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Contents

PUNCHING SHEAR AT INNER, EDGE AND CORNER COLUMNS

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Preface

The research on punching shear in reinforced concrete slabs was carried out by the Institute TNO for Building Materials and Building Structures (IBBC-TNO) with the financial support of the Netherlands Committee for Concrete Research (CUR).

The problem of punching shear around axially or eccentrically loaded inner columns was studied first. That part of the research, sponsored by CUR Committee A 18, was published in HERON, Vol. 20 (1974 No. 2). The formula derived in it has been adopted in the Netherlands Code of Practice for Concrete.

Further research on punching shear is reported in the present publication. The research was sponsored by CUR Committee A 25, which was entrusted with drawing up a proposal for the calculation of punching shear resistance at edge and corner columns, so as to supplement the Code of Practice.

Committee A 25 was constituted as follows:

J. Buijs, Chairman
M. Dragosavić, Secretary
J. Brakel
H. van Tongeren
G. Vlas
Ch. J. Vos, Mentor

The research on punching shear at edge and corner columns was carried out by Ir. A. van den Beukel, assisted by Ir. M. Dragosavić, both on the staff of IBBC-TNO.

Thanks are due to the Netherlands Committee for Concrete Research for financing this work.

The translation from the original report no. BI-75-87 (in Dutch) into English is by Ir. C. van Amerongen, MICE.

NOTATION

a	length of side of a square column section
a_b	shorter length of side of a rectangular column section
a_l	longer length of side of a rectangular column section
a_x	length of side of a rectangular column section in x-direction
a_y	length of side of a rectangular column section in y-direction
b	width of a beam
c	distance from axis of column to edge of slab
d	diameter of an (imaginary) round column
e, e_x, e_y	eccentricity of the column load in relation to the centroid of the column section
e'	eccentricity of the column load in relation to the centroid of the periphery
e_p	eccentricity of the centroid of the periphery (sectional area) with respect to the axis of the column
f_c	average cylinder strength of the concrete
f_{cm}	average cube strength of the concrete
f_{ct}	splitting tensile strength of the concrete
f_{ctd}	design value of the tensile strength of the concrete
f_e	yield stress of the steel
h	effective depth of slab
p	perimeter (peripheral length)
A	area (in general)
F	column force to be transmitted
F_d	design value of the column force
F_{ut}	punching shear force
$I_{x'}, I_{y'}$	principal moment of inertia of the periphery
M	external moment due to F
M_{ut}	external moment due to F_{ut}
T	shear force
W	section modulus for an extreme fibre of the periphery
$\alpha, \alpha_x, \alpha_y$	geometry factor
α_t	eccentricity factor
β	moment coefficient (indicating what proportion (βM) of the moment equilibrates the vertical shear stresses)
σ	stress (in general)
τ	nominal shear stress at the periphery
τ_{max}	maximum nominal shear stress at the periphery

PUNCHING SHEAR AT INNER, EDGE AND CORNER COLUMNS

Summary

This report describes how the punching shear force (failure load) of a slab around a column can be calculated and also fills in the background to this analysis procedure.

The analysis for inner columns as well as for edge and corner columns starts from a simple basic formula. The effect of the eccentricity of the column force to be transmitted is taken into account by means of an eccentricity factor. This factor is in part dependent on the geometry factor, numerical values of which are given in tables relating to various cases. The whole analysis procedure is summarized in Chapter 3.7, and the method is illustrated with reference to two worked examples in Chapter 4.

Punching shear at inner, edge and corner columns

CHAPTER 1

INTRODUCTION

With regard to the failure of a slab around a column it is usual to distinguish between two possible causes:

a. the cause may be that the moment resisting capacity of the slab is reached; in that case yielding of the reinforcement occurs;
b. the shear force that has to be transmitted from the slab to the column may attain the maximum that can be structurally resisted before yielding of the slab reinforcement occurs; in that case the term punching shear is applicable.

By punching shear force is understood the force that causes the slab to fail in shear.

In HERON, Vol. 20 "Punching shear" (1974, No. 2), it has already been indicated how the shear resistance capacity or punching shear force of a slab at an inner column can be calculated. On the basis of recent research it is now possible to give a method of analysis applicable also to edge columns and corner columns. It has furthermore been shown that a uniform method can be applied to inner as well as to edge and corner columns. The results of these investigations are embodied in this report.

A more detailed account of the punching shear tests performed on edge and corner columns is given in IBBC-TNO Report No. BI-75-55 "Shear resistance capacity of slabs on point-type supports" dated 2 September 1975. The results of those tests are given in condensed form in the appendix to the present report.

Chapter 2

FORMS OF FAILURE

Before indicating how a practical analysis of punching shear may be carried out il will be helpful to examine some failure patterns.

The transmission of load from the slab to the column is associated with a complex pattern of forces around the column. With increasing magnitude of the load, radial and tangential cracks will generally develop at the surface of the slab; the stage of failure is characterized by more or less conical shearing around the column. This conical shape is found to occur most distinctly around an axially (concentrically) loaded centre column: see Fig. 1. It is to be noted that this cone manifests itself both when bending moment failure and when shear failure (as defined in Chapter 1) occurs. At most, the difference is that in the case of pronounced bending moment failure a number of the cracks are relatively wider (yield lines, radial and directly around the column).

If the force acting in the column is eccentric, the conical shape associated with failure is often not so readily recognizable at the surface of the slab. This is true also of an edge column or corner column because in such cases there is almost invariably eccentric loading. By way of illustration some photographs of test specimens after failure, reproduced from the IBBC-TNO report referred to in Chapter 1, are given in Fig. 2. These specimens correspond to the situation at an edge column. It should be conceived that the semicircular edge, which served as the bearing in the tests, corresponds to the line of zero bending moment in a slab supported by several columns. The two specimens are alike as regards their dimensions, reinforcement and quality of concrete. From that same report it also appears that both specimens failed in shear. They differ, however, in the magnitude of the eccentricity e of the force acting in the column. This difference manifested itself not only in a difference in the failure load but also in distinctly different cracking patterns. The conical shape is discernible in the side views of the specimens, but the orientation is reversed. This is due to the fact that the column load produces bending moments in the slab which are of opposite algebraic sign in these examples. It is further explained in Chapter 3.

Some cracking patterns at a corner column are illustrated in Fig. 3. Further particulars concerning these specimens are given in the appendix.

