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Preface

In many cases building structures are subjected to rapidly varying loads. Examples are impact loads, vibrations caused by machinery or traffic and wind loads. In tall or slender structures the importance of the dynamic phenomena caused by these actions is relatively great. This has been the reason why in the Netherlands several aspects of dynamic problems have been studied by the Institute TNO for Building Materials and Building Structures with the financial support of the Netherlands Committee for Concrete Research (CUR). Several reports have been published previously, viz.

CUR report no. 17 "Vibration problems in prestressed concrete" (in English)
CUR report no. 35 "Constructieve aspecten van tafelfundamenten voor roterende machines (in Dutch)*
CUR report no. 57 "Dynamische problemen in de bouw" (in Dutch)**
CUR report no. 61 "Richtlijnen voor ontwerp en berekening machinefundamenten" (in Dutch)***

Summaries of these reports have been attached as an appendix to this report.

In assessing the importance of a vibration phenomenon, the stresses caused by it and the amplitude of the movements are important parameters. Especially in the case of resonance these may increase appreciably. The magnitude in that case is determined mainly by the damping. Thus far, sufficient data on damping were lacking. A committee was set up to study this problem. Modern developments in measuring technique made a new approach possible. The Committee was constituted as follows:

B. W. van der Vlugt, Chairman
W. Nijenhuis, Secretary
N. J. Cuperus
J. G. Hageman
H. van Koten

The research on damping was carried out by IBBC under the responsibility of H. van Koten. The result is thought to be of wider than national interest. This is the reason why an English translation is published in HERON.

Thanks are due to the Netherlands Committee for Concrete Research for financing the work.

This article is based on CUR Report 75 "Demping van bouwconstructies" (in Dutch).

* The structural consequences of dynamic influences upon table foundations of rotating machines.
** Dynamic problems associated with civil engineering structures.
*** Recommandations for the design and analysis of machine rotations.

STRUCTURAL DAMPING

Summary

The damping of building structures is determined by the dissipation of energy in the material, by absorbtion of energy at the supports and to the free air or liquid.

The various causes of damping are treated in the respective sections of this report.

Several combinations of dampers are examined, so that the damping can be calculated for various kinds of structure. Some examples are included.

The increase in structural damping due to artificial dampers is discussed. Some types of these dampers are described and examples given.

The influence of non-linear material properties is investigated theoretically.

LIST OF SYMBOLS

a	amplitude	m
	velocity	m/s
A	energy	Nm
	coefficient	
b	width	m
B	coefficient	
c	damping constant	Ns/m
C_{cr}	critical damping constant	Ns/m
C_D	drag coefficient	
D	damping/diameter	–/m
e	factor for oil damping	
E	modulus of elasticity	N/m^2
f	frequency	s^{-1}
f_0	resonance frequency	s^{-1}
F	surface	m^2
g	earth acceleration	m/s^2
h	height	m
H	length	m
Hz	Hertz	s^{-1}
I	moment of inertia	m^4
k	spring constant	N/m
l	length	m
L	beam length	m
m	mass	kg
p	$= (h/b) \cdot (\varrho_1/\varrho)$	
P	force	N
Q	magnification	
r	radius of circle	m
R	damping constant to the frequency	
t	time	s
V	volume	m^3
v	velocity	m/s
x	motion amplitude	m
y_0	distance	m
z	distance	m
α	exponent	
δ	exponent	s^{-1}
$\varkappa$	$= (\omega/\nu) y_0^2$	
λ	exponent/log decrement	
ν	viscosity	m^2/s

φ	phase angle	
ϱ	density	kg/m^3
σ	stress	N/m^2
ω	circular frequency	rad/s
ω_0	circular frequency at resonance	rad/s

Structural damping

1 Introduction

Just as the vibrations of a string of a musical instrument die out after being struck, the vibrations of a structure die out after it has been subjected to a sudden impulse. This dying-out of vibrations, i.e., decreasing motion, results from the dissipation of the energy that had been introduced into the structure by the impact.The sooner this dying-out of the vibration ends, the more strongly the structure is damped.

This damping limits the motion of a structure when it is loaded periodically at its natural frequency. This limitation of the motion, or dying-out of the vibration, has a favourable effect on the stresses in many dynamically loaded structures. It is therefore important to know the value of the damping and, if possible, to increase this value.

In this report the effect of damping will be described with the aid of simple dynamic models. The damping behavious of several materials and structures will be discussed. In some cases damping can be increased quite simply. Examples are presented in the last chapter.

2 Damping of the single-degree-of-freedom system

Some simple experiments

As an illustration of vibration, some cantilevers were loaded by a pulse. The motions resulting from the pulse were measured. The experimental rig is shown in Fig. 1. The recordings are given in Fig. 2. The vibration is a measure for the damping; it can be expressed as the ratio between two successive peak values of the amplitude $a_2/a_1 = e^{-\lambda}$ or by the envelope of the peak values. This envelope has approximately the form of an exponential function ($e^{-\delta t}$). This and other methods demonstrating damping are also mentioned in CUR report no. 57 "Dynamic problems in building" page 45, lit. [1] (see also Fig. 9). From experiments on cantilevers made of wood, concrete and steel a mean value was found of respectively:

Table 1. Values for the damping of the beams of Fig. 2.

damping	symbol	wood	concrete	steel
ratio a_2/a_1		0.79	0.81	0.97
exponent	δ	92	33	13
log. decrement	λ	0.23	0.20	0.03

These values show how large the damping was in these simple examples.

For calculating these values we proceeded from a mathematical model from which the behaviour of the structure can be derived approximately.

Schematically this model consists of a single-degree-of-freedom system with damp-

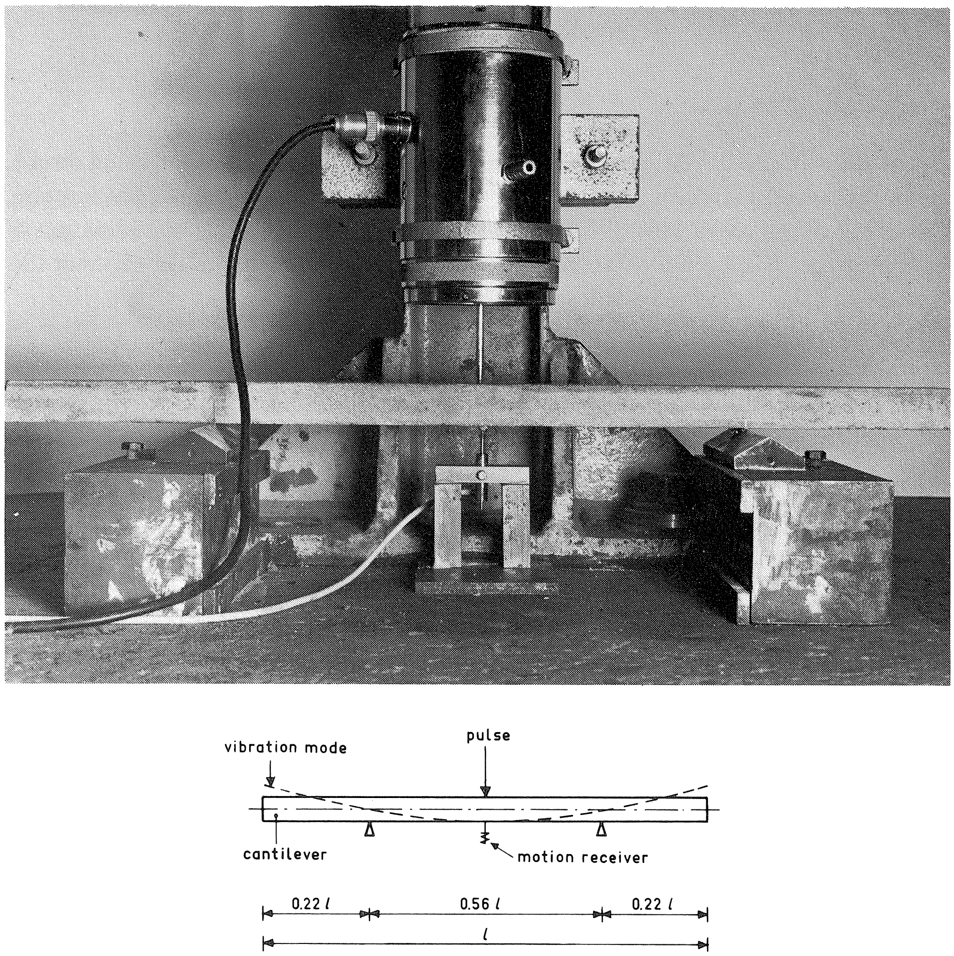


Fig. 1. The experimental rig.

ing. The mathematical description enables us to determine some characteristic and extreme values which may be instructive.

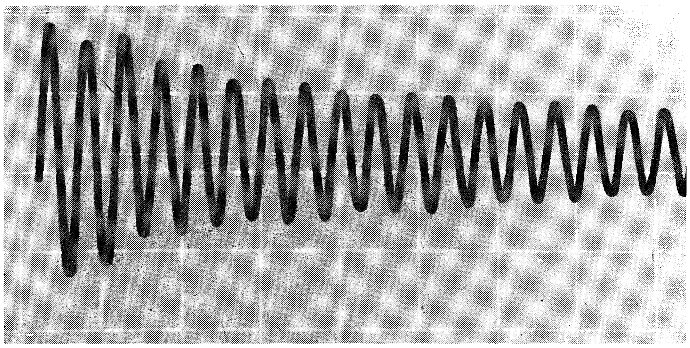
In order to clarity this report with simple formulas, a few calculations of the single-degree-of-freedom system will now be presented.

The single-degree-of-freedom system

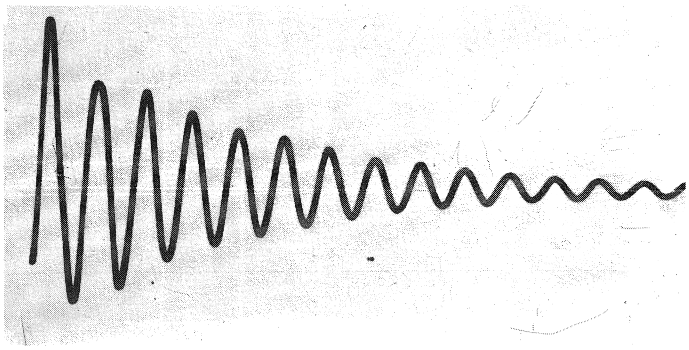
Mass will be devoted by m, the spring used by k and the damper by c (see Fig. 3).

If a mass moves down through a distance x, the spring produces an upward force $k \cdot x$. The inertia force (Newton's Law) also gives an upward force $m(\mathrm{d}^2x/\mathrm{d}t^2)$.

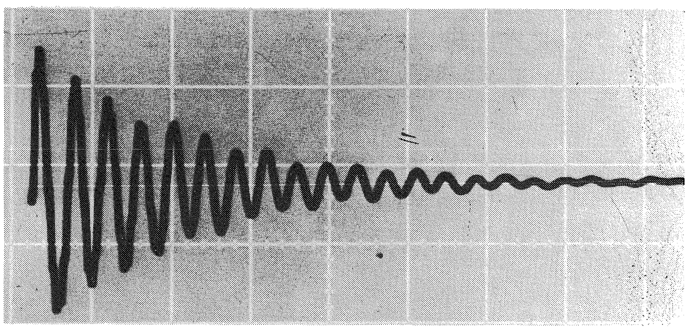
The damping force acts in the opposite direction to the motion. This force may be



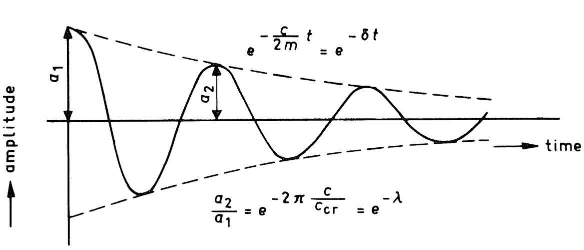
a. Beech wood dimensions $0.72 \times 0.05 \times 0.05$ m^3 Natural frequency following the test 400 Hz (calculation 390 Hz).



b. Microconcrete dimensions $0.72 \times 0.03 \times 0.05$ m^3. Natural frequency following the test 160 Hz (calculation 160 Hz).



c. Steel dimensions $0.72 \times 0.02 \times 0.04$ m^3. Natural frequency following the test 220 Hz (calculation 215 Hz).



d. Vibration dying-out after a pulse.

Fig. 2. The dying-out of vibration of a cantilever after a pulse.

constant, as for example a frictional force, but it may also be proportional to the velocity $c(\mathrm{d}x/\mathrm{d}t)$, or proportional to the square of the velocity $c(\mathrm{d}x/\mathrm{d}t)^2$, or to the product of velocity and displacement $c \cdot x(\mathrm{d}x/\mathrm{d}t)$, etc.

A useful damper for building structures is the second one: $c(\mathrm{d}x/\mathrm{d}t)$. A damper is characterized in that a force is necessary to achieve a displacement $K = c(\mathrm{d}x/\mathrm{d}t)$. This displacement, however, is not restored without an external force. The energy needed to produce the displacement is dissipated. The energy introduced into a spring will be recovered.

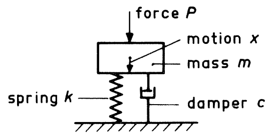


Fig. 3. The single-degree-of-freedom system.

For equilibrium of the forces acting on the mass the equation is:

$$m\frac{d^2x}{\mathrm{d}t^2} + c\frac{\mathrm{d}x}{\mathrm{d}t} + kx = 0 \tag{1}$$

if there is moreover an external force acting on the mass, then:

$$m\frac{d^2x}{\mathrm{d}t^2} + c\frac{\mathrm{d}x}{\mathrm{d}t} + kx = P \tag{2}$$

The solution of (1) is:

$$x = a_1 e^{\alpha_1 t} + a_2 e^{\alpha_2 t}$$

with

$$\alpha_1 = \frac{1}{2m}\left(-c + \sqrt{c^2 - 4\,km}\right)$$

$$\alpha_2 = \frac{1}{2m}\left(-c - \sqrt{c^2 - 4\,km}\right)$$

If the damper c has no significant influence, the expression under the root sign is negative. The solution of (1) can be wiitten as:

$$x = x_1\, e^{-\frac{c}{2m}t} \sin \omega_0 t + x_2 e^{-\frac{c}{2m}t} \cos \omega_0 t \tag{3}$$

with

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}} \approx \sqrt{\frac{k}{m}} \tag{4}$$

($f_0 = \omega_0/2\pi$ is the natural frequency of the system).

If mass m is slightly displaced from its equilibrium position, then at $t=0$,

$$x = x_2$$

and

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0; \quad \text{so} \quad x_1 = 0$$

The equation of motion will then be:

$$x = x_2 e^{-\frac{c}{2m}t} \cos \omega_0 t$$

This expression satisfactorily describes the free vibration as determined from the experiments (Fig. 2a).

The peak values are found to occur for $\cos \omega_0 t \approx 1$; hence $t = n2\pi/\omega_0$. The proportion between two successive maxima is, since $t_{n+1} - t_n = 2\pi/\omega_0$:

$$\frac{a_{n+1}}{a_n} = e^{-\frac{2\pi c}{2m\,\omega_0}} \tag{5}$$

The value of c can be defined by means of the equation:

$$\lambda = \frac{\pi c}{m\omega_0} = \ln \frac{a_n}{a_{n+1}}$$

$$c = \frac{m\omega_0}{\pi} \ln \frac{a_n}{a_{n+1}} \tag{6}$$

$\lambda =$ the logarithmic decrement

The value $c = 2\sqrt{km}$ was found to be a limit value in deriving equation (3).

When c exceeds $2\sqrt{km}$, the motion of the mass is damped so heavily that the mass slowly returns to the position of rest after undergoing a deflection. (See Fig. 4).
$c = 2\sqrt{km}$ is called the critical damping:

$$c_{cr} = 2\sqrt{km}$$

The ratio $D = c/c_{cr}$ is the damping rate.

For slender structures, often $D \ll 1$ (so $c \ll 2\sqrt{km}$).

The energy in the damper will be dissipated during the process of free vibration.

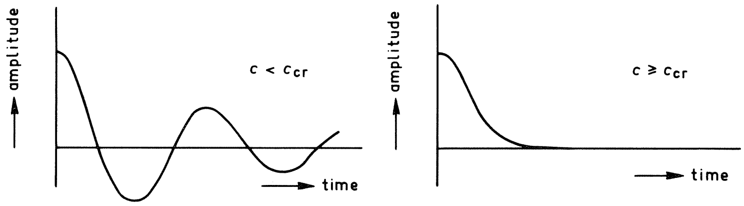


Fig. 4. Non-periodic dying-out of vibration.

This is the potential energy of the mass in the extreme position. The energy required to give mass m a displacement x_0 is:

$$\int_0^{x_0} K\,\mathrm{d}x = \int_0^{x_0} kx\,\mathrm{d}x = \tfrac{1}{2}kx_0^2$$

This energy has been lost (dissipated) when the mass is at rest again after the motion. The energy dissipated (i.e., the work done) by the damper is:

$$A = \int_0^{x_0} c\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}x = \int_0^{\infty} c\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 \mathrm{d}t = \tfrac{1}{2}kx_0^2$$

If a force P is acting on the mass, a motion due to that force must be added to the above-mentioned motion. If the force acts continuously, e.g., an alternating force

$$P = P_1 \sin \omega t \tag{8}$$

then, after some time, the influence of the free vibrating mass will have disappeared because of the damping. Then only the motion due to the alternating force is perceptible:

This motion is expressed in a formula according to (2):

$$x = x_1 \sin \omega t - x_2 \cos \omega t \tag{9}$$

where:

$$x_1 = P_1 \frac{k - m\omega^2}{(k - m\omega^2)^2 + c^2\omega^2} \tag{10a}$$

and

$$x_2 = P_1 \frac{c\omega}{(k - m\omega^2)^2 + c^2\omega^2} \tag{10b}$$

(If $c = 0$, then $x_2 = 0$)

The time can be eliminated from expressions (8) and (9); a relation between P and x is then found.

This, however, is not necessary because (8) and (9) are together the parameter function of an ellipse. This ellipse is shown in Fig. 5. Motion x reaches its maxima or 0-value a little later than the force does.

There is a phase angle between force and displacement. This angle is $\varphi = bg\,\text{tg}\,(x_2/x_1)$.

The phase angle is 90°, if $k = m\omega^2$ or $\omega = \sqrt{k/m}$. For small values of c/c_{cr}, this frequency corresponds to the natural frequency of the system. The ellipse in that case has a vertical principal axis ($x_1 = 0$).

Motions x_1 and x_2 together represent the response of the single-degree-of-freedom system to the alternating force. It is useful to consider these two motion components in order to account for some of the phenomena occurring in the experiments.

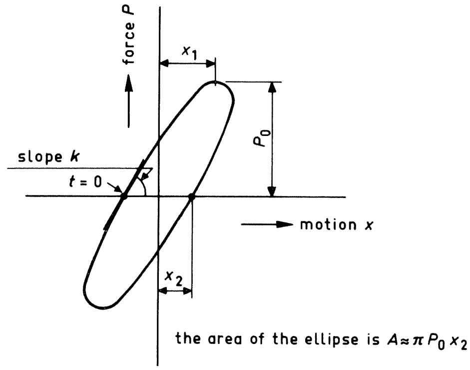


Fig. 5. The area of the ellipse is $A \approx \pi \cdot P_0 \cdot x_2$ the relation x, P.

The maximum amplitudes

The largest displacement of the mass is

$$x = \sqrt{x_1^2 + x_2^2}$$

This is according to formulas (10a) and (10b).

$$x = \frac{P_1}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}}$$

or in dimensionless form:

$$\frac{xk}{P_1} = \frac{x_0}{\frac{P_0}{k}} = \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \frac{4c^2\omega^2}{c_{cr}^2\omega_0^2}}} \qquad \left(\omega_0 = \sqrt{\frac{k}{m}}\right)$$

This expression gives the well-known response diagram (Fig. 6). The influence of the damping on the amplitude is clearly apparent at the resonant frequency.

