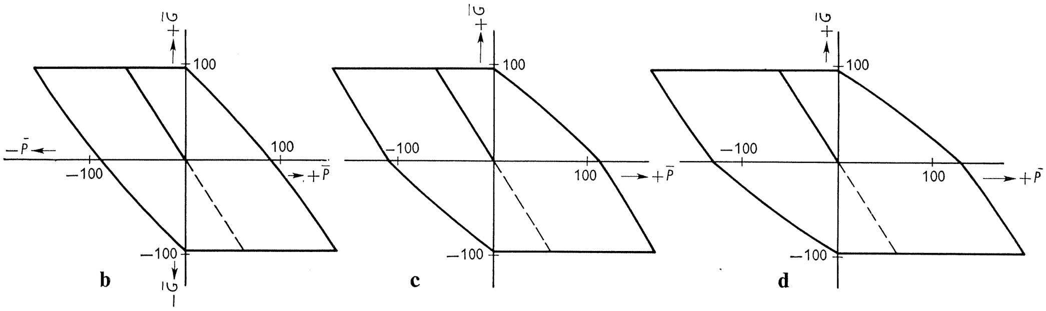

b c d

applied. However, this statement as to shortening is not quite correct; the assumption

$$\bar{S}_t = \bar{S}_0 - \frac{1-\alpha_g}{\alpha_g}\bar{G} - \frac{1-\alpha_p}{\alpha_p}\bar{P} \qquad \text{(see formula 2)}$$

would in effect, for opposite algebraic signs of $\bar{S}_0$ and of $\bar{G}$ or $\bar{P}$, signify an increase in strength.

Now it seems illogical that, for example, the compressive strength of a member should increase as a result of having been loaded in tension for a certain length of time. This consideration leads to the supplementary condition that the strength increase is caused only by loading of the same sign as the strength.[14]) With regard to the above-mentioned shortening or contraction of the stress region in the axial directions it is therefore necessary to take this into account; this shortening should occur only in so far as the total load has the same sign as the compressive strength. In the quadrants $(+G; +P)$ and $(-G; -P)$ this presents no difficulties: in the G-direction the

[14]) It is conceivable that connections are adversely affected by such variations in the direction of the load.

values become t_g times as small, and in the P-direction they become t_p times as small (see Fig. 28). In the quadrants where G and P are of different sign, G can be regarded as a prestress which has to be exceeded by P before the strength S undergoes a reduction. The straight line through O, valid for $\bar{S} = 0$, indicates the value of P which – with the probability determined by f_{st} – will cancel the values of the prestress due to G. So the shortening or contraction of the diagram begins only from this line. The shortened stress region has now been determined.

By way of example these stress regions have, in Fig. 29, been drawn for the undermentioned cases, where it has been assumed that $f_{st} = 2.5$ and $v_p = 0.20$:

$\bar{S}_0$ (kg/cm^2)	v_s	t_g	t_p	remarks
400	0.2	1	1	material without time effects
400	0.2	2	2	material with time effects; dead loads and live loads treated alike: $t_g = t_p = 2$
400	0.2	2	1.6	as above; time factor for dead loads again taken as $t_g = 2$; time factor for live (or superimposed) loads, e.g., snow, has been taken as $t_p = 1.6$.
400	0.2	2	1.4	as above, but time factor for live loads, e.g., wind, has been taken as $t_p = 1.4$

The considerable reduction of the permissible stresses in consequence of the time factor is clearly manifested, while a further striking feature is the altered shape of the diagrams when the time factors t_g and t_p are not equal. As a result of this latter situation the various boundary lines in the quadrants $(+\bar{G}; +\bar{P})$ and $(-\bar{G}; -\bar{P})$ will slope less steeply according as the live load is of shorter duration, i.e., according as t_p becomes smaller. The "kinks" at the horizontal axis are due to the assumption that a load cannot exercise a favourable influence upon the strength.

For materials with a time effect, such as timber, it might be desirable that with regard to wind loading, for example, a distinction is made between a virtually permanent wind load (i.e., almost a "dead" load) and a maximum anticipated value thereof. This is the case only when:

$$t_w W < t'_w W'$$

where W is the wind load in accordance with the definition and W' is a "permanent" wind load.