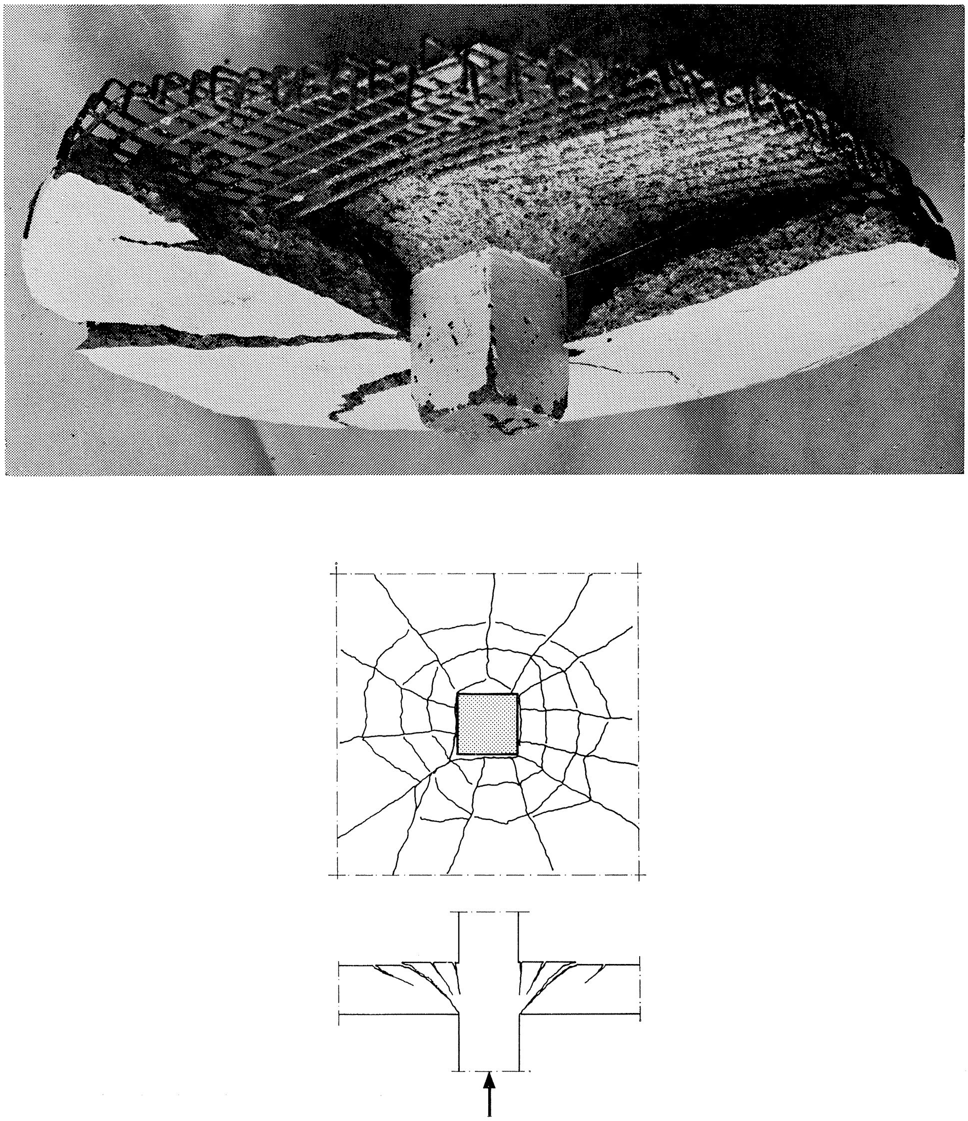
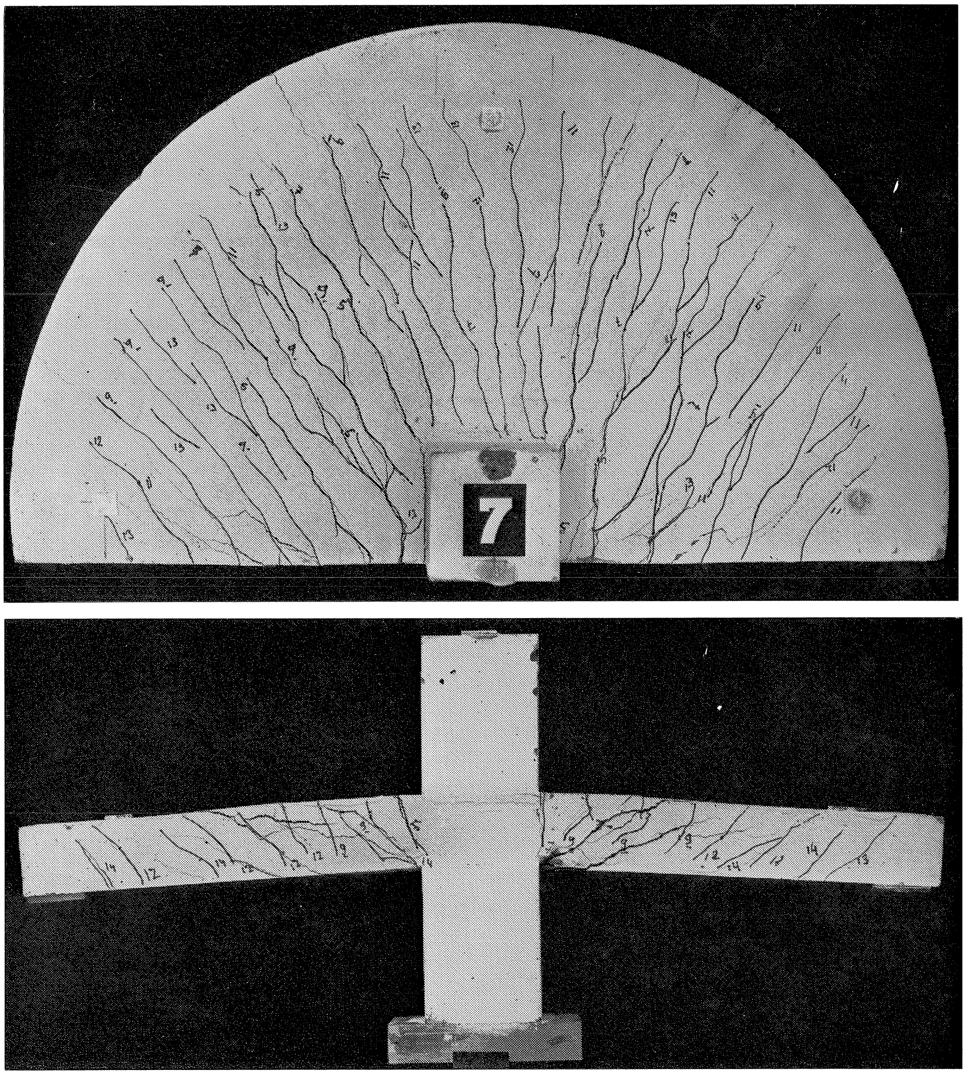


Fig. 1. Failure pattern at an axially loaded inner column.