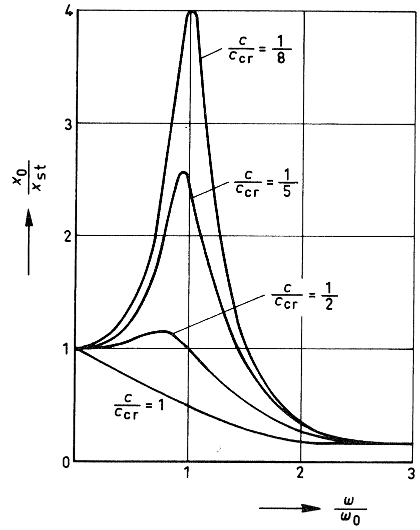


Fig. 6. Maximum displacement amplitude as a function of the frequency.

The maximum value is $xk/P = Q = c_{cr}/2c$ (magnification).

The width of the peak also gives information about the ratio c/c_{cr}. $xk/P = Q/\sqrt{2}$ is the value with the frequency

$$1 - \left(\frac{\omega}{\omega_0}\right)^2 = \pm \frac{2c}{c_{cr}} \frac{\omega}{\omega_0}$$

The difference between the two positive roots of the quadratic equation in ω/ω_0 is $2c/c_{cr}$. At the height $xk/P = Q/\sqrt{2}$ (Fig. 6) $2c/c_{cr}$ is equal to $\omega_1 - \omega_2/\omega_0$. This expression has been used to determine the damping in some cases.

The in-phase part x_1

Dimensionless:

$$\frac{x_1}{x_{st}} = \frac{x_1}{\frac{P_0}{k}} = \frac{1 - \left(\frac{\omega}{\omega_0}\right)^2}{\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \frac{4c^2\omega^2}{c_{cr}^2\omega_0^2}}$$

This function has been plotted in Fig. 7. x_1 increases when the frequency approaches the resonance frequency and suddenly reverses at this frequency. The peak value is situated at

$$\left(\frac{\omega}{\omega_0}\right)^2 = 1 \pm \frac{2c}{c_{cr}} = 1 \pm 2D$$

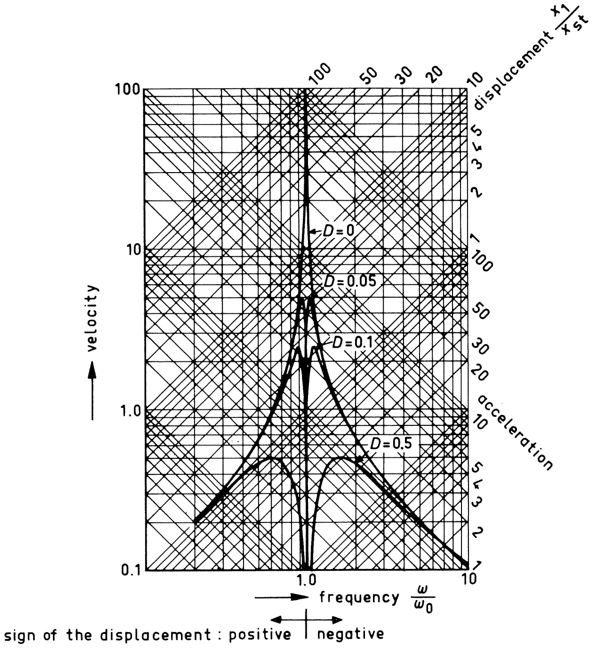


Fig. 7. The part of the displacement in phase with the force ($\varphi = 0°$).

The out-of-phase part x_2

Dimensionless:

$$\frac{x_2}{x_{st}} = \frac{x_2}{\frac{P_0}{k}} = \frac{\frac{2c\omega}{c_{kr}\omega_0}}{\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \frac{4c^2\omega^2}{c_{cr}^2\omega_0^2}}$$

Fig. 8 presents this function. The maximum value is

$$\frac{x_2}{P} = \frac{1}{\frac{2c}{c_{cr}}} = \frac{1}{2D}$$

The work done per period by force P is:

$$A = \int_0^{2\pi/\omega} P\frac{dx}{dt}dt = \int_0^{2\pi/\omega} \omega P_0 x_1 \sin\omega t \cos\omega t \, dt +$$

$$+ \int_0^{2\pi/\omega} \omega P_0 x_2 \sin\omega t \sin\omega t \, dt = 0 + \tfrac{1}{2}\omega P_0 x_2 \frac{2\pi}{\omega} = \pi P_0 x_2$$

In the case of resonance,

$$A = \frac{\pi P_0^2}{c\omega_0} = \pi c \omega_0 x_2^2$$

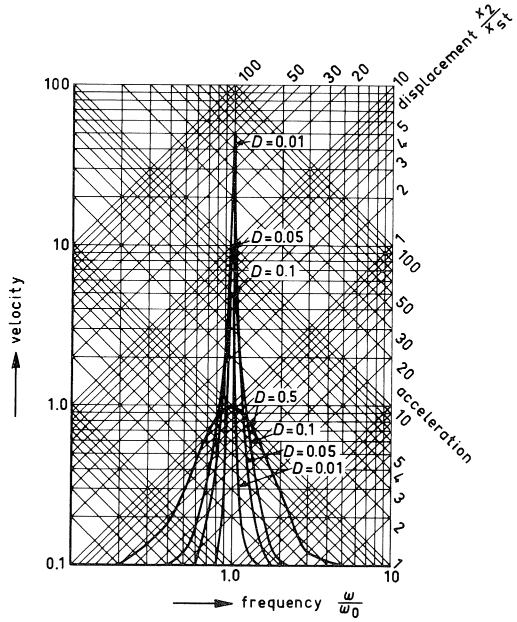


Fig. 8. The part x_2 of the displacement with a phase angle of 90° with the force ($\varphi = 90°$).

Only that part of the motion with a phase displacement of 90° with respect to the force does work. This work is equal to the area of the ellipse in Fig. 5.

The energy dissipated through the damper is:

$$A = \int_0^{2\pi/\omega} c\left(\frac{dx}{dt}\right)^2 dt = \int_0^{2\pi/\omega} c\omega^2(x_1 \cos \omega t + x_2 \sin \omega t)^2 \, dt =$$

$$= \tfrac{1}{2}c\omega^2 \frac{2\pi}{\omega}(x_1^2 + x_2^2) = \pi c\omega(x_1^2 + x_2^2)$$

This energy is equal to the work done by the force; therefore

$$\pi P_0 x_2 = \pi c\omega(x_1^2 + x_2^2)$$

To summarize, the following relations between the dynamic motions of a single-degree-of-freedom system and the damping are presented:

1. The dying-out after a pulse:

$$c = \frac{m\omega_0}{\pi} \ln \frac{a_n}{a_{n+1}} \quad \text{or} \quad \frac{c}{c_{cr}} = D = \frac{1}{2\pi} \ln \frac{a_n}{a_{n+1}} \qquad \text{(Fig. 2a)}$$

2. The energy on dying-out:

$$A = \int_0^{\infty} c\left(\frac{dx}{dt}\right)^2 dt = \tfrac{1}{2}kx_0^2$$

3. The shape of the ellipse of motion for periodic loading:

$$\frac{x_2}{x_1} = \frac{c\omega}{k - m\omega^2} \quad \text{(Fig. 5)}$$

4. The phase angle between force and motion for periodic loading:

$$\varphi = \arctan\frac{x_2}{x_1} \quad \text{or} \quad \varphi = \arctan\frac{c\omega}{k - m\omega^2}$$

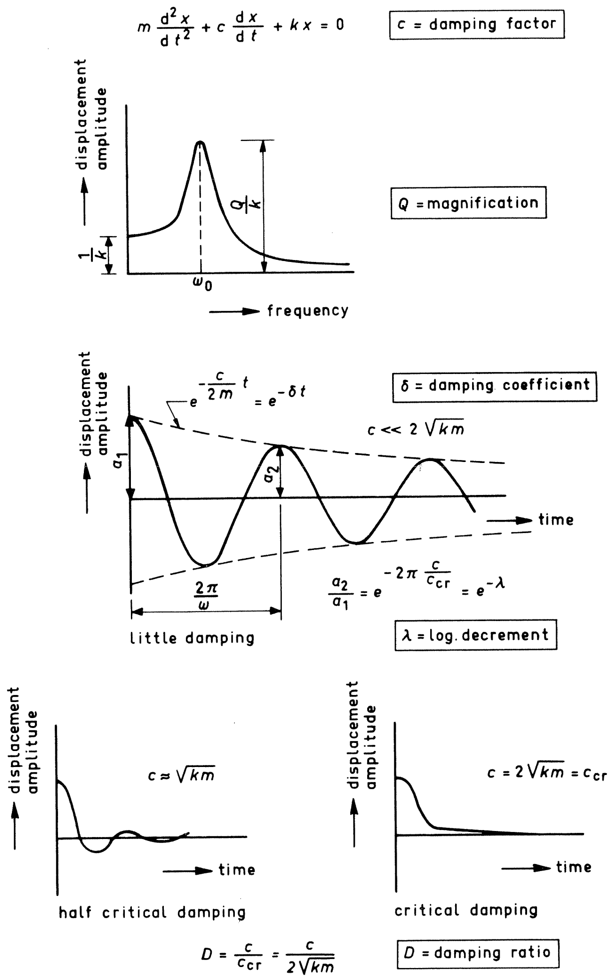


Fig. 9. Different definitions of the damping.

5. The energy input for periodic loading:

$$A = \pi P_0 x_2 = \frac{\pi P_0^2 c\omega}{(k-m\omega^2)^2+c^2\omega^2} = \pi c\omega(x_1^2+x_2^2)$$

If $\omega = \sqrt{k/m}$, then:

$$A = \frac{\pi P_0^2}{c\omega} \quad \text{or} \quad \frac{c}{c_{cr}} = \frac{c\omega}{2k} = \frac{\pi P_0^2}{2kA} = \frac{\pi P_0 x_{st}}{2A}$$

6. The shape of the response diagram for periodic loading:

$$\frac{c}{c_{cr}} = \frac{1}{2Q} = \frac{f_2-f_1}{2f_0} \qquad \text{(Fig. 6)}$$

The various damping constants are reviewed in Fig. 9.

3 Material damping

A material loaded by a compressive or a tensile force will deform like the spring of a single-degree-of-freedom system, but it will also behave more or less like a damper. This appears from the deformation due to an alternating load.

If the force is harmonic

$$P = P_1 \sin \omega t$$

the deformation due to this force has a small phase angle

$$x = x_1 \sin \omega t - x_2 \cos \omega t$$

or

$$x = x_0 \sin(\omega t - \varphi)$$

The ellipse is once more found as a relation between deformation and force. The steel cantilever in Fig. 2c was excited by an excitator to show this relation (Fig. 1). The force-deformation diagrams have been measured for a number of frequencies (Fig. 10).

The elliptical shape can be distinctly seen, specially at frequencies in the neighbourhood of the resonant frequency. The ellipse is somewhat disturbed at 100 Hz. This was caused by a beam in the mounting structure of the excitator; the beam joined in the vibration.

The diagrams show much resemblance with the single-degree-of-freedom system. x_1 increases when the frequency approaches the resonant frequency and then suddenly reverses; x_2 has a maximum for $\omega = \omega_0$.

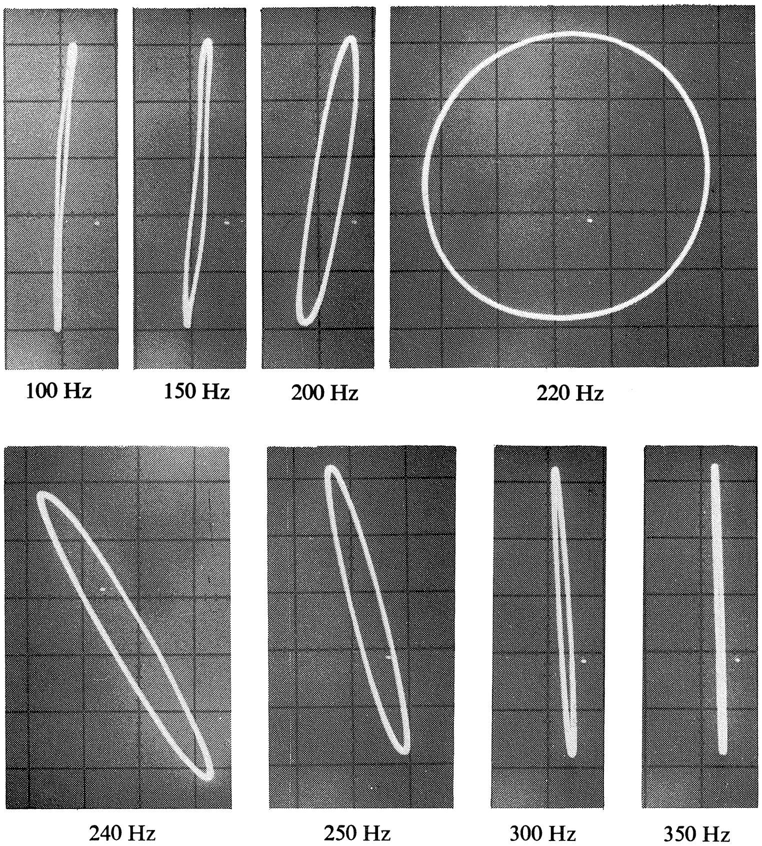


Fig. 10. Force-deformation diagrams for harmonic loading of steel cantilever shown in Fig. 2c.

The dynamic behaviour of the cantilever is satisfactorily described by the single-degree-of-freedom system.

A problem is still whether the expression c/c_{cr} can in fact be used to demonstrate the damping of the material, since c_{cr} varies with the mass or the spring constant (stiffness):

$$c_{cr} = 2\sqrt{km} = \frac{2k}{\omega_0} = 2\omega m_0$$

To obtain information on this point, a steel strip was rigidly clamped at one end and attached to a mass m at the other end.

The damping of this structure is only the damping of the steel strip. With chang mass m, c_{cr} changes without changing the damping. The arrangement is shown in Fig. 11.

The dying-out of the vibration of the strip with a mass of 1 kg, and a mass of 4 kg, is shown in Fig. 12. In both cases $c/c_{cr} = 0.00061$.

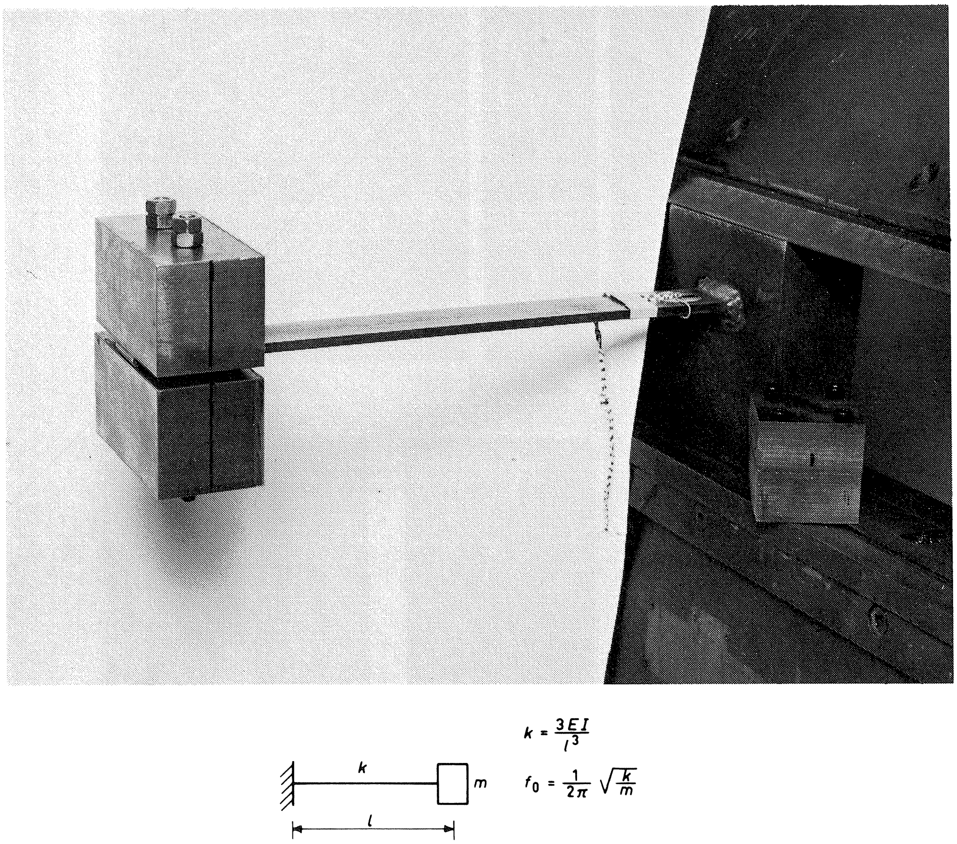


Fig. 11. Steel strip with mass for the investigation of material damping.

The value of c_{cr} in the second case, however, is half that in the first test. The value of c has been halved, and so has the frequency ω. The value $c/m\omega_0$ has remained constant. The damping can be expressed in $c/m\omega_0$ or in c/c_{cr}. This was found to be an acceptable assumption also for concrete, see CUR-report no. 17, lit. [2].

If the damping of an elementary part c_e of a structure is assumed to be proportional to $c_e/\omega_0 = R$, the total damping is

$$c = \alpha V c_e = \alpha V R \omega_0$$

V is the total volume of the structure and α is a factor determined by the amplitude ratio of the mass. Schematising to a single-degree-of-freedom system, the vibrating mass is

$$m = \alpha \varrho_1$$

The damping at resonance is

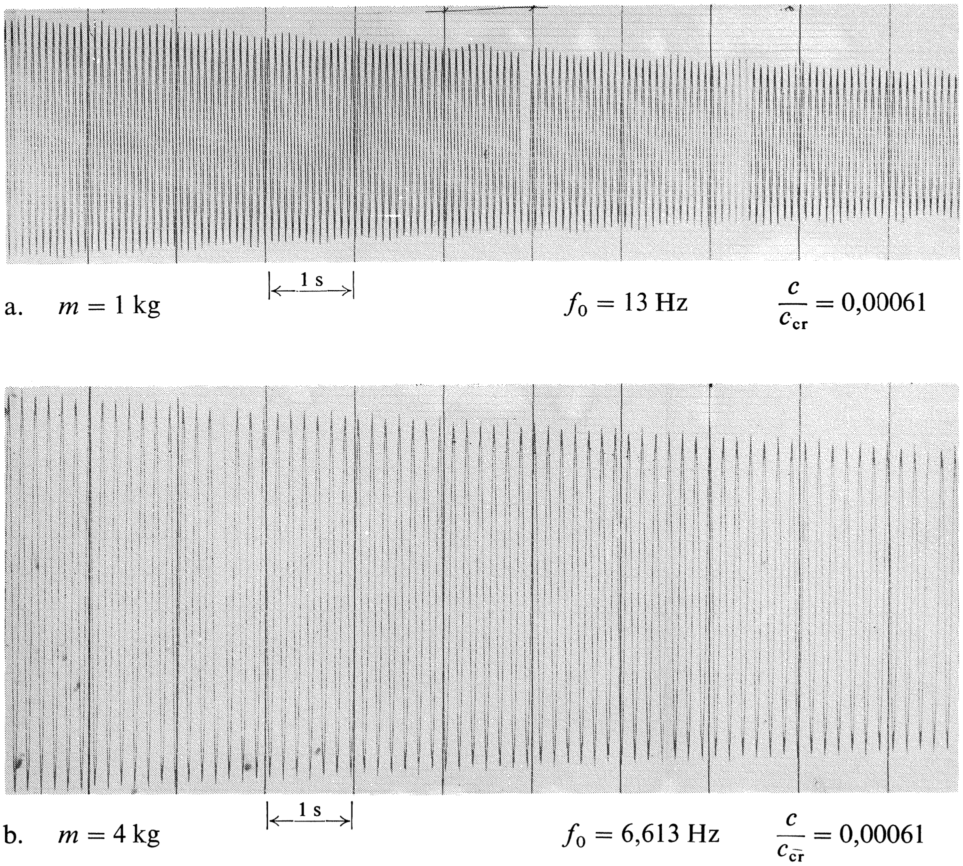


Fig. 12. Dying-out of vibration of experimental rig shown in Fig. 11 with $m = 1$ kg and $m = 4$ kg.

$$D = \frac{c}{c_{cr}} = \frac{c}{2m\omega_0} = \frac{\alpha V R \omega_0}{2\alpha \varrho_1 V \omega_0} = \frac{R}{2\varrho_1}$$

In this expression, the dimensions of the structure are no longer present. For the same value of R, the damping of a material increases if the density decreases. From the test, and the expression $c/c_{cr} = R/2\varrho_1$, it emerges that the damping of a rigid block mounted on an elastic and damping material will be determined by the damping c/c_{cr} of this supporting material, irrespective of the mass of the block.

