Preface

In the construction of reinforced concrete structures such as tunnels, shipping locks and basements it often occurs that the floor slab is concreted first and the walls and roof are added after an interval of time. In consequence of this construction procedure, cracks frequently develop in the walls (and possibly in the roof), since the floor restrains the temperature and shrinkage deformations tending to occur in the walls. Not enough is yet known as regards possible measures for restricting this type of cracking. Accordingly, a Committee undertook research into the effect of one of the most familiar measures of this kind, namely, the application of additional reinforcement disposed at right angles to the direction of cracking. The research comprised a theoretical and an experimental part.

The Committee was constituted as follows:

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CRACKING DUE TO SHRINKAGE AND TEMPERATURE VARIATION IN WALLS

Summary and conclusions

In general, the choice of the correct measures for the prevention of cracking in reinforced concrete structures calls for a great deal of careful attention. The difficulty is that in most cases little is known about the effectiveness of a measure applied. A commonly employed precaution consists in providing extra reinforcement. Although this will not prevent cracking, it could perhaps limit the widths of the cracks that develop. In the present report it has been endeavoured to find out to what extent the provision of reinforcement can be effective as a means of limiting the widths of cracks due to deformation restraint.

Such cracks frequently occur in structures such as tunnels, shipping locks and basements, i.e., consisting of rigidly interconnected parts which have been concreted in various stages. The walls of such structures are usually concreted after the floor slab has hardened. Because of this procedure, the floor and the wall are, as it were, out of phase with regard to shrinkage and temperature behaviour (due to liberation of heat of hydration). Differences in deformation thus tend to develop between floor and wall, but they are prevented from actually developing by the rigid interconnection of these structural parts. This restraint is liable to give rise to cracking of the concrete.

Deformation restraint also often causes cracking in cantilevered structures such as balconies and in cantilevered cycle tracks and footpaths made of concrete.

An introductory chapter is followed by four chapters (Nos. 2 to 5) explaining the types of structure in which cracking due to restraint may occur, what the consequences of such cracking may be, and what counter-measures can be taken to limit the crack widths. Next, Chapter 6 gives the results of linear-elastic analysis of stresses and strains in walls joined to a floor slab, as published in the literature. The next chapter presents a theory whereby the magnitude of the curvature can be calculated for a structure subjected to restrained deformation due to shrinkage or some similar cause. It emerges that this curvature considerably affects the resultant stresses and strains.

Since the literature gives hardly any information on the crack development process and on the widths of the cracks that occur in structures subjected to restrained deformation, some experimental research was carried out on wall models made of micro-concrete (concrete with scaled-down granulometric composition). The effect of the quantity of reinforcement on crack width was also investigated in this research, which is described in Chapter 8.

A theory of cracking is developed in Chapter 9, presenting formulas for the relationships between the quantity of reinforcement, the deformation restraint, the quality of the concrete, etc. The validity of this theory is examined in Chapter 10 with reference to the results of the tests performed on the wall models in the above-mentioned experimental research. There is found to be good agreement between the theory and the experiments.

In order to obtain information, inter alia, on crack widths developing over long periods of time, the widths of cracks in actual structures were measured and are reported in Chapter 11. The cracking and curvature theory presented here was verified with reference to these measured values, and it is inferred that it is possible with the aid of the theory to determine how much reinforcement is needed for limiting the widths of cracks to acceptable values.

Chapter 12 presents considerations on the effect of the subgrade or underlayer on the curvature behaviour of tunnels, locks and basements. It appears that in most cases this effect is negligible, so that the curvature formulas given in this report can be used unmodified.

The application of the theory of cracking in practice is explained in Chapter 13. An important part of this procedure is the choice of the average value for the crack width, which is dealt with in Section 13.2. The cracks due to deformation restraint, as envisaged in this report, and the possible resulting corrosion of the (distribution) reinforcement will not in principle directly affect the strength (loadbearing capacity) of the structure. In view of this, and having regard to the development of available repair techniques, the choice of the average crack width has been made co-dependent on economic considerations.

Finally, three worked examples are presented in Chapter 14.

The following general conclusions can be drawn from the research presented in this report:

- The effect of reinforcement on crack width is greater in structures which preserve their straightness than in curved ones, this being bound up with factors mentioned below. The effect of reinforcement on crack width is greater according as the crack-distributing effect of the connected floor slab is less.
- The extent to which a structural component undergoes curvature within its own plane is found to be an important factor affecting the development of stresses and strains. Thus, for example, in structures which remain straight the cracks are wider than in curved ones. The linear-elastic considerations presented in the report and the experimental research conducted – described in Chapters 6 and 8 respectively – provide the evidence for this. The degree of curvature of a structure depends on the flexural stiffness ratios of the rigidly interconnected parts of the structure.
- The floor slab has a crack-causing and a crack-distributing effect on the wall or other component to which it is connected. Both these effects are greatest close to the floor and diminish linearly to zero up to a certain distance from the floor.
- Reinforcement installed close to the floor is found to have hardly any limiting effect on the width of cracks occurring in the vicinity thereof.

NOTATIONS

The following subscripts may be added to a symbol (here exemplified by a fictitious quantity i):

i_d	the quantity i relates to the roof of a tunnel, basement, etc.
i_v	the quantity i relates to the floor of a tunnel, basement, etc.
i_w	the quantity i relates to the wall of a tunnel, basement, etc.
A_a	cross-sectional area of longitudinal reinforcement
A_b	gross cross-sectional area of concrete
a	uncracked depth (height) of the concrete section of the wall
b	(effective) width of the concrete section
d	overall thickness of the concrete section
c	concrete cover to longitudinal reinforcement
E_a	modulus of elasticity of reinforcing steel
E_b	modulus of elasticity of concrete in tension
E_b'	modulus of elasticity of concrete in compression
f_a	design value of tensile strength of reinforcing steel
f_b	design value of tensile strength of concrete
f_{ck}'	characteristic cube strength of concrete
f_{bu}	ultimate tensile strength of concrete
f_d	bond strength between steel and concrete
f_{du}	ultimate bond strength
h_t	overall depth (height) of the concrete section
h_w	overall depth (height) of the part of the wall situated above the floor of a tunnel, basement, etc.
h_{wb}	depth (height) of the concrete section of a strip with reduced reinforcement at the top of the wall
h_{wo}	depth (height) of the concrete section of a strip with reduced reinforcement at the base of the wall
I	moment of inertia of a section
L	length of a member
Δl_y	average crack spacing at a distance y from the floor
M	bending moment
N	normal tensile force
N'	normal compressive force
N_s	normal tensile force at which cracking occurs
n	modular ratio of steel and concrete
w	average crack width (short duration)
$\bar{w}$	permissible average crack width
X	number of cracks
y	distance to floor
z	length of zone with bond stresses on each side of a crack

ε	strain
ε_b'	compressive strain of concrete at top of wall
ε_{sv}	strain at which the crack pattern has been completed
ε_u	maximum strain of concrete
$\Delta\varepsilon_v$	difference in strain due to shrinkage and/or temperature
ε_w	restrained strain of wall at junction with floor
ε_y	strain in wall at distance y from floor
σ_{as}	normal tensile stress in steel at a crack
σ_b	normal tensile stress in concrete between two cracks
$\bar{\omega}$	relative quantity of longitudinal reinforcement in an overall rectangular section $= A_a/bh_t$
$\bar{\omega}_0$	$\bar{\omega}$ expressed as a percentage
$\bar{\omega}_{min}$	minimum required relative quantity of longitudinal reinforcement in an overall rectangular section
$\bar{\omega}_r$	relative quantity of longitudinal reinforcement in the concrete section of the wall strips designated by h_{wo} and h_{wb}
$\varnothing_k$	characteristic diameter of a plain or a deformed bar

Cracking due to shrinkage and temperature variation in walls

1 Introduction

Choosing appropriate measures for the prevention, restriction or repair of cracking in concrete structures constitutes a constantly recurring problem. In some cases there may be differences of opinion as to the need for such measures, but these differences can be avoided by applying requirements – based on technical grounds – as to the amount of cracking that can be tolerated. It will then of course be desirable to have some clear insight into the effect of the measures adopted. Quite often such insight is lacking, especially in cases where cracks are caused by shrinkage and temperature variation. In practice such cases are often encountered more particularly in tunnels, shipping locks and basements. These structures mostly develop cracks in the walls and possibly in the roof during construction (see Fig. 1). But in the service stage, too, i.e., when the structure is in actual use, this type of cracking is not infrequently encountered in cantilevered balconies (see Fig. 2) and in cantilevered footways and cycle tracks on bridges and flyovers.

A frequently employed measure in these structures is the provision of additional reinforcement at right angles to the direction of the expected cracks. Although this reinforcement cannot prevent cracking, it is considered capable of limiting the crack widths to acceptable values. Opinions differ as to the effectiveness of the measure, which is apparent, inter alia, from the fact that the percentages of reinforcement installed for the purpose in practice range from 0,1–0,2% to more than 1,0% of the concrete cross-sectional area.

In order to obtain a clearer picture of the effect attained, the Committee more particularly studied the relationship between the quantity of reinforcement and the widths of the cracks that occur. To this end, a literature research project was carried out and experimental investigations were conducted on microconcrete models which

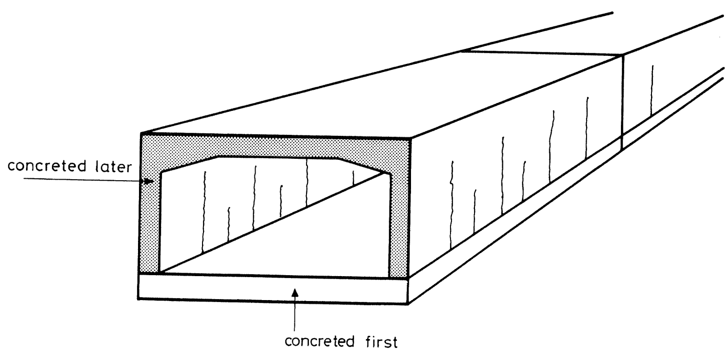


Fig. 1. Cracks in tunnel walls.

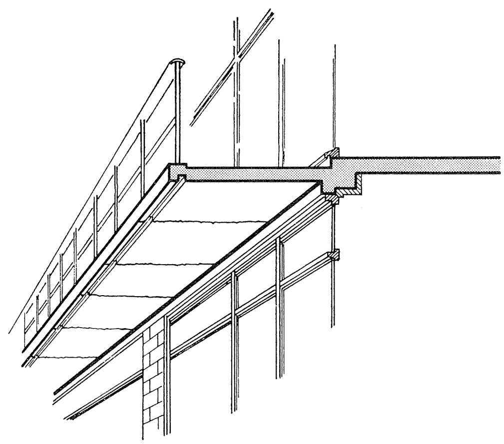


Fig. 2. Cracking in a cantilevering balcony.

were subjected to imposed deformations. The quantity of reinforcement was varied in order to ascertain the effect thereof upon crack distribution and crack width. Finally, a theoretical analysis was established, enabling the quantity of reinforcement resulting in acceptable crack widths to be calculated in advance.

2 Cracking in walls of tunnels, basements, etc.

In structures such as tunnels, shipping locks, basements, etc. is normal practice to concrete the floor, the walls and, in certain cases, also the roof in a number of successive stages. As a result of this working procedure, the hardening of the various parts of the structure is subject to phase differences, as also are the development of shrinkage and possibly the rate of temperature rise due to the release of heat of hydration. Since the parts in question are rigidly interconnected, such phase differences give rise to stresses and often to cracking.

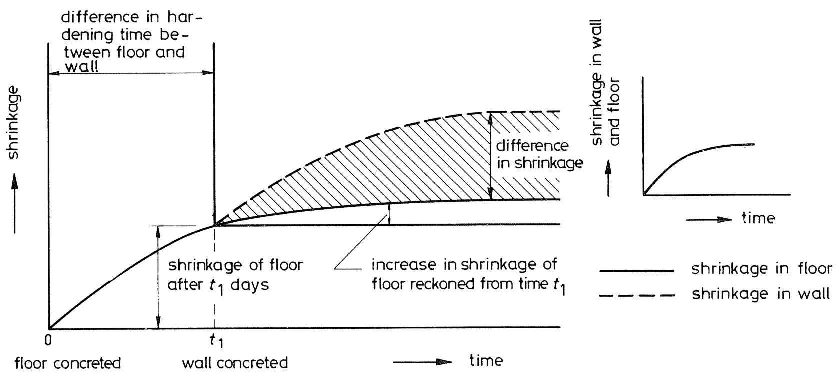


Fig. 3. Schematic representation of shrinkage behaviour in a floor and in a wall subsequently concreted onto it, and of the differential shrinkage.

By way of example, the development of hardening and drying shrinkage in the floor of a tunnel, lock or basement and in a wall subsequently cast onto the floor is illustrated schematically in Fig. 3. The shrinkage is marked on the vertical, the time on the horizontal axis. The curves indicate the shrinkage behaviour of the floor and of the wall, respectively, in the absence of shrinkage restraint. After a hardening period t_1 of the floor, the wall is concreted. During this length of time the floor has undergone unrestrained shrinkage, and subsequent shortening due to this cause will not amount to much. On the other hand, the increase in shrinkage in the wall, from that point of time t_1 onwards, is much greater than in the floor. Hence a difference in shortening between wall and floor develops, at least if the shrinkage process could take place without restraint. The rigid interconnection between floor and wall resists this difference in shortening, however, so that stresses are formed in these two parts of the structure. The magnitude of these stresses will depend on the difference in shortening and on the strain and flexural stiffness ratio of the interconnected parts. Tunnels, locks, etc. generally undergo a certain amount of curvature in consequence of the differential shortening.

Broadly speaking, tensile stresses develop longitudinally in the walls, and compressive stresses in the floors. The walls often display vertical cracks due to the tensile stresses. This is schematically exemplified in Fig. 4. The cracks are generally most numerous in the proximity of the floor but they are more finely distributed there, and the average crack width is smaller. Just above the construction joint connecting the floor and the wall the cracks in the latter are indeed often invisible.

A phenomenon similar to that of differential shrinkage may occur in consequence of temperature influences, e.g., if the temperature in the wall due to heat of hydration becomes higher than that in the floor joined to it and subsequently goes down to ambient temperature. Particularly in thick walls, such as those of tunnels and locks, this phenomenon is more important than that due to shrinkage. By way of example, Fig. 5 shows the temperature curves for a wall and a floor respectively. Directly after concreting, the temperature in the wall rises rapidly and, in thick walls, sometimes

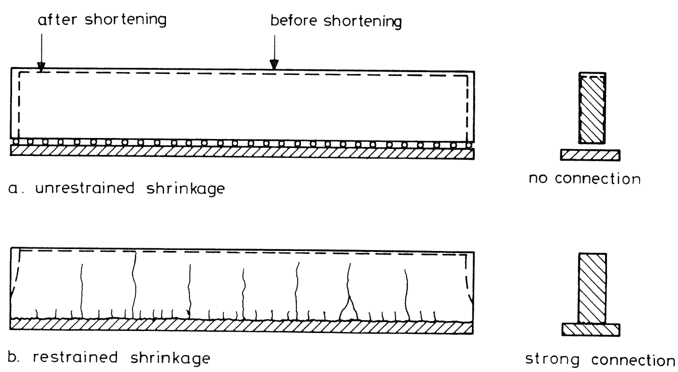


Fig. 4. Cracking due to restrained shrinkage in a wall concreted onto a floor that had already hardened.

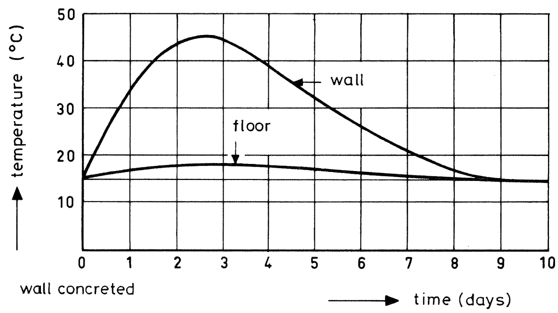


Fig. 5. Temperature-time relation (schematic) in a thick tunnel wall and in a tunnel floor, reckoned from the time when the wall was concreted onto the floor.

reaches values above 50 °C. The increase in length (thermal expansion) associated with this rise can develop virtually stress-free, since the concrete at this stage is stil plastic or is at most in its first stage of hardening, when the modulus of elasticity is still very low. During cooling, which generally begins after one or two days and which proceeds at a much slower rate than heating, the concrete has meanwhile hardened more and now has a much higher modulus of elasticity, with the result that restraint of deformation gives rise to such high tensile stresses that cracking occurs in the wall. These cracks are often already present at the time of formwork removal. In the concrete walls and floors of dwellings built with the aid of so-called tunnel forms, in which accelerated hardening of the concrete has been achieved by heating, it is not unusual to find cracks which likewise belong to the category of temperature cracks. These, too, are often already present at the time of removing the formwork.

3 Cracking in cantilevered balconies, etc.

Cracking occurs after a time in some structures because parts of them are unequally affected by external conditions or respond with a difference in time lag to such conditions.

Thus, in consequence of unequal external conditions, cracks may occur in cantilevered balconies in the service stage (see Fig. 2). This is because the balcony structure is situated partly inside and partly outside the building. The cantilevered outer part tends to vary in length under the influence of the daily variations in temperature, whereas the inner part opposes this because its temperature varies much less. The deformations of the cantilevered outer part are therefore resisted, so that stresses occur which may result in cracking perpendicularly to the outer edge. Although shrinkage also takes place, the differences in shrinkage remain small, since the whole balcony structure was concreted in a single operation.

Cracking similar to that described above also often occurs in cantilevered footways and cycle tracks. These cracks are likewise due to restrained deformations, but the latter are now not caused by a difference in external conditions, but to a difference in the (rate of) response to these conditions. More particularly, this difference is due to the fact that a cantilevered member of usually quite light or slender construction is rigidly connected to a heavy main structural member. Under the influence of solar radiation, for example, the lighter member will be heated up more rapidly and also cool more rapidly than the main structural member, so that the temperature behaviour in the respective parts of the structure will be subject to a difference in phase and amplitude. In consequence of this, there occurs restraint of deformation, resulting in the above-mentioned effects.

4 Consequences of cracking

Besides aesthetic objections, objections of a technical nature may necessitate controlling the cracking process in structures of the kind referred to in the foregoing chapters. These technical objections are:
- increased water permeability of the wall, a factor which is especially important in tunnels and basements;
- increasing hazard of corrosion of the reinforcement, which is a factor to reckon with in many kinds of structure.

Since a wall cracks through its entire thickness, its permeability may become quite considerable. Even small amounts of water seeping through cracks will moreover increase the corrosion hazard.

In the Netherlands, requirements relating to the restriction of crack widths are mainly concerned with the danger of corrosion. Although the crack width is not the only factor affecting the occurrence of corrosion of the reinforcement – the concrete cover and the density of the concrete also play a part – it is nevertheless realistic to impose limiting values. Corrosion reduces the effective cross-sectional area of the reinforcement. Besides, corroded bars may cause spalling of the concrete cover, which is aesthetically unacceptable and accelerates the corrosion process. Additional objectionable consequences may be: the need to remove inleaked water by pumping, and slipperiness due to freezing of the water. It is therefore desirable to impose limits on the width of cracks.

5 Measures for the restriction of cracking

5.1 *Additional reinforcement*

The use of additional reinforcement to limit the width of cracks is a measure which, in the Netherlands, is applied often more or less as a matter of course. The fact that in the Netherlands code of practice for concrete construction (VB 1974) a direct relationship is indicated between reinforcement and crack width has undoubtedly contributed to this approach.