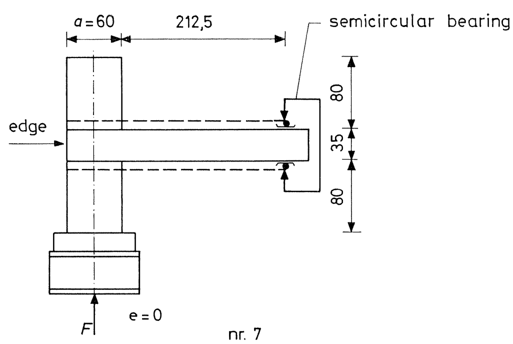
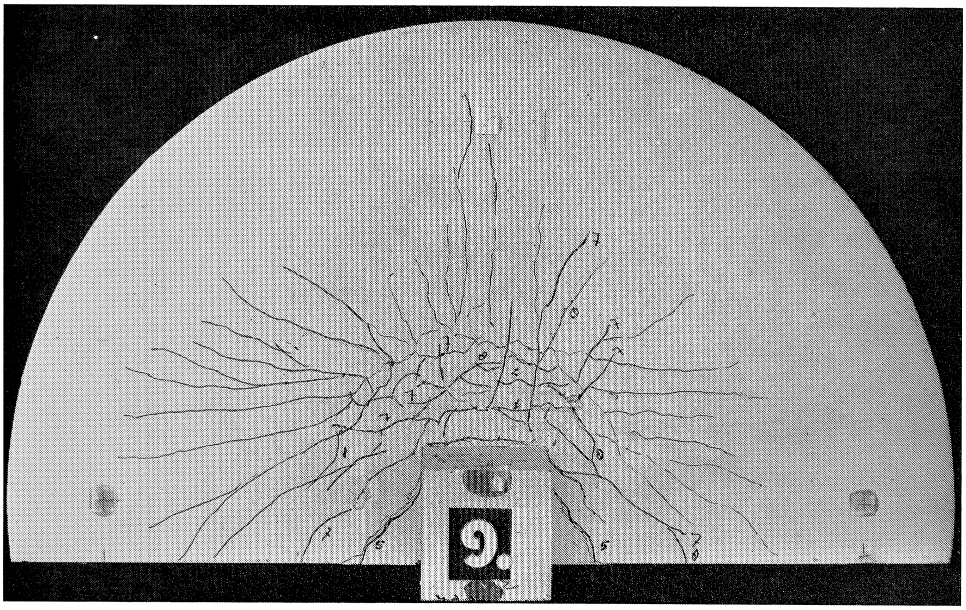


Fig. 2.
Failure patterns of two slab/edge column test specimens.

reinforcement: orthogonal mesh, top and bottom, with $\omega_0 = 1,5$ %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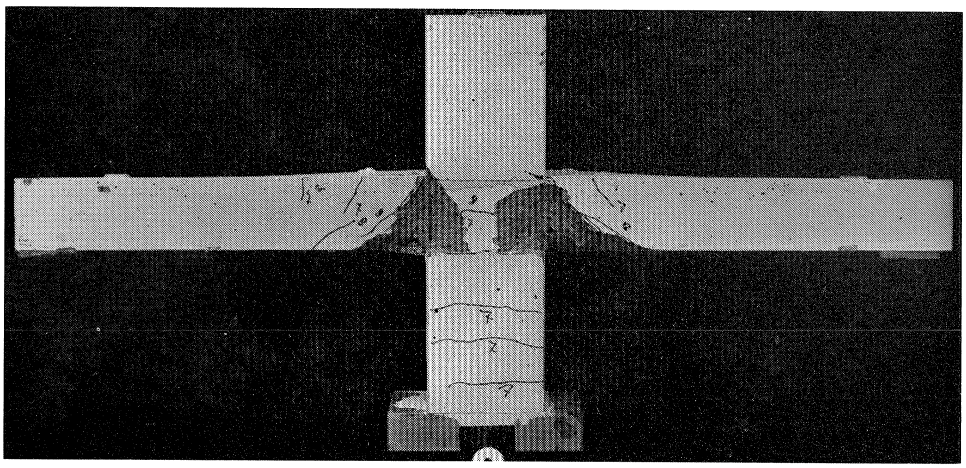
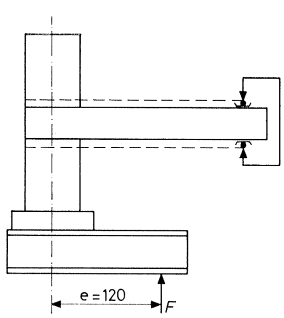



e = 120
F

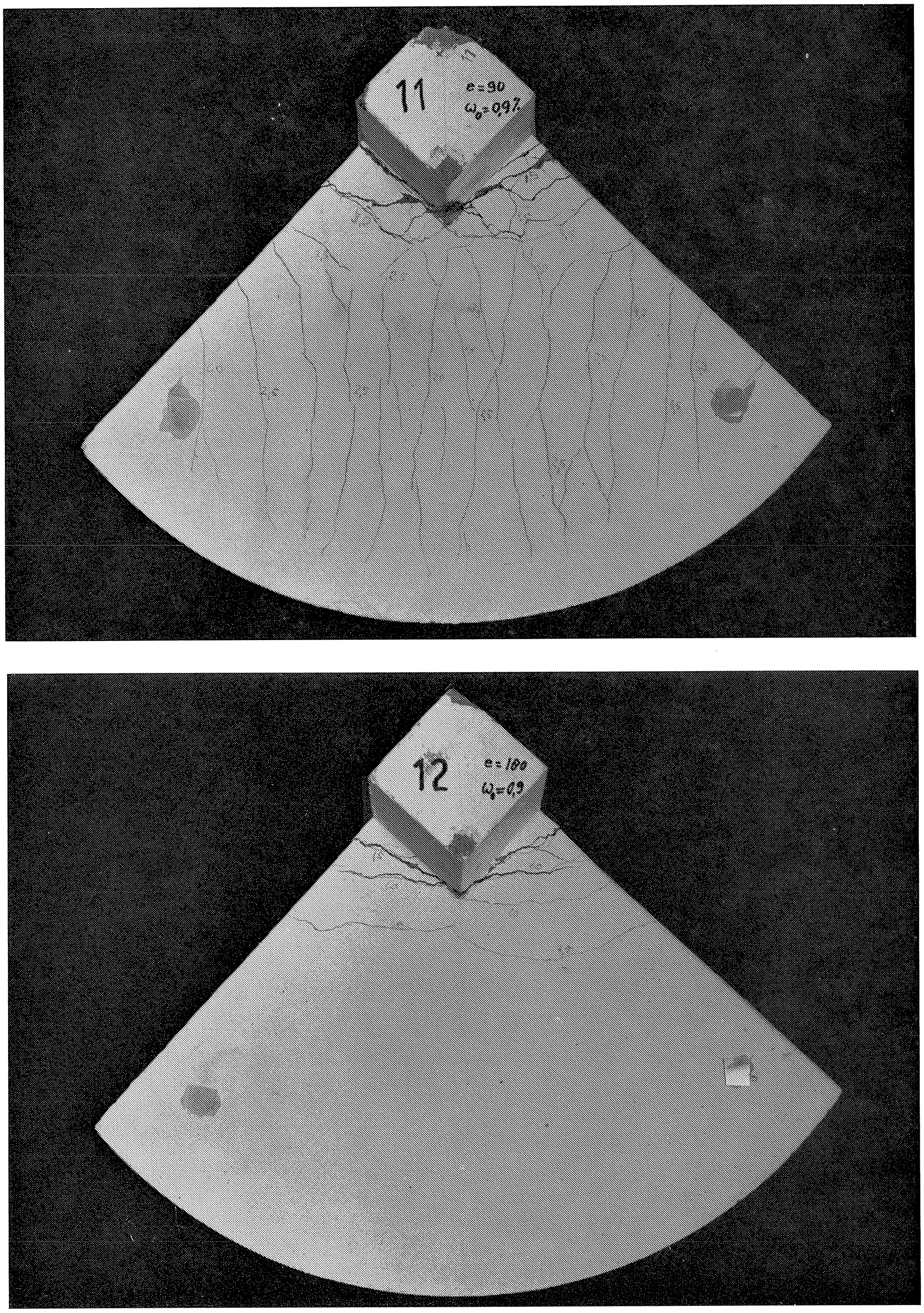


Fig. 3. Failure patterns of two slab/corner column test specimens.