The damping of a structure of any particular material, as well as that of a rigid structure which rests on that material, are found to be equal to c/c_{cr}. To determine the damping of structures, or parts of structures, we compiled a list of values for c/c_{cr}.

These values were partly determined from "dying-out" tests and partly from response ellipses.

material	c/c_{cr}	remarks
steel	0.004	
reinforced concrete	0.009	before and after cracking
prestressed concerte	0.009	
pine wood	0.021	
beech wood	0.025	
natural rubber	0.03	for frequencies above 10 Hz
natural rubber with canvas	0.08	for frequencies above 1 Hz
aluminium	0.018	
glass	0.06	
masonry	0.04	

The influence of strain

For increasing stresses, a more than proportional increase in deformation occurs in many materials. The stress in steel at which this happens is called the limit of proportionality. For many other materials, e.g., wood and concrete, no such a limit be indicated because even at low stresses this limit of proportionality hardly exists.

These deviations from proportionality between stresses and strains can cause extra damping. Due to these more than proportional increases in strain, the area of the hysterisis loop (ellipse) increases.This increase in damping, or the increase in the area of the force-deformation diagram, is distinctly manifest for steel at loads aboven the yield stress. The results of the investigation of damping at increasing stress for some metals are given in Fig. 13.

Alternating loading up to strains above the proportionality limit seldom occurs, because it rapidly causes fatigue. Structures may be loaded dynamically with a relatively small force, however, in such a way that alternation of the force in the natural frequency causes magnification,so that the stresses can increase up to the yield stress. Because of yielding, the damping will be greater for this frequency than for other frequencies. The increase in damping at strains in excess of the yielding strain is so great that the amplitude hardly becomes greater that at which yielding occurs (see Fig. 14).

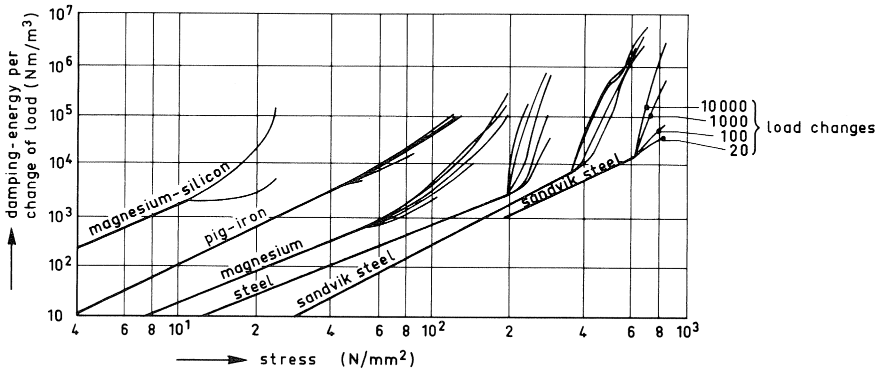


Fig. 13. Damping-energy measured for various metals as a function of stress.

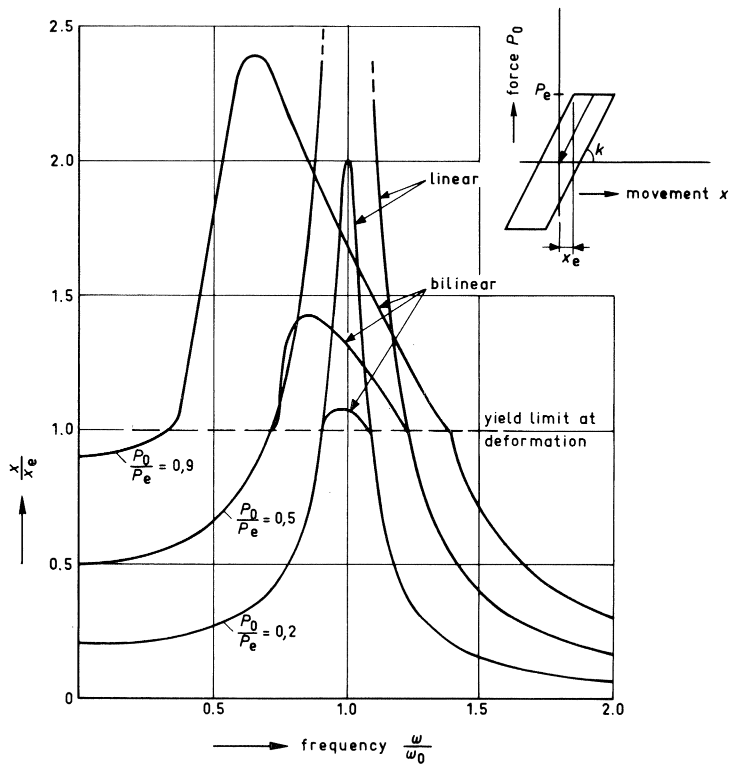


Fig. 14. Calculated frequency response.

The magnification is in this case

$$Q = \frac{\sigma_v}{\sigma_0}$$

The damping can then be expressed as:

$$\frac{c}{c_{cr}} = \frac{1}{2Q} = \frac{\sigma_0}{2\sigma_v}$$

This result has been obtained from calculations with a bilinear stress-strain diagram (see Fig. 14).

If the initial strain is large, motions can occur which are considerably larger than those at which yielding starts.

Fig. 15 shows the peak values of the response curves as functions of the loading for different values of stiffness after yielding (k_2, Fig. 15). The damping for the frequencies at which this maximum occurs can be represented as

$$\frac{c}{c_{cr}} = \frac{1}{2Q};$$

Q can be read from Fig. 15.

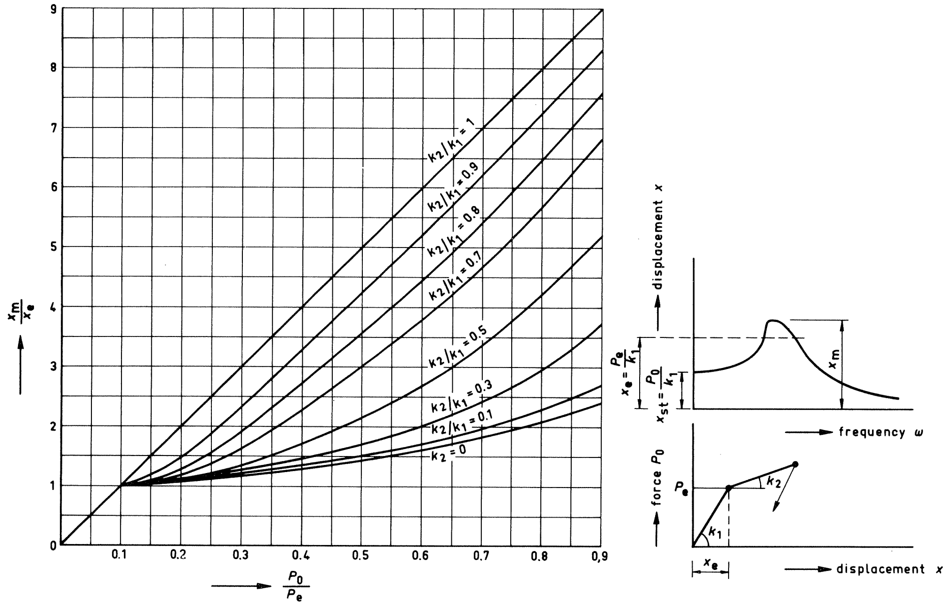


Fig. 15. The maximum magnification of a mass on a bilinear spring.

4 Geometric damping

The energy of motion or stress in a structure can decrease not only by conversion of the energy into heat, but also by its removal via the supports. This removal has the same effect as material damping and is called geometric damping.

As an example for this damping, an alternating force $P = P_0 \sin \omega t$ is applied to one end of a very long beam. The displacement x of the loaded end is found by integration of the strains:

$$x = \int_0^z \frac{P_0 \sin \omega t}{EF} \mathrm{d}z$$

z is the distance which the stress wave has travelled from the start of loading. If the initial velocity of the stress is a m/s, then $z = a \cdot t$. The displacement thus becomes:

$$x = \int_0^t \frac{P_0 \sin \omega t}{EF} a \, \mathrm{d}t = \frac{P_0 a}{EF\omega}(1 - \cos \omega t)$$

The displacement consists of a constant part $P_0 a/EF\omega$ and a variable part $P_0 a/EF\omega$ which has been shifted 90° in phase with the force. This part does work and also acts as a damper. The energy is removed. There is no motion in phase with the force.

In comparison with the single-degree-of-freedom, this is the situation in which the damping force is in equilibrium with the load:

$$c\frac{\mathrm{d}x}{\mathrm{d}t} = P \quad \text{or} \quad c\omega x_0 = P_0$$

In this case, $x_0 = P_0 a/EF\omega$, so that

$$c\omega\frac{P_0 a}{EF\omega} = P_0 \quad \text{or} \quad c = \frac{EF}{a}$$

This is the damping constant of a very long beam which is loaded at one end. The beam is assumed to be considerably longer than the distance covered in one period of the stress wave

$$l \gg z = \frac{2\pi a}{\omega}$$

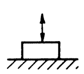
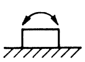
This length is of no practical significance, except at very high frequencies, and this damping is of no importance for beams, but it does have significance for small structures connected to a large one, like a window to a building or a building to the earth.

Vibrations of a structure on a half-space

The damping of a structure on a half-space is mainly caused by dissipation of energy. The dynamic behaviour of such a structure has been described in the literature (see lit. [3], [4]). The results of the theoretical calculations show the damping of the structure to be similar to that mentioned above for the long beam.

The spring force and damping force of the half-space depend on the shape of the contact area and on the method of loading.

A description and derivation of the several formulas for spring constant and damper are given in lit. [3]. The formulas used are given in the following table.

		k	c/c_{cr}
	m	$\approx E\sqrt{F}$	$\approx \dfrac{\frac{EF}{a}}{2\sqrt{km}} = \dfrac{F^{\frac{3}{4}}}{2}\sqrt{\dfrac{\varrho}{m}}$
	$\varrho \cdot l = F$	$\approx \dfrac{G}{2(1-\nu)} b^2 l$	$\approx \dfrac{0.15}{(1+B)\sqrt{B}}$ with $B = \dfrac{6(1-\nu)I}{\varrho b^4 l}$

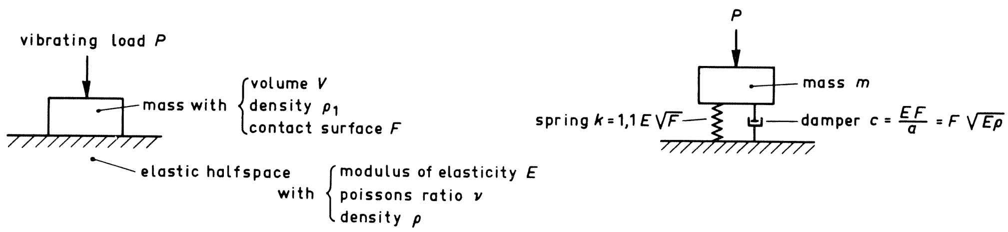


Fig. 16. Schematization of a mass on a halfspace.

In these formulas:

m = the mass on the half-space
F = the contact area
I = the rotational inertia of the mass
b = the width of the contact area
l = the length of the contact area
E = the modulus of elasticity of the half-space
$G = E/2(1+\nu)$ = the shear modulus of the half-space
ν = Poissons ratio of the half-space
ϱ = the density of the half-space

The damping is very large for small structures, as appears from the following calculation:

$$\frac{c}{c_{cr}} = \frac{\frac{EF}{a}}{2\sqrt{km}}$$

Where

$a = \sqrt{E/\varrho}$ (half-space with specific density ϱ)
$k \approx E\sqrt{F}$
$m = \varrho_1 Fh = \varrho_1 V$ (mass with density ϱ_1 and volume V)

$$\frac{c}{c_{cr}} = \frac{\frac{EF\sqrt{\varrho}}{\sqrt{E}}}{2\sqrt{E\sqrt{F}m}} = \frac{F^{\frac{3}{4}}}{2}\sqrt{\frac{\varrho}{m}} = \frac{F^{\frac{3}{4}}}{2V^{\frac{1}{2}}}\sqrt{\frac{\varrho}{\varrho_1}}$$

For a cube with edge length b:

$F = b^2$
$V = b^3$

$$\frac{c}{c_{cr}} = \tfrac{1}{2}\sqrt{\frac{\varrho}{\varrho_1}}$$

The damping for $\varrho = \varrho_1$ is half the critical damping.

For an average house, the following approximations are valid:

$$F = b^2$$
$$V = b^3$$
$$\varrho = 1800\ \text{kg/m}^3$$
$$\varrho_1 = 300\ \text{kg/m}^3$$
$$\frac{c}{c_{cr}} = \tfrac{1}{2}\sqrt{6} = 1{,}2$$

This damping is even greater than the critical damping. Small buildings will show no magnification in the vertical direction.

This damping decreases if the building is higher. For height h and bases $b \times l$, for example,

$$\frac{c}{c_{cr}} = \tfrac{1}{2}\sqrt{\frac{\varrho}{\varrho_1}\frac{(bl)^{\frac{3}{2}}}{hbl}} = \tfrac{1}{2}\sqrt{\frac{1}{p}\left(\frac{l}{b}\right)^{\frac{1}{2}}}$$

where

$$p = \frac{h}{b}\frac{\varrho_1}{\varrho}$$

(see Fig. 17)

From these calculations it emerges that buildings and machine foundations are considerably damped in the vertical direction.

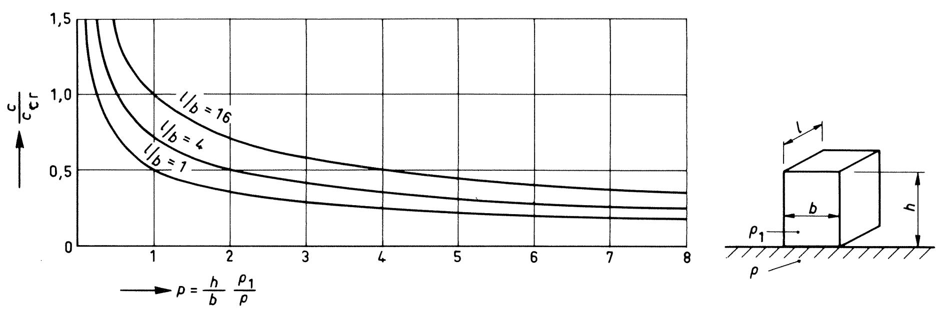


Fig. 17. The damping of a building in the vertical direction.

Rotational vibrations of a structure in a half-space

Approximate spring and damping constants for the "rocking" of a foundation, i.e., its rotation about a horizontal axis, can be indicated, as can also be indicated for its vertical motion, lit. [3].

The spring constant is dependent of the length-width ratio $l{:}b$ of the rigid foundation slab.

An approximation of this constant is:

$$k_\varphi = \frac{G}{2(1-\nu)} b^2 l$$

The rotation takes place about an axis parallel to the side l. G is the modulus of shear of the soil. k_φ has the dimension Nm, which is the moment needed for rotation through one radian.

The critical damping is now:

$$c_{cr} = 2\sqrt{I k_\varphi}$$

where:

$I =$ the rotational moment of inertia of the mass m

$$I = m\left(\frac{h^2}{3} + \frac{b^2}{12}\right)$$

$$m = \varrho_1 b l h$$

The damping constant is dependent on the ratio of the rotational moment of inertia of the mass and the part of the half-space under the mass:

$$B = \frac{6(1-\nu)I}{\varrho b^4 l}$$

$$\frac{c}{c_{cr}} = \frac{0.15}{(1+B)\sqrt{B}}$$

The expression was found by adjustment of the motions of a single-degree-of-freedom system to the accurately calculated motions of a mass on a half-space, lit. :3;.

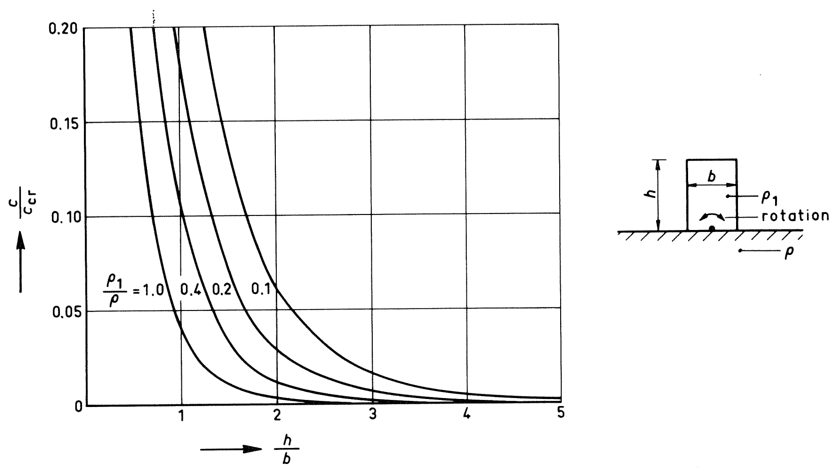


Fig. 18. The damping of a building in the horizontal direction (rocking).

The damping c/c_{cr} can be calculated as a function of the ratio ϱ_1/ϱ and $h/b = p$.

$$B = \frac{6(1-\nu)}{\varrho} \frac{\varrho_1 bhl\left(\frac{h^2}{3}+\frac{b^2}{12}\right)}{b^4 l} = 0.35\frac{\varrho_1}{\varrho} p'(4p'^2+1) \qquad (\nu = 0.3)$$

With this expression, c/c_{cr} can be calculated. Fig. 18 shows the value c/c_{cr} for $\varrho_1/\varrho = 0.1; 0.2; 0.4$ and 1.0.

5 Aerodynamic damping

Energy can be dissipated through the air in which an object vibrates. This is important for objects with low density and large motions. Damping through air is called aerodynamic damping; it is proportional the velocity of the structure relative the velocity of the air stream V, the density of air ϱ, the area F and the drag coefficient C_D:

$$c = \varrho V F C_D$$

For a high-voltage cable (with diameter D) in a strong wind (for example, $\overline{V} = 30$ m/sec) this damping is important. Then:

$$\frac{c}{c_{cr}} = \frac{\varrho V D C_D}{2\omega_0 m} = \frac{\varrho V D C_D}{2\omega_0 \varrho_1 \frac{\pi D^2}{4}}$$

for example, with:

$\varrho = 1.25$ kg/m^3 (air)
$\varrho_1 = 2500$ kg/m^3 (steel+aluminium)
$\omega_0 = 1$ s^{-1}
$D = 0.03$ m
$C_D = 1$

$$\frac{c}{c_{cr}} = \frac{1.25 \cdot 30}{2 \cdot 1 \cdot 2500 \cdot \frac{\pi \cdot 0.03}{4}} = 0.30$$

Cables with a low natural frequency are damped considerably at high wind velocities. This damping is appreciably less for buildings.

For example, for a square tall building:

$\varrho = 1.25$ kg/m^3
$\varrho_1 = 200$ kg/m^3
$\omega_0 = 1$ s^{-1}
$D = 15$ m
$\overline{V} = 20$ m/s
$C_D = 1$

$$\frac{c}{c_{cr}} = \frac{1.25 \cdot 20 \cdot 15}{2 \cdot 1 \cdot 200 \cdot 15^2} = 0.0042$$

This value is small, but it is of the same order of magnitude as the internal damping.

For aerodynamic as well as for hydrodynamic damping it should be noted that the values given, which are partly derived from theoretical considerations, see lit. [5], [6], [7] and [8], are only valid for laminar flow. In turbulent gas or liquid streams, damping may deviate considerably from the given value. For example, in the case of shedding of vortices along cylinders at high Reynolds-numbers the damping is negative (instability) if the frequency approaches the Strouhal-frequency, lit. [9]. Calculation of the response of such cylinders is complicated.