If $W = 70$ kg/m^2 and $t_w = 1.4$, while t'_w is taken as being equal to 1.8, introduction of the permanent wind load would be meaningful if its value exceeded $1.4 \times 70/1.8 \approx 55$ kg/m^2. Such a value of the wind pressure, corresponding to a wind velocity of 29 m/s, i.e., wind force 11–12 on the Beaufort scale, can certainly not be regarded a as permanent, however. Hence, there would seem to be little point in introducing permanent wind load of this kind.

3.4 *Application in regulations*

The stress regions established in the foregoing can as such constitute a complete set of regulations as to permissible stresses for a particular material. They provide a clear and convenient indication of what stress combinations can and what stress combinations cannot be permitted in conjunction with certain values of the strength (positive and/or negative)[15].

If it is not desired to include such graphs in the regulations for various materials but, instead, to define permissible stresses, etc. in clauses, a fairly obvious step will be to adopt straight lines as approximations to the theoretical curves. Having regard to the fact that the numerical values of the variables concerned are only roughly known, there can be no objection to such approximation.

Starting from some extreme values of the variables, the "true" stress region is indicated in Fig. 30. In this diagram the permissible stress for dead load alone, i.e., for $\bar{\sigma}_p = 0$, has always been taken as $\bar{\sigma}_g = 100$. In that case the boundary lines are indicated for four cases, namely, for $f_{st} = 2.5$ and $f_{st} = 3$, in both cases for $v_s = 0$ and $v_s = 0.25$. If these chosen values are regarded as extremes, then the boundary lines will range within the zones defined by them. (In all four cases the values chosen for v_g and v_p are 0.10 and 0.20 respectively.)

The slope of the line through the origin, for which $\bar{S} = 0$, varies very little for the values chosen for f_{st}; the equation of this line is approximately $0.6\sigma_g + \sigma_p = 0$.

In view of the rather steep slope of this line, it is uneconomical to give all the boundary lines this same slope; it will therefore be necessary to make a distinction

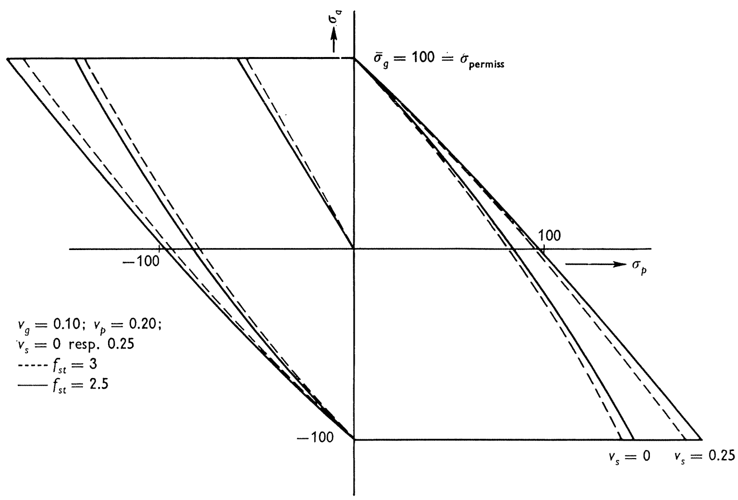

Fig. 30. „True" stress region; all values in % of $\bar{\sigma}_g$.

[15]) As it is stated in 1.3 the "forces" G and P represent "internal actions of forces", e.g. forces, moments or particularly stresses. In the following reference is made to Codes of Practice; therefore the notations G and P are replaced by σ_g and σ_p, while instead of the strength S a part of the ultimate stress is used, named σ_{permiss}.

between the quadrants where σ_g and σ_p have the same algebraic sign and those where these signs are different. It is proposed that in the quadrants I and III the course of the boundary lines be adapted to the variation in strength and that in the other two quadrants the slope of the boundary lines always be chosen parallel to that for $\bar{S} = 0$.

Then stress regions as indicated in Fig. 31 will be obtained. The differences between Fig. 30 and Fig. 31 will thus, in extreme cases, not exceed about 10%. The equations of the straight boundary lines obtained in this way are as follows:

- for σ_g and σ_p having the same sign: $\sigma_g + c\sigma_p = \sigma_{\text{permiss}}$
- for σ_g and σ_p having different signs: $c(0.6\sigma_g + \sigma_p) = \sigma_{\text{permiss}}$

For steel with a coefficient of variation $v_s = 0.10$ to 0.15 for the strength, the value of c can be taken as 1.15; for timber with $v_s = 0.15$ to 0.20 a value of 1.10 is appropriate for c. These assumptions are valid only in so far as no time effects have to be allowed for.