CHAPTER 3

ANALYSIS

3.1 Tie-up with the conventional methods of analysis

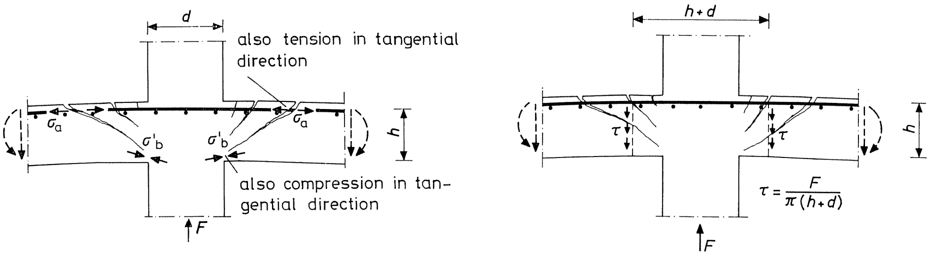
In the shear design of beams it is usual to adopt a nominal shear stress τ in the calculations, defined by:

$$\tau = \frac{T}{bh}$$

T denotes the shear force in the beam cross-section concerned.

Taking this as the point of departure, and having regard to the problems associated with a description of the actually quite complex state of stress concerned, a similar procedure will be adopted for the present purpose.

In the analysis of punching shear a nominal vertical shear stress acting in a vertical section through the slab at some distance from the column is considered (see Fig. 4). This section is called the *periphery*; its length along the periphery is called the *perimeter*.



a. Actual stress distribution

b. Stress distribution assumed for the analysis

Fig. 4. Stress distributions.

If there is no eccentricity of the column force in relation to the centroid of the periphery, the shear stress is uniformly distributed along the latter. The nominal shear stress is then:

$$\tau = \frac{F}{ph}$$

where

F = the column force to be transmitted
p = the perimeter
h = the average effective depth of the slab

If the force F has an eccentricity e' in relation to the centroid of the periphery, there will also be a bending moment equal to $M = Fe'$. This moment was referred to at the end of Chapter 2. The external moment M gives rise to a complex state of stress (bending, shear, torsion) in the slab around the column. For the purpose of punching shear analysis it will be presupposed that a certain fraction $\beta(< 1)$ of the external moment produces only vertical shear stresses in the periphery. β is called the moment coefficient. In analogy with the well-known formula

$$\sigma = \frac{F}{A} \pm \frac{M}{W}$$

the maximum nominal shear stress is calculated from:

$$\tau_{max} = \frac{F}{ph} + \frac{\beta F e'}{W} \tag{1}$$

where

W = section modulus = I/u
I = principal moment of inertia ($I_{x'}$, or $I_{y'}$) of the periphery
u = distance from the extreme fibre of the periphery to the controid thereof

In equation (1) the eccentricity e' is assumed to be directed along one of both principal axis.

The derivation of the moment coefficient involves some rather complicated expressions and will therefore not be given here. Suffice it to mention that β is based on the elastic stress distribution in a rectangular periphery and that it is dependent only on the geometry of the perimeter. The section modulus W is likewise a geometric quantity.

3.2 Punching shear force

From various tests it has been found that (the design value of) the punching shear force can be calculated by establishing the condition that at failure the maximum nominal shear is just equal to (the design value of) the tensile strength f_{ctd} of the concrete. The corresponding column force is the punching shear force F_{ut} (u denotes ultimate, t denotes shear). Equation (1) then becomes:

$$f_{ctd} = \frac{F_{ut}}{ph} + \frac{F_{ut}}{ph} \frac{\beta e' ph}{W}$$

or

$$F_{ut} = phf_{ctd} \left(\frac{1}{1 + \dfrac{\beta e' ph}{W}} \right) \tag{2}$$

The factor in parentheses is called the eccentricity factor α_t. The expression for punching shear force is thus:

$$\boxed{F_{ut} = \alpha_t phf_{ctd}} \tag{3}$$

with

$$\alpha_t = \frac{1}{1 + \frac{\beta e' ph}{W}}$$

3.3 Perimeter and periphery

It is evident that the perimeter p, i.e., the peripheral length, plays an important part because both α_t and β are dependent on p. From literature research and from experimental investigations a reasonably good degree of agreement has been found to exist between the punching shear force calculated from equation (3) and the experimentally determined punching shear force, if the periphery is defined as follows.

The periphery is the vertical section through the slab with depth h situated at a distance $\frac{1}{2}h$ from an (imaginary) round column and terminating at right angles to an edge (if any) of the slab. This is illustrated in Fig. 5. If the column in question is rectangular, it is conceived as replaced by a round column with the same circumferential cross-sectional length as that of the original column. Hence the diameter of the round column is expressed by:

$$d = \frac{2}{\pi}(a_l + a_b)$$

where α_l is the longer and a_b the shorter side of the rectangular column. A further requirement for applying this approximation is that the rectangular shape of the column does not deviate too much from a square. If the ratio a_l/a_b exceeds a value of 2,

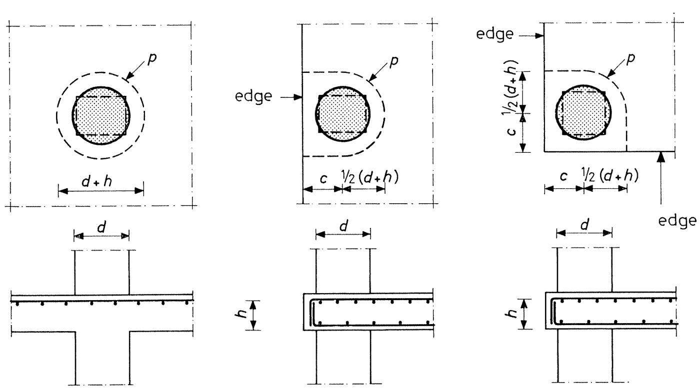


Fig. 5. Peripheral shapes; the periphery is designated by p.

the analysis presented here can no longer be regarded as sufficiently accurate, for then the load transmitting action will tend to concentrate at the two shorter column faces.

It may seem somewhat odd that the punching shear analysis is based on a (partly) circular perimeter, whereas the moment coefficient β envisaged in 3.1 is based on a rectangular perimeter. The reason is that, in the first place, the determination of this coefficient is a very complicated procedure for a circular as opposed to a rectangular perimeter and that, secondly, a rectangular perimeter at a distance $\frac{1}{2}h$ from the column will, for fairly large slab thicknesses, result in some over-estimation of the punching shear force, as experimental research has shown.