The formulas for the calculation of crack width which are given in that code relate to cracking caused by (external) loads. When the width has been calculated with the aid of these formulas, the effect, if any, of shrinkage and/or temperature variations upon the crack width can be allowed for. The cracks with which the present report is concerned, however, are not caused by (external) loads, but by shrinkage and/or temperature variations. This being so, the required quantity of reinforcement tends to be determined in widely varying ways. Sometimes the quantity is small because cost aspects play a dominant part, in other cases a large quantity is used in order to comply with stringent crack-restricting requirements.

In general, it can be stated that the quantity of reinforcement provided with a view to restricting crack formation due to shrinkage and/or temperature variation is determined by subjective judgment of by intuition, this being in part due to the absence of a clear-cut method of analysis. Because of this, too, opinions differ with regard to the effect achieved by this reinforcement. The widely varying percentages of steel employed in practice reflect these differences. For the sake of completeness it should be mentioned (see also Chapter 1) that cracking cannot be prevented by reinforcement, but that the judicious use of reinforcement will limit the width of the cracks.

5.2 *Technological and other arrangements*

Since this report is more particularly concerned with the results of research into the relationship between reinforcement and cracking, other measures for the restriction of cracking will only be briefly considered here.

For structures in which cracks develop in consequence of the manner of construction, measures which result in limiting the difference in shortening between two component parts are most effective. Such measures which limit the difference in shortening are, for example:

- Reduction of the cement content. The amount of heat evolved by hydration is reduced in consequence, and so is the shrinkage (for equal water-cement ratio). The cement content cannot be reduced indefinitely, however, because other factors also play a part, such as the required strength of the concrete and the protection of the reinforcement from corrosion.
- Using a slow-hardening cement. This causes the heat of hydration to be evolved more gradually, so that it can more effectively be dissipated to the surroundings. A slowhardening cement moreover reduces shrinkage.
- Using less water in the mix. The lower limit of the water content is determined by the required workability of the fresh concrete. The addition of a plasticizer can improve workability.
- Concreting the two parts of the structure in a single operation or with the shortest possible interval of time between them.
- Retarding the drying shrinkage of the part concreted first. This can be done by keeping it wet or by using a curing compound. In the case of thick walls cast onto floors of tunnels or locks the effect of drying shrinkage is, however negligible in relation to the shrinkage due to heat of hydration.
- Reduction of the deformations by cooling the part concreted last. Cooling should be applied for the first few days after the concrete has been placed. It prevents excessive rise in temperature due to the heat of hydration.

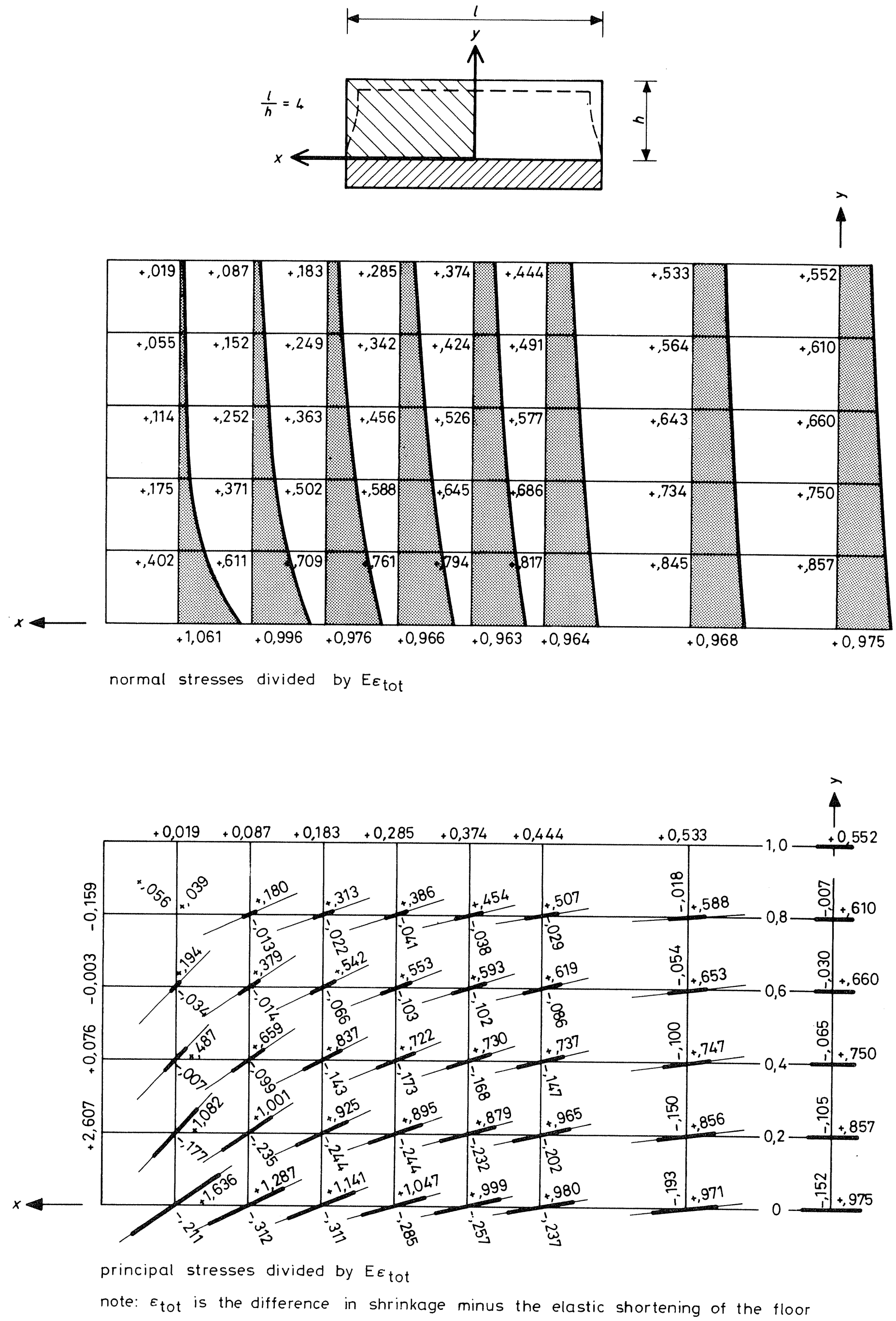


Fig. 6. Stress distribution in a wall remaining straight (curvature = 0) according to SCHLEEH [4].

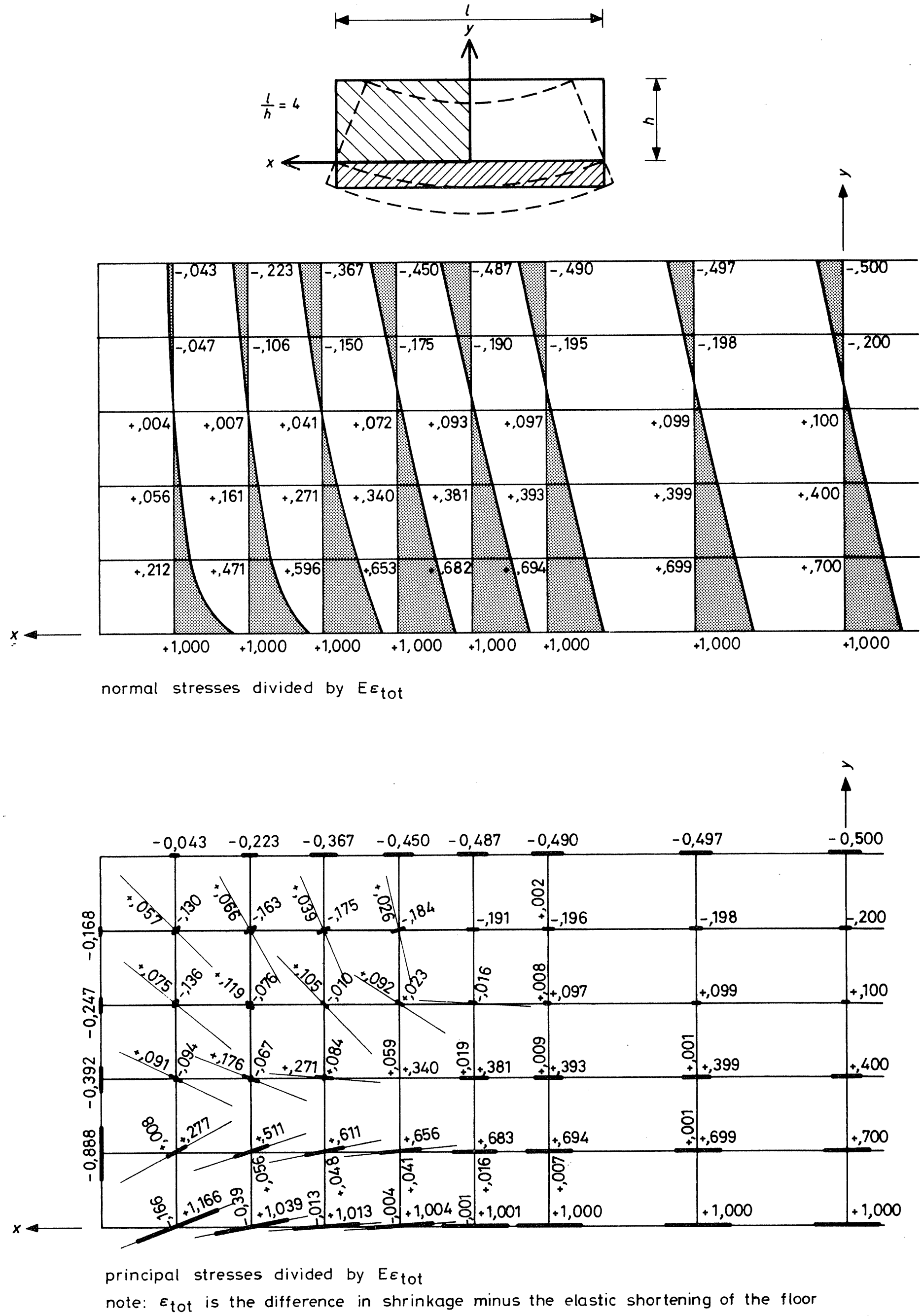


Fig. 7. Stress distribution in a wall connected to a flexurally non-rigid floor according to SCHLEEH [4].

– Applying a prestress, in successive stages, to the part concreted first. Such prestressing, which should be applied after the completion of the part concreted last, suppresses the difference in shortening between the two parts. With this procedure it is possible, in principle, to obtain a (practically) uncracked structure.

Cracking caused by external factors cannot be prevented by the above-mentioned measures, except prestressing.

In the case of cantilevered balconies, footways and cycle tracks it is possible to reduce differences in shortening by means of thermal insulation, provided that the cantilever lengths are not too large. Crack widths can also be limited by the provision of expansion joints (artificial cracks).

6 Stress distribution in a wall in the uncracked stage

Various investigators have studied the stresses and strains occurring in the uncracked stage in a wall connected to a floor. In all cases they based themselves on linear-elastic behaviour, either adopting a purely analytical approach or having recourse to research on linear-elastic models.

SCHLEEH [4] analysed the stress distribution in a wall for various ratios of wall length and height. He also investigated the effect of the development or non-development of curvature of the wall. In Fig. 6, for a wall with a ratio $l/h = 4$, the calculated distribution of the principal tensile stresses and the normal tensile stresses has been plotted for the case where the wall cannot curve (i.e., remains straight). The stress

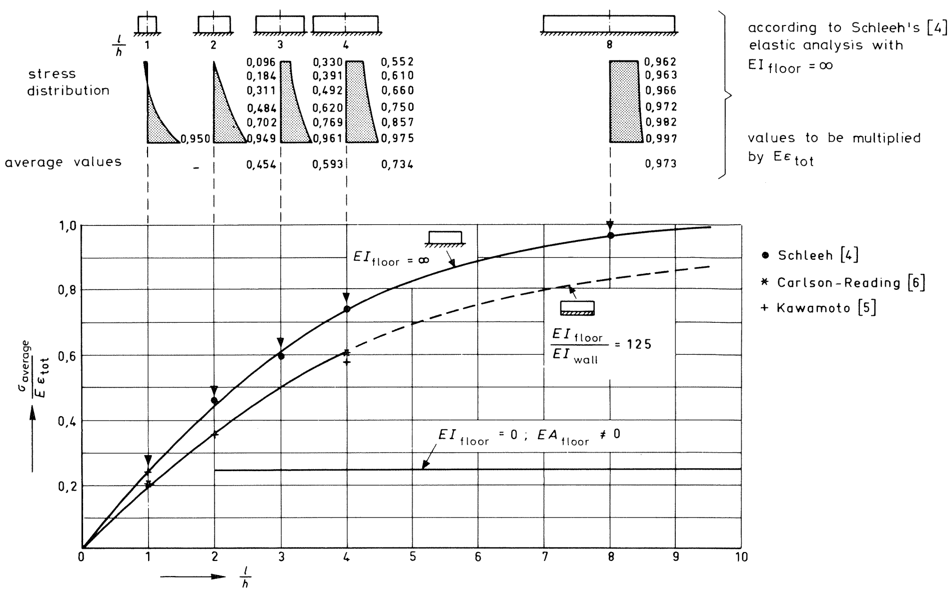


Fig. 8. Stress distribution in the cross-section in the middle of the wall, in the elastic range, for various values of l/h.

distribution in the case where the wall, likewise with $l/h = 4$, can freely undergo curvature is indicated in Fig. 7.

Particularly in the wall which remains straight there occur large "disturbance areas" at the ends, i.e., the principal tensile stress directions deviate considerably from the horizontal in these areas, so that cracks extending in a direction deviating from the vertical can be expected to develop here. Another notable feature of this wall is that, for a ratio $l/h = 4$, there are no constant tensile stresses over the height of the cross-section in the middle of the wall. For $EI_{floor} =$ such stress conditions are approximately attained only for higher values of l/h namely, of about 8–10 (see Fig. 8).

For the case where $EI_{floor} \neq \infty$ Kawamoto [5] determined the stress distribution with the aid of photoelastic analysis. A tensile force was applied to a wall made of synthetic resin and was "frozen in"; the wall was then glued to the floor. After the glue had set, the force was "thawed out". As a result of this the wall strove to shorten, but was prevented by the floor. The results of this research are also included in Fig. 8.

Similar investigations were conducted by Carlson and Reading [6] and their results are likewise given in Fig. 8.

7 Curvature theory

From the linear-elastic investigations described in Chapter 6 it emerges that the curving of the structure has a considerable effect on the resulting stresses and strains (see Figs. 6 and 7). The degree of cracking and the crack widths can be presumed to be closely associated with this. Hence it is, in connection with possible crack-restricting measures, important to gain insight into the magnitude of the curvature that the structure can be expected to develop. On the basis of the interaction of forces between wall and floor, formulas for calculating the curvature have accordingly been derived.

At the junction between floor and wall the shear forces S and bending moments M will occur at the ends in consequence of the restraining action of the floor. If the structure is not subjected to external loading, these forces and moments will be of such magnitude as to satisfy three conditions (see Figs. 9 and 10):

- there is horizontal equilibrium between the normal forces in the structural parts situated below and above the floor-to-wall junction respectively;
- the curvatures of the structural parts below and above the junction are equal;
- at the junction the sum of the shortening of the floor and the lengthening of the wall is equal to the difference in strain $\Delta\varepsilon_v$ between wall and floor in the case of unrestrained deformation.

With reference to the state of strain in the wall, floor and roof (if any), as indicated in Fig. 10, it is possible to establish equations for the normal force, curvature and strain occurring in each of these parts at the junction. Next, on the basis of three above-mentioned conditions, the relations between the quantities in the equations can be indicated and the unknowns solved. In establishing the equations it was presumed that the roof slab (if any) was concreted simultaneously with the walls and would

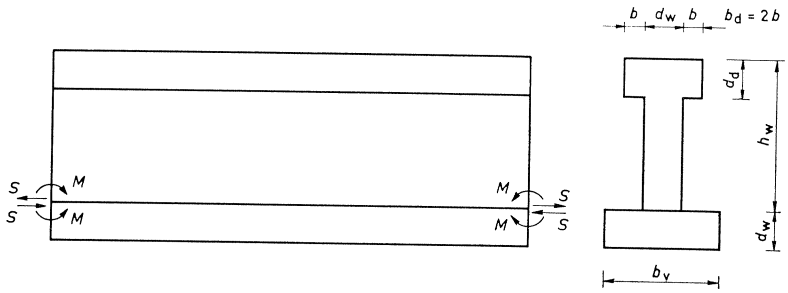


Fig. 9. Schematic representation of the assumed pattern of forces in the floor and in the wall.

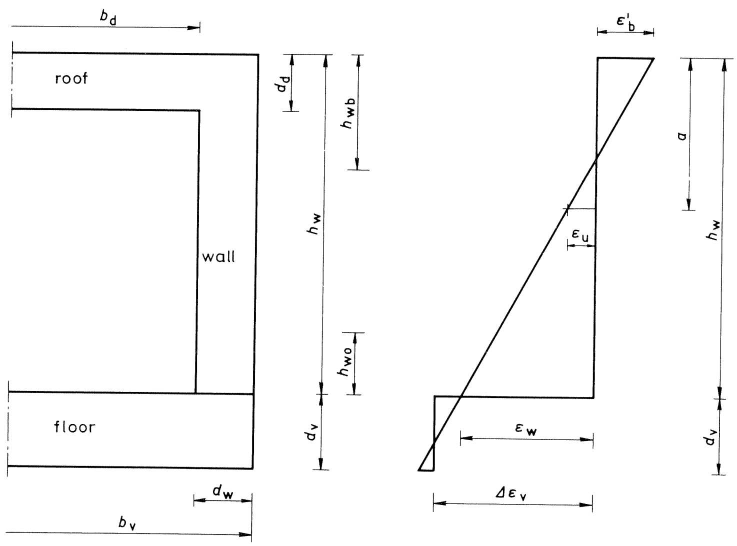


Fig. 10. Assumed state of deformation.

therefore strive to undergo the same amount of shortening as the walls. For the sake of completeness, the effect of the reinforcement in the roof ($\bar{\omega}_d$) and in the floor ($\bar{\omega}_v$) has been incorporated in the equations. In most cases, however this effect is negligible, so that the calculations can be based on $\bar{\omega}_d = 0$ and $\bar{\omega}_v = 0$. The formulas which have been derived are presented in Appendix A, where the intermediate stages of their derivations have been omitted, however.