6 Hydrodynamic damping

The damping of a structure in a non-flowing fluid is dependent on the viscosity of the fluid, the shape of the structure and the depth to which the structure is immersed in the fluid. For structures of simple shape (cylinder and globe) there is a theory for

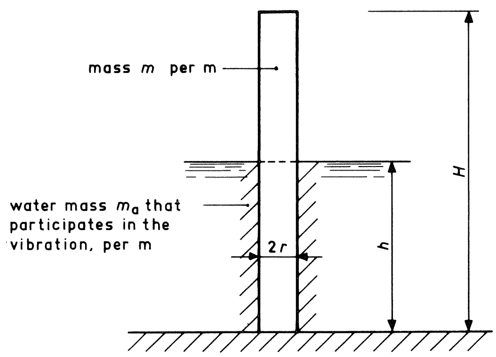


Fig. 19.

calculating the damping, see lit. [5] and [6]. If follows from this theory that for a vertical cylinder completely immersed in water the following approximation holds:

$$\frac{c}{c_{cr}} = \frac{m_a}{m+m_a}\left(\frac{h}{H}\right)^3 \sqrt{\frac{2.2 \cdot 10^{-4}}{r^2 f_0}} + \frac{c_0}{c_{cr}} \qquad (11)$$

Where (see also Fig. 19):

m_a = the vibrating water mass ($m_a \approx \pi r^2 \cdot 1000$ kg/m')
m = the mass of the cylinder
r = the radius of the cylinder
f_0 = the lowest natural frequency of the cylinder in water (in Hz)

2.2×10^{-4} = a constant with dimension m^2/s, and
c_0 = the damping without water
h = the water depth
H = the length of the cylinder

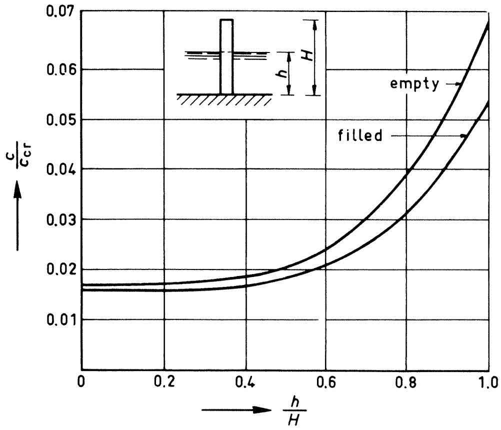


Fig. 20. The hydrodynamic damping of a tube.

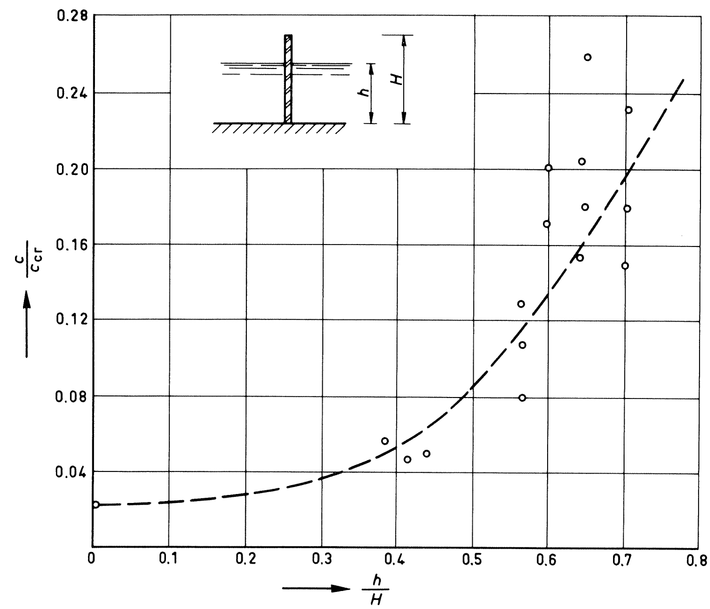


Fig. 21. The hydrodynamic damping of a wall.

If the cylinder is not completely immersed, the damping will be less.

The results for such a cylinder are given in Fig. 20. A wall in water has similar damping. Test results are given in Fig. 21. This damping can be approximated by the formula:

$$\frac{c}{c_{cr}} = \frac{m_a}{m+m_a}\left(\frac{h}{H}\right)^3 \sqrt{\frac{2}{f_0}}$$

Where

$$m_a \approx 0.5h \times 1000 \text{ kg/m}^2$$

For damping in a flowing fluid should be added damping $c = \varrho VFC_D$. This damping is especially important in fluids. If a structure is partly immersed in a fluid, the damping is reduced.

As an approximation the factor $(h/H)^3$ can be used, so

$$\frac{c}{c_{cr}} = \frac{\varrho VFC_D}{2\varrho_1 V\omega_0}\left(\frac{h}{H}\right)^3$$

7 Combination of two different dampers

Many structures are damped by a combination of dampers, such as material-damping in the structure itself, system-damping at the supports and aerodynamic damping by wind.

It is often useful to schematise the structure as a system with two degrees of freedom. Then the dampers can be introduced separately and the damping of the main part of the structure can be calculated. To illustrate this, some structures will be schematised as a two-degrees-of-freedom system.

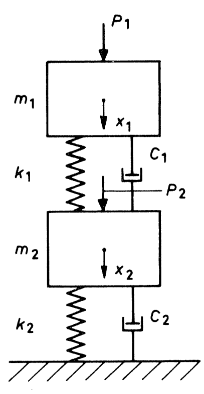


Fig. 22. The two degrees of freedom system.

1. A beam supported on spring:

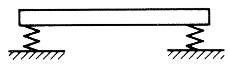


Fig. 23.

The beam supported on fixed hinges has as its lowest resonance frequency:

$$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI}{\mu}}$$

This beam can be regarded as a one-degree-of-freedom system by concentrating a mass in the middle and determining the spring constant from the deflection. Hence this constant is

$$k_1 = \frac{48EI}{l^3}$$

The mass can be found by making the resonance frequency of the system

$$\frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}$$

equal to

$$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI}{\mu}}$$

Thus

$$\frac{1}{4\mu^2}\frac{48EI}{l^3 m_1} = \frac{\pi^2}{4l^4}\frac{EI}{\mu}$$

or

$$m_1 = \frac{48}{\pi^4}\mu l \approx 0.5\mu l$$

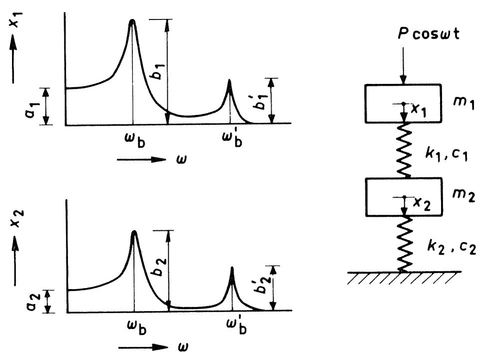


Fig. 24. Amplitudes for a two-degrees-of-freedom system with little damping.

The mass m_1 is approximately half the mass of the beam. The middle of the beam has the same dynamic response as the mass m_1 of the system.

For the beam on springs the two-degrees-of-freedom system is chosen with k_2 as the spring constant of the supports. The mass m_2 has been found from $m_1 + m_2 = \mu l$.

Therefore

$$m_2 = \mu l - \frac{48}{\pi^4}\mu l \approx 0.5\mu l$$

The movements of the mass m_1 and m_2 under harmonic loading of the system are given in Fig. 24. The resonance frequencies of this system can be calculated from the equilibrium equations.

2. An example of a rectangular building founded on sand:
The lowest resonance frequency is associated with a mode of oscillation where, the top of the building undergoes maximum displacement by bending of the building and rotation of the foundation (see Fig. 25).

The damping of this structure is the material damping of the building together with the aerodynamic damping and the system damping of the subsoil by rotation.

The-two-degrees-of-freedom system for this building is determined by the bending of the building (as a spring) together with the corresponding mass and the rotational stiffness of the foundation on sand as the second spring with the rotational inertia as the mass.

The mass m_1 has been determined in the same way as in the first example.

The resonance frequency of the cantilever with length h is:

$$f_1 = \frac{3.52}{2\pi h^2}\frac{EI}{\mu} = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}$$

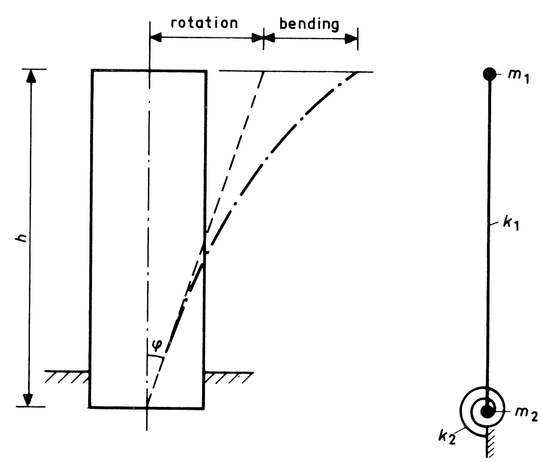
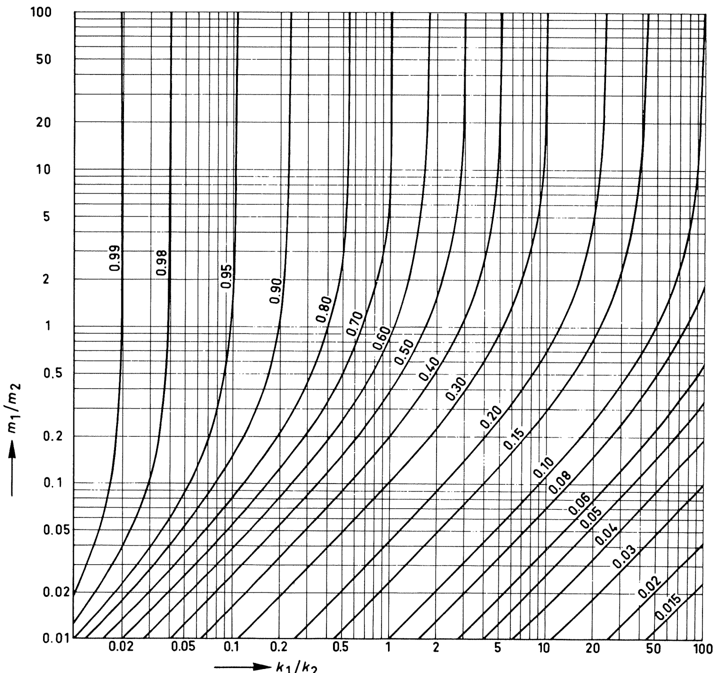
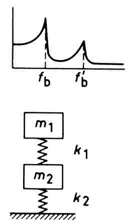


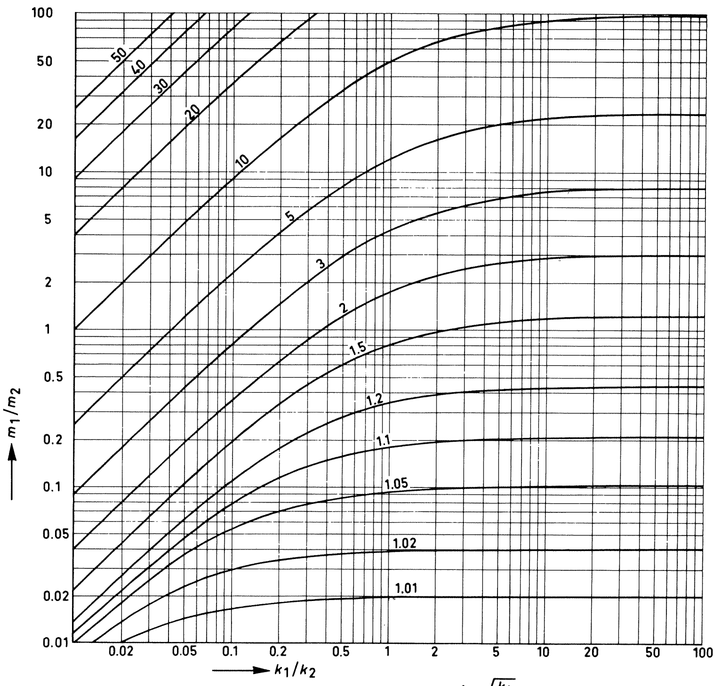
Fig. 25. Schematization of a tall building as a two-degrees-of-freedom system.



a. the lowest natural frequency, f_b, expressed in $\frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}$



after interchange of axes, the natural frequency is expressed in : $\frac{1}{2\pi}\sqrt{\frac{k_2}{m_2}}$



b. the highest natural frequency, f_b', expressed in $\frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}$

Fig. 26.

From the deflection of the top it follows that:

$$k_1 = \frac{3EI}{h^3}$$

Hence:

$$\frac{k_1}{m_1} = \frac{3.52^2}{h^4}\frac{EI}{\mu}$$

and

$$m_1 = \frac{\dfrac{3EI}{h^3}}{\dfrac{3.52^2 EI}{h^4 \mu}} = 0.24\mu h \approx \tfrac{1}{4}\mu h$$

The rotational spring has the stiffness:

$$k_\varphi = \frac{G}{2(1-\nu)} b^2 l \qquad \text{(see page 28)}$$

Where:

G is the shear modulus of the subsoil
b and l are the dimensions of the contact area of the foundation

The rotational inertia is:

$$I = \mu h\left(\frac{h^2}{3} + \frac{b^2}{12}\right) \approx \tfrac{1}{3}\mu h^3$$

The spring constant k_2 and the mass m_2 of the two-degrees-of-freedom system, as shown in Fig. 22, can be found from the equilibrium equations. For the mass at the top of the building in the horizontal direction follows:

$$m_1 \frac{d^2 w_1}{dt^2} + k_1(w_1 - \varphi h) = 0$$

For the rotation of the foundation:

$$I \frac{d^2\varphi}{dt^2} + k_\varphi \varphi - k_1\left(\frac{w_1}{h} - \varphi\right) = 0$$

These equations are transformed into those of the two-degrees-of-freedom system if (Fig. 22):

$$\varphi h = w_2 \qquad m_2 = \frac{I}{h^2} \qquad k_2 = \frac{k_\varphi}{h^2}$$

Summarizing:

$$m_1 \approx \tfrac{1}{4}\mu h \qquad k_1 = \frac{3EI}{h^3}$$

$$m_2 \approx \tfrac{1}{3}\mu h \qquad k_2 = \frac{G}{2(1-\nu)} \frac{b^2 l}{h^2}$$

3. A comparable example is a vertical cylinder embedded in the soil.
In this case is again:

$$m_1 \approx \tfrac{1}{4}\mu h \qquad k_1 = \frac{3EI}{h^3} \qquad m_2 \approx \tfrac{1}{3}\mu h$$

Now the spring constant of the part of the cylinder fixed in the soil is determined by the stiffness of the cylinder and the modulus of elasticity of the soil:

$$k_\psi \approx 1.5 \frac{E_p I_p}{d} \left(\frac{E_g}{E_p}\right)^{\frac{1}{4}}$$

E_p of the cylinder
E_g of the soil

The spring constant k_2 in the system is:

$$k_2 = \frac{k_\psi}{h^2} \approx 1.5 \frac{E_p I_p}{dh^2} \left(\frac{E_g}{E_p}\right)^{\frac{1}{4}}$$

The damping of the system m_2, k_2 is approximately:

$$\frac{c_2}{c_{2cr}} \approx 0.2 \sqrt[4]{\frac{E_g\, \varrho_g^2}{E_p\, \varrho_p^2}}$$

The damped two-degrees-of-freedom system

To determine the damping the displacements are expressed in the spring and damping constants.

The equations of motion are (see Fig. 22):

$$m_1 \frac{d^2 x_1}{dt^2} + c_1 \left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + k_1(x_1 - x_2) = P_1$$

and

$$m_2 \frac{d^2 x_2}{dt^2} - c_1 \left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + c_2 \frac{dx_2}{dt} - k_1(x_1 - x_2) + k_2 x_2 = P_2$$

x_1 and x_2 can be calculated by substitution:

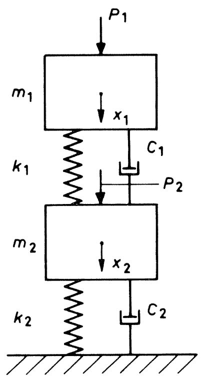
$$x_1 = \bar{x}_1 e^{i\omega t}$$
$$x_2 = \bar{x}_2 e^{i\omega t}$$
$$P_1 = \bar{P}_1 e^{i\omega t}$$
$$P_2 = \bar{P}_2 e^{i\omega t}$$

The solution of the equations is obtained after some rearrangement:

$$|x_1|^2 = \frac{\{P_1(-m_2\omega^2+k_1+k_2)+P_2k_2\}^2+\{P_1(c_1+c_2)\omega+P_2c_1\omega\}^2}{\{(-m_1\omega^2+k_1)(-m_2\omega^2+k_1+k_2)-k_1^2-c_1c_2\omega^2\}^2+\{c_1\omega(-m_2\omega^2-k_1+k_2)+(c_1+c_2)\omega(-m_1\omega^2+k_1)\}^2}$$

$$|x_2|^2 = \frac{\{P_1k_1+P_2(-m_1\omega^2+k_1)\}^2+\{P_1c_1\omega+P_2c_1\omega\}^2}{\{(-m_1\omega^2+k_1)(-m_2\omega^2+k_1+k_2)-k_1^2-c_1c_2\omega^2\}^2+\{c_1\omega(-m_2\omega^2-k_1+k_2)+(c_1+c_2)\omega(-m_1\omega^2+k_1)\}^2}$$

This response is illustrated schematically in Fig. 24.



If the damping is small, the maximum amplitude is found at the resonance frequencies of the undamped system. These follow from the zero values of the denominator as $c_1 = c_2 = 0$.

Hence:

$$(-m_1\omega^2+k_1)(-m_2\omega^2+k_1+k_2)-k_1^2 = 0$$

The values of ω as functions of m_1/m_2 and k_1/k_2 can be read from Fig. 26.

If the damping is small, the amplitudes x_1 and x_2 for the resonance frequencies (ω_b) are:

$$|x_1|_{res} \approx \frac{P_1(-m_2\omega_b^2+k_1+k_2)+P_2k_1}{c_1\omega_b(-m_2\omega_b^2-k_1+k_2)+(c_1+c_2)\omega_b(-m_1\omega_b^2+k_1)}$$

$$= \frac{P_1+P_2\left(1-\dfrac{m_1\omega_b^2}{k_1}\right)}{c_1\omega_b\left(\dfrac{m_1\omega_b^2}{k_1}\right)^2+c_2\omega_b\left(1-\dfrac{m_1\omega_b^2}{k_1}\right)^2}$$

and

$$|x_2|_{res} \approx \frac{P_1 k_1 + P_2(-m_1\omega_b^2 + k_1)}{c_1\omega_b(-m_2\omega_b^2 - k_1 + k_2) + (c_1 + c_2)\omega_b(-m_1\omega_b^2 + k_1)}$$

$$= \frac{P_1\left(1 - \dfrac{m_1\omega_b^2}{k_1}\right) + P_2\left(1 - \dfrac{m_1\omega_b^2}{k_1}\right)^2}{c_1\omega_b\left(\dfrac{m_1\omega_b^2}{k_1}\right)^2 + c_2\omega_b\left(1 - \dfrac{m_1\omega_b^2}{k_1}\right)^2}$$

Using these approximate formulas, the maximum amplitudes can be calculated.

The damping can be written as:

$$\frac{c}{c_{cr}} = \frac{1}{2Q} = \frac{x_{st}}{2x_{res}} \qquad \text{(see Fig. 9)}$$

For x_1 with $P_2 = 0$:

$$x_{st} = P_1\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\frac{c}{c_{cr}} = \frac{P_1\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)}{2x_{1_{res}}} = \tfrac{1}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)\left\{c_1\omega_b\left(\frac{m_1\omega_b^2}{k_1}\right)^2 + c_2\omega_b\left(1 - \frac{m_1\omega_b^2}{k_1}\right)^2\right\}$$

Where:

$$c_{1cr} = 2\sqrt{k_1 m_1} = \frac{2k_1}{\omega_1} \qquad \omega_1 = \sqrt{\frac{k_1}{m_1}}$$

and

$$c_{2cr} = 2\sqrt{k_2 m_2} = \frac{2k_2}{\omega_2} \qquad \omega_2 = \sqrt{\frac{k_2}{m_2}}$$

can be written as:

$$\frac{c}{c_{cr}} = \left(1 + \frac{k_1}{k_2}\right)\left\{\frac{c_1}{c_{1cr}}\left(\frac{\omega_b}{\omega_1}\right)^5 + \frac{c_2}{c_{2cr}}\frac{k_2}{k_1}\frac{\omega_b}{\omega_2}\left(1 - \frac{\omega_b^2}{\omega_1^2}\right)^2\right\}$$

$$= A\frac{c_1}{c_{1cr}} + B\frac{c_2}{c_{2cr}}$$

The damping of the system is composed of the sum of the functions of the individual dampers.