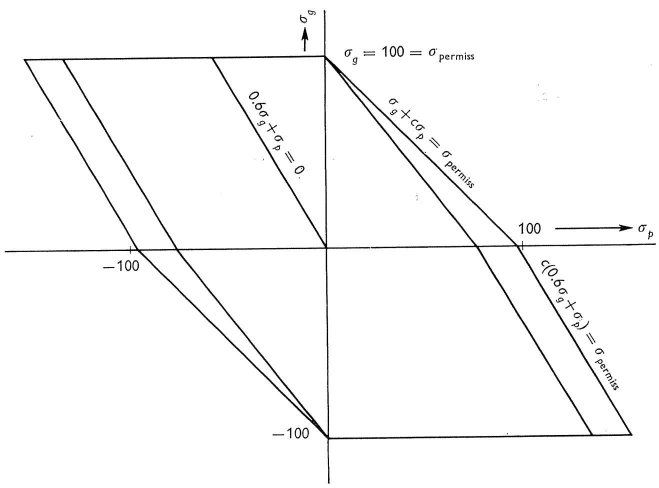

Fig. 31. Approximation of fig. 30 by straight lines.

For timber, however, it is necessary to consider the time effect: in the quadrants I and III (see Fig. 32) the equation of the boundary line is $t_g\sigma_g + ct_p\sigma_p = \bar{\sigma}_0$, while the corresponding equation in the other two quadrants is:

$$ct_p(0.6\sigma_g + \sigma_p) = \bar{\sigma}_0$$

The values of $\sigma_g = \bar{\sigma}_0/t_g$ therefore correspond to the usual permissible stresses.

3.5 *Combination of three load components*

The statistical approach to the problem of structural safety has so far confined itself to the combination of two load components. Combinations of more than two different

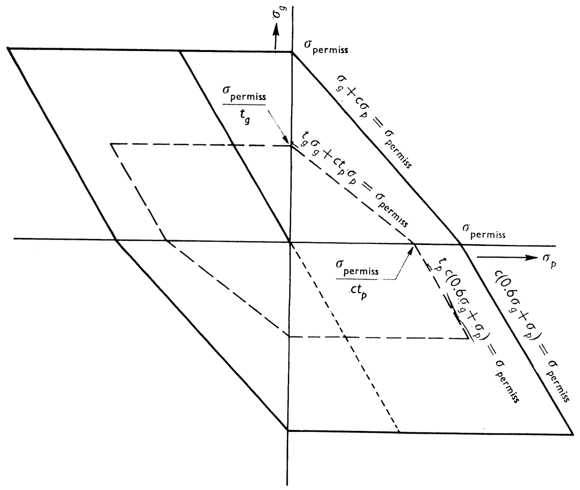

Fig. 32. Deformation of a stress region as a result of introducing time factors t_g and t_p.
——— stress region for non-time-sensitive material
– – – – – stress region for time-sensitive material
(chosen: $t_g = 2$; $t_p = 1.4$)

loads are not inconceivable, however, so that such cases deserve some attention. From the statistical point of view they make the problem more complicated because now the possibility of the simultaneous occurrence of these loads will play an important part. No data concerning this are available, though some sets of regulations do give rules which allow the live loads to be reduced if they are considered to occur in combination with one another. Just as the permissible combinations of two load or stress components σ_g and σ_p can be represented as a plane (two-dimensional) stress region, so a spatial (three-dimensional) stress region can serve to indicate what combinations of three stresses – e.g., σ_g, σ_p and σ_w – are permissible. Any linear relationship between these three stresses will represent a plane in the three-dimensional diagram. For example, the condition $\sigma_g+\sigma_p+\sigma_w \leqslant \bar{\sigma}$ determines the boundary plane indicated in Fig. 33. This requirement may be imposed if the risk of simultaneous occurrence of the maximum values of σ_p and σ_w is to be regarded as considerable.

The other extreme case, namely, that no such risk exists, determines the boundary planes shown by dotted lines in Fig. 33, these being associated with $\sigma_g+\sigma_p = \bar{\sigma}$ and $\sigma_g+\sigma_w = \bar{\sigma}$ respectively. Together these planes form a pyramid. For the normal structure, for which the probability of the simultaneous occurrence of σ_p and σ_w will be greater or less, a boundary figure will be situated somewhere between these two extremes shown in the diagram.

The spatial stress region may be useful in judging the limitations imposed by various regulations.