Since the formulas are unwieldy, a simpler procedure was sought. It was found that in many cases the following can be assumed:

Table 1. Strains occurring in some concrete structures, calculated with the formulas given in Appendix A*

structure	d_v (m)	b_v (m)	d_w (m)	h_w (m)	d_d (m)	b_d (m)	strain difference $\Delta\varepsilon_v$ (‰)	strains in the wall ε'_b (‰)	ε_w (‰)
1	0,30	2,90	0,25	2,85	0,25	2,40	0,2	−0,045	0,182
							0,3	−0,052	0,276
							0,4	−0,060	0,369
							0,5	−0,068	0,462
							0,6	−0,071	0,561
2	0,20	3,00	0,20	2,50	–	–	0,2	−0,045	0,188
							0,3	−0,052	0,283
							0,4	−0,055	0,374
							0,5	−0,065	0,474
							0,6	−0,073	0,569
3	0,20	3,00	0,20	2,50	0,20	2,60	0,2	0,000	0,178
							0,3	0,049	0,283
							0,4	0,053	0,374
							0,5	0,059	0,468
							0,6	0,062	0,563
4	0,40	6,00	0,40	2,50	0,20	5,20	0,2	0,009	0,179
							0,3	0,000	0,272
							0,4	0,044	0,368
							0,5	−0,011	0,457
							0,6	−0,016	0,549
5	0,40	6,00	0,40	2,50	0,40	5,20	0,2	0,050	0,187
							0,3	0,087	0,279
							0,4	0,049	0,370
							0,5	0,049	0,462
							0,6	0,048	0,554
6	0,50	6,00	0,50	4,00	–	–	0,2	−0,044	0,183
							0,3	−0,049	0,277
							0,4	−0,054	0,370
							0,5	−0,060	0,463
							0,6	−0,066	0,556
7	0,50	6,00	0,50	4,00	0,50	5,00	0,2	0,049	0,187
							0,3	0,068	0,277
							0,4	0,075	0,371
							0,5	0,047	0,465
							0,6	0,046	0,558
8	1,29	18,60	3,00	7,34	1,20	15,60	0,2	−0,004	0,168
							0,3	0,062	0,269
							0,4	0,046	0,362
							0,5	0,043	0,452
							0,6	0,040	0,542
9	0,60	29,50	3,50	5,70	0,70	26,00	0,2	0,040	0,152
							0,3	0,046	0,270
							0,4	0,057	0,364
							0,5	0,063	0,459
							0,6	0,043	0,555
10	1,10	20,30	3,00	6,20	0,90	17,30	0,2	−0,002	0,169
							0,3	0,060	0,270
							0,4	0,046	0,363
							0,5	0,044	0,453
							0,6	0,042	0,543

* the following values have been used in the formulas: $n=7$, $\varnothing_k=15$ mm, $f_{bu}=1{,}5$ N/mm², $\bar{\omega}_w=0{,}005$, $\bar{\omega}_d=0{,}0$ and $\bar{\omega}_v=0{,}0$

– at the wall-to-floor junction:

$$\varepsilon_w \cong 0{,}9\Delta\varepsilon_v \tag{7-1}$$

– at the top of the wall:

$$\varepsilon_b' \cong 0{,}1\Delta\varepsilon_v \tag{7-2}$$

These values were adopted after the curvature formulas given in Appendix A had been applied to ten structures with specific dimensions (see Table 1). Values of $\Delta\varepsilon_v$ ranging from 0,02% to 0,06% were introduced into these calculations, the results of which are likewise given in the table.

Since the ten structures envisaged in the table constitute a reasonably good average of structures occurring in actual practice, the range of validity of the formulas (7-1) and (7-2) is a faily wide one.

In practice it can be verified, by comparison with the dimensions listed in Table 1, whether any particular structure is within the range of validity. If not, the curvature formulas given in Appendix A will have to applied.

8 Experimental research

8.1 *Introduction*

The linear-elastic approach and the investigations on linear-elastic models, as described in Chapter 6, provide only a broad outline of information on the crack development process. Since the effect of the reinforcement becomes manifest only in the cracking stage, experimental research is needed.

For this purpose 18 model walls were constructed from micro-concrete, i.e., concrete with scaled-down granulometric composition (see Fig. 11). These models were cast against a rolled steel section which was under compressive prestress (and had thus undergone a shortening deformation) and which represented the floor. After the concrete had hardened, the prestress in the steel section was reduced in stages, so that a certain elongation was imposed upon the concrete wall.

The "substitution" of a rolled steel section for a reinforced concrete floor does not, in principle, bring about any change in the behaviour of the wall; the stress and cracking phenomena are the same as in a wall cast onto a reinforced concrete floor. A steel section, however, has the advantage that no shrinkage and creep effects occur. The tests do not simulate the situation where the temperature and/or the shrinkage varies through the thickness of the wall, but the test results can be expected to be reasonably applicable to such cases as well. In practice, tunnel walls, etc., have been found nearly always to have cracks which go all the way through them. Surface cracks, which could more particularly be expected in the case of non-uniform temperature and shrinkage distribution through the wall thickness, actually very rarely occur.

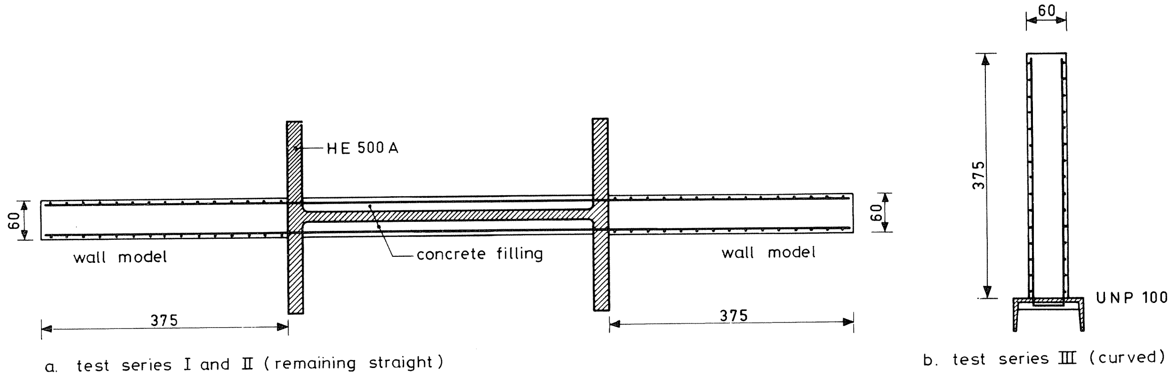


Fig. 11. Cross-sections of the wall models.

8.2 *Test series*

In all, three series of wall models were tested. In series I and II the occurrence of curvature was prevented, whereas in series III it was not. Series I was performed in duplicate in order to verify the suitability of the testing technique and in order to obtain a (rough) indication of the effect of reinforcement on crack width. The chosen dimensions of the walls corresponded approximately to 1/10 of those of a tunnel wall and to 1/5 of those of a basement wall.

The wall models comprised in this experimental research are summarized in

Table 2. Summary of the wall models investigated (cross-sectional dimensions 60 mm × 375 mm)

			longitudinal reinforcement		
			per wall face		
test series	wall model	length (mm)	$\varnothing_k$ (mm)	bar spacing (mm)	$\bar{\omega}_{0w}$ (%)
I*	1	3000	–	–	0
structures	2	3000	2,5	22	0,75
remaining straight	3	3000	3,0	16	1,50
II	4	2500	2,0	43	0,25
structures	5	2500	1,4	21	0,25
remaining straight	6	2500	3,0	47	0,50
	7	2500	1,4	10	0,50
	8	2500	2,0	14	0,75
	9	2500	2,5	22	0,75
	10	2500	2,0	21	0,50
	11	2500	2,0	21	0,50**
III	12	2500	–	–	0
structures	13	2500	1,4	21	0,25
undergoing	14	2500	1,4	10	0,50
curvature	15	2500	2,5	22	0,75

* performed in duplicate
** no reinforcement in $\frac{1}{3}$ of wall depth (height) adjacent to rolled steel section

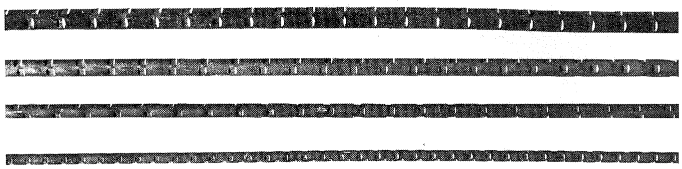


Fig. 12. Model reinforcing bars (actual size).

Table 2. In the series of tests the reinforcement percentage in the wall was varied and also, for each percentage the distribution of the reinforcing bars. The reinforcement consisted of deformed model bars of steel grade FeB 400 HW (f_a = 400 N/mm^2). Some of these bars of various diameters are shown in Fig. 12. The micro-concrete had an average compressive strength of about 30 N/mm^2.

The wall models in series I and II were cast symmetrically against a rolled steel

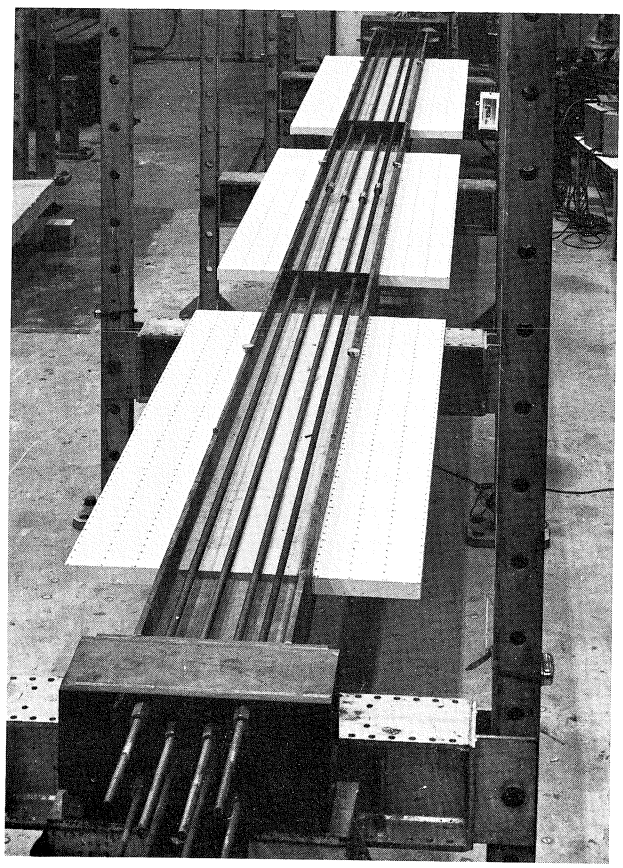


Fig. 13. General view of the wall models in test series I.

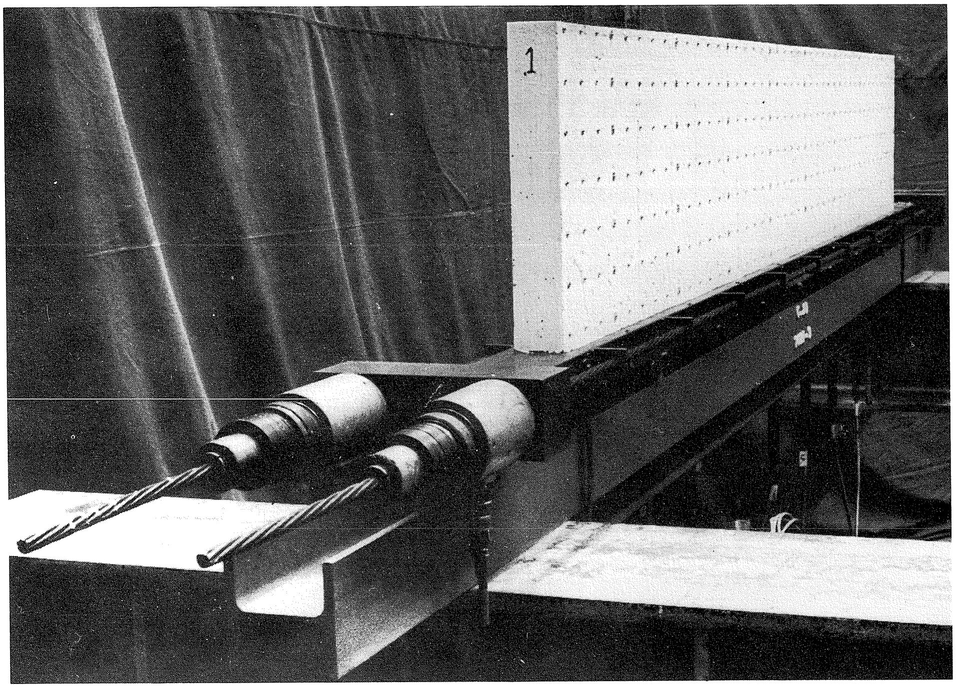


Fig. 14. Wall model 14 in test series III.

section (HE 500 A). Thanks to the symmetrical arrangement, all possibility of curvature was obviated. The wall models in series III were cast against one side of a rolled steel channel section (UNP 100), so that in this case curvature could develop. These two arrangements are shown in cross-section in Fig. 11. As already stated, the rolled steel section had been given a compressive prestress and had thus undergone preliminary shortening before the wall models were cast onto them. The shortening strain was about 0,075% (steel stress about 160 N/mm^2). Before the concrete was cast, the surface of the steel section coming into contact with it was treated with a pneumatic-powered chisel. A comparative investigation had shown that, as a result of this treatment, the bond achieved would be approximately equal to that of concrete to concrete in normal construction practice. Some more wall models before the start of testing are shown in Figs. 13 and 14.

8.3 *Method of testing*

Prior to testing, i.e., before the compressive prestress on the rolled steel sections was released, the wall models were measured in order to ascertain the amount of shrinkage deformation that had already occurred. By measuring not only the wall models themselves, but also dummy models without attached rolled steel sections, it was deduced how much restraint of shrinkage shortening had occurred in the wall models.

The degree of cracking was also determined. Next, the wall models were tested by releasing the prestress in stages. After each stage of prestress reduction, deformation measurements were performed and the positions of the cracks were ascertained. In test series III the deflection and overall curvature were, in addition, measured after each stage.

8.4 *Test results*

8.4.1 Deformations

The shrinkage before testing was ascertained at 0,030%, 0,026% and about 0,020% for the wall models of series I, II and III respectively. In series I and II the restraint of shortening was found to be uniformly distributed over the full height of the wall; in series III this restraint was almost complete near the rolled steel section and diminished over the height of the wall. In this last-mentioned test series the curvature that occurred was of such magnitude that the shortening at the top of the wall exceeded the shrinkage, i.e., a compressive zone developed there. The elongation measured after each stage of prestress release is listed for series I and II in Table 3.

Table 3. Summary of the strains measured in the walls in series I and II, per loading stage

	strain (‰)	
loading stage	series I	series II
0	0,30 shrinkage	0,26 shrinkage
1	0,35	0,31
2	0,39	0,35
3	0,44	0,40
4	0,48	0,44
5	0,55	0,51
6	0,62	0,58
7	0,70	0,66
8	0,79	0,75

Table 4. Summary of the strains measured in the walls, close to the junction with the rolled steel section, in series III, per loading stage

	strain (‰)			
loading stage	wall model 12	wall model 13	wall model 14	wall model 15
0	ca. 0,20 shrinkage	ca. 0,20 shrinkage	ca. 0,20 shrinkage	ca. 0,20 shrinkage
1	0,26	0,25	0,19	0,23
2	0,30	0,32	0,23	0,30
3	0,34	0,40	0,28	0,38
4	0,42	0,46	0,33	0,44
5	0,52	0,53	0,40	0,50
6	0,60	0,61	0,48	0,57
7	0,74	0,72	0,57	0,65
8	0,82	0,79	0,70	0,75
9	0,90	0,87	0,75	0,83
10	0,99	0,94	0,86	0,89

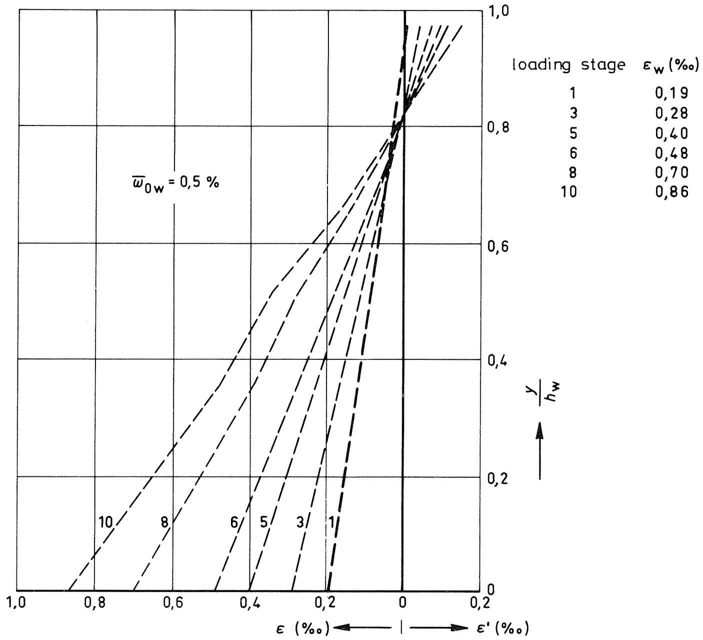


Fig. 15. Measured deformation distribution in the wall as a function of the distance to the steel section, for some loading stages, in wall model 14 in test series III.

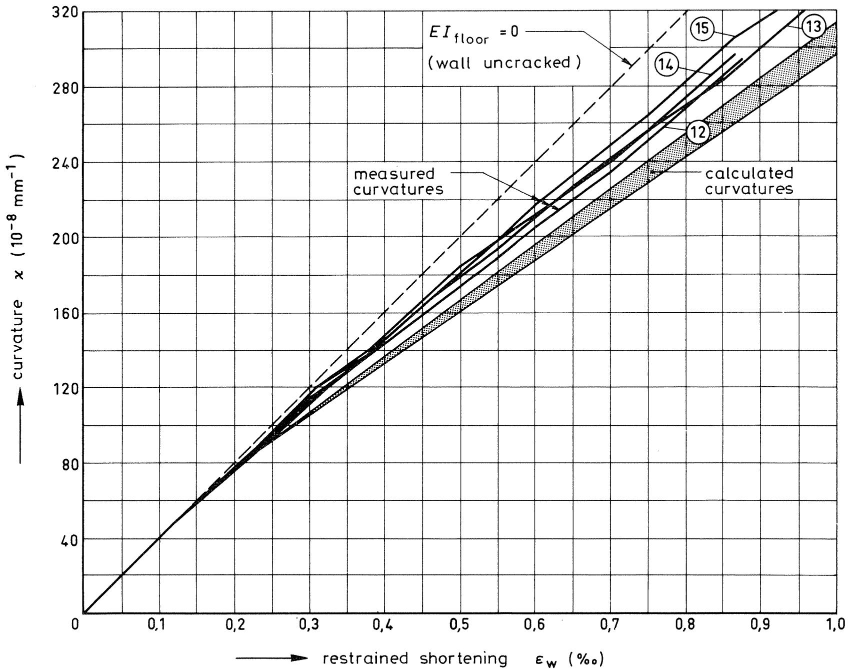


Fig. 16. Curvature as a function of restrained shortening at the junction between the wall and the steel section, in the wall models in test series III.

The final values measured were 0,079% for series I and 0,075% for series II. Such high values, which do not occur in actual practice, were intentionally aimed at in these tests, in order to avoid as much as possible having to extrapolate the results obtained. In series III, when the prestress was released, the elongation of the wall close to the rolled steel section increased (see Table 4), but the curvature became so much greater that a compressive zone with increased compressive strain continued in existence at the top of the wall models. This occurred even in the wall model containing no longitudinal reinforcement. As a representative example the deformation distribution of wall model 14 in series III is presented in Fig. 15. It appears that the strains decrease linearly over the height of the wall. The curvature distribution in the walls of series III has been plotted in Fig. 16, which also includes the curvature for the extreme case of an uncracked wall joined to an infinitely flexible floor. It appears from this diagram that the behaviour of the wall models under investigation is rather close to this extreme case and that the reinforcement hardly has any effect on the curvature.