Coefficients A and B are given in Fig. 27 as functions of k_1/k_2 and m_1/m_2. With the aid of this diagram, the magnification of m_1 at the lowest natural frequency can be easily determined. In this way a possibility has been created to combine two dampers into one.

For example: the influence of the damping and rotation of a foundation of a building on the damping of the whole structure can be estimated in this way.

Similarly, for x_2, with $P_1 = 0$:

$$x_{st} = P_2 \frac{1}{k_2}$$

and

$$\frac{c}{c_{cr}} = \frac{c_1}{c_{1\,cr}} \frac{k_1}{k_2} \left(\frac{\omega_b}{\omega_1}\right)^5 \frac{1}{\left(1 - \frac{\omega_b^2}{\omega_1^2}\right)^2} + \frac{c_2}{c_{2\,cr}} \frac{\omega_b}{\omega_2} = A' \frac{c_1}{c_{1\,cr}} + B' \frac{c_2}{c_{2\,cr}}$$

The values for A' and B' are plotted in Fig. 28.

8 Artificial damping of structures

Introduction

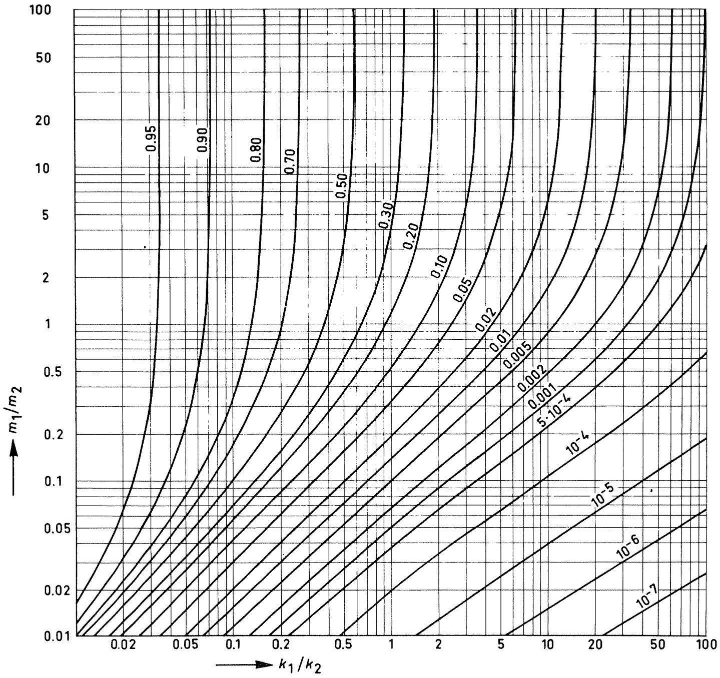
In the foregoing, various natural sources of damping of structures have been discussed. Besides these types of damping that are always present, a structure can also be damped by arrangements specially introduced for the purpose. This happens by the application of forces generated by the movement of the structure and directed counter to the vibration. An example of this is the introduction of suspended chains which oscillate with the movements of the structure and beat against the structure (see "the impact damper"). Another example which has been very fully treated in the literature is that of the added mass-spring system. In that case, a second mass on springs is fixed to the structure that is schematized to a single-degree-of-freedom system.

This second mass is damped through constructed (viscous) dampers. The damping and the movement of this added mass are used to give the structure itself more damping. Some of these "artificial" dampers will be described below.

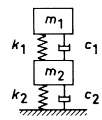
Specially made viscous dampers, e.g. shock absorbers, are often used for such artificial damping.

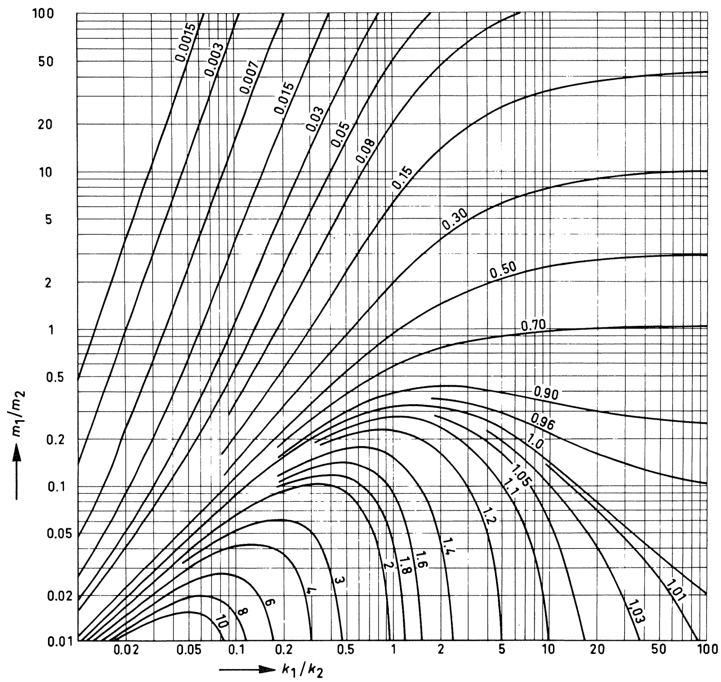
Viscous dampers

A well-known example of these dampers is the car shock absorber. The shock absorber derives its damping action from its internal construction, where oil is forced through a narrow opening, taking time to get through it. The shock absorber can be compressed at a constant speed by an applied force. This force per m/s is the damping constant c of the damper. A schematic section through such a damper is shown in Fig. 29. Shock absorbers for cars develop a damping force that is not constant over the whole stroke of the damper. This force has been plotted in Fig. 30. Usual values for the damping value c of car shock absorbers are:



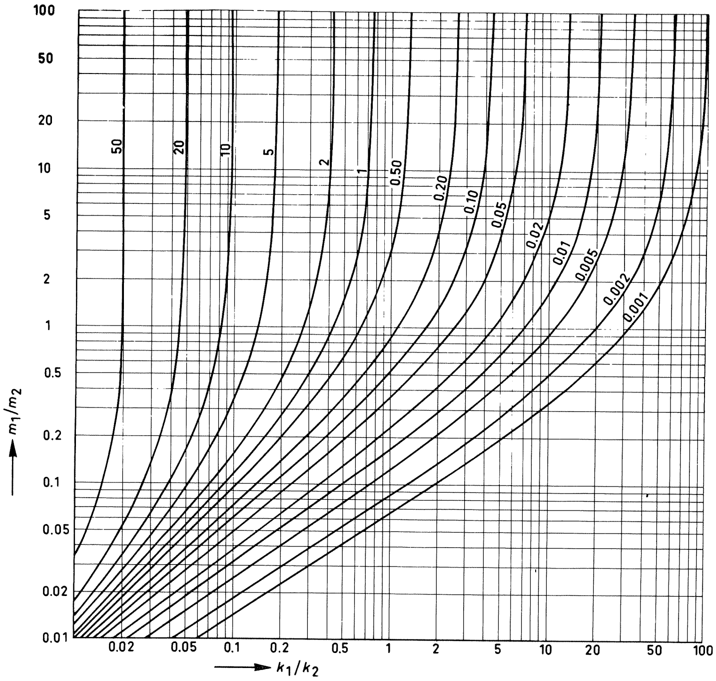
a. coefficient A



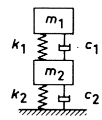


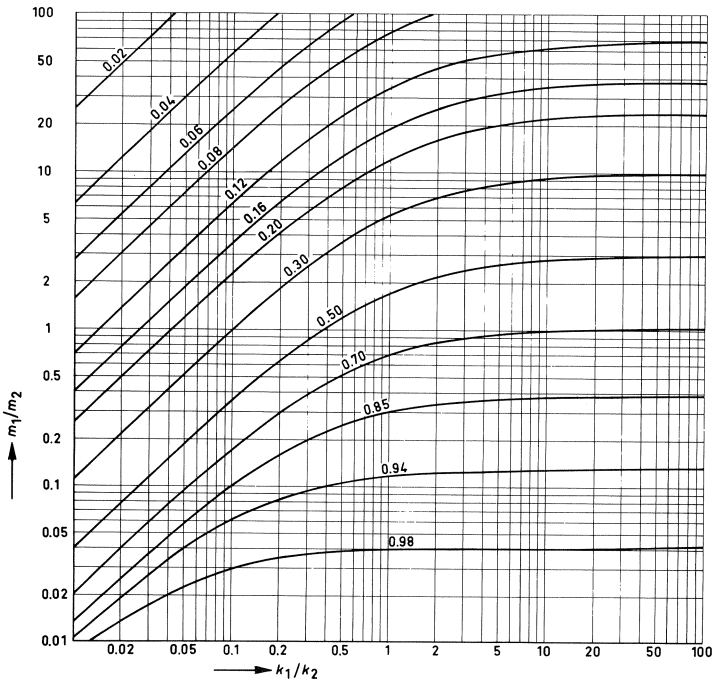
b. coefficient B

Fig. 27.



a. coefficient A'





b. coefficient B'

Fig. 28.

$c = 1000$ Ns/m to $c = 3000$ Ns/m

Special forms of construction for building structures are possible.

Auxiliary masses

In some cases a structure clearly behaves as a single-degree-of-freedom system, for instance a chimney, can be extended to a two-degrees-of-freedom system by the addition of a mass at the top. By providing the second mass with well chosen springs and dampers, the main structure (chimney) can be strongly damped. For designing an auxiliary mass-spring system to damp a structure, the formulas for the response of

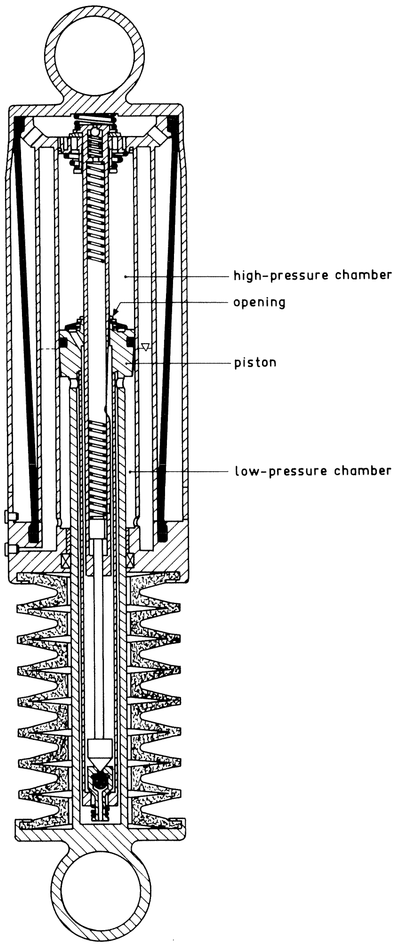


Fig. 29. Section through adjustable shock absorber (KONI) for motorcar.

the two-degrees-of-freedom system must be further analyzed. For this analysis, two cases are to be distinguished:

1. The system $m_2 - k_2$ is the schematization of the structure ($c_2 = 0$) and $m_1 - k_1$ is the auxiliary mass-spring system with the damper $c_1(P_1 = 0)$. This system $m_1 - k_1 - c_1$ should be so designed that the amplitude x_2 remains as small as possible.
2. The system $m_1 - k_1$ is the schematization of the structure (suppose $c_1 = 0$) and $m_2 - k_2 - c_2$ is the auxiliary system ($P_2 = 0$).

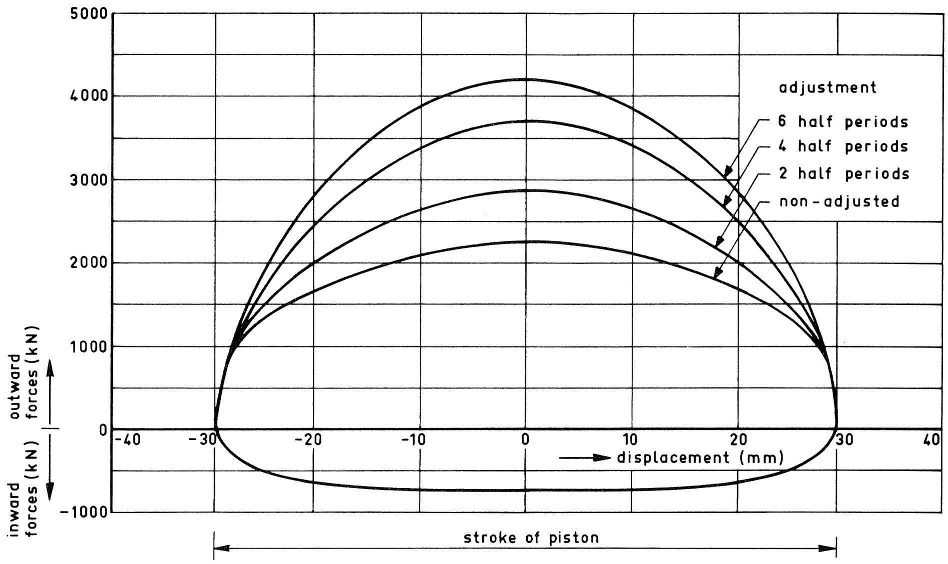


Fig. 30. Force-displacement diagram for shock absorber (KONI).

The values of x_1 and x_2 can immediately be calculated for these two cases from the formulas given on page 38. It appears that the formula for the response consists of the quotient of two expressions with a real (in-phase) part and an imaginary (90° out-of-phase) part.

$$x = \frac{a + ib}{c + id} \quad \text{hence} \quad |x|^2 = \frac{a^2 + b^2}{c^2 + d^2} \quad \text{(page 38)}$$

If the frequency ω is so chosen that $a/c = b/d$, then $x = a/c$; it is therefore independent of the imaginary (damping) part. This proves to be possible for two values of ω. The amplitude $x(a/c)$ cannot become smaller through adaptation of the damping. The response curve for one of these cases has been given as an illustration in Fig. 31. The damping can best be so chosen for the smallest amplitude that the tangent to the response curve is horizontal at the lowest frequency for which $x = a/c$.

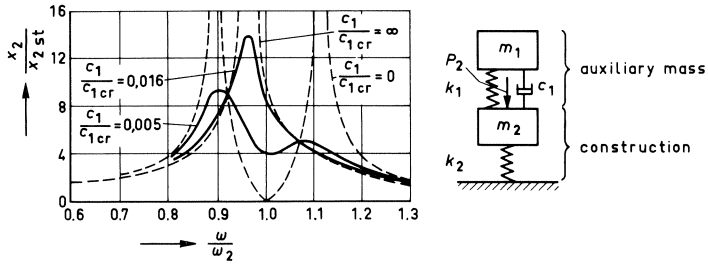


Fig. 31. Response of a structure with auxiliary mass.

This value of the damper is given for the two cases in Fig. 32 and 33. The magnification of the vibration for case 1 is given in Fig. 34. For designing an auxiliary mass the condition can be stated that the amplitudes at the two frequencies for which $x = a/c$ are as small as possible. This results in a relationship between m_1, k_1, m_2 and k_2 namely for the first case:

$$\frac{k_1}{k_2} = \frac{\frac{m_1}{m_2}}{\left(1 + \frac{m_1}{m_2}\right)^2} \qquad \text{(see Fig. 32)}$$

and for the second case:

$$\frac{k_1}{k_2} = \frac{\frac{m_1}{m_2}}{1 - \frac{m_1}{m_2}} \qquad \text{(see Fig. 33)}$$

If these conditions are satisfied and the damping has been suitably chosen, the amplitude of the main structure will have its maximum magnitude:

First case:

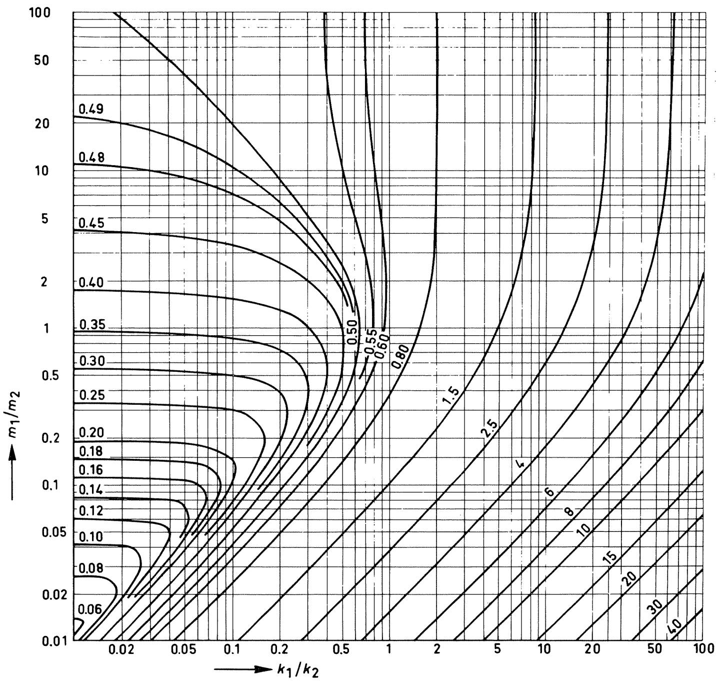
$$\frac{x_2}{x_{2st}} = \sqrt{1 + 2\frac{m_2}{m_1}}$$

Second case:

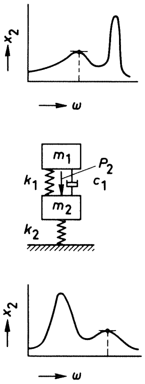
$$\frac{x_1}{x_{1st}} = \left(1 + \frac{k_1}{k_2}\right)\sqrt{2\left(1 + \frac{k_2}{k_1}\right)}$$

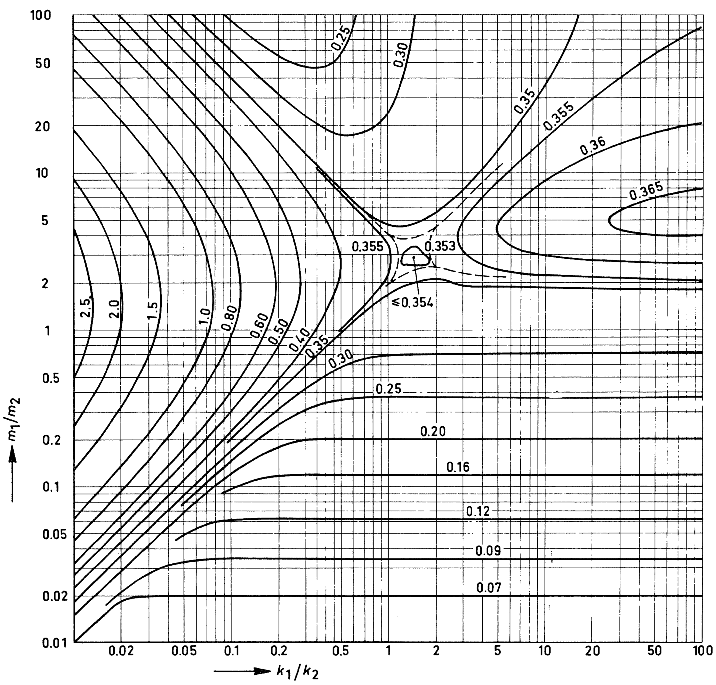
These formulas are given in Figs. 35 and 36.

The design of an auxiliary mass according to the first case, namely by adding a damped mass-spring system to the structure, has several applications such as oil-dampers and impact-dampers.