As already stated, the eccentricity e' of the column load should be reckoned in relation to the centroid of the periphery. Normally the eccentricity e in relation to the axis of the column is adopted. If e_p is the distance between the centroid of the periphery and the (centroidal) axis of the column, then:

$$e' = e - e_p$$

The eccentricity factor α_t can now be written as:

$$\boxed{\alpha_t = \frac{1}{1 + \alpha \dfrac{e - e_p}{h + d}}} \tag{4}$$

where the geometric factor α is equal to:

$$\alpha = \frac{\beta p h (h + d)}{W} \tag{5}$$

This latter factor is determined entirely by geometric quantities. In the IBBC-TNO report already referred to, values of α for various cases have been calculated; they are listed in tables given at the end of the present chapter. As a consequence of the above-mentioned difference in periphery in so far as the calculation of p and β is concerned, for a round column the value of α (in which β has been taken into account) should be determined as though for a square column.

3.4 Maximum perimeter

In all the punching shear tests relating to edge and corner columns the face or faces of the column always coincided with the edge or (for a corner column) edges of the slab. It is, however, unlikely that an essentially different method of analysis will be necessary in a case where the column is not at the actual edge of the slab but is instead set back some distance from the edge. In this latter case the analysis is indeed still valid, provided that the value adopted for the perimeter does not exceed the circumference of a completely circular perimeter, i.e., always $p \leqslant \pi(h+d)$.

3.5 Double eccentricity

So far it has tacitly been presupposed that the eccentricity e or e' is directed along one principal axis of inertia of the perimeter. In general, however, there may be eccentricity in the directions of both the principal axes of inertia x' and y' (see Fig. 6 and 7). The eccentricity e_p of the periphery, already referred to, is therefore always directed along the x'-axis. Bearing in mind that the calculation of the punching shear force comprises in effect the calculation of the largest nominal shear stress at the periphery by means of an algebraic summation of stresses, it follows, for given eccentricities e_x and e_y, from the general expression for τ_{max}:

$$\tau_{max} = \frac{F}{ph} + \left(\frac{\beta F e'}{W}\right)_x + \left(\frac{\beta F e'}{W}\right)_y$$

$$= \frac{F}{ph}\left(1 + \alpha_x \frac{|e_x - e_p|}{h+d} + \alpha_y \frac{|e_y|}{h+d}\right)$$

As before, the expression for the punching shear force is:

$$F_{ut} = \alpha_t phf_{ctd} \tag{3}$$

where, however:

$$\alpha_t = \frac{1}{1 + \alpha_x \frac{|e_x - e_p|}{h+d} + \alpha_y \frac{|e_y|}{h+d}} \tag{6}$$

The absolute value signs in the expression for α_t are necessary if the given eccentricities e_x and e_y are reckoned algebraically with respect to the $x-y$ co-ordinate system. Admittedly the section modulus W depends on the cases $e_x \geqq e_p$ and $e_x \leqslant e_p$, but this is already taken into account in the values of α.

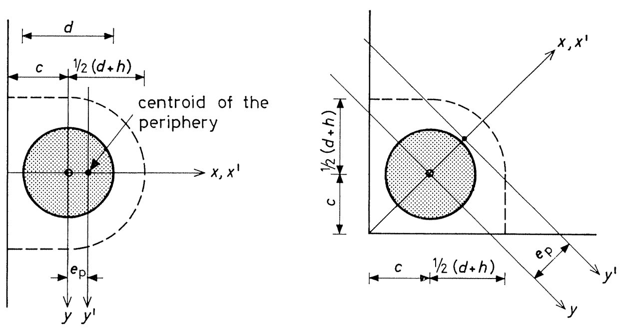


Fig. 6. Principal axes of inertia x', y' of the periphery.

The values of α as given in the tables at the end of this chapter actually relate to the extreme fibres of the periphery which correspond to the smallest value of the section modulus. If the punching force is calculated from a stress analysis at an arbitrary point C of the periphery (Fig. 7), the requisite values of α_x and α_y are deducible from:

$$\alpha_{xC} = \alpha_{xA} \frac{x'_C}{x'_A}$$

$$\alpha_{yC} = \alpha_{yB} \frac{y'_C}{y'_B}$$

These relationships are based on the fact that the section modulus increases linearly with the distance to the origin of the $x'-y'$ co-ordinate system.

If it is not immediately evident which point of the perimeter is the determinative one, the eccentricity factors should be calculated for a number of points. The lowest value of α_t will then of cours be determinative.

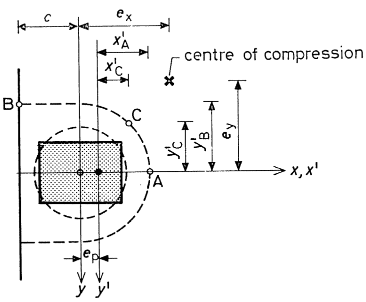


Fig. 7. Geometric quantities for eccentricity in two directions.

3.6 Maximum eccentricity factor

If the value $|e-e_p|$ alone is varied in the expression for α_t, there is found to be a pronounced peak value (maximum) of F_{ut} for $e-e_p=0$. In the vicinity of this peak value the punching shear force F_{ut} is very sensitive to minor variations in the eccentricity. Since the values of e and e_p are never accurately known, it is necessary to truncate the peak. For this reason the maximum value of α_t to be adopted in the calculations is taken as 0,9.

3.7 Procedure for calculating the punching shear force

Summarizing, the following procedure for calculating the punching shear force F_{ut} can now be established:

– $F_{ut} = \alpha_t p h f_{ctd}$

– A rectangular column with cross-sectional dimensions a_l and a_b is conceived as replaced by a round column with a diameter:

$$d = \frac{2}{\pi}(a_l + a_b)$$

The calculation applies when: $a_l \leqslant 2a_b$.

– The perimeter is:
for an inner column: $p = \pi(h+d)$
for an edge column: $p = \frac{1}{2}\pi(h+d)+2c$
for a corner column: $p = \frac{1}{4}\pi(h+d)+2c$

– The eccentricity factor is:

$$\alpha_t = \frac{1}{1+\alpha_x \dfrac{|e_x - e_p|}{h+d} + \alpha_y \dfrac{|e_y|}{h+d}} \leqslant 0{,}9$$

where:
for an inner column:

$$e_p = 0$$

for an edge column:

$$e_p = \frac{\frac{1}{2}(h+d)^2 - c^2}{p}$$

for a corner column:

$$e_p = \tfrac{1}{4}\sqrt{2}\,\frac{(h+d)^2 + 2c(h+d) - 2c^2}{p}$$

The eccentricities e_x and e_y with respect to the centroid of the column are measured along the principal axes of inertia of the periphery (see also Fig. 7).