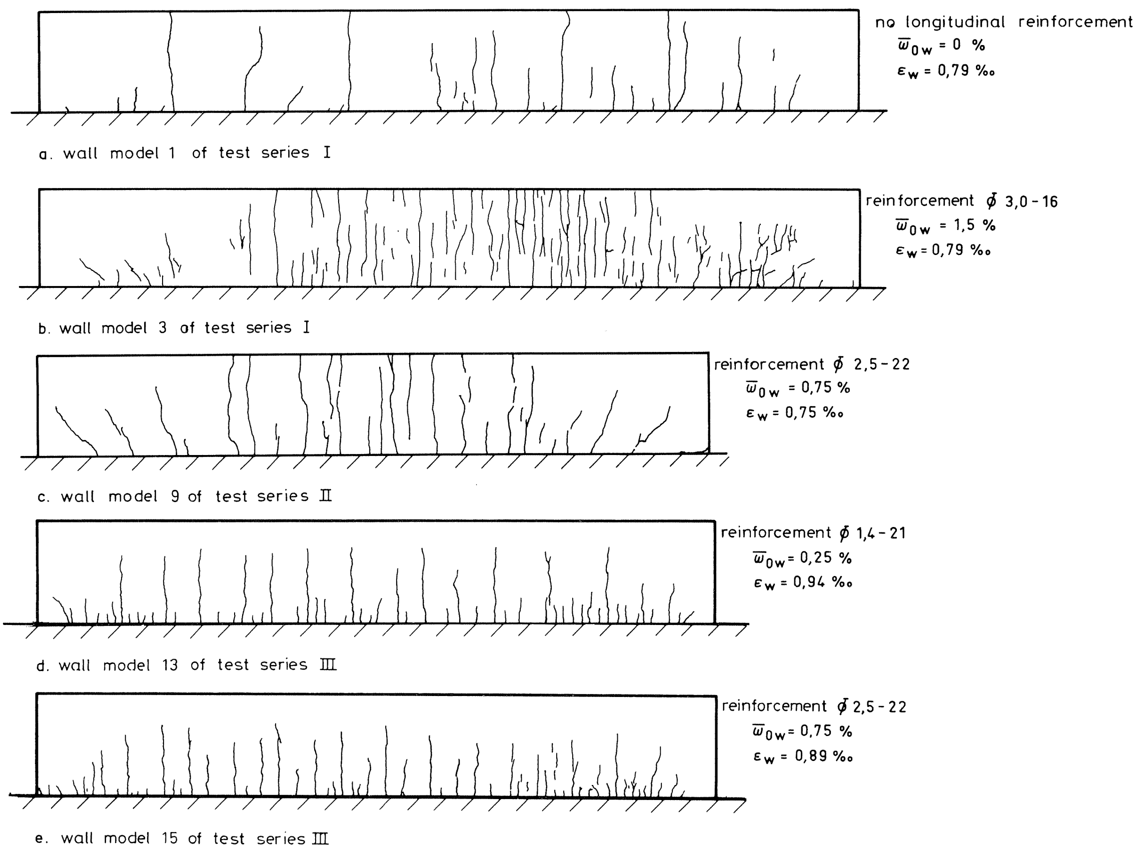
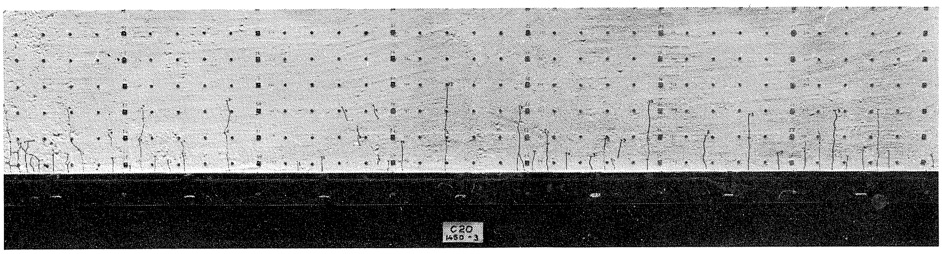


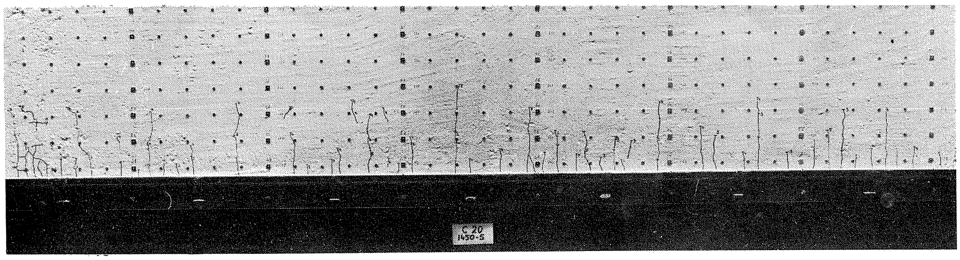
Fig. 17. Crack patterns of some wall models after complete release of the prestress in the steel section.

8.4.2 Cracking

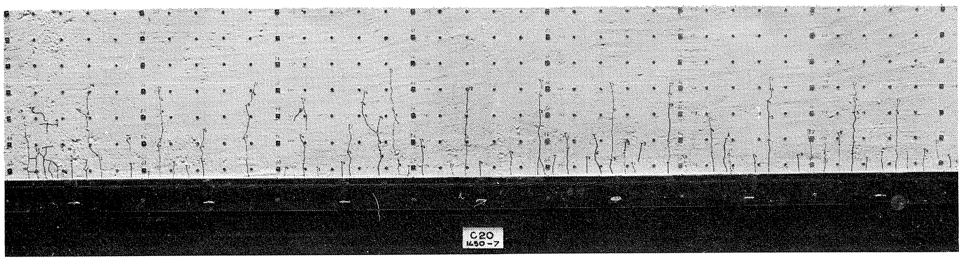
Fig. 17 shows the crack patterns of some of the models investigated, after complete release of the prestress. Both in the non-curved walls of series I and II (Figs. 17a,b,c) and in the curved walls of series III (Figs. 17d,e) the development of cracking proceeded in a regular manner. By way of example, Fig. 18 illustrates the development of



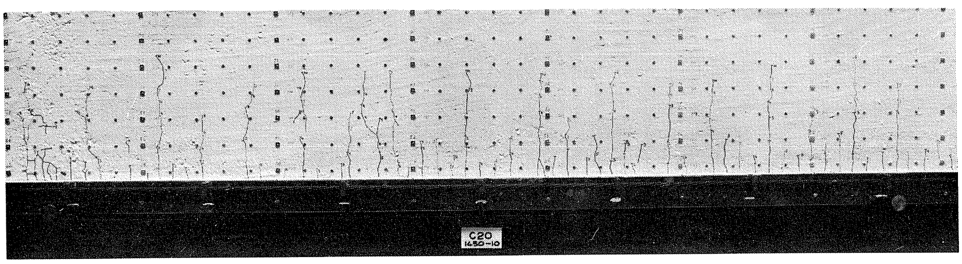
a. $\varepsilon_w = 0{,}28‰$



b. $\varepsilon_w = 0{,}39‰$



c. $\varepsilon_w = 0{,}57‰$

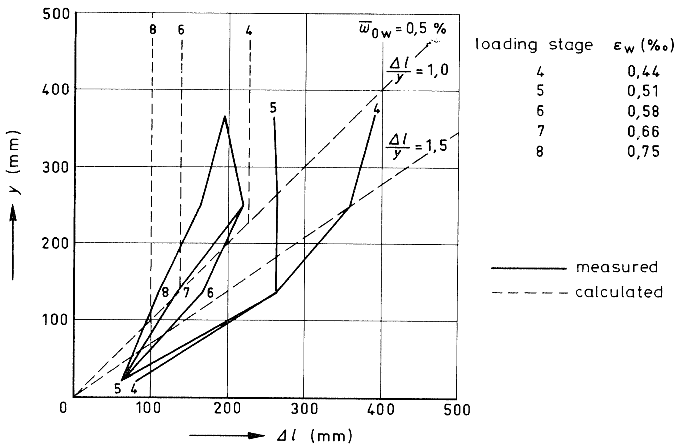


d. $\varepsilon_w = 0{,}84‰$

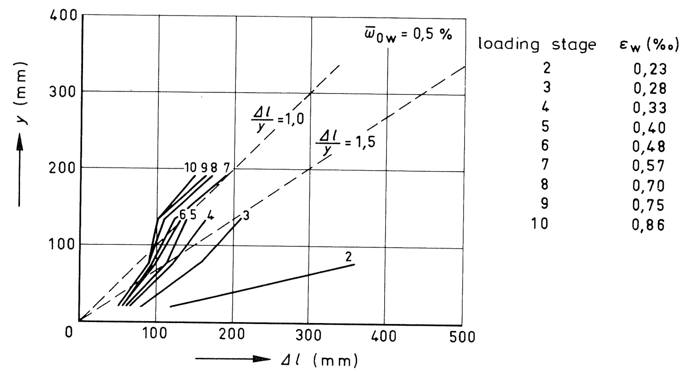
Fig. 18. Development of the crack pattern in wall model 14 in test series III.

the crack pattern in wall model 14 (series III) at four stages of loading. It appears that the cracks steadily increase in number (particularly in the direct vicinity of the rolled steel section) and in length. This latter effect was to be expected on the evidence of the measured deformations, inasmuch as the cracks extend to the level where the strain is equal to the ultimate strain (elongation at fracture) of the concrete. This level is higher up in the wall according as the elongation of the rolled steel section increases (see Fig. 15).

The effect of the reinforcement is clearly manifest in the crack patterns, more particularly in the wall models in series I and II (values of $\bar{\omega}_{0w}$ above approx. 0,5%). For low reinforcement percentages this effect is less pronounced or indeed absent. The effect of the rolled steel section (the floor) on cracking is also clearly manifest: close to the steel section the number of cracks is greater than at some distance from it. At the junction of the wall and the steel section there were a very large number of small



a. wall model 7 of test series II



b. wall model 14 of test series III

Fig. 19. Average crack spacing as a function of the distance to the steel section for some loading stages, in wall models 7 and 14.

cracks. By way of illustration, the average crack width has been plotted as a function of the distance to the steel section for wall models 7 and 14 in Fig. 19.

8.4.3 Crack width

Whether or not the wall could develop curvature was found to be of major influence on the width of the cracks. In the non-curved walls of series I and II the average crack width* increased steadily with increasing distance to the rolled steel section (Fig. 20). The largest value occurred at the top of the wall. On the other hand, in the walls of series III the average crack width initially increased in the upward direction, reached its largest value at a distance of about 200 mm from the steel section, and then decreased (Fig. 21). In Table 5 the largest values for the average crack widths are summarized for all the wall models. This table also gives the calculated widths. They will be further considered in Chapter 10.

In the following comparisons between the measured largest values the corresponding distances y to the rolled steel section have not been taken into consideration, since this is of no interest with regard to these comparisons.

For all the values of $\bar{\omega}_{0w}$ and $\Delta\varepsilon_v$ that were investigated it appears that the crack widths and the effect of the quantity of reinforcement upon these widths are appreciably smaller for curved models than for non-curved ones. For example, non-curved

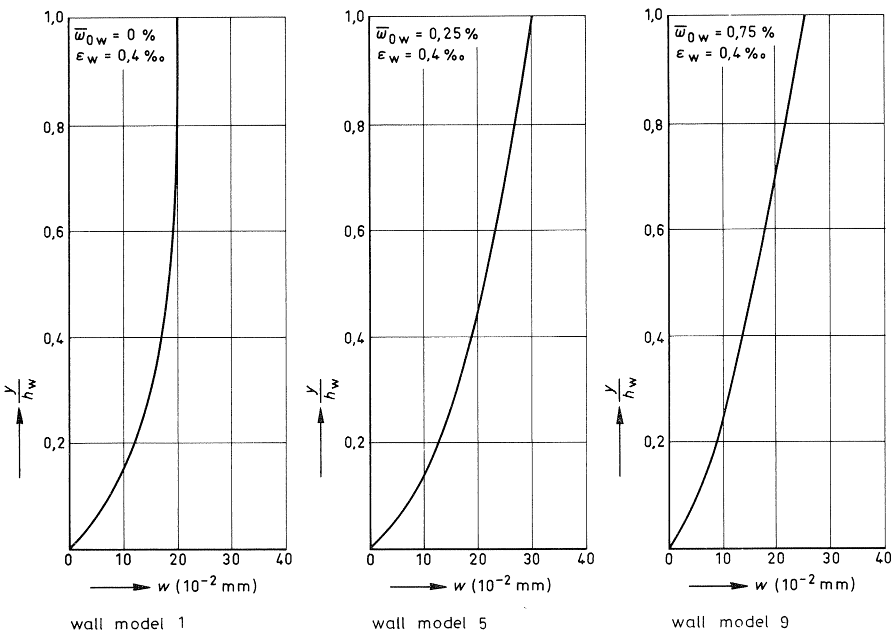


Fig. 20. Average crack width as a function of the distance to the steel section, in some walls in test series I and II.

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* The concept "average crack width" is further explained in Chapter 10.

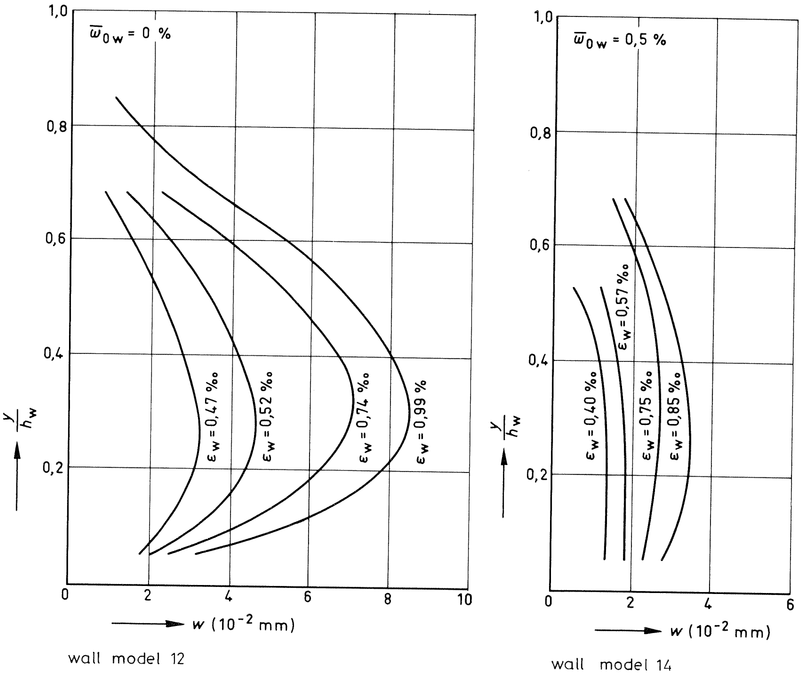

Fig. 21. Average crack width as a function of the distance to the steel section, in wall models 12 and 14 in test series III.

Table 5. Summary of the measured and calculated maximum values of the average crack widths

series	wall model	reinforcement $\varnothing_k$ (mm)	bar spacing (mm)	$\bar{\omega}_{0w}$ (%)	$\Delta\varepsilon_v = 0{,}4‰$ w_{gem} (mm)	w_{ber} (mm)	$\frac{w_{ber}}{w_{gem}}$	$\Delta\varepsilon_v = 0{,}6‰$ w_{gem} (mm)	w_{ber} (mm)	$\frac{w_{ber}}{w_{gem}}$	$\Delta\varepsilon_v = 0{,}8‰$ w_{gem} (mm)	w_{ber} (mm)	$\frac{w_{ber}}{w_{gem}}$
I	1			0	0,174	0,150	0,86	0,342	0,225	0,66	>0,405	0,300	<0,75
	2	2,5	22	0,75	0,042	0,060	1,43	0,050	0,060	1,20	0,050	0,060	1,20
	3	3,0	16	1,50	0,016	0,019	1,19	0,024	0,019	0,79	0,027	0,019	1,20
II	4	2,0	43	0,25	0,110	0,134	1,22	0,184	0,206	1,12	0,195	0,279	1,43
	5	1,4	21	0,25	0,149	1,30	0,87	0,273	0,201	0,74	0,259	0,273	1,05
	6	3,0	47	0,50	0,106	0,127	1,20	0,125	0,184	1,47	0,138	0,184	1,33
	7	1,4	10	0,50	0,077	0,072	0,94	0,077	0,072	0,94	0,105	0,072	0,69
	8	2,0	14	0,75	0,039	0,051	1,31	0,042	0,051	1,21	0,069	0,051	0,74
	9	2,5	22	0,75	0,045	0,067	1,49	0,054	0,067	1,24	0,057	0,067	1,18
	10	2,0	21	0,50	0,090	0,113	1,26	0,114	0,113	0,99	0,132	0,113	0,86
	11*	2,0	21	0,50	0,094	0,113	1,20	0,110	0,113	1,03	0,128	0,113	0,88
III	12			0	0,022	0,022	1,00	0,050	0,045	0,90	0,075	0,069	0,92
	13	1,4	21	0,25	0,019	0,020	1,05	0,037	0,040	1,08	0,053	0,063	1,19
	14	1,4	10	0,50	0,015	0,018	1,20	0,026	0,037	1,42	0,035	0,058	1,66
	15	2,5	22	0,75	0,019	0,018	0,95	0,028	0,036	1,29	0,042	0,057	1,36

* no reinforcement in $\frac{1}{3}$ of wall depth (height) adjacent to rolled steel section

wall models 6, 7 and 10 the average crack width for $\Delta\varepsilon_v = 0{,}04\%$ and $\bar{\omega}_{Ow} = 0{,}5\%$ found to be only half that in the similar model without reinforcement. For a larger difference in strain this ratio is even greater (for $\Delta\varepsilon_v = 0{,}06$ and $\bar{\omega}_{Ow} = 0{,}5\%$ the crack width ratio with respect to the unreinforced model is about 1:3,5). On the other hand, in the curved wall models the effect of the reinforcement for $\Delta\varepsilon_v = 0{,}04\%$ is negligible. In the case of a larger strain difference some effect of reinforcement is indeed discernible, but it remains much smaller than in non-curved wall models.

8.4.4 Maximum crack width

In connection with the corrosion hazard of the reinforcement not only the average crack width, but also the maximum crack width*, is of importance. The tests have shown that the ratio between maximum and average crack width in all the test specimens was about 1,8, with a 5% probability of this value being exceeded.

8.5 *Conclusions*

The results of the tests do not provide a direct answer to the question as to how great an effect the longitudinal reinforcement has upon the width of shrinkage and temperature cracks. In fact, the cracking process is affected by a number of factors which are difficult to separate from one another. An inspection of Table 5 indicates these factors to be:

- whether or not a curvature can develop;
- the magnitude of the deformations suppressed by restraint;
- the degree of curvature;
- the quantity of reinforcement;
- the distribution of the reinforcement.

Particularly the development or absence of curvature is found to have a considerable effect. In practice there is a wide variety of structures with different amounts of curvature, so that it is not reliably possible to make direct use of the information yielded by the tests. First, a theoretical examination of the whole process of curvature and cracking is needed. The method of curvature calculation given Chapter 7 already makes a contibution towards this. In addition, in Chapter 9 a theory of cracking has been developed which ties up with this curvature analysis. For the sake of completeness it is to be noted that the experimental research and the theory of cracking do not provide any indication as to the maximum crack width that can be expected to develop in the long run. This aspect will be further considered in Chapter 11. The effect of the subsoil on the curvature of tunnels, basements, shipping locks, etc, will be examined in Chapter 12.

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* The concept "maximum crack width" is further explained in Chapter 10.

9 Theory of cracking

9.1 *General*

In this chapter a theory of cracking is described, on the basis of which the average crack width which is expected to occur in a wall can be calculated. The state of strain of the wall is assumed to be known, e.g., from a curvature analysis as outlined in Chapter 7. In this theory of cracking the wall is conceived as a member restained at both ends and striving to undergo a certain deformation.