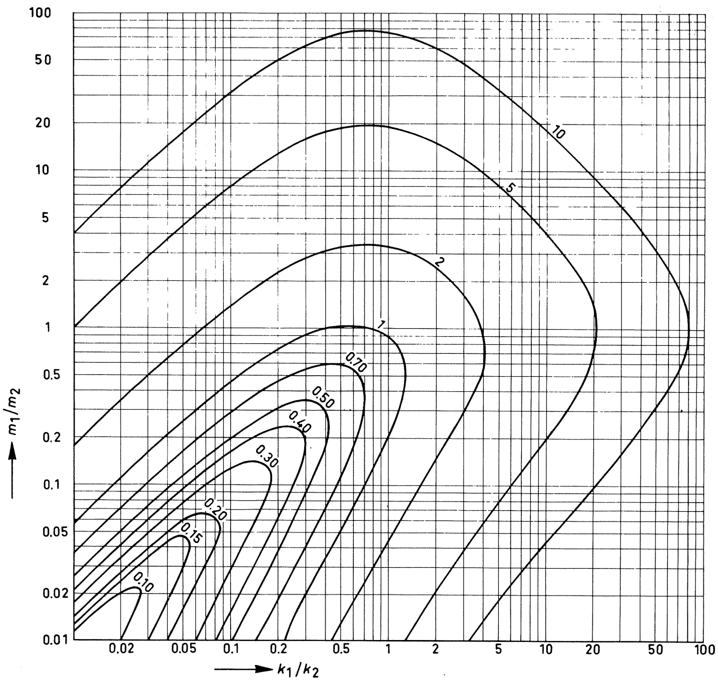
a. optimal value of the damping c_1/c_{1cr} for the first natural frequency



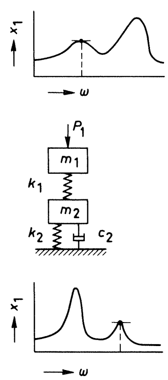


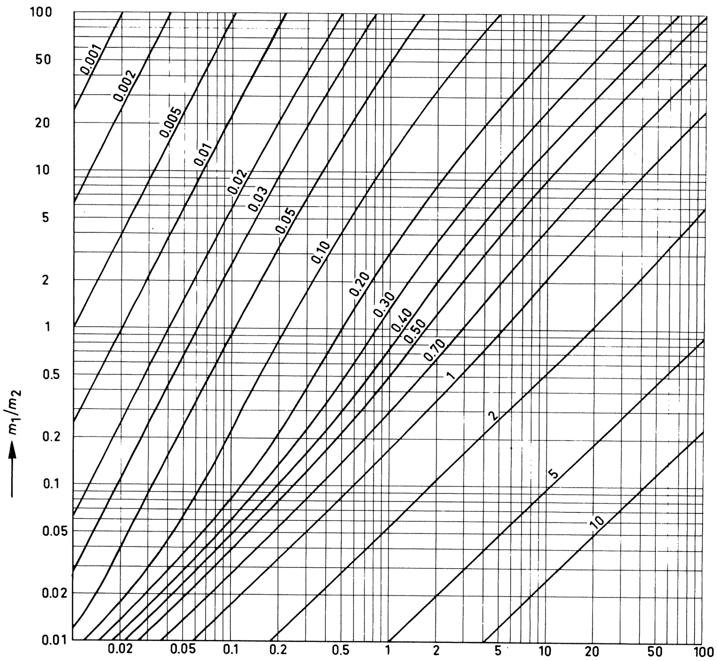
b. optimal value of the damping c_1/c_{1cr} for the second natural frequency

Fig. 32.



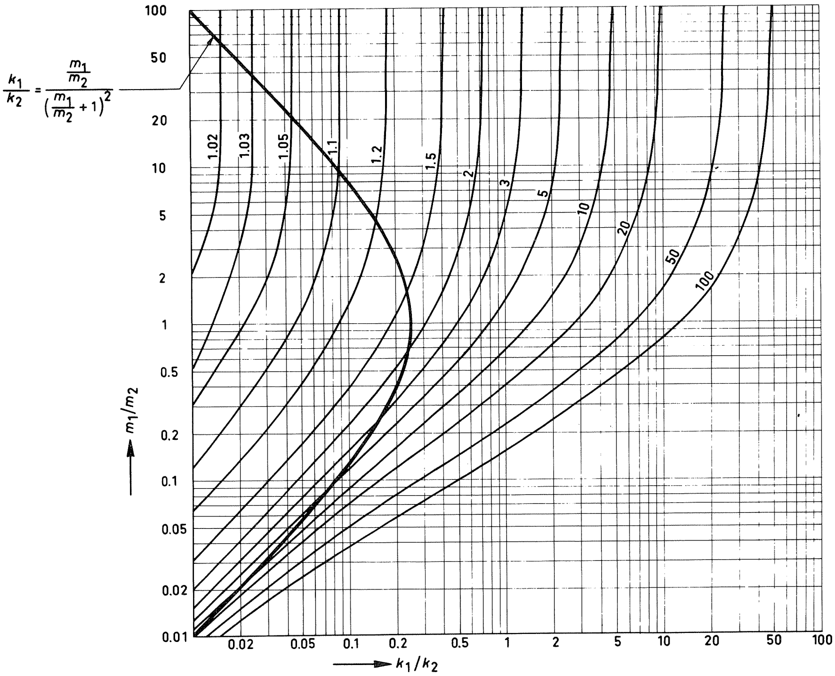
a. optimal value of the damping $c_2/c_{2\,cr}$ for the first natural frequency



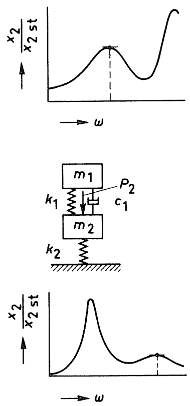


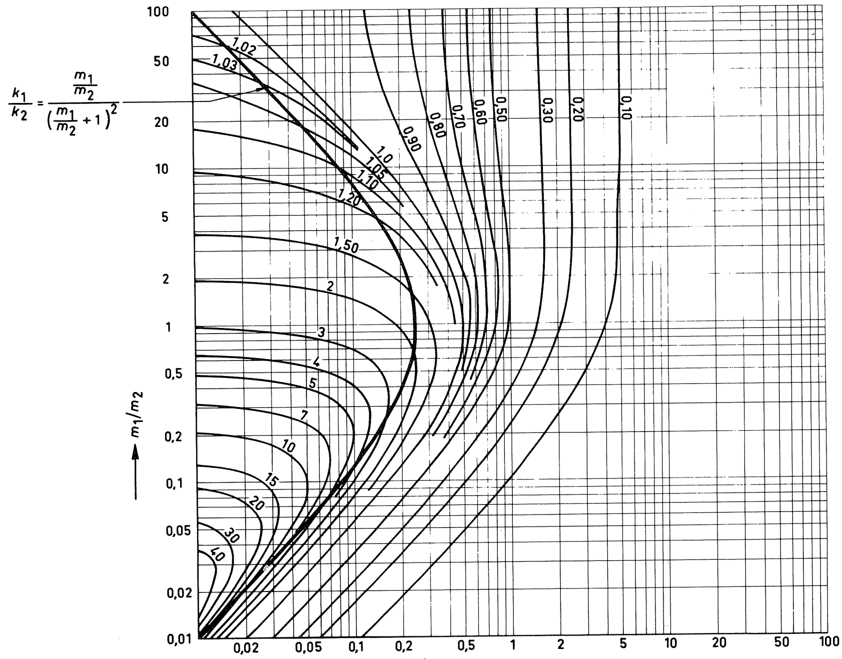
b. optimal value of the damping $c_2/c_{2\,cr}$ for the second natural frequency

Fig. 33.



a. magnification $x_2/x_{2\,st}$ for the first natural frequency





b. magnification $x_2/x_{2\,st}$ for the second natural frequency

Fig. 34.

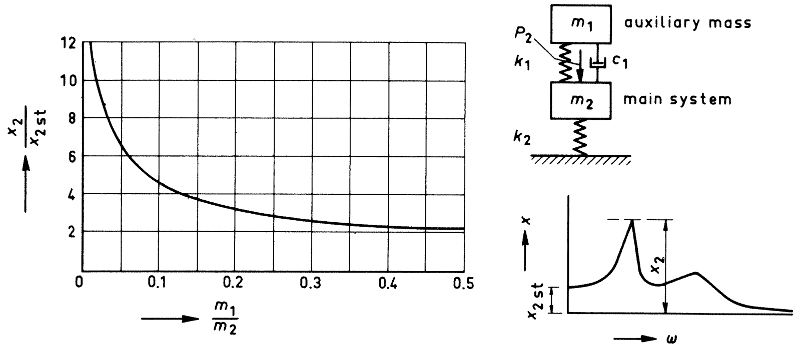


Fig. 35. Maximum amplitude of a structure with auxiliary mass.

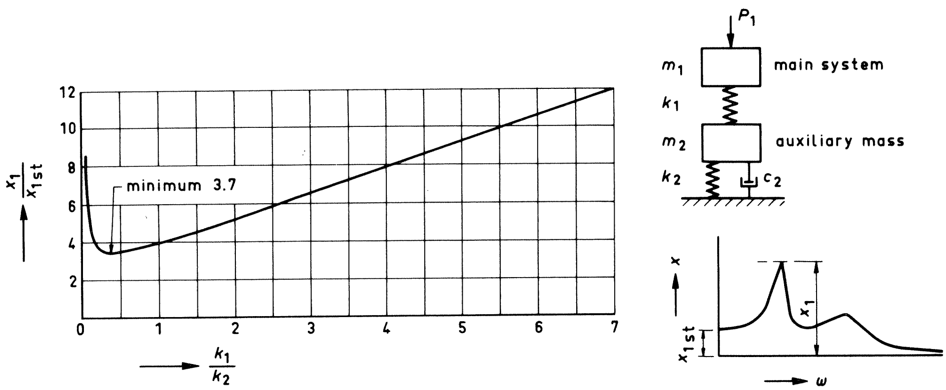


Fig. 36. Maximum amplitude of a structure on a dynamic optimally dimensioned foundation.

The oil-damper

A damper that has been applied to some chimneys is the oil-damper. Such a damper is mounted at the top of a chimney. It consists of a big tank with oil and, in it, horizontally installed plates. The oil will not or hardly shift upon movement of the chimney, but the plates will. The frictional forces between the oil and the plates provide the required damping. This damping is proportional to the viscosity of the oil, the quantity of oil and plates, and the velocity of the movement.

The energy A dissipated per period is:

$$A = m_1 x_2^2 \omega^2 e$$

Where:

m_1 = the mass of the oil
x_2 = the amplitude of the movement
ω = the circular frequency of the movement
e = a factor dependent on the plate spacing $2y_0$ and viscosity ν

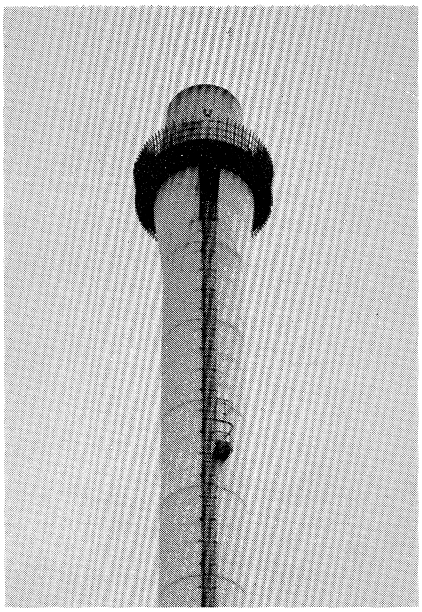


Fig. 37. Oil-damper mounted on smoke stack.

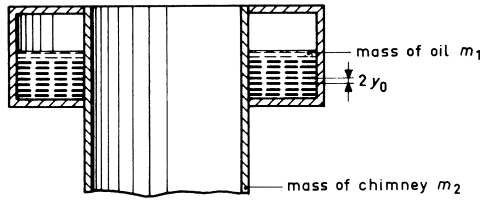


Fig. 38. Principle of an oil-damper.

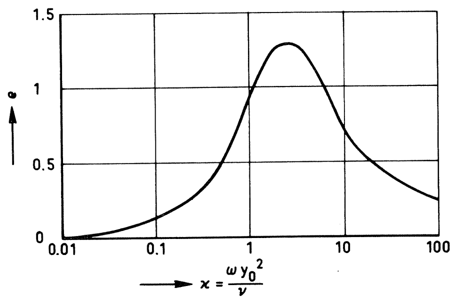


Fig. 39. Constant e for an oil-damper as a function of x.

This factor is given in Fig. 39 as a function of

$$\varkappa = \frac{\omega}{\nu} y_0^2$$

Where:

ν = the viscosity of the oil used, in m^2/s
y_0 = the spacing of the plates

From the value of A we obtain for the damping of a one-mass-spring system with $A = \pi c \omega_0 x_2^2$ (see page 16):

$$c = \frac{A}{\pi \omega_0 x_2^2} = \frac{m_1 \omega_0 e}{\pi}$$

With $c_{cr} = 2 m_2 \omega_0$ we obtain

$$\frac{c}{c_{cr}} = \frac{m_1}{m_2} \frac{e}{2\pi}$$

Where:

m_1 = the mass of the oil
m_2 = the mass of the system
f_0 = the natural frequency of the system
e = the factor given in Fig. 39

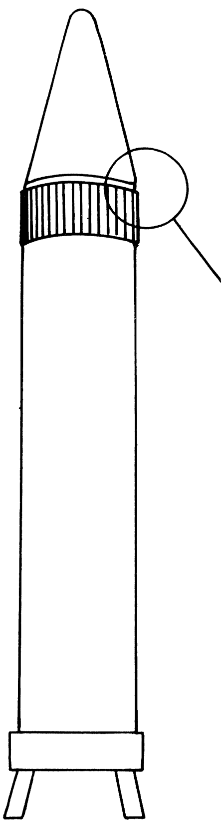
An oil-damper can be designed with the aid of this expression and that for the auxiliary mass-spring-system. The maximum amplitude follows from:

$$\frac{x_{max}}{x_{st}} = \frac{1}{2\dfrac{c}{c_{cr}}} = \frac{\pi}{e}\frac{m_2}{m_1}$$

This formula is comparable with that for the auxiliary mass. Fig. 37 gives an example of such a damper.

The impact damper

A damper which is often of simple construction is the "impact damper". The harmonic movements of a (light) structure are influenced by the force exerted by a freely suspended or rolling mass.



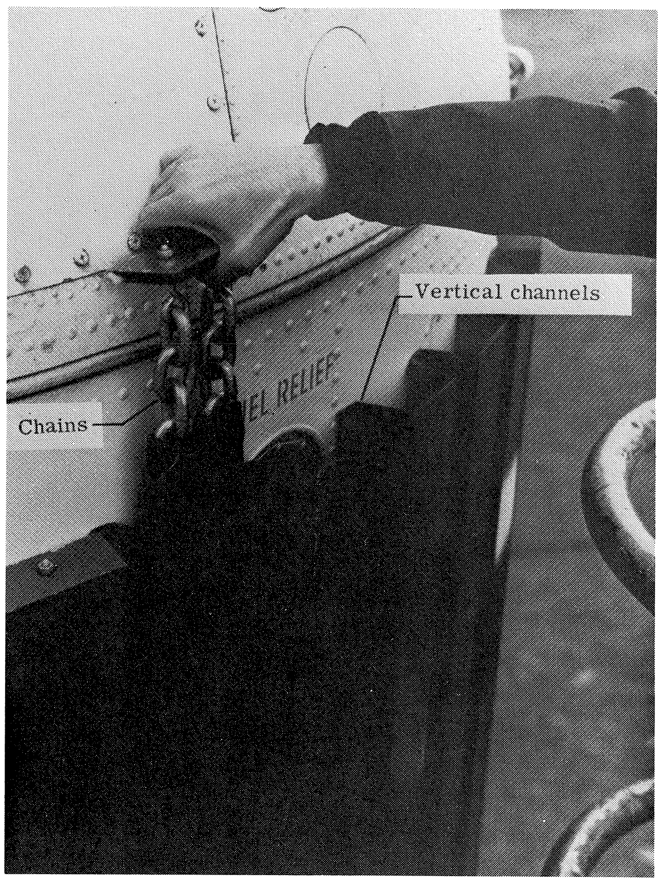


Fig. 40. Example of a chain-damper.

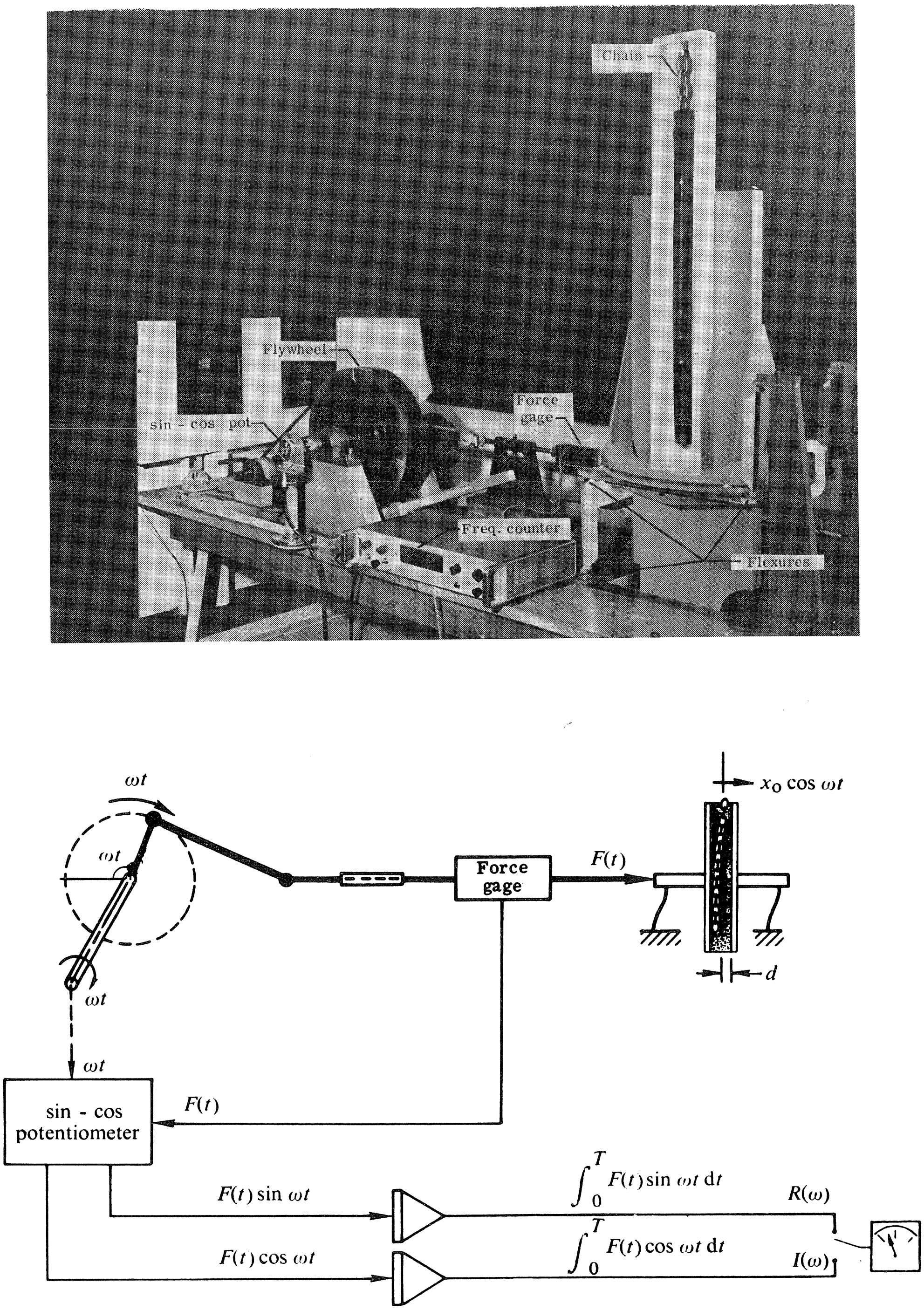


Fig. 41. Test rig for chain-damper.

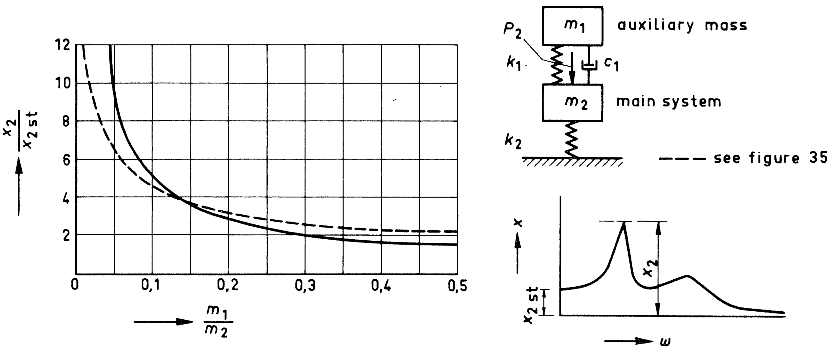


Fig. 42.

This may be a chain or a steel ball in a tank or a simple pendulum. Fig. 40 gives an example of a such a damper.

The theory for these dampers is rather complicated and approximative. Their damping action is of course dependent on the relation between the mass m_2 that is damped and the mass of the damper m_1. Since $m_1/m_2 = \mu$, then, with suitably chosen dimensions of the impact space and the frequencies, c/c_{cr} is approximately equal to

$$\frac{c}{c_{cr}} = \frac{\frac{m_1}{m_2}}{1 + \frac{m_1}{m_2}} \quad \text{(see Fig. 35)}$$

With $m_1 = 0.1 m_2$, c/c_{cr} is therefore approximation $0.1/1.1 = 0.09$.