– The maximum value of c to be taken into account in the calculations is:
for an edge column:

$$c = \tfrac{1}{4}\pi(h+d)$$

for a corner column:

$$c = \tfrac{3}{8}\pi(h+d)$$

– The values of α, α_x and α_y for various cases are indicated within the accompanying tables. Linear interpolation is permissible for intermediate values of the geometric

variables concerned. The tables have been established for a rectangular column; for a round column the value of α is found by substitution of $a_l = a_b = \frac{1}{4}\pi d$ or $a_x = a_y = \frac{1}{4}\pi d$ as stated at the end of 3.3. In this latter expression a_x and a_y are the side lengths (cross-sectional dimensions) of the column perpendicular and parallel to the edge of the slab respectively.
For a rectangular corner column with sides a_x and a_y it is permissible, as an approximation, to consider a square column with sides $a = \frac{1}{2}(a_x + a_y)$.

- It may occur that the punching shear force of an edge column or a corner column, calculated by this procedure, is larger than the punching shear force of an identical inner column having the same eccentricity e of the column force. This should be checked, and the lower value thus found should be adopted as the punching shear force of the edge column or corner column.

Table 1. Values of α for a rectangular inner column

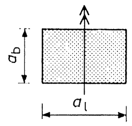
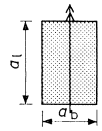
$\frac{a_l+h}{a_b+h}$		
1,0	2,00	2,00
1,2	2,23	1,77
1,4	2,42	1,58
1,6	2,58	1,42
1,8	2,71	1,29
2,0	2,82	1,18

Table 2. Values of α_x for an edge column with moment vector parallel to the edge

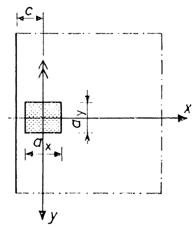
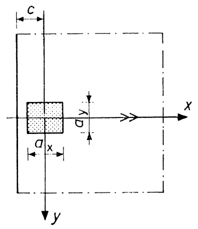
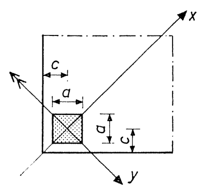
		$\frac{2c}{h+d}$							
	$\frac{a_x+h+2c}{2(a_y+h)}$	0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6
$e_x \geq e_p$	0,4	0,38	0,31	0,30	0,29	0,30	0,31	0,32	0,34
	0,5	0,57	0,47	0,45	0,44	0,45	0,47	0,49	0,51
	0,6	0,78	0,65	0,61	0,61	0,62	0,64	0,67	0,69
	0,7	0,99	0,82	0,78	0,77	0,79	0,81	0,85	0,88
	0,8	1,20	0,99	0,94	0,93	0,95	0,98	1,02	1,06
	0,9	1,40	1,16	1,09	1,08	1,11	1,14	1,19	1,24
	1,0	1,58	1,31	1,23	1,22	1,25	1,29	1,34	1,40
	1,1	1,75	1,45	1,36	1,36	1,38	1,43	1,49	1,55
	1,2	1,90	1,57	1,48	1,47	1,50	1,55	1,62	1,68
	1,3	2,04	1,69	1,59	1,58	1,61	1,67	1,73	1,81
	1,4	2,17	1,79	1,69	1,68	1,71	1,77	1,84	1,92
	1,5	2,28	1,89	1,78	1,77	1,81	1,87	1,94	2,02
$e_x < e_p$	0,4	0,64	0,51	0,46	0,45	0,44	0,45	0,46	0,47
	0,5	0,97	0,77	0,70	0,68	0,67	0,68	0,69	0,70
	0,6	1,32	1,05	0,96	0,92	0,92	0,93	0,94	0,96
	0,7	1,68	1,34	1,22	1,17	1,17	1,18	1,20	1,22
	0,8	2,03	1,62	1,47	1,42	1,41	1,42	1,45	1,48
	0,9	2,36	1,88	1,71	1,65	1,64	1,65	1,68	1,72
	1,0	2,66	2,13	1,94	1,87	1,85	1,87	1,90	1,94
	1,1	2,95	2,35	2,14	2,07	2,05	2,07	2,10	2,15
	1,2	3,21	2,56	2,33	2,25	2,23	2,25	2,29	2,34
	1,3	3,44	2,75	2,50	2,41	2,39	2,42	2,46	2,51
	1,4	3,66	2,92	2,66	2,56	2,54	2,56	2,61	2,66
	1,5	3,85	3,07	2,80	2,70	2,68	2,70	2,75	2,80

Table 3. Values of α_y for an edge column with moment vector perpendicular to the edge



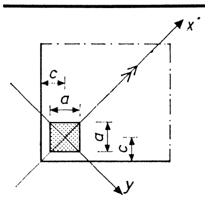
	$\frac{a_y+h}{a_x+h+2c}$	$\frac{2c}{h+d}$: 0,2	0,4	0,6	0,8	1,0	1,2	$\geqslant$1,4
$e_p=0$	0,3	0,67	0,62	0,58	0,56	0,54	0,52	0,50
	0,4	0,87	0,81	0,76	0,72	0,70	0,68	0,66
	0,5	1,06	0,98	0,93	0,88	0,85	0,82	0,80
	0,6	1,24	1,15	1,08	1,03	0,99	0,96	0,94
	0,7	1,40	1,29	1,22	1,17	1,12	1,09	1,06
	0,8	1,54	1,43	1,34	1,28	1,24	1,20	1,17
	0,9	1,68	1,55	1,46	1,40	1,34	1,30	1,27
	1,0	1,80	1,66	1,57	1,50	1,44	1,40	1,36

Table 4. Values of α_x for a square corner column with moment vector perpendicular to bisectrix



	$\frac{2c}{h+d}$: 0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8
$e_x \geqslant e_p$	5,05	3,84	3,15	2,69	2,37	2,12	1,93	1,77	1,64
$e_x < e_p$	9,28	6,51	5,04	4,10	3,47	3,00	2,65	2,38	2,16

Table 5. Values of α_y for a square corner column with moment vector parallel to bisectrix



	$\frac{2c}{h+d}$: 0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8
$e_p=0$	1,85	1,58	1,39	1,24	1,13	1,03	0,95	0,88	0,83

CHAPTER 4

WORKED EXAMPLES

Example 1

A junction of a floor slab with a corner column as illustrated in Fig. 8 is characterized by the following data:

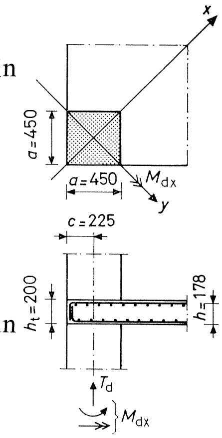


Fig. 8.
Data concerning example 1.

$a\ = 450$ mm
$h\ = 178$ mm (average effective depth)
$h_t = 200$ mm
$c\ = \frac{1}{2}a = 225$ mm

The design values of the loads to be transmitted from the column to the slab are:

$T_d\ \ = 145$ kN
$M_{dx} = \ 27$ kNm
$M_{dy} = \ \ 0$ kNm

The concrete of the slab is of quality class B 17,5 ($f_{ctd} = 1{,}1$ N/mm^2).

Check

Wether the shear resistance capacity of the slab is adequate.