The floor exercises an influence on the cracking that occurs in the wall. It is presupposed that this influence decreases linearly with increasing distance to the floor. The procedure consists in first calculating the crack width without taking the effect of the floor into account. Next, the effect of the floor is added.

9.2 *Cracking in an end-restrained reinforced concrete member*

A reinforced concrete member which is restrained at its ends and which strives to shorten, e.g., in consequence of shrinkage, will be loaded in tension and may crack. The first crack is formed at the weakest section when the tensile strength of the concrete is exceeded. The extensional rigidity of the member is thereby reduced, and the tensile force is therefore also reduced. If the member then strives to shorten still more the tensile force increases again. The next crack occurs when the tensile strength of the concrete is once again exceeded. During this process, conditions relating to the equilibrium of forces and to deformations are applicable. On the basis of these conditions it is possible to derive formulas which describe the cracking process. Consider the element shown in Fig. 22; it has a gross concrete cross-sectional area A_b, a length L and a reinforcement A_a uniformly distributed over the concrete cross-section. In the case of complete restraint of the strain ε the normal force in the as yet uncracked member ($\varepsilon < \varepsilon_u$) is:

$$N = E_b A_b \{1 + (n-1)\varpi\}\varepsilon \qquad (9\text{-}1)$$

where:

n = modular ratio of steel and concrete

With progressive shortening strain ε the maximum strain ε_u in the concrete will be attained and the first crack will occur. The normal tensile force N_s at which cracking takes place is therefore:

$$N_s = A_b \{1 + (n-1)\varpi\} f_{bu} \qquad (9\text{-}2)$$

where:

f_{bu} = uniaxial ultimate tensile strength of the concrete

N_s is the largest force that can occur during the development of the crack pattern. This force will always be attained just before each successive crack is formed. The number of cracks will depend on the magnitude of the restrained shortening strain.

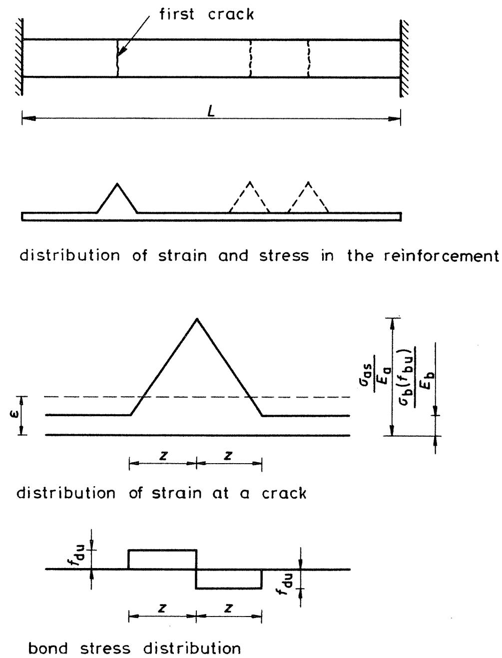


Fig. 22. Schematized distribution of the stress and deformation in a reinforced concrete member restrained at the ends.

For a particular value of this strain, designated as ε_{sv}, the crack pattern will have become fully developed, i.e., further shortening will not give rise to any more new cracks. Just before a crack is formed the steel stress at the cracks already present in the concrete is:

$$\sigma_{as} = \frac{1+(n-1)\overline{\omega}}{\overline{\omega}} f_{bu} \qquad (9\text{-}3)$$

On each side of a crack, part of the tensile force in the steel is transmitted through bond to the concrete, until – at some distance from the crack – a point is reached where the two materials fully co-operate again. The bond strength between the reinforcement and the concrete in the transmission zone can justifiably be assumed constant. The ultimate value of the bond strength is f_{du}. The following formula for the length of the zone with bond stresses on each side of a crack can be derived:

$$z = \frac{\varnothing_k}{4} \cdot \frac{\sigma_{as}}{f_{du}} \cdot \frac{1-\overline{\omega}}{1+(n-1)\overline{\omega}} \qquad (9\text{-}4a)$$

or:

$$z = \frac{\varnothing_{\mathrm{k}} f_{\mathrm{bu}}}{4 f_{\mathrm{du}}} \cdot \frac{1-\bar{\omega}}{\bar{\omega}} \tag{9-4b}$$

The values of σ_{as} and z are therefore independent of ε, at least in the stage while the crack pattern is developing. This incomplete crack pattern occurs in practice nearly always in the case of imposed deformations due to shrinkage and/or temperature variations*. The average crack width is obtained from:

$$w = \frac{\sigma_{\mathrm{as}}}{2E_{\mathrm{a}}} 2z = \frac{\varnothing_{\mathrm{k}} f_{\mathrm{bu}}^2}{4E_{\mathrm{a}} f_{\mathrm{du}}} \{1+(n-1)\bar{\omega}\} \frac{1-\bar{\omega}}{\bar{\omega}^2} \tag{9-5}$$

or, approximately (rounded off):

$$w = \frac{\varnothing_{\mathrm{k}} f_{\mathrm{bu}}^2}{4E_{\mathrm{a}} f_{\mathrm{du}} \bar{\omega}^2} \tag{9-5a}$$

This width w of the cracks already present will occur just before the formation of each subsequent crack. Immediately after each successive new crack is formed, the crack width decreases. This case is therefore not determinative and will not be further considered here.
Besides the crack width, the anticipated number of cracks as a function of ε may be of importance. For a member subjected to tensile force acting at its ends this follows from the condition that the average strain of the reinforcement is equal to ε.

If X is the number of cracks just before the formation of the next crack, i.e., on attainment of the maximum strain ε_{u} of the concrete between two cracks, the following expression is valid (see Fig. 22):

$$X\left(\frac{\sigma_{\mathrm{as}}}{E_{\mathrm{a}}} - \varepsilon_{\mathrm{u}}\right) z + \varepsilon_{\mathrm{u}} L = \varepsilon L$$

Substitution of the formulas (9-3) and (9-4) into this expression gives:

$$\frac{L}{X} = \frac{\varnothing_{\mathrm{k}} f_{\mathrm{bu}}^2}{4E_{\mathrm{a}} f_{\mathrm{du}} \bar{\omega}^2} \cdot \frac{(1-\bar{\omega})^2}{\varepsilon - \varepsilon_{\mathrm{u}}} \tag{9-6}$$

and then with the aid of formula (9-5a):

$$\frac{L}{X} = w \frac{(1-\bar{\omega})^2}{\varepsilon - \varepsilon_{\mathrm{u}}} \tag{9-6a}$$

---

* If the deformations are caused by external loads, a complete crack pattern will generally develop.

where L/X can be conceived as an average crack spacing. From formula (9-6) it appears that, for equal value of ε, the number of cracks is proportional to the length of the member. In Fig. 23 the average crack width and average crack spacing have been plotted as functions of ε. As already stated, the strain ε_{sv} at which the crack pattern is complete is not normally attained in practice.

The crack pattern is complete when the crack spacing has become less than $2z$. When this occurs, there is no longer sufficient bond length available to enable the reinforcement to transmit so much tensile force to the concrete that the maximum strain ε_u will again be attained. The magnitude of ε_{sv} follows, with $L/X = 2z$, from:

$$\varepsilon_{sv} = \frac{1+(2n-1)\varpi}{2n\varpi} \cdot \frac{f_{bu}}{E_b} \approx \frac{f_{bu}}{2n\varpi E_b} = \frac{f_{bu}}{2\varpi E_a} \tag{9-7}$$

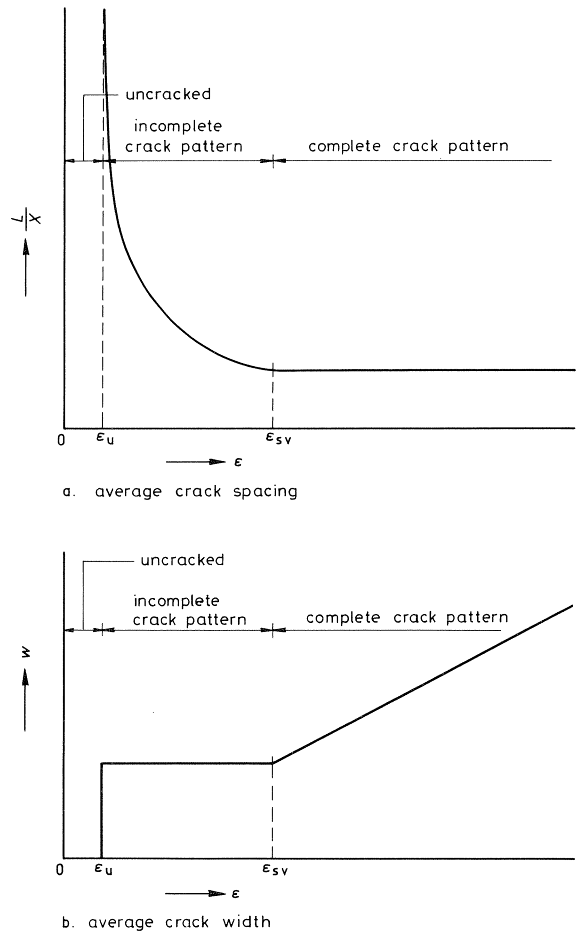


Fig. 23. Schematic representation of the crack pattern development in a member restrained at the ends.

9.3 *Effect of the floor upon cracking*

It is difficult to derive theoretically the effect of the floor upon the crack pattern and crack width occurring in the wall. For this reason, merely the trends that emerged from observations during the execution of the tests will be indicated here, as well as results of the linear elastic considerations described in Chapter 6.

With reference to the test results (see Figs. 17, 18 and 19) it can be stated that the number of cracks in the wall increases with decreasing distance to the floor. Directly above the floor there are very numerous small cracks. A schematized crack pattern is presented in Fig. 24. The spacing of the so-called primary cracks calls for some comment on the basis of research by SCHLEEH [4] and others. Thus it emerges from Fig. 8 for non-curved walls that for $l/h > 10$ the stress in the middle cross-section of the wall remains constant over the entire height. This means that for $l/h = 10$ an imposed deformation equal to the maximum strain ε_u of the concrete will produce a continuous vertical crack (extending the full height) in the middle of the wall. On the assumption that cracking close to the floor has little effect on the stresses near the upper edge, it can be inferred from the values indicated in Fig. 8 that in the middle of a wall with $l/h = 4$ there will occur a continuous crack when the strain becomes equal to:

$$\varepsilon = \frac{0{,}975}{0{,}552}\varepsilon_u = 1{,}77\varepsilon_u$$

In walls with $l/h = 1{,}0$–$1{,}5$ there are no tensile stresses at the upper edge, so that no continuous cracks can develop in such walls. Broadly speaking, it can be assumed that for walls that remain straight the expected spacing of the primary cracks, due to the effect of the floor, will be equal to about 1,0–1,5 times the height of the wall. This assumption is also valid for intermediate cracks, on the understanding that the length must then be conceived as "wall height" (see Fig. 24). The assumptions are, in the main, confirmed by the experimental results. Fig. 19 gives the measured average crack spacing in the wall models 14 and 7 as a function of the distance to the floor. The

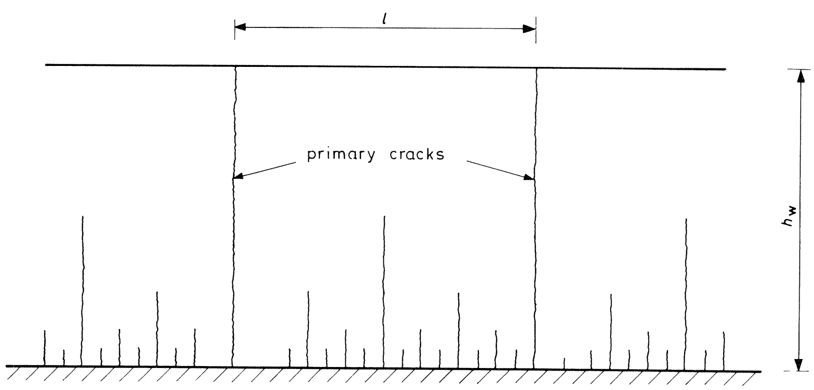


Fig. 24. Schematized crack pattern due to the effect of the floor.

theoretical lines for $\Delta l = 1{,}5\ y$ and $\Delta l = 1{,}0\ y$, likewise shown, are found to be in reasonably good agreement with the measured values. The deflection of the lines to the vertical direction, as observed in the case of wall model 7, indicates that the formation of cracks in that range is determined by the reinforcement and no longer by the floor. It is apparent from Fig. 19 that also for the curved wall the theoretical lines are in good agreement with the measured ones. On the evidence of the foregoing it can be stated that the reducing effect that the floor has upon the crack spacing Δl decreases linearly with the distance y to the floor. For convenience the following will be adopted to express the effect of the floor:

$$\Delta l_y = y \tag{9-8}$$

According to Fig. 19, on applying this equation in the case of small strain differences $\Delta\varepsilon_v$, the values thus obtained for the theoretical crack spacing are too small. However, as will be shown in Chapter 10, the crack widths calculated in this way are in good agreement with the measured values.

To summarize, it can therefore be stated that *in the vicinity* of the floor the average crack spacing Δl is equal to y. This spacing is determined by the effect of the floor and not by any reinforcement that may be present in the wall. Nevertheless a certain amount of stress will exist in the reinforcement, and this stress will be greatest at a crack. In the following, formulas have been derived with which the steel stress at a crack, determined by the floor, can be calculated.

If in a wall the crack spacing Δl_y at a distance y from the floor is equal to y, for an average strain ε_y, then the following equation applies to the reinforcement (see Fig. 22):

$$z\left(\frac{\sigma_{asy}}{E_a} - \frac{\sigma_b}{E_b}\right) + \frac{\sigma_b}{E_b} y = y\varepsilon_y \tag{9-9}$$

where, in accordance with formula (9-3), we may put:

$$\sigma_b = \sigma_{as}\frac{\bar{\omega}}{1+(n-1)\bar{\omega}} \approx \sigma_{as}\bar{\omega}$$

With formula (9-4a) and $\sigma_{as} = \sigma_{asy}$ formula (9-9) becomes:

$$\sigma_{asy} = -2f_{du}\frac{n\bar{\omega}y}{\varnothing_k} + \sqrt{\left(\frac{2f_{du}n\bar{\omega}y}{\varnothing_k}\right)^2 + \frac{4f_{du}E_a y\varepsilon_y}{\varnothing_k}} \tag{9-10}$$

where $\bar{\omega} \neq 0$ and $\varnothing_k \neq 0$.

The crack width w_y is obtained from:

$$w_y = \frac{\varnothing_k\sigma_{asy}^2}{4f_{du}E_a}\cdot\frac{1-\bar{\omega}}{1+(n-1)\bar{\omega}} \tag{9-11}$$

Formula (9-10) is valid for $y \geqslant 2\,z$, or with the aid of formula (9-4a):

$$y \geqq \frac{\varnothing_k \sigma_{asy}}{2 f_{du}} \cdot \frac{1-\bar{\omega}}{1+(n-1)\bar{\omega}} \approx \frac{\varnothing_k \sigma_{asy}}{2 f_{du}} \qquad (9\text{-}12)$$

For practical reasons no formula for calculating the steel stress in a case where $y < 2z$ has been derived. In such cases, should they occur, it is advisable to calculate the crack width for small values of y on the assumption that there is no reinforcement at this distance from the floor. Thus, all strain will be concentrated in the cracks. The crack width w_y is then:

$$w_y = y \varepsilon_y \qquad (9\text{-}13)$$

The crack spacing and the crack width are now entirely determned by the effect of the floor. Since it can, for curved structures of normal type (see Chapter 7), justifiably be assumed that $\varepsilon_w \cong 0{,}9 \Delta\varepsilon_v$ and $\varepsilon'_b \cong 0{,}1 \Delta\varepsilon_v$, formula (9-13) can be further developed with regard to ε_y we can write:

$$\varepsilon_y = \left(0{,}9 - \frac{y}{h_w}\right) \Delta\varepsilon_v \qquad (9\text{-}14)$$

On substitution of this into formula (9-13) we obtain:

$$w_y = y\left(0{,}9 - \frac{y}{h_w}\right) \Delta\varepsilon_v \qquad (9\text{-}15)$$

In this formula the crack width w_y attains a maximum value for $y = 0{,}45\,h_w$. The associated value for ε_y is $0{,}45 \Delta\varepsilon_v$. The maximum value for the average crack width is then:

$$w_{y\,max} = 0{,}20 h_w \Delta\varepsilon_v \qquad (9\text{-}16)$$

This theoretically expected maximum therefore occurs in the wall of a curved structure of normal type without longitudinal reinforcement.
For structures which remain straight the maximum value of the average crack width will occur at the top of the wall, so that the following can be substituted into formula (9-13): $y = h_w$ and $\varepsilon_y = \Delta\varepsilon_v$. Hence we obtain:

$$w = h_w \Delta\varepsilon_v \qquad (9\text{-}17)$$

This theoretically expected maximum therefore occurs in the wall of a non-curved structure without longitudinal reinforcement.

9.4 *Validity of the theory*

In the derivation of the formulas in 9.2 and 9.3 it has implicitly been assumed that the quantity of reinforcement provided is such that no yielding of the steel will occur during cracking of the concrete, i.e.:

$$\sigma_{asy} \leqq f_a$$

In the theory presented in 9.2, relating to a member restrained at its ends, this assumption is necessary in order to ensure the presupposed crack-distributing action of the reinforcement.

In the case of cracking governed by the effect of the floor the requirement $\sigma_{asy} \leqq f_a$ is not so stringently applicable, because in the absence of reinforcement the crack distribution, and therefore the crack width, will be determined only by the effect of the floor. Nevertheless, in both cases the following principle holds:

If reinforcement is applied as a measure to restrict cracking, the steel stress that occurs in it should remain below the yield point.

This is so because if the steel yields, the certainty as to the permanent effect of the reinforcement upon the crack width will have gone. A required minimum reinforcement percentage $\bar{\omega}_{min}$ can be deduced on the basis of this consideration. For this purpose σ_{as} in formula (9-3) is replaced by f_a, giving:

$$\bar{\omega}_{min} = \frac{f_{bu}}{f_a - (n-1) f_{bu}} \approx \frac{f_{bu}}{f_a} \qquad (9\text{-}18)$$

To apply the formulas derived in 9.2 is therefore permissible only of formula (9-18) is satisfied. For the formulas derived in 9.3 the requirement that $\sigma_{asy} \leqq f_{as}$ should likewise be fulfilled. By putting $\sigma_{asy} = f_a$ and $f_{du} = 2{,}5 f_{bu}$ we obtain on combining the formulas (9-10) and (9-11):

$$\bar{\omega}_{min} = \frac{E_b \varepsilon_y}{f_a} - \frac{\varnothing_k f_a}{4 f_{du} n y} \qquad (9\text{-}19)$$

$$y \geqq \frac{\varnothing_k f_a}{2 f_{du}} \qquad (9\text{-}20)$$

If the lower limit is taken as:

$$y = \frac{\varnothing_k f_a}{2 f_{du}}$$

and this value of y is substituted into formula (9-19), we obtain:

$$\bar{\omega}_{min} = \frac{2 E_a \varepsilon_y - f_a}{2 n f_a} \qquad (9\text{-}21)$$

In this formula: $\varepsilon_y = \Delta\varepsilon_v$ for structures which remain straight and $\varepsilon_y = 0{,}45\Delta\varepsilon_v$ for curved structures of normal type. For structures whose curvature is calculated with the aid of the formulas given in Appendix A, the value of ε_w is, inter alia, obtained from that calculation. In formula (9-21) we should put:

$$\varepsilon_y = 0{,}5\varepsilon_w$$

Hence it is necessary to satisfy formula (9-21) before the formulas derived in 9.3 can permissibly be applied.