Fig. 43 presents the test results of a chain as a damper. The natural frequency of the damper $\omega_1 = 1.2\sqrt{g/l}$ must be 3 to 4 times the excitation frequency to achieve good impact between structure and chain. This determines the length l of the chain.

With the help of Fig. 43, the mass and oscillation space can be established. For an impact damper comprising a ball in a tank the damping is also approximately:

$$\frac{c}{c_{cr}} = \frac{\frac{m_1}{m_2}}{1 + \frac{m_1}{m_2}}$$

The free space of the ball must be 20 to 50 times the static amplitude of the mass-spring system, see lit. [12].

9 Non-viscous damping

In the foregoing chapters we have always based ourselves on a damping force that is

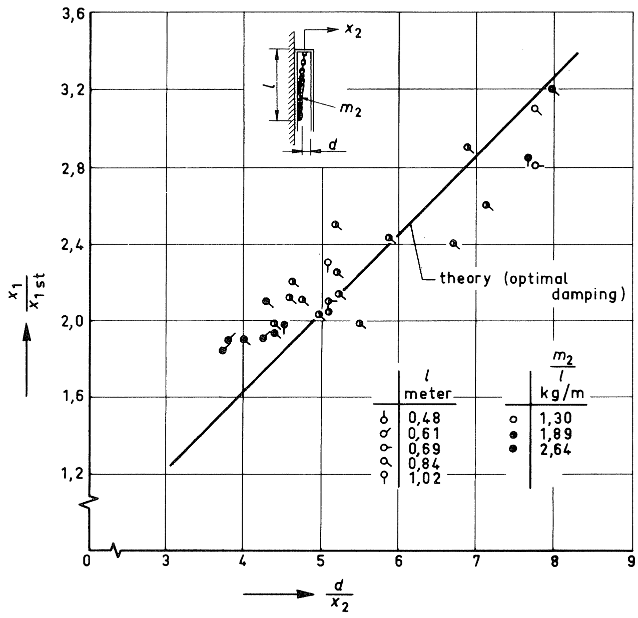


Fig. 43. Damping as a function of oscillation space for impact-chain.

proportional to the velocity $c(\mathrm{d}x/\mathrm{d}t)$. This damping corresponds to that of a damper with a viscous liquid. There are, however, also other damping forces, for example, the frictional or Coulomb damping.

The damping force is then constant and directed counter to the movement. Also non-linear forms such as $c(\mathrm{d}x/\mathrm{d}t)^2$ and $c(\mathrm{d}x/\mathrm{d}t)x$ occur.

Fig. 44 gives some damping forces as a function of time. These damping forces will influence the movements of a vibrating structure, and they will not be completely harmonic. In many cases, however, a viscous damper will suffice (see lit. [13], page 362).

10 Examples

Although the damping of a number of simple structures can be defined with the formulas given, it will be difficult in many cases to indicate the damping without measurement. Particularly with heavily damped structures, the damping cannot be well defined without measurement. However, those cases are often less interesting as regards vibrations.

The cause of damping in slightly damped strucures can mostly be adequately indicated; it can be calculated with the formulas given. To get an impression of the accuracy of those calculations, we shall now give a number of examples of calculated and measured damping.

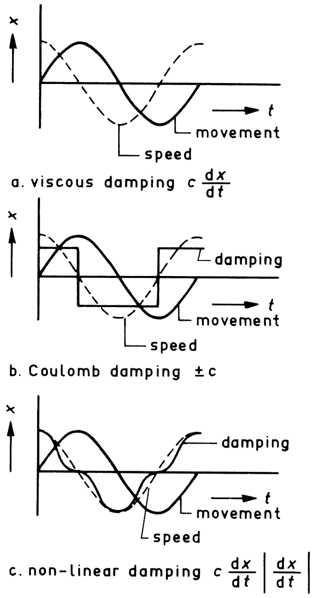


Fig. 44. Viscous and other types of damping.

Material damping

The damping which is always present is the material damping. For convenience, the table given on page 22 is repeated below.

material	c/c_{cr}	remarks
steel	0.004	
reinforced concrete	0.009	before and after cracking
prestressed concrete	0.009	
pine wood	0.021	
beech wood	0.025	
natural rubber	0.03	for frequencies above 10 Hz
natural rubber with canvas	0.08	for frequencies above 1 Hz
aluminium	0.018	
glass	0.06	
masonry	0.04	

Geometric damping

Parts of buildings composed of materials as given in the table, will have as much or more damping than the material alone. At the supports or attachments the energy can be removed (geometric damping), so that extra damping is available.

The following structural components were tested for damping:

A window-pane

The response of a pane to an impact load is given in Fig. 45. The damping was found to be:

$$\frac{c}{c_{cr'}} = 0.09$$

This value is greater than the material damping; some energy is removed at the supports. The pane tested was carefully fixed in its frame, so that energy dissipation was limited, however.

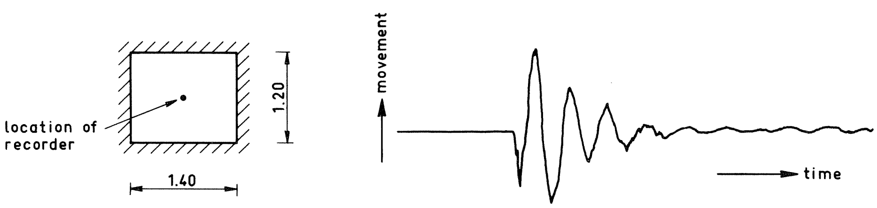


Fig. 45. Window-pane.

A masonry wall

The response to an impact load of a masonry wall, 22 cm thick, plastered and without cracks is given in Fig. 46. The damping is:

$$\frac{c}{c_{cr}} = 0.043$$

This value is also close to the material damping. The wall was founded on a heavy concrete slab and connected to an uncracked masonry wall.

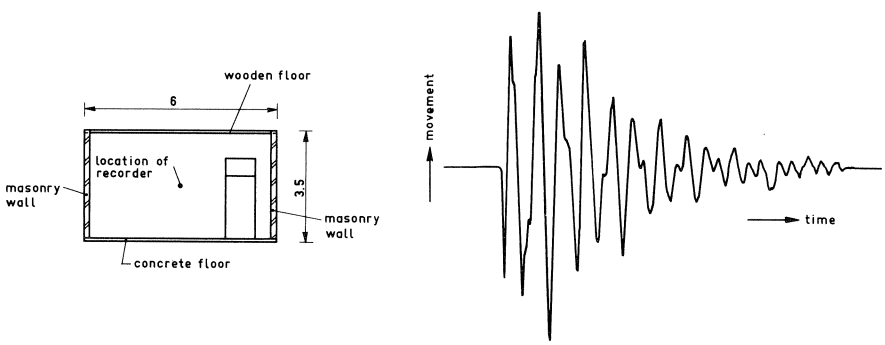


Fig. 46. Dying-out of vibration of wall-masonry.

A concrete floor on cellar walls

The response of a wide concrete slab fixed to cellar walls is given in Fig. 47. The damping is approx.:

$$\frac{c}{c_{cr}} = 0{,}055$$

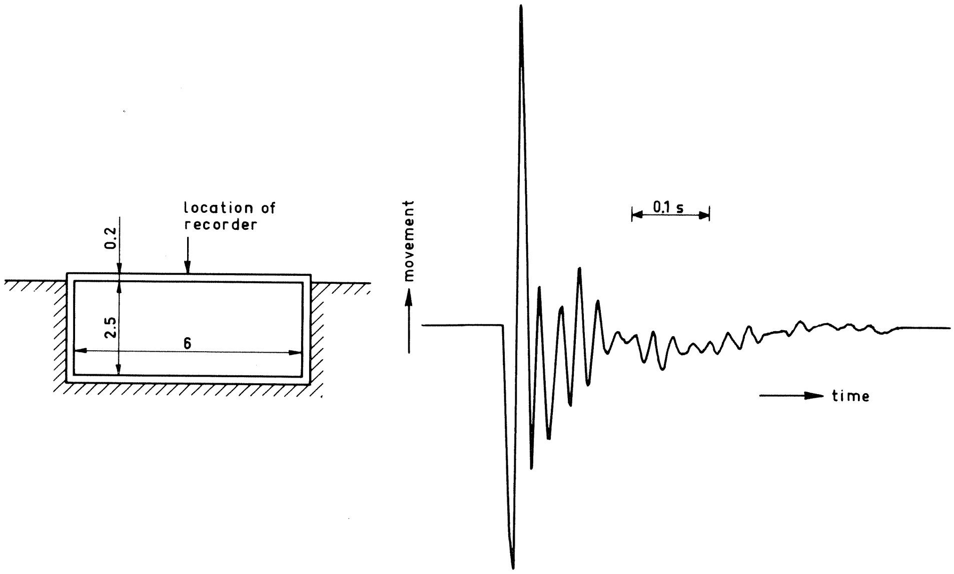


Fig. 47. Movements of concrete floor.

This value is considerably higher than the material damping. The energy can be removed via the materials piled on the floor and through the cellar walls. A rough estimate of the energy transport to the cellar walls supported by the ground can be made with a two-mass-spring system. It is, however, by no means, simple to find a good schematization of this case. One possibility is shown in Fig. 48, where the system m_1, k_1 is the schematization of the built-in (fixed) slab. The two-mass-spring system m_1, k_1, m_2 and k_2 represents the rotatably supported slab. The spring k_2' and the damper c_2' indicate the degree of fixity (restraint). If $k_2' = 0$, then the lowest natural frequency is that of the rotatably supported slab. If $k_2' = \infty$, then the scheme for the built-in slab remains.

The ratio of the natural frequency of the rotatably supported slab and the built-in slab is

$$\frac{\omega_0}{\omega_b} = \frac{9.87}{22.4} = 0.44$$

In Fig. 26, therefore, $\omega_b = 0.44\omega_1$.

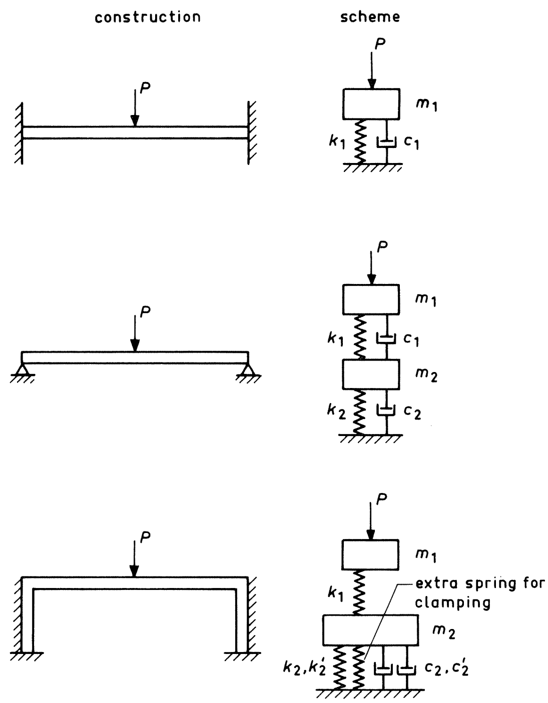


Fig. 48.

Besides (statically)

$$\frac{1}{k_1}+\frac{1}{k_2}=\frac{5}{k_1}$$

so that

$$k_2=\tfrac{1}{4}k_1$$

From these two values, and Fig. 26, it follows that $m_1/m_2 \approx 10$.

The spring k_2' and the damper c_2' must now be estimated. If it is assumed that the static deflection is half that of the simply supported slab, then:

$$\frac{1}{k_1}+\frac{1}{k_2+k_2'}=\frac{2.5}{k_1}$$

therefore:

$$k_2+k_2'=0.67k_1$$

Then (Fig. 26) $\omega_b/\omega_1=0.6$.

The damping c_2' of the cellar wall against the ground is taken as $c/c_{cr} = 0.5$. From Fig. 27 follows:

$$\frac{c}{c_{cr}} = A\frac{c_1}{c_{1cr}} + B\frac{c_2}{c_{2cr}}$$

$$= 0.22 \times 0.009 + 0.15 \times 0.5 = 0.002 + 0.075 = 0.077$$

The damping of the cellar wall here has a considerable share in the total damping of the floor.

A timber floor
The response of a timber floor to an impact load is given in Fig. 49. The damping is:

$$\frac{c}{c_{cr}} = 0{,}14$$

This damping will be caused by friction between the various parts of which the floor consists. The material damping of wood is much smaller.

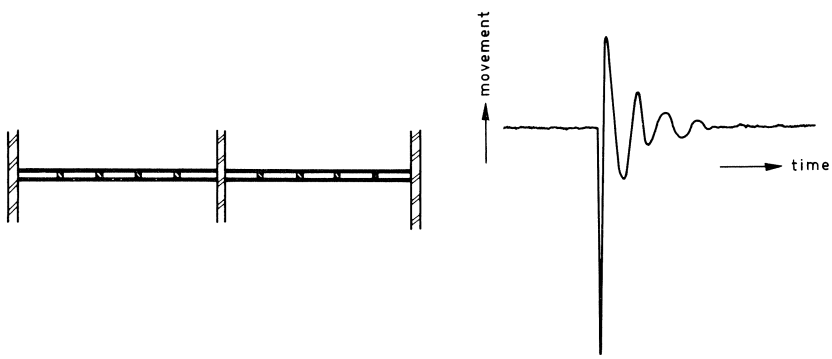


Fig. 49. Wooden floor.

A high-tension power cable
The accelerations of a cable with a diameter of 32 mm were measured at a wind velocity of 10 m/s. A part of the recorded movements is shown in Fig. 50. At these accelerations, many (high) frequencies are found to occur. The damping associated with the lowest natural frequency of the calbe was determined from the decrease in autocorrelation.

The damping was approximately:

$$\frac{c}{c_{cr}} = 0.05$$

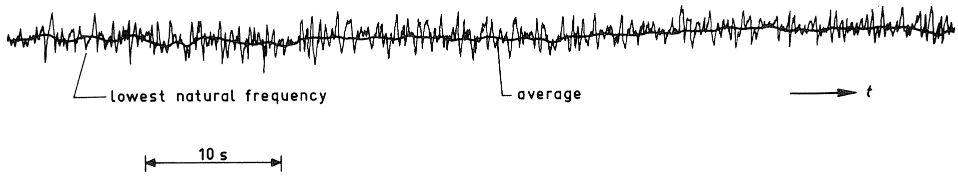


Fig. 50. Movements of high-voltage cable in wind.

The natural frequency of the cable can be estimated with the formula

$$f_0 = \frac{62}{L}$$

where

L = the distance between the supports

In this case, L equalled 300 m; f_0 was 0.2 Hz ($\omega_0 = 1.3\ \text{sec}^{-1}$). According to the formula on page 29, the damping is:

$$\frac{c}{c_{cr}} = \frac{\varrho V D C_D}{2\omega_0 \varrho_1 \frac{\pi D^2}{4}} = \frac{1.25 \cdot 10 \cdot 0.032 \cdot 1.1}{2 \cdot 2\pi \cdot 0.2 \cdot 2500 \cdot \frac{\pi}{4} \cdot 0.032^2} = 0.084$$

High buildings

Besides the investigations on the structural components mentioned above, a number of measurements were done on high buildings, masts, etc. Some results are given below.

The motions in the wind direction of a building with a concrete frame were measured. The external dimensions of the building are:

width 33 m (see Fig. 51)
length 79 m
height 104 m

The lowest natural frequency was 0.48 Hz. The damping was found to be

$$\frac{c}{c_{cr}} = 0.021$$

for motion parallel to the short sides and

$$\frac{c}{c_{cr}} = 0.036$$

for motion parallel to the long sides.

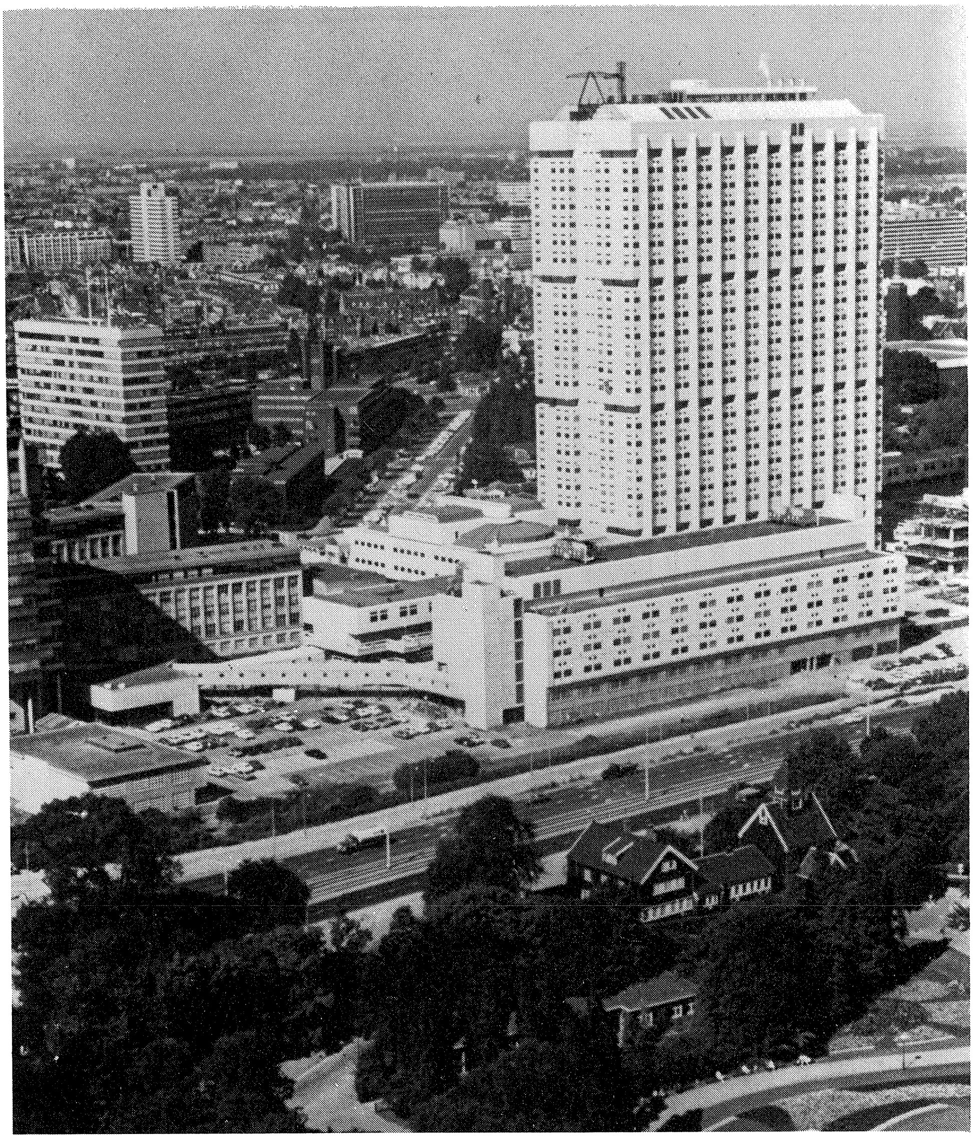


Fig. 51. Building of Erasmus University, medical faculty, Rotterdam.

The mass was $\mu = 305$ kg/m^3. The wind velocity at which the measurements were made was 18 m/s.

For calculating the damping, three causes of damping are considered:

1. material damping of concrete
2. geometric damping of the foundation
3. aerodynamic damping.

The material damping is taken as

$$\frac{c}{c_{cr}} = 0.030$$

in view of the construction using prefabricated elements.
The geometric damping follows from Fig. 18; $c/c_{cr} = 0.01$. The aerodynamic damping is, according to the equation on page 29:

$$\frac{c}{c_{cr}} = \frac{\varrho V D C_D}{2\omega_0 \varrho_1 D B} = \frac{1.25 \cdot 18 \cdot 79 \cdot 1.2}{2 \cdot 2\pi \cdot 0.48 \cdot 305 \cdot 79 \cdot 33} = 0.0004$$

The material damping and the geometric damping must be combined to a two-mass-spring system.

The structure above the foundation is schematized as a one-mass-spring system with a mass $m_1 = \frac{1}{4}m$ and spring constant $k_1 = 3EI/h^3$. (Calculated from the movement of the top of the building).