Solution

Perimeter

$$p = \tfrac{1}{4}\pi(h+d)+2c$$

$$d = \frac{4a}{\pi} = \frac{4\cdot 450}{\pi} = 573 \text{ mm (= diameter of imaginary round column)}$$

$$p = \tfrac{1}{4}\pi(178+573)+2\cdot 225 = 1040 \text{ mm}$$

Eccentricities

$$e_p = \tfrac{1}{4}\sqrt{2}\,\frac{(h+d)^2+2c(h+d)-2c^2}{p}$$

$$= \tfrac{1}{4}\sqrt{2}\,\frac{(178+573)^2+2\cdot 225(178+573)-2\cdot 225^2}{1040} = 272 \text{ mm}$$

$$e_x = \frac{M_{dx}}{T_d} = \frac{27000}{145} = 186 \text{ mm } (< e_p)$$

Value of α

$$\frac{2c}{h+d} = \frac{2 \cdot 225}{178+573} = 0{,}60$$

From Table 4:

$$\alpha_x = 5{,}04$$

Eccentricity factor

$$\alpha_t = \frac{1}{1+\alpha_x \dfrac{|e_x - e_p|}{h+d}} = \frac{1}{1+5{,}04 \dfrac{|186-272|}{178+573}} = 0{,}634$$

Punching shear force

$$F_{ut} = \alpha_t phf_{ctd} = 0{,}634 \cdot 1040 \cdot 178 \cdot 1{,}1 \cdot 10^{-3} = 129 \text{ kN}$$

Punching shear force of an identical inner column

$$p = \pi(h+d) = \pi(178+573) = 2359 \text{ mm} \quad e_p = 0 \quad \alpha = 2{,}00 \text{ (Table 1)}$$

$$\alpha_t = \frac{1}{1+\alpha \dfrac{e}{h+d}} = \frac{1}{1+2{,}00 \dfrac{186}{178+573}} = 0{,}669$$

$$F_{ut} \text{ (inner column)} = \alpha_t phf_{ctd} = 0{,}669 \cdot 2359 \cdot 178 \cdot 1{,}1 \cdot 10^{-3} = 309 \text{ kN}$$

This value is not determinative.

Conclusion

The punching shear force F_{ut}(= 129 kN) is less than the design value T_d of the column force to be transmitted. The design calculations will therefore have to be revised.

Possibilities for modifying the design

F_{ut} can be increased in various ways. Some of these are indicated below. Combinations are also possible.

a. Using a better quality for the slab, e.g., B 22,5 ($f_{ctd} = 1{,}3$ N/mm^2). Then:

$$F_{ut} = 0{,}634 \cdot 1040 \cdot 178 \cdot 1{,}3 \cdot 10^{-3} = 152 \text{ kN } (> T_d).$$

b. Increasing the thickness of the slab. With $h_t = 240$ mm and $h = 218$ mm follows: $p = 1071$ mm, $e_p = 291$ mm, $\alpha_x = 5{,}27$, $\alpha_t = 0{,}588$, so that:
$F_{ut} = 0{,}588 \cdot 1071 \cdot 218 \cdot 1{,}1 \cdot 10^{-3} = 151$ kN ($> T_d$).
c. Increasing the perimeter by keeping the edges of the slab some distance away from the faces of the column. Suppose that the edge is 35 mm from the column faces. Then: $c = 225 + 35 = 260$ mm, $p = 1110$ mm, $e_p = 261$ mm, $\alpha_x = 4{,}61$, $\alpha_t = 0{,}685$ and
$F_{ut} = 0{,}685 \cdot 1110 \cdot 178 \cdot 1{,}1 \cdot 10^{-3} = 149$ kN ($> T_d$).
d. Increasing the dimensions of the column. Suppose $a = 630$ mm. Then: $p = 1400$ mm, $e_p = 348$ mm, $\alpha_x = 4{,}84$, $\alpha_t = 0{,}556$ and $F_{ut} = 0{,}556 \cdot 1400 \cdot 178 \cdot 1{,}1 \cdot 10^{-3} = 152$ kN ($> T_d$).

Another possible solution could consist in installing special punching shear reinforcement. The effectiveness of such reinforcement has not yet been conclusively demonstrated, however.

Example 2

A junction of a floor slab with an edge column as illustrated in Fig. 9 is characterized by the following data:

$d = 600$ mm
$h = 222$ mm (average effective depth)
$h_t = 250$ mm
$c = 830$ mm

The design values of the loads to be transmitted from the column slab are:

$T_d = 731$ kN
$M_{dx} = 148{,}95$ kNm
$M_{dy} = 54{,}10$ kNm

The concrete of the slab is of quality class B 45 ($f_{ctd} = 2$ N/mm^2).

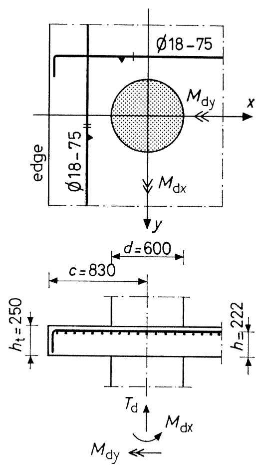


Fig. 9.
Data concerning example 2.

Calculate

The punching shear force F_{ut} in order to check that $F_{ut} \geqslant T_d$.

Solution

Perimeter
$p = \frac{1}{2}\pi(h+d) + 2c$ with the condition $c \leqslant \frac{1}{4}\pi(h+d)$.
$\frac{1}{4}\pi(h+d) = \frac{1}{4}\pi(222+600) = 645{,}5$ mm; this is less than the given value of 830 mm so that $c = 645{,}5$ should be adopted. Then:

$$p = \tfrac{1}{2}\pi(222+600) + 2 \cdot 645{,}5 = 2582 \text{ mm}$$

Eccentricities

$$e_p = \frac{\frac{1}{2}(h+d)^2 - c^2}{p} = \frac{\frac{1}{2}(222+600)^2 - 645{,}5^2}{2582} = -31 \text{ mm } (!)$$

$$e_x = \frac{M_{dx}}{T_d} = \frac{148950}{731} = 204 \text{ mm } (> e_p)$$

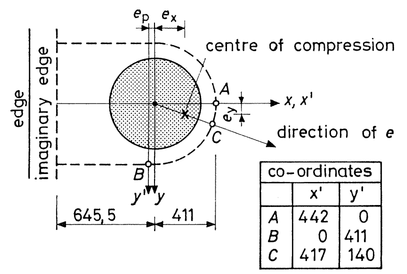
$$e_y = \frac{M_{dy}}{T_d} = \frac{54100}{731} = 74 \text{ mm}$$

Values of α

These are obtained from tables 2 and 3, substituting:

$$a_x = a_y = \tfrac{1}{4}\pi d = \tfrac{1}{4}\pi \cdot 600 = 471 \text{ mm}$$

The value $c = 645{,}5$ mm should be introduced for the edge distance; an imaginary edge is in fact adopted in the calculation (see Fig. 10).



co-ordinates	x'	y'
A	442	0
B	0	411
C	417	140

Fig. 10. Geometric quantities, example 2.