10 Verification of the theories of curvature and cracking

The theories presented in Chapters 7 and 9 have been verified against the results obtained with the wall models tested in the experimental research. The characteristic features of these models have already been summarized in Table 2. With these data the curvature of the wall models in series III was first calculated as a function of the strain at the wall-to-floor junction. The formulas given in Appendix A were employed for the purpose. For the wall models in series I and II, which remain straight, no curvature calculation is of course required.

The curvature calculation results as well as the measured curvatures have been plotted in Fig. 16. There is fairly good agreement. The calculations moreover indicate, as the measurements do, that there is practically 100% restraint at the junction with the floor. Therefore it is, within certain limits, permissible to assume complete restraint even for walls joined to relatively flexible floors.

After the curvature calculations were performed, the average crack widths* were calculated for all the wall models in series I, II and III for strain differences of 0,04%, 0,06% and 0,08% respectively. The formulas given in Chapter 9 were used for the purpose. For the bond strength f_{du} between concrete and steel the approximate value $f_{du} = 2{,}5\ f_{bu}$ was introduced. The results of the calculations and also the measured crack widths are summarized in Table 5. The ratios between the calculated and the measured widths are also indicated. The average of the ratios wf_{cal}/w_{meas} is 1,10, the standard deviation being $s = 0{,}239$ and the coefficient of variation v.c. = 0,217 (21,7%). In view of the erratic character of the phenomenon of cracking in concrete, there can be said to exist fairly good agreement between the calculated and the measured values. This being so, it appears reasonable to suppose that the crack widths can be calculated with satisfactory accuracy also for intermediate cases.

Definitions

In describing, among other phenomena, the crack widths which we found to occur in the experimental investigations some concepts have been used which will now be defined.

* The concept of average crack width is explained at the end of this chapter.

Average crack width
At an (arbitrary distance y from the floor the widths of all the cracks which are present in the wall at that level are measured. Next, these widths are added together and their average is determined. The value calculated in this way is referred as the average crack width and is therefore linked to a distance y from the floor.

The average crack width is not constant over the height of the wall. At a certain value of the distance y the average crack width attains its *maximum value*. In walls which remain straight this maximum occurs at the top of the wall. In walls which undergo curvature the maximum occurs somewhere intermediately between the floor and the top of the wall.

Maximum crack width
When the widths of all the cracks in the wall which are present at an (arbitrary) distance y from the floor have been measured, one of these widths will have the largest value. This is referred to as the maximum crack width and is therefore likewise linked to a distance y from the floor.

The maximum crack width is not constant over the height of the wall. However, the ratio of maximum to average crack width is approximately constant over the height of the wall.

11 Long-term crack width

In determining the required quantity of reinforcement it is necessary also to take account of the crack width that develops in the long run. Little information on this is available, however. Some indications have been obtained from inspections carried out by the Committee in some tunnels constructed in the Netherlands, namely, the Heinenoord, the Benelux and the Schiphol tunnel. At the time of the inspection these tunnels had already been in use for some years. The quantities of reinforcement installed in them, and the measured maximum crack widths*, are listed in Table 6. The maximum values which actually occurred are found to remain reasonably well within the limits laid down in the Netherlands code of practice for concrete construction, although relatively low reinforcement percentages were employed. In order to verify that the theory of cracking and curvature presented in this report is in agreement with the measured values, appropriate analyses of these tunnels were carried out. Since there was insufficient information on the temperature and shrinkage behaviour in the tunnels values ranging from 0,02% to 0,06% were adopted for the strain difference $\Delta\varepsilon_v$ in these calculations.
Application of the curvature formulas given in Appendix A showed the calculated curvature of the tunnels to be in good agreement with the assumed values

$$\varepsilon_b' = -0{,}1\Delta\varepsilon_v \quad \text{and} \quad \varepsilon_w = 0{,}9\Delta\varepsilon_v$$

* This concept is explained in Chapter 10.

Table 6. Summary of the measured and calculated maximum values of the average crack widths in some tunnels

tunnel	longitudinal reinforcement FeB 400 $\varnothing_K$ (mm)	$\bar{\omega}_{0w}$ (%)	w_{max} measured (mm)	strain difference $\Delta\varepsilon_v$ (‰)	w_{max} calculated (mm)
Heinenoordtunnel	16	0,18	0,40	0,2	0,25
				0,3	0,40
				0,4	0,54
				0,5	0,69
				0,6	0,84
Beneluxtunnel	19	0,28	0,45	0,2	0,21
				0,3	0,33
				0,4	0,45
				0,5	0,56
				0,6	0,68
Schipholtunnel	16	0,30	0,45	0,2	0,19
				0,3	0,30
				0,4	0,42
				0,5	0,54
				0,6	0,66

With regard to the crack width calculations it can be stated that, in connection with the relatively low percentages of reinforcement installed, the calculation procedure presented in 9.2 could not be applied. The crack width distribution has accordingly been determined with the formulas (9-10) and (9-11). The calculated average crack widths* are given in Table 6 (see, inter alia, Fig. 21).

If it is assumed that the ratio $w_{max}/w_{average}$ is approximately equal to 2 also for the above-mentioned tunnels, the calculated average crack widths for $\Delta\varepsilon_v = 0{,}02\%$ are in reasonably good agreement with the measured values. This could indicate that a temperature difference of about 20 °C had occurred between the walls (+ roof) and the floor, which is an acceptable value. It would, however, also mean that the crack width had undergone little or no increase as a function of time. This conclusion ties up well with the results of crack measurements that the Committee carried out over period of some months during the construction of a lock for yachts near the outfall sluices in the Volkerak estuary. In that case, too, it was found that the cracks, once they had formed, underwent no further appreciable increase in width. On the basis of this experience it can justifiably be concluded that the width of shrinkage and temperature cracks in tunnels, shipping locks and basements undergoes little or no subsequent increase.

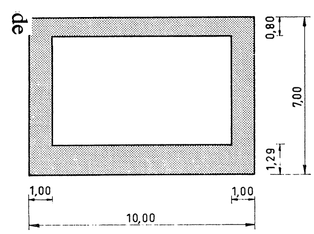
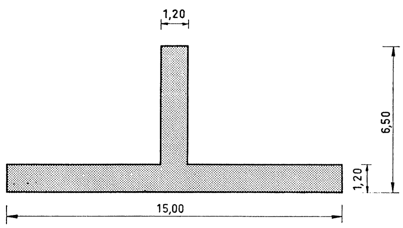
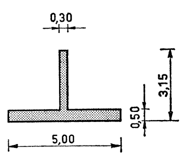
As for the long-term crack width in cantilevered balconies, footways and cycle tracks there are no grounds for arriving at a different conclusion. Although in these cases the ambient temperature may vary more than in tunnels, this will not necessarily result in a permanent increase in crack width.

12 Effect of the subgrade on the curvature

The curvatures measured in the experimental research on models reported in Chapter 8, and the curvature formulas therefrom and mentioned in Chapter 7, relate in principle to structures which can freely undergoo curvature, i.e., they do not take account of the possible effect of the subgrade (foundation soil) under, for example, basements, locks and tunnels.

In order to obtain more insight into the effect of the subgrade upon the curvature, some supplementary calculations were done. Elastically supported beams varying in length and flexural stiffness were analysed. The modulus of subgrade reaction was also varied for each beam in these calculations. The cases analysed are summarized in Table 7. Just as in the experiment research, the beams were assumed to be loaded by an externally applied bending moment M_0. In a beam which can curve freely the curvature and the bending moment remain constant along the entire length; in an elastically supported beam they do not. For this latter case it was investigated to

Table 7. Calculated effect of subgrade upon curvature

section *	flexural stiffness EI** (Nmm²)	length (m)	M_0/M_m modulus of subgrade reaction k_0 (N/mm³) 0,0025	0,005	0,010	0,050
	$6034{,}8\cdot10^{15}$	5	1,0000	1,0000	1,0000	1,0001
		10	1,0001	1,0002	1,0004	1,0022
		15	1,0005	1,0011	1,0022	1,0109
		20	1,0017	1,0035	1,0069	1,0348
		25	1,0042	1,0084	1,0169	1,0859
		30	1,0088	1,0175	1,0352	1.1818
	$1999{,}5\cdot10^{15}$	5	1,0000	1,0001	1,0001	1,0006
		10	1,0005	1,0010	1,0020	1,0098
		15	1,0025	1,0050	1,0099	1,0500
		20	1,0078	1,0157	1.0315	1,1619
		25	1,0192	1,0385	1,0776	1,4167
		30	1,0399	1,0805	1,1640	1,9566
	$60{,}8\cdot10^{15}$	5	1,0003	1,0007	1,0013	1,0067
		10	1,0054	1,0107	1,0215	1,1097
		15	1,0273	1,0549	1,1111	1,6156
		20	1,0873	1,1781	1,3708	3,6948
		25	1,2193	1,4609	2,0234	25,1315
		30	1,4796	2,0696	2,7471	−16,1227

* dimensions in m
** $E = 30000$ N/mm²

what extent the bending moments which occur along the length of the beam differ from M_0, as this provides a direct indication of the curvature of the beam.

The results of the calculations are summarized in Table 7. M_m denotes the moments occurring midway along the beam. From the values of the ratio M_m/M_0 it appears that in most cases M_0 and M_m are of the same order of magnitude. Hence it can be inferred that usually the curvature of the structure is little affected by the forces developed by the subgrade. Only in relatively long structures ($l = 30$ m or more) supported on a base with a modulus of subgrade reaction of around 0,05 N/mm^3 – a high value for our country – do larger differences occur. In such cases the assumption that no curvature of the structure will develop is a safe one. It is to be noted that structures on piled foundation also more or less freely develop curvature. This applies to basements, locks and tunnels supported on piles, at least if the piles are of normal size and normally spaced.

13 Application of the theory of cracking in practice

13.1 *Material properties to be adopted*

13.1.1 Tensile strength and bond strength

In order to use the formulas derived from the theory of cracking in Chapter 9, the values to be adopted for the tensile strength f_{bu} and bond strength f_{du} must first be decided. Two questions must be considered in connection with this:

a. In what stage of the hardening process do the expected deformations occur and what, at that instant, is the average tensile strength of the concrete.
b. What value is to be adopted for the tensile strength f_{bu} to enable, inter alia, $\bar{\omega}_{min}$ to be calculated with formula (9–18).

These questions are considered in the present chapter. Although the circumstances are different from one case to another, it is possible to give approximate guidelines for the tensile strength to be adopted.

Re a. Deformations arising from rises in temperature due to hydration of the cement and causing the concrete to crack will occur in, for example, tunnel and lock walls about 3–7 days after concreting. In this stage of deformation the tensile strength of the concrete will – depending on the composition of the concrete, the grade of cement, etc. – be about 40–80% of the standard 28-day tensile strength.

Deformations due to temperature variations caused by climatic conditions play a part in cantilevered balconies and in cantilevered footways and cycle tracks on bridges, flyovers, etc. In these cases the standard tensile strength may already have been fully attained before the anticipated deformations occur. Early cracking must then have been prevented by the application of appropriate measures during execution of the work.

Deformations due to shrinkage of the concrete develop at a relatively slow rate. In this case it can, for the purpose of cracking analyses, likewise be assumed that the concrete has fully attained its standard tensile strength.

Re b. The criterion with regard to the calculation of $\overline{\omega}_{min}$ in accordance with formula (9-18) is that when the tensile strength in the concrete is reached, the steel should not yet be at its yield point, for only then can be crack-distributing effect of the reinforcement be ensured. Therefore the value of the concrete tensile strength is a major deciding factor with reference to the quantity of reinforcement required.

Now the tensile strength of concrete is something of a stochastic quantity displaying a rather considerable scatter (dispersion). For the determination of $\overline{\omega}_{min}$ it makes a good of deal of difference whether the characteristic lower limit, the average or the characteristic upper limit of the tensile strength is adopted.

Before a reasonable value can be determined for the tensile strength to be adopted, it will therefore be necessary consider in somewhat more depth the process and the theory of cracking. In formula (9-2) it has been presupposed that normal force N_s remains unchanged during the development of the crack pattern. However, because of the scatter in the tensile strength of the concrete, this assumption is not quite correct. The first crack occurs at the section where the tensile strength is lowest. The next crack occurs at a section with higher tensile strength, and so on, until finally the upper limit of the tensile strength has been reached. The normal tensile force N_s will therefore increase during cracking. It will accordingly depend on the expected deformations whether $\overline{\omega}_{min}$ in formula (9-18) will be based on the characteristic lower limit, the average or the characteristic upper limit of the tensile strength.

So although the tensile strength depends in principle on a number of factors and although each case must be considered individually, it is quite appropriate to give general guidelines for the values to be adopted.

- Tunnel, lock and basement walls:

To be adopted for f_{bu}:

$$f_{bu} = f_b$$

To be adopted for f_{du}:

$$f_{du} = 2{,}5 f_{bu} = 2{,}5 f_b$$

In these formulas f_b denotes a lower limit for the concrete tensile strength, associated with the quality (strength class) of the concrete concerned, as laid down in the Netherlands code of practice for concrete construction.

- Cantilevered, balconies, footways and cycle tracks:

To be adopted for f_{bu}:

$$f_{bu} = \frac{f'_{cm}}{20} + 1{,}0$$

where:

f_{bu} = average tensile strength of concrete in N/mm^2
f'_{cm} = average 28-day cube strength in N/mm^2

To be adopted for f_{du}:

$$f_{du} = 2{,}5 f_{bu}$$

13.1.2 Modulus of elasticity
In a number of formulas the modulus of elasticity E_b of the concrete in tension is used. This may be taken as equal to the modulus of elasticity in compression.

13.2 *Permissible crack width*

The requirements as to crack width restriction are very important in connection with calculating the quantity of reinforcement to be provided.

If cracking is liable to endanger the loadbearing capacity of a structure, the limiting values for the crack widths laid down in the code of practice will have to be conformed to. On the other hand, the cracks with which this report is concerned are, generally speaking, not structural cracks. Although certain requirements may be laid down on the basis of the serviceability of the structure, the criteria for crack width restriction are somewhat subjective. This being so, the limiting value imposed for the crack width will vary from one case to another.

There is a further aspect to be considered. Techniques for the repair of cracks are developing fairly rapidly. Synthetic resins are increasingly used for the purpose, often successfull. In view of this, it may be appropriate to consider whether, depending on the type of cracking, it may justifiably be preferable to accept a certain probability of a limited number of wider cracks instead of complying with a predetermined maximum crack width. If such cracks do indeed occur, they can be repaired.

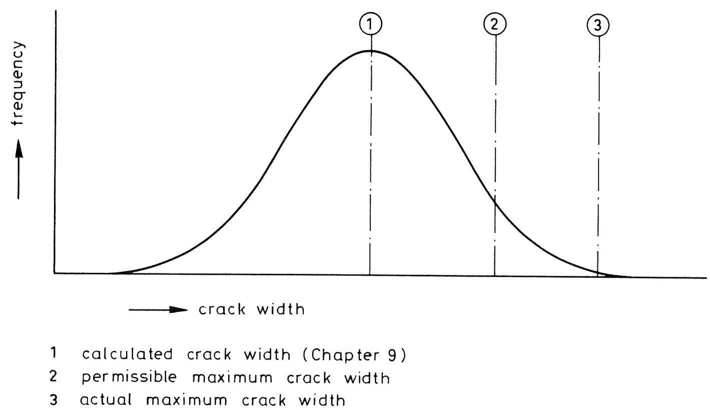


Fig. 25. Relationship between the calculated and the maximum permissible crack width.

This approach is primarily a cost-saving one, as less reinforcement is used, but it may subsequently become necessary to spend more money on repairs. These considerations lead to a – partly economic – assessment of what is acceptable in principle.

In practical terms this philosophy means that the calculated crack width (which is the *expected average crack width*) is allowed to be closer to the permissible (maximum) width according as the permissible value can acceptably be exceeded by a greater amount. These aspects are further elucidated in Fig. 25. In principle, it is possible to establish a scale of crack widths whose frequency distribution can be represented in the form of a Gaussian curve. The theory of crack width as set forth in Chapter 9 relates to the average width (see Fig. 25). If this calculated value is equated to the permissible maximum crack width, it means that 50% of the cracks that occur will be wider than the permissible maximum. If crack widths are in principle not allowed to exceed the permissible maximum, the calculated crack width (= average width) will have to be not more than about half this maximum width.

The procedure consists in determining the permissible average crack width $\bar{w}$ by dividing the specified maximum width by a factor which depends on the acceptable percentage of cracks that may exceed the permissible value. Some percentages with their associated factors are listed in Table 8. The values for $\bar{w}$ obtained in this way should be used in the crack width formulas of Chapter 9.

Table 8. Factors for taking account of permissible maximum crack width

percentage exceeding	factor
5	1,80
10	1,62
15	1,51
20	1,41
30	1,26
40	1,12
50	1,0

13.3 *Calculation of the reinforcement*

After the expected deformation, the material properties, the crack width and the curvature of the structure have successively been determined, the required quantity of reinforcement can be calculated. The further stages of the analysis will now be described in their appropriate sequence:

a. First, it must be investigated whether the calculated crack width of the structure remains below the specified limiting value even without reinforcement.