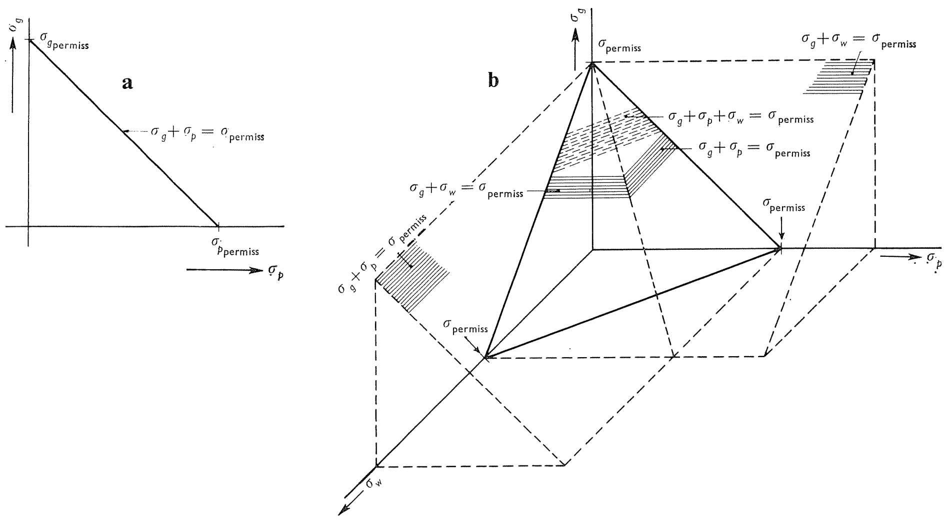

Fig. 33. a. First quadrant of plane stress region.
b. First octant of spatial stress region.

4 Summary

In the first part of this paper it is demonstrated that the application of a safety factor in determining the permissible magnitude of forces, stresses, etc. provides only in a limited sense a criterion of structural safety. Some idea of the degree of safety can, however, be obtained by utilizing the concept "probability of unserviceability" of the structure. This concept leads to the application of statistical methods to the study of this problem, as has indeed generally been done in recent investigations in this field.

The probability of unserviceability is determined, on the one hand, by the reliability with which the available strength can be predicted and, on the other, by the accuracy with which the maximum forces and stresses that may occur can be calculated; a certain difference between the mean values of strength and load is associated with a probability. The "statistical safety index" f_{st} is introduced as a criterion of this probability.

It has furthermore been endeavoured to give a good definition of the quantities involved in the investigation of structural safety. A proper understanding of these appears to be very necessary in discussions on the present subject. In the second part of the paper an assessment is first made of the variations in the loads. Data on the actually occurring loads are scarce. Because of this, it is not possible properly to determine the true probabilities. Nor would there, for the time being, seem to be much point in striving to achieve great refinement in the calculations employed. This lack of adequate information does, however, raise the question whether a larger share of the investigations relating to structures ought not to be aimed at collecting more knowledge concerning the loads that actually occur. The present author is of the opinion that this question must be answered in the affirmative, because the accuracy with which the forces and loads liable to act on a structure are known is not comparable with the accuracy with which information on the strength is obtained.

Having made the above-mentioned assessment of the variability in the loads, it is then investigated what values the statistical safety index may assume for various construction materials when certain foreign or Dutch codes of practice are applied. Fig. 20 presents a survey of this; it appears that f_{st} is between approximately 2.5 and 3.5. Part of the dispersion (scatter) in the values of f_{st} is due to the fact that in some sets of regulations no distinction is made between dead (permanent) load and live (variable) load, whereas the present paper does make such a distinction.

In Chapter 3 the definition formula for the statistical safety index is graphically elaborated, whereby so-called "stress regions" are determined in which the permissibility of load combinations is represented.

The stress region is interpreted into formulae with which the permissibility of the stresses, etc. can be judged. As a result of some simplifications and approximations these formulae are reduced to fairly simple and conveniently manageable expressions.

Several times particular attention is called to the need to give sufficient consideration to structural members in which live load produces forces of a different algebraic sign from the forces due to dead load. Caution is necessary more particularly if the prestress produced by the dead load alone must serve to resist the live load forces acting in the opposite direction and the member itself lacks the strength properties needed for this.

Finally, the possibility of combination of three load components is considered. This problem is not dealt with statistically, but it is indicated how, with the aid of a spatial stress region, it is possible to gain insight into this case.

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