In the schematized system the rotational stiffness of the foundation will be the second spring and the rotational inertia of the building the second mass. To this spring and mass corresponds the rotation as a movement. For adjustment of the horizontal displacement of the top of the building it is necessary to divide by h^2:

$$m_2 = \frac{I}{h^2} = \frac{M(\frac{1}{3}h^2 + \frac{1}{12}b^2)}{h^2} \approx \frac{1}{3}M$$

k_2 is determined by the spring stiffness of the pile foundation and was in this case found to be the cause of a movement at the top that was 25% of the total movement:

$$\frac{m_1}{m_2} = \frac{\frac{1}{4}m}{\frac{1}{3}m} = 0.75$$

$$\frac{k_1}{k_2} = \frac{0.25}{0.75} = 0.33$$

The damping of m_1 can be determined from:

$$\frac{c}{c_{cr}} = A\frac{c_1}{c_{1cr}} + B\frac{c_2}{c_{2cr}}$$

A and B follow from Fig. 27:

$$\frac{c}{c_{cr}} = 0.5 \times 0.030 + 0.25 \times 0.010 = 0.015 + 0.0025 = 0.0175$$

Together with the aerodynamic damping,

$$\frac{c}{c_{cr}} = 0.0177$$

In the longitudinal direction of the building a damping of 0.036 was measured. In that direction, the geometric damping follows from Fig. 18, with $h/b = 104/79 = 1.32$:

$$\frac{c_2}{c_{2cr}} = 0.12$$

The ratio of the stiffness of the foundation and the structure in the longitudinal direction was found to be $k_1/k_2 = 0.15$. Then

$$\frac{c}{c_{cr}} = 0.75 \times 0.030 + 0.08 \times 0.12 = 0.022 + 0.010 = 0.032$$

The damping in the longitudinal directions is increased more particularly by the geometric damping.

A steel-framed building
Dimensions of the building:

width 50 m (see Fig. 52)
length 80 m
height 330 m

The lowest natural frequency was 0.147 Hz. The mass was 160 kg/m^3. The damping was determined as

$$\frac{c}{c_{cr}} = 0.0055$$

for a wind velocity of 18 m/s. In this case:

$$\frac{m_1}{m_2} = 0.75$$

$$\frac{k_1}{k_2} = 0.10$$

$$\frac{c_1}{c_{1cr}} = 0.004 \qquad \text{(steel)}$$

$$\frac{c_2}{c_{2cr}} = 0.001 \qquad \text{(Fig. 18)}$$

From Fig. 27:

$$A = 0.85$$

$$B = 0.04$$

$$\frac{c}{c_{\text{cr}}} = 0.85 \times 0.004 + 0.04 \times 0.001 = 0.0034 + 0.00004 = 0.0034$$

The aerodynamic damping is:

$$\frac{c}{c_{\text{cr}}} = \frac{\varrho V D C_{\text{D}}}{2\omega_0 \varrho_1 D B} = \frac{1.25 \cdot 18 \cdot 80 \cdot 1.2}{2 \cdot 2\pi \cdot 0.147 \cdot 160 \cdot 80 \cdot 50} = 0.0018$$

The calculated total damping is thus 0.0052.

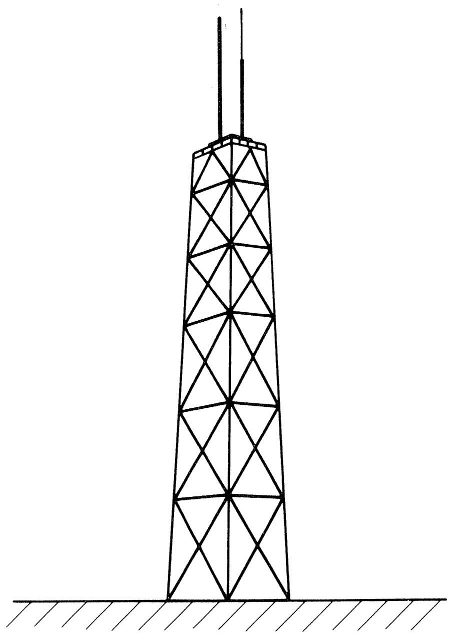


Fig. 52. John Hancock building, Chicago.

A pylon of a high-voltage cable (see Fig. 53)
The damping of a high-voltage pylon was measured as 0.008. Measuring was done at a wind velocity of 6 m/s. The mass consists of a lattice structure with a total height of 61 m.

The spring stiffness of the foundation (piles) is so great in comparison with that of the mass that the geometric damping has no effect on the mass damping. The total damping is the sum of material and aerodynamic damping. For the material:

$$\frac{c}{c_{cr}} = 0.004$$

The aerodynamic damping per m height of about ten 50 mm × 50 mm × 7 mm rolled steel angle sections is:

$$\frac{c}{c_{cr}} = \frac{1.25 \cdot 6 \cdot 10 \cdot 0.05 \cdot 2}{2 \cdot 2\pi \cdot 1 \cdot 7800 \cdot 10 \cdot 0.05^2} = 0.004$$

The total damping is therefore:

$$\frac{c}{c_{cr}} = 0.004 + 0.004 = 0.008$$

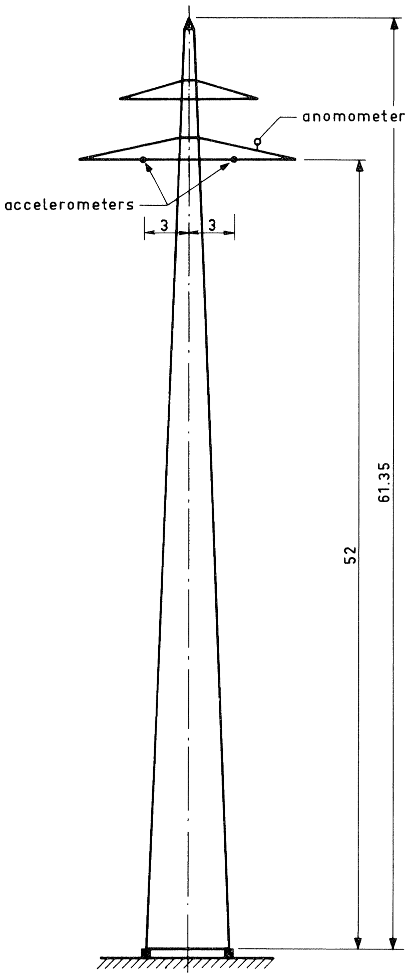


Fig. 53. Pylon for high-voltage cable.

Hydrodynamic damping

As an example of hydrodynamic damping, the damping is calculated for a steel tube in water. Measuring was done on an open steel tube with a diameter of 3.60 m and a wallthickness of 4 cm. The tube had a length of 36 m above the fixed section, 20 m of which was under water. The natural frequency was 0.68 Hz:

$$m_a = \pi \cdot 1.8^2 \cdot 1000 = 10200 \text{ kg/m}'$$

$$m = 2 \cdot \pi \cdot 1.8 \cdot 0.04 \cdot 7800 + m_a \text{ (water in the tube)} =$$

$$= 3500 + 10200 = 13700 \text{ kg/m}'$$

$$\frac{c}{c_{cr}} = \frac{10200}{13700+10200}\left(\frac{20}{36}\right)^3 \sqrt{\frac{2.2\cdot 10^{-4}}{1.8^2 \cdot 0.68}} + \frac{c_0}{c_{cr}}$$

$$= 0.00073 + 0.004 = 0,0047$$

$$\frac{c}{c_{cr}} = 0.014$$

was measured, which is considerably more than the calculated value.

The difference was caused by the damping due to the flow velocity of the water and the geometric damping. The flow velocity at the time of measurement was 2 m/s.

The damping caused by flow is, with $C_D = 0.6$:

$$\frac{c}{c_{cr}} = \frac{10200 \cdot 2 \cdot 4 \cdot 0.6}{2 \cdot 13700 \cdot \frac{1}{4} \cdot \pi \cdot 4^2 \cdot 2\pi \cdot 0.68} \cdot \left(\frac{20}{36}\right)^3 = 0.0064$$

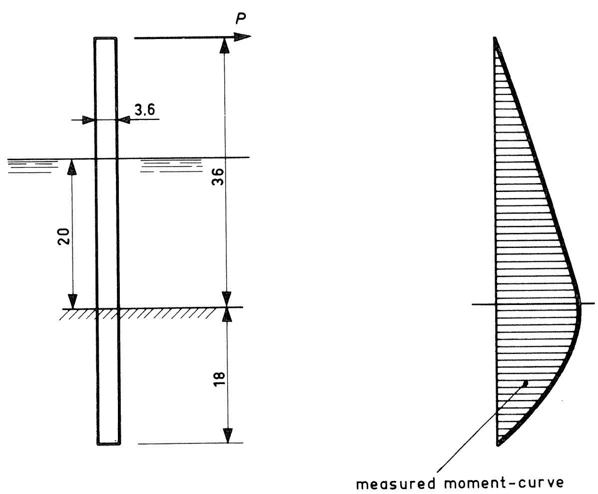


Fig. 54. Steel tube in water.

$$\frac{c_1}{c_{1cr}} = 0.0047 + 0.0064 = 0.0111$$

The damping caused by energy transport can be found with the formulas in 7.1, example 3.

According to those formulas the ratio between the mass m_1 and m_2 is:

$$\frac{m_1}{m_1} = \frac{\frac{1}{4}M}{\frac{1}{3}M} = 0.75$$

and the ratio between the spring constant k_1 and k_2 is:

$$\frac{k_1}{k_2} = \frac{\dfrac{3EI}{h^3}}{1.5\dfrac{EI}{Dh^2}\left(\dfrac{E_g}{E}\right)^4} = \frac{2D}{h}\left(\frac{E}{E_g}\right)^{\frac{1}{4}} = \frac{2 \cdot 3.6}{36}\left(\frac{2.1 \cdot 10^{11}}{2 \cdot 10^8}\right)^{\frac{1}{4}} = 1.1$$

The geometric damping according to formula (26) is:

$$\frac{c_2}{c_{2cr}} = 0.2\left(\frac{E_g}{E}\frac{\varrho_g^2}{\varrho_1^2}\right)^{\frac{1}{4}} \approx 0.2\left(\frac{E_g}{E}\right)^{\frac{1}{4}} = 0.035$$

According to Fig. 25, $A = 0.15$; $B = 0.60$:

$$\frac{c}{c_{cr}} = 0.15 \cdot 0.0111 + 0.60 \cdot 0.035 = 0.0017 + 0.021 = 0.023$$

Summary of examples

structure	calculated damping: material	geometric	aerodynamic	total	measured damping
window pane	0.06	0.03 *	–	0.09	0.09
concrete floor on cellar walls	0.002	0.075	–	0.077	0.055
concrete floor on cantilevers and columns	0.009	–	–	0.009	–
masonry wall	0.04	–	–	0.04	0.043
wooden floor	0.021	0.12 *	–	0.141	0.14
high-tension cable	0.004	–	0.042	0.046	0.05
building with concrete frame					
(H = 100 m) short side	0.015	0.0025	0.0004	0.018	0.021
long side	0.022	0.010	0.0004	0.032	0.036
building with steel frame (H = 330)	0.0034	0.00004	0.0018	0.0052	0.0055
mast with high-voltage cable	0.004	–	0.004	0.008	0.008
steel pipe in water (H = 40 m)	0.0007	0.021	0.0010	0.023	0.014

* not calculated

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Summaries

VIBRATION PROBLEMS IN PRESTRESSED CONCRETE (CUR report no. 17)

Vibration problems are more likely to arise in connection with prestressed concrete structures than with reinforced concrete structures. The natural frequencies of the former class of structures, which are often of slender proportions, are relatively low, and mechanical causes (as, for instance, live loads with frequencies of 1–5 cycles per second) are liable to set up vibrations corresponding to these frequencies. The prestress furthermore prevents the formation of cracks in the concrete, so that an oscillatory motion, once it has started, will not be damped out so rapidly as in the case of reinforced concrete.

A comprehensive theory is available for the study of these vibration phenomena. In the present Report the essentials of this theory are given in a brief survey, which is intended as an introduction to the relevant literature. This theory can, however, be applied only if sufficient information is available regarding the material properties of prestressed concrete. From the existing literature and from some tests carried out it was inferred that the mass per cm^3 is $2.55 \cdot 10^{-6}$ kg sec^2/cm (the kilogramme is the unit of force) and that the dynamic modulus of elasticity is $E_{dyn} = 370.000$ kg/cm^2.

It was not possible, however, to obtain an adequate insight into the phenomenon of damping from information given in the literature nor from a few simple tests. For this reason an exhaustive investigation into the phenomena associated with damping was undertaken. In this connection a series of tests was carried out, in which with the aid of steel columns a small simple prestressed concrete beam was incorporated into a portal-frame type of structure. By changing the columns it was possible to obtain several different portals, so that one and the same prestressed concrete beam could thereby be made to vibrate according to many different frequencies and modes.

From the observed shapes of the deflection curves and from the displacements it was possible for each case to obtain an equation expressing that the energy added to the system was distributed over three types of internal damping in the beam and two types of damping in the support or clamped connections. From a fairly large number of such equations (viz., 15 and 11 for two different beams respectively) it could be inferred that the angle of lag between stress and strain in prestressed concrete is approximately constant (= 0.018 rad) and independent of the frequency. This means that the resonance factor (if there is only internal damping) is likewise independent of the frequency and has a value of approximately 55.

Naturally, in many cases the supports etc. will absorb so much energy that deflections amounting to 55 times the static deflections are not reached by a long way. In this connection Chapter VII of this Report gives some examples of how it could be investigated whether vibration problems are liable to occur in a given prestressed concrete structure. The constructional possibilities for combating and obviating objectionable or dangerous vibrations are discussed in Chapter VIII. In conclusion, a number of simple worked examples are given.

THE STRUCTURAL CONSEQUENCES OF DYNAMIC INFLUENCES UPON TABLE FOUNDATIONS OF ROTATING MACHINES (CUR report no. 35, in Dutch)

In this report it has been endeavoured to gain insight into the influences exercised upon the vibration behaviour of a machine foundation by the dimensions of the various structural components. The investigation is based on a table-type foundation on which the machine is mounted on a top slab supported by columns which in turn stand on a bottom slab.

This investigation is important because, on account of the continually increasing size of turbo-generators and suchlike machines, it is becoming progressively impossible to build "rigid" foundations in which hardly any natural frequencies below the normal running speed of the machine occur. Hence it is necessary for the designer to have an insight into the vibration behaviour.

First, the behaviour of a flat frame was investigated by means of calculations and experiments. In these investigations the cross-sectional dimensions were varied between very slender and very stout while, in addition, either an extra heavy bottom slab or a heavy top slab was employed. These calculations showed that the various natural frequencies can easily be estimated on the basis of a schematisation to a one-mass or two-mass spring system. A conclusion of practical importance was, furthermore, that a heavy bottom slab had no demonstrably favourable effect upon the vibration behaviour.

On studying a three-dimensional foundation it was found to be possible to deduce the behaviour of the foundation as a whole from the behaviour of the transverse frames. On account of the interconnection of the various transverse frames by the longitudinal frames the number of natural frequencies is considerably increased.

As a result of the investigation, the designer can gain insight into the vibration behaviour of a foundation so that he can judge the effect of modifications to the transverse or longitudinal frames upon that behaviour. Hence the structural design can, more than was formerly possible, be carried out in a purposive manner. Of course, once a foundation has been provisionally designed with the aid of approximate calculations, the structure as a whole can then be analysed with an electronic computer. In addition to the natural frequencies, it is necessary also to determine the amplitudes in order to be able to ascertain whether a natural frequency is likely to be dangerous. For this purpose it is necessary to start from an assumed damping effect, because as yet not enough is known about damping to enable the amplitude associated with resonance to be accurately calculated.

From the investigation it emerged that it is highly desirable that the mechanical engineers should make available to the foundation designer more reliable data concerning the machine. In this way it is possible to avoid having to impose excessively severe requirements upon the foundation.

In certain circumstances the fact that, in consequence of a fault in the balancing of the machine, a flexible foundation will start vibrating more noticeably may provide an early warning of defects in the machine.

DYNAMIC PROBLEMS ASSOCIATED WITH CIVIL ENGINEERING STRUCTURES
(CUR report no. 57, in Dutch)

Dynamic loads acting on civil engineering structures will often not be of a periodic character. An obvious example of this is wind loading. The design of structures subject to non-periodic loads calls for a different technique from that adopted in design calculations for periodic loading conditions. In the present report this technique is briefly outlined, and applications thereof to impact loads and arbitrarily varying loads are indicated.

Besides this short theoretical treatment of the subject, a number of frequently encountered loading conditions are examined. These are subdivided as follows:

- periodic loads;
- impact loads;
- arbitrarily varying loads.

The types of loading indicated relate to

for periodic loads:
- piston-operated machinery;
- rotary machinery;
- rotary printing presses;
- church bells;
- walking, dancing and jumping;
- wind or water currents;

for impact loads:
- falling objects;
- slamming doors;
- gas explosion;
- vessels berthing;
- bumping against a wall;

for arbitrarily varying loads (random loading):
- wave action;
- gusts of wind;
- eddies;
- traffic;
- subsoil movements.

The calculation of the response to these types of loading for simple structures is presented.

Finally, the behaviour of the materials steel and concrete under alternating loads is dealt with, and the limits for movements of buildings, both with regard to the users and with regard to the structure itself, are indicated.

RECOMMENDATIONS FOR THE DESIGN AND ANALYSIS OF MACHINE FOUNDATIONS (CUR report no. 61, in Dutch)

In 1967, Hoogovens (Royal Netherlands Blastfurnaces and Steelworks), IJmuiden, set up a study committee which was entrusted with drawing up a set of recommendations for the design of foundation structures for machinery. These recommendations, which were completed in 1972, have been examined and adopted by CUR Committee A 20 "Dynamic problems in construction".

The recommendations are concerned with the following subjects:

Sect. I *Notation, units, definitions*

This section comprises all the symbols used in the dynamic calculations with the associated units in accordance with the S.I. system and with definitions of their meanings. Furthermore the concepts of dynamic forces, damping, stress, elasticity and fatigue are explained. The subscripts required in conjunction with the symbols are indicated, as also the co-ordinate system with the sign conventions. Finally, the various types of loading are defined.

It is intended that all who are concerned with the design and construction of a foundation should base themselves on the information given in this section. This will ensure that they all speak the same language, as it were.

Sect. II *Data to be provided by the supplier of the machine*

This section comprises a number of forms to be completed by the machinery supplier. This information will of course have to be supplemented with drawings and diagrams. The required data are subdivided as follows:
- general data;
- permanent loads;

- periodically varying loads for;
  unbalanced machines;
  balanced rotary machines;
- impact loads (separate form for forge hammers);
- temporary loads;
- heat evolution;
- supplementary information, including wishes as to choice of materials;
- criteria specified by the machinery supplier (see also section IV).

For dealing with cases where, for example, the unbalanced forces exerted by rotary machines are not stated by the supplier this section gives appropriate values that the foundation designer can adopt.

Sect. III *Structural data to be provided*

This section bases itself on the principle that the machine foundation is designed by a specialised designer. Of course, all essential information must be made available to him, namely:

- general information;
- soil conditions;
- data relating to piled and/or shallow foundation structure;
- additional loads;
- data on sources of disturbance in the vicinity;
- data on sensitive equipment in the vicinity;
- supplementary information, including wishes as to choice of materials;
- criteria specified by the construction department (see also Section IV).

In this section, too, appropriate design values are given, e.g., with regard to the resilient properties of piles. These are based on empirical data.

It may occur that there is no separate designer for the machine foundation and that, instead, the building together with the machine foundation is dealt with by one and the same designer. In that case the present section can serve as a check-list.

Sect. IV *Criteria to be satisfied by the foundation*

In Sections II and III it is stated that both the machinery supplier and the construction department can specify certain criteria. In order to arrive at a practical approach with regard to this, criteria are laid down in Section IV. Only in cases where it is desired to depart from these need this can be stated on the relevant form envisaged in Section II or III.

Criteria are established or elucidated for the following subjects:

- strength;
- vibrations (displacement amplitude as a function of the disturbance frequency);
- rigidity;
- experimental verification (amplitudes, rigidity).

Sect. V *Construction*

This section gives practical hints and suggestions, both for the machinery supplier and for the foundation designer, with regard to:

- structural foundation to support the actual machine foundation;
- choice of material;
- choice of foundation type;
- connection of machine to foundation.