   For structures which remain straight, with the largest crack widths at the top of the wall, we have in accordance with formula (9-17):

$$w = h_w \Delta\varepsilon_v \leqq \bar{w} \tag{13-1}$$

   For curved structures of normal type we have in accordance with formula (9-16):

$$w = 0{,}20 h_w \Delta\varepsilon_v \leqq \bar{w} \tag{13-2}$$

For structures whose curvature has been calculated with the aid of the formulas given in Appendix A (see Fig. 10):

$$w = y \left\{ \varepsilon_w - (\varepsilon_w - \varepsilon_b') \frac{y}{h_w} \right\} \leqq \bar{w}$$

The crack width according to this formula attains a maximum for:

$$y = \frac{h_w}{2} \frac{\varepsilon_w}{\varepsilon_w - \varepsilon_b'}$$

The maximum value of the crack width is then:

$$w = \frac{h_w}{4} \frac{\varepsilon_w^2}{\varepsilon_w - \varepsilon_b'} \tag{13-3}$$

If it indeed turns out that the calculated crack width remains below the specified limit even if no reinforcement is provided, the analysis will have been completed. In actual practice a certain amount of nominal reinforcement will of course nevertheless be installed.

b. If the permissible average crack width is only slightly exceeded, it can be investigated whether, with the stated minimum reinforcement percentage $\bar{\omega}_{min}$, the calculated crack width remains below the specified permissible value. This minimum percentage of reinforcement should in the first instance conform to formula (9-21), i.e.:

$$\bar{\omega}_{min} \geqq \frac{2E_a\varepsilon_y - f_a}{2nf_a} \tag{13-4}$$

For cantilevered balconies, etc. it is necessary to substitute $\varepsilon_y = \Delta\varepsilon_v$ in this formula, and for curved structures of normal type the value to be substituted is $\varepsilon_y = 0{,}45\Delta\varepsilon_v$. For structures whose curvature is calculated with the aid of the formulas in Appendix A the value of ε_w follows, inter alia, from the calculation. Then $\varepsilon_y = 0{,}5\varepsilon_w$ should be substituted into formula (13-4). If this formula is not satisfied, the quantity of reinforcement should be increased until it is. Next, the crack widths can be calculated with formulas (9-10) and (9-11).
Depending on the nature of the structure, substitutions for y and ε_y can be introduced into formula (9-10). In accordance with 13.1 furthermore:

$$f_{du} = 2{,}5 f_{bu}$$

For cantilevered balconies, etc.:

$$y = h_w \quad \text{en} \quad \varepsilon_y = \Delta\varepsilon_v$$

so that:

$$\sigma_{asy} = -5f_{bu}\frac{n\bar{\omega}_w h_w}{\varnothing_k} + 2\sqrt{\left(\frac{2{,}5f_{bu}n\bar{\omega}_w h_w}{\varnothing_k}\right)^2 + \frac{2{,}5f_{bu}E_a h_w \Delta\varepsilon_y}{\varnothing_k}} \tag{13-5}$$

On applying formula (9-10) for curved structures of normal type, the following can be substituted into this formula:

$$y = 0{,}45h_w \quad \text{en} \quad \varepsilon_y = 0{,}45\Delta\varepsilon_v$$

so that:

$$\sigma_{asy} = -2{,}25f_{bu}\frac{n\bar{\omega}_w h_w}{\varnothing_k} + 2\sqrt{\left(\frac{1{,}125f_{bu}n\bar{\omega}_w h_w}{\varnothing_k}\right)^2 + \frac{0{,}506f_{bu}E_a h_w \Delta\varepsilon_v}{\varnothing_k}} \tag{13-6}$$

For structures whose curvature is calculated with the aid of the formulas in Appendix A, the following should be substituted into formula (9-10):

$$y = \frac{\varepsilon_w}{\varepsilon_w - \varepsilon_b'}\cdot\frac{h_w}{2} \quad \text{en} \quad \varepsilon_y = 0{,}5\varepsilon_w \tag{13-7}$$

When σ_{asy} has been calculated, the calculation of the crack width can be performed on the basis of formula (9-11), into which can be substituted according to 13.1:

$$f_{du} = 2{,}5f_{bu}$$

so that:

$$w_y = \frac{\varnothing_k \sigma_{asy}^2}{10f_{bu}E_a}\cdot\frac{1-\bar{\omega}_w}{1+(n-1)\bar{\omega}_w} \leqq \bar{w} \tag{13-8}$$

c. Non-compliance with formula (13-8) means that the floor does not have a sufficiently good crack-distributing effect to keep the crack width below the specified average value. In that case the distributing effect will have to be achieved by means of reinforcement. The formulas derived in 9.2 will have to be used for this analysis. Formula (9-5a) gives the expected average crack width, inter alia, as a function of $\bar{\omega}$. On substitution of $\bar{\omega} = \bar{\omega}_w$, $w = \bar{w}$ and $f_{du} = 2{,}5\,f_{bu}$ in accordance with 13,1 and rearrangement, we obtain for the required percentage of reinforcement:

$$\bar{\omega}_w = 0{,}5\sqrt{\frac{\varnothing_k f_{bu}}{2{,}5E_a\bar{w}}} \tag{13-9}$$

This formula is valid in the cracking stage, i.e., in which the strain that occurs must not exceed ε_{sv} in accordance with formula (9-7). On substitution of $nE_b = E_a$ and $\bar{\omega} = \bar{\omega}_w$ into this formula, we obtain ,with formula (13-9), the following expression for the strain ε_y:

$$\varepsilon_y \leqq \sqrt{\frac{2{,}5 f_{bu} \bar{w}}{\varnothing_k E_a}} \tag{13-10}$$

For cantilevered balconies, etc. we must again put $\varepsilon_y = \Delta\varepsilon_v$, for curved structures of normal type $\varepsilon_y = 0{,}45\ \Delta\varepsilon_v$, and for the other structures $\varepsilon_y = 0{,}5\ \varepsilon_w$.
If formula (13-10) is satisfied, it remains to check that for the calculated percentage of reinforcement the stress in the steel will remain below the yield point. The required minimum reinforcement percentage for this is indicated in 9.4. Formula (9-18) is applicable, whence we obtain:

$$\bar{\omega}_w \geqq \frac{f_{bu}}{f_a} \tag{13-11}$$

If this condition is satisfied, the calculation will have been completed. If not, then $\bar{\omega}_w$ will have to be increased to the required reinforcement percentage based on formula (13-11). If formula (13-10) is not satisfied, it can first be attempted to satisfy it by, for example, decreasing $\varnothing_k$, which results in a reduction of $\bar{\omega}_w$; in accordance with formula (13-9). If this attempt does not succeed, it means that a completed crack pattern exists (see also Fig. 23). For the sake of simplicity, this case has not been taken into consideration in the theory of cracking presented in 9.2 and 9.3 because the crack pattern can in practice be expected mostly to be in the stage of development. Nevertheless, it will now be explained how to proceed in the case of a complete crack pattern, i.e., for which the condition expressed in formula (3-10) is not satisfied. The crack width must then be calculated from:

$$w_y = \Delta l \varepsilon_y \tag{13-12}$$

where:

$$\Delta l = \left(2c + 0{,}15\ \frac{\varnothing_k}{\bar{\omega}_w}\right) \tag{13-13}$$

In this formula c denotes the concrete cover to the reinforcement. On substitition of formula (13-13) and $w_y = \bar{w}$ into formula (13-1) we obtain, after rearrangement, the following expression for the required percentage of reinforcement:

$$\bar{\omega}_w = \frac{0{,}15 \varnothing_k \varepsilon_y}{\bar{w} - 2c\varepsilon_y} \tag{13-14}$$

In order to ensure that the stress in the reinforcement remains below the yield point of the steel, the following condition must be satisfied in order to apply the last-mentioned formula:

$$\varepsilon_y \leqq \frac{\bar{\omega}_w \varnothing_k f_a - 2{,}5(2c\bar{\omega}_w + 0{,}15\varnothing_k) f_{bu}}{\bar{\omega}_w E_a} \qquad (13\text{-}15)$$

If this condition is not satisfied, $\bar{\omega}_w$ will have to be increased until it is. The usual substitutions for ε_y can again be introduced into the formulas (13-12) and (13-14), namely:

$\varepsilon_y = 0{,}5\varepsilon_w$ for curved structures in general

$\varepsilon_y = 0{,}45\Delta\varepsilon_v$ for curved structures of normal type

$\varepsilon_y = \Delta\varepsilon_v$ for structures which remain straight

13.4 *Distribution of reinforcement over the height of the wall*

After the required quantity of reinforcement has been calculated its distribution over the height of the wall has to be determined. In general, it will not be uniformly distributed. Thus, less reinforcement need be installed just above the floor and, in curved walls, also at the top of the wall. The width of the strip in which such reduction of reinforcement is permitted will depend on how much reinforcement is placed in the strip. Thus the width of the strip will be smaller according as the percentage of reinforcement is lower. According to the Netherlands code of practice for concrete construction (VB 1974) a certain minimum percentage of reinforcement should at least be provided, however.

By combining the formulas (9-10) and (9-11) it will now be determined at what distance y_v from the floor the crack width attains a particular permissible value $\bar{w}$. For the purpose of this derivation a lower reinforcement percentage has been introduced than follows from 13.3.

With regard to formula (9-11) we may put:

$$w_y \leqq \bar{w}$$

On neglecting the term

$$\frac{1-\bar{\omega}}{1+(n-1)\bar{\omega}} \quad (\cong 1{,}0)$$

and substitution of $w_y = \bar{w}$, formula (9-11) becomes:

$$\sigma_{asy} = 2\sqrt{\frac{2{,}5 f_{bu} E_a \bar{w}}{\varnothing_k}} \qquad (13\text{-}16)$$

As already stated, when this formula is applied, it must be ensured that the yield point f_a of the reinforcement is not exceeded:

$$\sigma_{asy} \leqq f_a \tag{13-16a}$$

If this condition is not satisfied, other values will have to be chosen for $\varnothing_k$ and/or $\bar{w}$ in formula (13-16). If formula (13-16a) is satisfied, σ_{asy} can be eliminated by equating formula (13-16) to formula (9-10); ε_y (see Fig. 10) is expressed in ε_w and ε'_b. After rearrangement and reduction, this results in second-degree equation for y. On substitution of $y = y_v$ we then obtain:

$$\frac{\varepsilon_w - \varepsilon'_b}{h_w} y_v^2 + \left\{ 2n\bar{\omega}_r \sqrt{\frac{2{,}5 f_{bu} \bar{w}}{E_a \varnothing_k}} - \varepsilon_w \right\} y_v + \bar{w} = 0 \tag{13-17}$$

Two values for y_v result from this equation. The smaller value y_{vmin} indicates the width h_{wo} of the strip, adjacent to the floor, in which the value adopted for $\bar{\omega}_r$ can be applied. The larger value y_{vmax} indicates the distance, measured from the floor, above which the value adopted for $\bar{\omega}_r$ will likewise suffice. The width of this upper strip h_{wb} is therefore:

$$h_{wb} = h_w - y_{v\,max}$$

From equation (13-17) it appears that increasing $\varnothing_k$ and/or reducing $\bar{w}$ will – in connection with the condition $\sigma_{asy} \leqq f_a$ – result in smaller strip widths h_{wo} and h_{wb}. The values of ε_w and ε'_b follow from the curvature calculation or, in the case of curved structures of normal type, from the assumptions:

$$\varepsilon_w = 0{,}9\Delta\varepsilon_v$$

For structures which remain straight:

$$\varepsilon'_b = -0{,}1\Delta\varepsilon_v$$

$$\varepsilon_w = \varepsilon'_b = \Delta\varepsilon_v$$

Summarizing, the calculation procedure for y_v is therefore as follows:

- curved structures in general: by solving equation (13-17)
- curved structures of normal type:

$$\frac{\Delta\varepsilon_v}{h_w} y_v^2 + \left\{ 2n\bar{\omega}_r \sqrt{\frac{2{,}5 f_{bu} \bar{w}}{E_a \varnothing_k}} - 0{,}9\Delta\varepsilon_v \right\} y_v + \bar{w} = 0 \tag{13-18}$$

- structures which remain straight:

$$y_v = \frac{\bar{w}}{\Delta\varepsilon_v - 2n\bar{\omega}_r\sqrt{\dfrac{2{,}5 f_{bu}\bar{w}}{E_a \varnothing_k}}} \qquad (13\text{-}19)$$

In this case only one value is obtained, namely:

$$y_v = y_{v\,min} = h_{wo}$$

When the widths of the strips have been calculated, it must be checked that y_{vmin} is not less than twice the transmission length z. By combination of the formulas (9-12) and (13-16) we obtain:

$$y_{v\,min} \geqq \sqrt{\frac{\varnothing_k E_a \bar{w}}{2{,}5 f_{bu}}} \qquad (13\text{-}20)$$

If this condition is satisfied, the operation of determining the widths of the strips has been completed. If it is not satisfied the strip width calculation will in principle have to be repeated i.e., starting from formula (13-16) and introducing different values for $\varnothing_k$ and/or $\bar{w}$.

If relatively low values are adopted for $\bar{\omega}_r$, the values of h_{wo} and h_{wb} will be little affected by $\bar{\omega}_r$. In that case it will suffice if y_v is calculated on the assumption that $\bar{\omega}_r = 0$. The calculated or the required quantity of reinforcement can then nevertheless be installed in the strips calculated in this way. Basing oneself on $\bar{\omega}_r = 0$, it is necessary in connection with passing to the limit values ($\bar{\omega}_r = 0$; $\varnothing_k = ?$) to replace the formulas (13-17), (13-18) and (13-19) by respectively:

– for curved structures in general:

$$\frac{\varepsilon_w - \varepsilon_b'}{h_w} y_v^2 - \varepsilon_w y_v + \bar{w} = 0 \qquad (13\text{-}21)$$

– for curved structures of normal type:

$$y_v = \left(0{,}45 \pm \sqrt{0{,}20 - \frac{\bar{w}}{\Delta\varepsilon_v h_w}}\right) h_w \qquad (13\text{-}22)$$

– for structures which remain straight:

$$y_v = \frac{\bar{w}}{\Delta\varepsilon_v} \qquad (13\text{-}23)$$

14 Examples

Three examples are presented here to illustrate how, having due regard to a limit value of 0,25 mm for the crack width, the required quantity of reinforcement can be calculated. For these examples so-called structures of normal type have been adopted (see Chapter 7), for which the expected curvature can with reasonable certainty be determined in a simple manner. In cases where the structures concerned are not of normal type the curvature will, as already stated, have to be calculated with the aid of the formulas given in Appendix A.

In the calculations the permissible crack width is based on considerations of corrosion of the reinforcing steel. Two cases are envisaged:

a. the permissible value of 0,25 mm for the crack width is adopted, with a 50% probability of being exceeded, i.e., a certain number of cracks will subsequently have to be repaired;
b. the permissible value of 0,25 mm for the crack width is adopted, with a 5% probability of being exceeded.

The quantity of reinforcement calculated in case (a) is smaller than in case (b), but the probability of having to execute repairs is greater in case (a) (see also 13.2).

14.1 *Cantilevered balcony slab, cycle track, etc.*

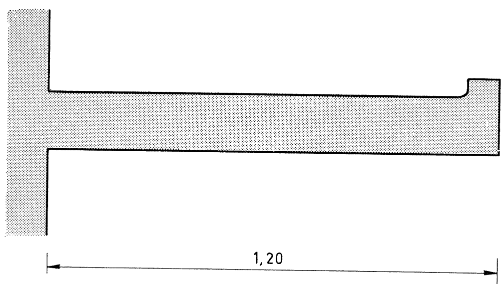


Fig. 26. Cantilevering balcony slab.

Given data:

$f'_{cm} = 35\ N/mm^2$
$E'_b = 28000\ N/mm^2$
$\varepsilon'_r = \Delta\varepsilon_v = 30 \cdot 10^{-5}$ (assumption)

steel grade FeB 400 ($f_a = 400\ N/mm^2$) with:
$E_a = 2{,}1 \cdot 10^5\ N/mm^2$
$\varnothing_k = 8\ mm$
$w = 0{,}25\ mm$

$$f_{bu} = \frac{35{,}0}{20} + 1{,}0 = 2{,}75\ N/mm^2 \text{ in accordance with 13.1.1}$$

$$n = \frac{2{,}1 \cdot 10^5}{28000} = 7{,}5$$

a. Calculation of the curvature:
Since the slab cannot curve in the horizontal plane, no calculation of curvature is required.
b. Calculation of crack width for a 50% probability of exceeding the permissible value:
The average crack width for a 50% probability of exceeding the permissible value is:

$$\bar{w} = \frac{w}{1{,}0} = \frac{0{,}25}{1{,}0} = 0{,}25 \text{ mm}$$

The largest crack width occurs at the outer edge of the cantilevered slab. It will be checked whether there is at this edge still sufficient crack-distributing effect from the floor located farther inwards. If there is no reinforcement in the structure, the expected crack width, as determined from formula (13-1), is:

$$w = h_w \Delta\varepsilon_v = 1200 \cdot 30 \cdot 10^{-5} = 0{,}36 \text{ mm}$$

This is substantially in excess of the chosen average of 0,25 mm. The crack-distributing effect of the floor located farther inwards thus turns out to be insufficient, so that the desired effect will have to be obtained by providing reinforcement. The required quantity $\bar{\omega}_w$ according to formula (13-9) is:

$$\bar{\omega}_w = 0{,}5 \sqrt{\frac{8 \cdot 2{,}75}{2{,}5 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}25}} = 0{,}0065$$

or:

$$\bar{\omega}_{0w} = 0{,}65\%$$

It must be checked that, for the expected strain $\Delta\varepsilon_v = 0{,}3 \times 10^{-3}$, there is not a complete crack pattern (see Fig. 23), for in that case formula (13-9) is not valid. The check is performed with formula (13-10):

$$\varepsilon_y \leqq \sqrt{\frac{2{,}5 \cdot 2{,}75 \cdot 0{,}25}{8 \cdot 2{,}1 \cdot 10^5}} = 1{,}01 \cdot 10^{-3}$$

Since the expected strain ε_y is equal to $\Delta\varepsilon_v = 0{,}3 \times 10^{-3}$, the condition is satisfied. Next, it must be checked that the steel stress in the calculated quantity of reinforcement $\bar{\omega}_w$ indeed remains below the yield point. This is ascertained with the formula:

$$\bar{\omega}_w \geqq \frac{2{,}75}{400} = 0{,}0069$$

Hence it emerges that the quantity of reinforcement $\bar{\omega}_w$ (= 0.0065) falls short of this value, i.e., the yield point is exceeded, which is not acceptable. Hence the quantity actually to be provided is:

$\bar{\omega}_w = 0{,}0069$

$\bar{\omega}_{0w} = 0{,}69\%$

Next, the width h_{wo} of the strip in which a reduced quantity of reinforcement can be used is determined. In order to obtain an indication of the effect of $\bar{\omega}_r$ upon the strip width h_{wo}, two cases $\bar{\omega}_r = 0$ and $\bar{\omega}_r = 0{,}0035$ have been considered, with $\varnothing_k = 8$ mm. Obviously, other values can also be adopted. For the case $\bar{\omega}_r = 0$ we have according to formula (13-23):

$$y_v = h_{wo} = \frac{0{,}25}{30 \cdot 10^{-5}} = 833 \text{ mm}$$

For the case $\bar{\omega}_r = 0{,}0035$ we have according to formula (13-16):

$$\sigma_{asy} = 2\sqrt{\frac{2{,}5 \cdot 2{,}75 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}25}{8}} = 424{,}8 \text{ N/mm}^2 > f_a = 400 \text{ N/mm}^2$$

Hence this does not satisfy the condition, but on introducing $\varnothing_k = 10$ mm instead of 8 mm we obtain $\sigma_{asy} = 380$ N/mm^2, which does satisfy the condition.

The strip width h_{wo} ($= y_v$) is now obtained from formula (13-19):

$$y_v = h_{wo} = \frac{0{,}25}{30 \cdot 10^{-5} - 2 \cdot 7{,}5 \cdot 0{,}0035\sqrt{\dfrac{2{,}5 \cdot 2{,}75 \cdot 0{,}25}{2{,}1 \cdot 10^5 \cdot 10}}} = 990 \text{ mm}$$

It appears therefore that the value of h_{wo} is only little affected by $\bar{\omega}_r$.

It remains to be checked that for $\bar{\omega}_r = 0{,}0035$ the crack spacing y_{vmin} (= strip width h_{wo}) is not less than twice the transmission length z, or according to formula (13-20):

$$y_{v\,min} \geqq \sqrt{\frac{10 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}25}{2{,}5 \cdot 2{,}75}} = 276 \text{ mm}$$

This condition is satisfied, since $y_{vmin} = 990$ mm.

It is apparent from the foregoing that the reinforcement percentage $\bar{\omega}_{0w} = 0{,}69\%$ calculated for the outer edge of the balcony should be provided over a width of approximately 0,30 m.

c. Calculation of crack width for a 5% probability of exceeding the permissible value:

The average crack width for a 5% probability of exceeding the permissible value is:

$$\overline{w} = \frac{w}{1,8} = \frac{0,25}{1,8} = 0,14 \text{ mm}$$

It has already been shown above that, in the absence of reinforcement in the structure, a crack width of 0,36 mm is to be expected. Since this greatly exceeds the chosen average value of 0,14, there is no point in investigating whether the provision of a certain minimum nominal reinforcement will keep the calculated crack width below the chosen average value.