For the x-direction:

$$\frac{a_x + h + 2c}{2(a_y + h)} = \frac{471 + 222 + 2 \cdot 645{,}5}{2(471+222)} = 1{,}43$$

$$\frac{2c}{h+d} = \frac{2 \cdot 645{,}5}{222+600} = 1{,}57$$

From table 2:

$$\alpha_{xA} = 1{,}94$$

Hence for point C:

$$\alpha_{xC} = \alpha_{xA} \frac{x'_C}{x'_A} = 1{,}94 \frac{417}{442} = 1{,}83$$

The co-ordinates relate to the $x' - y'$ co-ordinate system with its origin at the centroid of the perimeter.

For the y-direction:

$$\frac{a_y+h}{a_x+h+2c} = \frac{471+222}{471+222+2\cdot 645{,}5} = 0{,}35$$

$$\frac{2c}{h+d} = 1{,}57$$

From table 3:

$$\alpha_{yB} = 0{,}58$$

For point C:

$$\alpha_{yC} = \alpha_{yB}\frac{y'_C}{y'_B} = 0{,}58\frac{140}{411} = 0{,}20$$

Eccentricity factors α_t

For point A:

$$\alpha_t = \frac{1}{1+\alpha_{xA}\dfrac{|e_x-e_p|}{h+d}} = \frac{1}{1+1{,}94\dfrac{204+31}{222+600}} = 0{,}643$$

For point B:

$$\alpha_t = \frac{1}{1+\alpha_{yB}\dfrac{|e_y|}{h+d}} = \frac{1}{1+0{,}58\dfrac{74}{222+600}} = 0{,}950$$

For point C:

$$\alpha_t = \frac{1}{1+\alpha_{xC}\dfrac{|e_x-e_p|}{h+d}+\alpha_{yC}\dfrac{|e_y|}{h+d}}$$

$$= \frac{1}{1+1{,}83\dfrac{204+31}{222+600}+0{,}20\dfrac{74}{222+600}} = 0{,}649$$

The lowest of these values is determinative; therefore $\alpha_t = 0{,}643$

Punching shear force

$$F_{ut} = \alpha_t phf_{ctd} = 0{,}643\cdot 2582\cdot 222\cdot 2\cdot 10^{-3} = 737\ \text{kN}$$

The punching shear force (= shear resistance capacity) is bigger than the design value of the shear force.

Punching shear force of an indentical inner column

$$p = \pi(h+d) = \pi(222+600) = 2582 \text{ mm} \quad e_{\text{p}} = 0 \quad e = \sqrt{e_{\text{x}}^2 + e_{\text{y}}^2} = 217 \text{ mm}$$

$$\alpha = 2{,}00 \text{ (tabel 1)} \qquad \alpha_{\text{t}} = \frac{1}{1+\alpha\dfrac{e}{h+d}} = \frac{1}{1+2{,}00\dfrac{217}{222+600}} = 0{,}654$$

$$F_{\text{ut}} \text{ (inner column)} = \alpha_{\text{t}} phf_{\text{ctd}} = 0{,}654 \cdot 2582 \cdot 222 \cdot 2 \cdot 10^{-3} = 750 \text{ kN}$$

This value is not determinative.

APPENDIX

TEST RESULTS

The experimental research which was carried out under the terms of reference of CUR Committee A 25 "Punching shear" comprised the testing of nine edge and six corner columns. These investigations should be regarded as continuing the research described in HERON, Vol. 20 "Punching shear". Here merely the principal data are presented (Table 6).

Table 6. Data and test results for edge and corner columns

1	2	3	4	5	6	7	8	9	10	11	12	13	14
type	nr.	f_{ct} (N/mm²)	ω_0 (%)	m_u (kN)	p (mm)	e (mm)	e_p (mm)	α	α_t	F_{ut} (kN)	F_{ub} (kN)	F_{test} (kN)	$\frac{F_{test}}{F_{u\,min}}$
edge column	1	2,57	0,9	3,21	226	0	20,8	1,58	0,763	(13,1)	10,4	10,7	1,03
	2	2,60	0,9	3,21	226	60	20,8	1,00	0,730	(12,7)	10,4	11,7	1,13
	3	2,60	0,9	3,21	226	120	20,8	1,00	0,423	(8,9)	6,4	7,7	1,20
	4	2,60	1,2	4,26	226	0	20,8	1,58	0,763	13,2	(13,8)	13,0	0,98
	5	2,60	1,2	4,26	226	60	20,8	1,00	0,730	12,7	(13,8)	13,5	1,06
	6	2,42	1,2	4,26	226	120	20,8	1,00	0,423	8,3	(8,5)	8,3	1,00
	7	2,42	1,5	4,90	226	0	20,8	1,58	0,763	12,3	(15,9)	15,1	1,23
	8	2,42	1,5	4,90	226	60	20,8	1,00	0,730	11,8	(15,9)	14,2	1,20
	9	2,63	1,5	4,90	226	120	20,8	1,00	0,516	9,0	(9,8)	8,9	0,99
corner column	10	2,53	0,9	3,21	143	0	38,9	5,26	0,341	3,6	(5,0)	4,3	1,19
	11	2,53	0,9	3,21	143	90	38,9	3,24	0,390	4,2	(4,8)	5,9	1,40
	12	2,53	0,9	3,21	143	180	38,9	3,24	0,188	2,0	(2,7)	2,8	1,40
	13	2,67	1,8	5,86	143	0	38,9	5,26	0,341	3,8	(9,2)	5,7	1,50
	14	2,67	1,8	5,86	143	90	38,9	3,24	0,390	4,4	(8,8)	9,5	2,16
	15	2,67	1,8	5,86	143	180	38,9	3,24	0,188	2,1	(4,9)	4,4	2,10

This table calls for the following comment.

Column 3. For obtaining the best possible prediction of the test result, in equation (3) not the design value f_{ctd} but the splitting tensile strength f_{ct} has been substituted, calculated from $f_{ct} = 1 + 0{,}05 f_{cm}$, where f_{cm} is the average cube strength on 40 mm cubes.

Column 4. The slab is reinforced with a top and a bottom orthogonal mesh, the same percentage of steel being provided in each direction in each mesh.

Column 5. m_u is the failure moment (determined by yielding of the steel) of the slab per unit width, calculated from

$$m_u = \frac{\omega_0}{100} f_e h^2 \left(1 - 0{,}56 \frac{\omega_0}{100} \frac{f_e}{f_c}\right)$$

Columns 6, 8, 9 and 10. The values have been calculated in accordance with, or have been taken from chapter 3.7.

Column 7. The bending moment vector at the edge columns is parallel to the edge; at the corner columns the moment vector is perpendicular to the bisectrix of the angle enclosed by the edges at the corner.

Columns 11 and 12. F_{ut} is the punching shear force, calculated in accordance with equation (3), but with f_{ct} as the tensile strength of the concrete. F_{ub} is the calculated column force at which bending moment failure occurs in the slab. The calculation of F_{ub}, not given here, was necessary in order to obtain an indication of the determinative failure criterion. A value in parentheses in columns 11 and 12 of the table means that the failure criterion in question is not determinative.

Column 13. F_{test} is the failure load measured during the test.