Hence the required quantity of reinforcement must be determined by means of formula (13-9):

$$\overline{\omega}_w = 0,5 \sqrt{\frac{8 \cdot 2,75}{2,5 \cdot 2,1 \cdot 10^5 \cdot 0,14}} = 0,0087$$

$$\overline{\omega}_{0w} = 0,87\%$$

Next, it is checked whether there is indeed a crack pattern in process of development (see Fig. 23), since formula (13-9) is based on this. This check is performed with formula (13-10):

$$\varepsilon_y (= \Delta\varepsilon_v) \leqq \sqrt{\frac{2,5 \cdot 2,75 \cdot 0,14}{8 \cdot 2,1 \cdot 10^5}} = 0,76 \cdot 10^{-3}$$

This condition is satisfied, because $\Delta\varepsilon_v = 0,3 \cdot 10^{-3} < 0,76 \cdot 10^{-3}$. The value of $\overline{\omega}_w$ should be larger than follows from formula (13-11), for otherwise the yield point of the reinforcement will be exceeded; hence:

$$\overline{\omega}_w \geqq \frac{2,75}{400} = 0,0069$$

Since $\overline{\omega}_w = 0,0087 > 0,0069$, this condition is satisfied.

For convenience, the strip width h_{wo} in which a reduced quantity of reinforcement can be used is determined for $\overline{\omega}_r = 0$. As already shown in section b, the introduction of a higher value for $\overline{\omega}_r$ will have very little effect on the value obtained for the strip width h_{wo}. For $\overline{\omega}_r = 0$ this width is calculated as follows from formula (13-23):

$$y_v = h_{wo} = \frac{0,14}{30 \cdot 10^{-5}} = 467 \text{ mm}$$

In this strip it is therefore permissible to use less reinforcement, e.g. the minimum quantity required by the Netherlands code of practice (VB 1974).

14.2 *Basement wall*

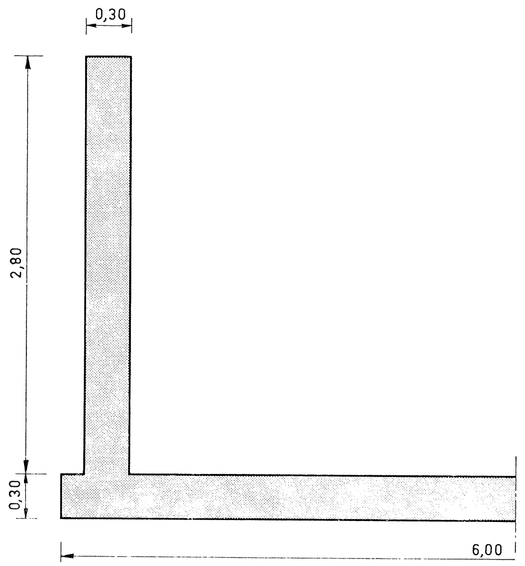


Fig. 27. Basement wall.

Given data:
$d_v = 0{,}30$ m
$b_v = 6{,}00$ m
$d_w = 0{,}30$ m
$h_w = 2{,}80$ m
grade of steel FeB 400 ($f_a = 400$ N/mm^2) with:
$E_a = 2{,}1 \cdot 10^5$ N/mm^2
$\varnothing_k = 12$ mm
$\Delta\varepsilon_v = 0{,}4 \cdot 10^{-3}$ (assumption)
$w = 0{,}25$ mm
$f_{bu} = f_b = 1{,}5$ N/mm^3 in accordance with 13.1.1

$$n = \frac{2{,}1 \cdot 10^5}{30500} = 6{,}9$$

a. Calculation of the curvature:
On comparing the given dimensions with those of the structures envisaged in Table 1, it is apparent that the basement in question is a structure of the normal type. As regards the expected curvature, the strain at the top of the wall is therefore:

$$\varepsilon_b' = -0{,}1 \cdot 0{,}4 \cdot 10^{-3} = -0{,}04 \cdot 10^{-3}$$

and at the junction with the floor:

$$\varepsilon_w = 0{,}9 \cdot 0{,}4 \cdot 10^{-3} = 0{,}36 \cdot 10^{-3}$$

Note: These values calculated with the "exact" formulas given in Appendix A become respectively:

$$\varepsilon_b' = -0{,}054 \cdot 10^{-3}$$

and

$$\varepsilon_w = 0{,}375 \cdot 10^{-3}$$

b. Calculation of crack width for a 50% probability of exceeding the permissible value:

The average crack width for a 50% probability of exceeding the permissible value is:

$$\bar{w} = \frac{w}{1{,}0} = \frac{0{,}25}{1{,}0} = 0{,}25 \text{ mm}$$

First, it is checked whether the crack width will remain below the specified average value even without reinforcement. In that case the crack-distributing effect of the floor will be determinative for the crack width according to formula (13-2):

$$w = 0{,}20 \cdot 2800 \cdot 0{,}4 \cdot 10^{-3} = 0{,}22 \text{ mm}$$

Hence it turns out that even in the absence of reinforcement the calculated crack width remains below the specified average of 0,25 mm. For practical reasons some longitudinal reinforcement will nevertheless be provided.

c. Calculation of crack width for a 5% probability of exceeding the permissible value:
The average crack width for 5% probability of exceeding the permissible value:

$$\bar{w} = \frac{w}{1{,}8} = \frac{0{,}25}{1{,}8} = 0{,}14 \text{ mm}$$

In the foregoing it has been calculated that without reinforcement a crack width of 0,22 mm is to be expected. This greatly exceeds the chosen average of 0,14 mm, so that there is no point in checking whether the minimum reinforcement to be provided in compliance with the Netherlands code of practice for concrete construction (VB 1974) will keep the calculated crack width below the specified average. The required quantity of reinforcement $\bar{\omega}_w$ is accordingly obtained from formula (13-9):

$$\bar{\omega}_w = 0{,}5 \sqrt{\frac{12 \cdot 1{,}5}{2{,}5 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}14}} = 0{,}0078$$

$$\bar{\omega}_{0w} = 0{,}78\%$$

In order to reduce the calculated crack width from 0,22 mm for $\bar{\omega}_{0w} = 0\%$ to 0,14 mm it is therefore necessary to provide 0,78% of reinforcement.

Now it must be checked whether, for the expected strain, there is indeed an incomplete crack pattern, this being the condition to permit formula (13-9) being applied. The check is performed with formula (13-10):

$$\varepsilon_y \leqq \sqrt{\frac{2{,}5\cdot 1{,}5\cdot 0{,}14}{12\cdot 2{,}1\cdot 10^5}} = 0{,}46\cdot 10^{-3}$$

Since $\varepsilon_y = 0{,}45\Delta\varepsilon_v = 0{,}45\cdot 0{,}4\cdot 10^{-3} = 0{,}18\cdot 10^{-3}$, this condition is indeed satisfied.

As a final check it must be verified that the stress in the calculated reinforcement $\bar{\omega}_w$ remains below the yield point. This check is performed with formula (13-11):

$$\bar{\omega}_w \geqq \frac{1{,}5}{400} = 0{,}00375$$

The yield point is not exceeded, since $\bar{\omega}_w = 0{,}0078 > 0{,}00375$.

Next, the width of the strip in which the calculated reinforcement is to be installed must be calculated. According to 13.4 a reduced quantity of reinforcement can be provided in the strip widths h_{wo} and h_{wb}, near the floor and at the top respectively. For $\bar{\omega}_r = 0$ these strip widths are obtained from formula (13-22):

$$y_v = \left(0{,}45 \pm \sqrt{0{,}20 - \frac{0{,}14}{0{,}4\cdot 10^{-3}\cdot 2800}}\right) 2800 = 1260 \pm 767 \text{ mm}$$

Whence:

$$h_{wo} = y_{v\,min} = 1260 - 767 = 493 \text{ mm}$$

$$h_{wb} = h_w - y_{v\,max} = 2800 - (1260 + 767) = 773 \text{ mm}$$

The relatively small value for h_{wo} is due to the chosen value of $\bar{w} = 0{,}14$ mm for the average crack width. For the average value $\bar{w} = 0{,}25$ mm required in section b it turns out that in principle the entire wall can remain unreinforced.

If the calculation of the strip widths is based on, for example $\bar{\omega}_r = 0{,}003$ with $\varnothing_k = 12$ mm instead of $\bar{\omega}_r = 0$, the values for the strip widths are found to undergo only little increase, namely, $h_{wo} = 531$ mm and $h_{wb} = 955$ mm.

To summarize, the arrangement adopted for restricting the crack width in the wall consists in installing the reinforcement in three strips, namely:

- close to the floor: a strip about 0,50 m wide, designated as h_{wo} and provided with nominal reinforcement;
- along the upper edge: a strip about 0,90 m wide, designated as h_{wb} and provided with nominal reinforcement;
- between these two strips: an intermediate zone about 1.40 m wide in which the calculated reinforcement $\bar{\omega}_{0w} = 0{,}78\%$. is provided

14.3 *Tunnel wall*

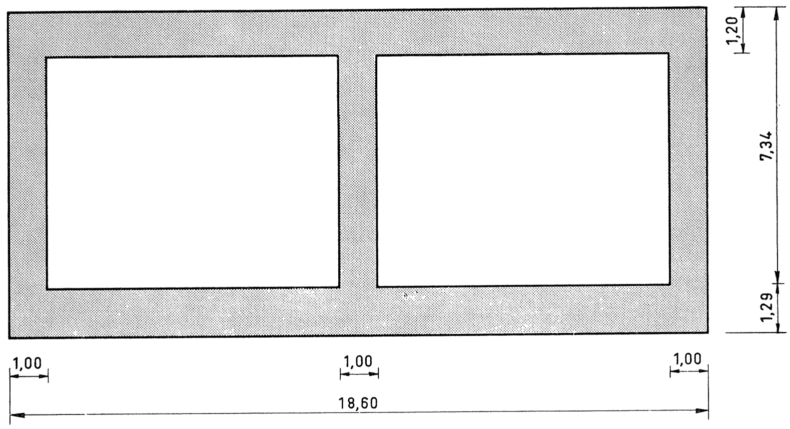


Fig. 28. Tunnel cross-section.

Given data:
$d_v = 1{,}29$ m
$b_v = 18{,}60$ m
$d_w = 3 \times 1{,}00$ m $= 3{,}00$ m
$h_w = 7{,}34$ m
$d_d = 1{,}20$ m
$b_d = 15{,}60$ m
steel grade FeB 400 ($f_a = 400$ N/mm^2) with:
$E_a = 2{,}1 \cdot 10^5$ N/mm^2
$\varnothing_k = 20$ mm
$\Delta\varepsilon_v = 0{,}6 \cdot 10^{-3}$ (assumption)
$w = 0{,}25$ mm
$f_{bu} = f_b = 1{,}5$ N/mm^2 in accordance with 13.1.1

Note
The wall thickness has been taken as the sum of all the wall thicknesses. The floor width is the overall width of the tunnel and the width of the roof is the tunnel widths minus the wall thicknesses.

a. Calculation of the curvature
On comparing the given dimensions with those of the structures in Table 1 it is apparent that the tunnel is a structure of the normal type. As regards the expected curvature, the strain at the top of the wall is:

$$\varepsilon_b' = -0{,}1 \cdot 0{,}6 \cdot 10^{-3} = -0{,}06 \cdot 10^{-3}$$

and at the junction with the floor:

$$\varepsilon_w = 0{,}9 \cdot 0{,}6 \cdot 10^{-3} = 0{,}54 \cdot 10^{-3}$$

Note. These values calculated with the "exact" formulas given in Appendix A become respectively:

$$\varepsilon_b' = -0{,}040 \cdot 10^{-3}$$

and

$$\varepsilon_w = 0{,}542 \cdot 10^{-3}$$

b. Calculation of crack width for a 50% probability of exceeding the permissible value:

The average crack width for a 50% probability of exceeding the permissible value is:

$$\bar{w} = \frac{w}{1{,}0} = \frac{0{,}25}{1{,}0} = 0{,}25 \text{ mm}$$

First, it is checked whether the crack width will remain below the specified average value even without reinforcement. In that case the crack-distributing effect of the floor will be determinative for the crack width according to formula (13-2):

$$w = 0{,}2 \Delta\varepsilon_v h_w = 0{,}2 \cdot 0{,}6 \cdot 10^{-3} \cdot 7340 = 0{,}88 \text{ mm}$$

This value greatly exceeds the chosen average value of 0,25 mm. Hence there is no point in checking whether the provision of nominal reinforcement will keep the calculated crack width below the specified average. The required quantity of reinforcement $\bar{\omega}_w$ is accordingly obtained from formula (13-9):

$$\bar{\omega}_w = 0{,}5 \sqrt{\frac{20 \cdot 1{,}5}{2{,}5 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}25}} = 0{,}0076$$

$$\bar{\omega}_{0w} = 0{,}76\%$$

Next, it must be checked whether, for the expected strain, there is an incomplete crack pattern (see Fig. 23), this being the condition to permit formula (13-9) being applied. The check is performed with formula (13-10):

$$\varepsilon_y \leqq \sqrt{\frac{2{,}5 \cdot 1{,}5 \cdot 0{,}25}{20 \cdot 2{,}1 \cdot 10^5}} = 0{,}47 \cdot 10^{-3}$$

Since $\varepsilon_y = 0{,}45\ \Delta\varepsilon_v = 0{,}45 \cdot 0{,}6 \cdot 10^{-3} = 0{,}27 \cdot 10^{-3}$, this condition is indeed satisfied

It must now be checked that the stress in the calculated reinforcement $\bar{\omega}_w$ remains below the yield point. This check is performed with formula (13-11):

$$\bar{\omega}_w \geqq \frac{1,5}{400} = 0,00375$$

This condition is likewise satisfied.

Now the width of the strip in which the calculated reinforcement is to be installed must be calculated. According to 13.4, a reduced quantity of reinforcement can be provided in the strip widths h_{wo} and h_{wb}, near the floor and at the top of the wall respectively. For $\bar{\omega}_r = 0$ these strip widths are obtained from formula (13-22):

$$y_v = \left(0,45 \pm \sqrt{0,20 - \frac{0,25}{0,6 \cdot 10^{-3} \cdot 7340}}\right) 7340 = 3303 \pm 2778 \text{ mm}$$

Whence:

$$h_{wo} = y_{v\,min} = 3303 - 2778 = 525 \text{ mm}$$

$$h_{wb} = h_w - y_{v\,max} = 7340 - (3303 + 2778) = 1259 \text{ mm}$$

If the calculation of the strip widths is based on, for example, $\bar{\omega}_r = 0,003$ with $\varnothing_k$ 14 mm instead of $\bar{\omega}_r = 0$, somewhat larger values for h_{wo} and h_{wb} are obtained, as appears from the following.

According to formula (13-16):

$$\sigma_{asy} = 2\sqrt{\frac{2,5 \cdot 1,5 \cdot 2,1 \cdot 10^5 \cdot 0,25}{14}} = 237,2 \text{ N/mm}^2 < 400 \text{ N/mm}^2$$

and from equation (13-18) it follows that:

$$\frac{0,6 \cdot 10^{-3}}{7340} y_v^2 + \left(2 \cdot 6,9 \cdot 0,003 \sqrt{\frac{2,5 \cdot 1,5 \cdot 0,25}{2,1 \cdot 10^5 \cdot 14}} - 0,9 \cdot 0,6 \cdot 10^{-3}\right) y_v + 0,25 = 0$$

whence:

$$0,82 y_v^2 - 5166,22 y_v + 25 \cdot 10^5 = 0$$

$$y_v = \frac{5166,22 \pm \sqrt{5166,22^2 - 4 \cdot 0,82 \cdot 25 \cdot 10^5}}{2 \cdot 0,82} = 3150 \pm 2622 \text{ mm}$$

from which are obtained:

$$h_{wo} = y_{v\,min} = 3150 - 2622 = 528 \text{ mm}$$

$$h_{wb} = h_w - y_{v\,max} = 7340 - (3150 + 2622) = 1564 \text{ mm}$$

The final check consists in verifying that the crack spacing y_{vmin} (= strip width h_{wo}) is not less than twice the transmission length z, or according to formula (13-20):

$$y_{v\,min} \geqq \sqrt{\frac{14 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}25}{2{,}5 \cdot 1{,}5}} = 443 \text{ mm}$$

This condition is satisfied, since $y_{vmin} = 528$ mm.

To summarize, the arrangement adopted for restricting the crack width in the wall consists in installing the reinforcement in three strips, namely:

- close to the floor: a strip about 0,50 m wide, designated a h_{wo} and provided with nominal reinforcement;
- along the upper edge: a strip about 1,50 m wide, designated as h_{wb} and provided with nominal reinforcement;
- between these two strips: an intermediate zone about 5,30 m wide in which the calculated reinforcement $\bar{\omega}_{0w} = 0{,}76\%$ is provided.

c. Calculation of crack width for a 5% probability of exceeding the permissible value:

The average crack width for a 5% probability of exceeding the permissible value is:

$$\bar{w} = \frac{w}{1{,}8} = \frac{0{,}25}{1{,}8} = 0{,}14 \text{ mm}$$

It has already been calculated that, in the absence of reinforcement, a crack width of 0,88 mm is to be expected. This is far too large, and the required quantity of longitudinal reinforcement $\bar{\omega}_w$ corresponding to $\bar{w} = 0{,}14$ mm must be calculated with the aid of formula (13-9):

$$\bar{\omega}_w = 0{,}5 \sqrt{\frac{20 \cdot 1{,}5}{2{,}5 \cdot 2{,}1 \cdot 10^5 \cdot 0{,}14}} = 0{,}0101$$

$$\bar{\omega}_{0w} = 1{,}01\%$$

Now it must be checked whether in this case there is an incomplete crack pattern (see Fig. 23), this being the condition to permit formula (13-9) being applied. The check is performed with formula (13-10):

$$\varepsilon_y \leqq \sqrt{\frac{2{,}5 \cdot 1{,}5 \cdot 0{,}14}{20 \cdot 2{,}1 \cdot 10^5}} = 0{,}35 \cdot 10^{-3}$$

Since $\varepsilon_y = 0{,}45 \Delta\varepsilon_v = 0{,}45 \cdot 0{,}6 \cdot 10^{-3} = 0{,}27 \cdot 10^{-3}$, this condition is indeed satisfied.

Next, it must be checked that the stress in the calculated reinforcement $\bar{\omega}_w$ remains below the yield point. This is done with the aid of formula (13-11):

$$\bar{\omega}_w \geqq \frac{1,5}{400} = 0,00375$$

This condition is amply satisfied.

Now the width of the strip in which the calculated reinforcement is to be installed must be calculated. According to 13.4, a reduced quantity of reinforcement can be provided in the strip widths h_{wo} and h_{wb}, near the floor and at the top of the wall respectively. For $\bar{\omega}_r = 0$ these strip widths are obtained from formula (13-22):

$$y_v = 0,45 \pm \sqrt{0,20 - \frac{0,14}{0,6 \cdot 10^{-3} \cdot 7340}} \, 7340 = 3303 \pm 3010 \text{ mm}$$

Whence:

$$h_{wo} = y_{v\,min} = 3303 - 3010 = 293 \text{ mm}$$

$$h_{wb} = h_w - y_{v\,max} = 7340 - (3303 + 3010) = 1027 \text{ mm}$$

h_{wo} is small because of the low value adopted for the average crack width $\bar{w} = 0,14$ mm and fairly large $\Delta\varepsilon_v$. If the calculation of the strip widths is based on, for example, $\bar{\omega}_r = 0,003$ with $\varnothing_k = 14$ mm instead of $\bar{\omega}_r = 0$, the values for the strip widths determined with formulas (13-16) and (13-18) are found to undergo only little increase namely:

$$h_{wo} = 295 \text{ mm}$$

$$h_{wb} = 1248 \text{ mm}$$

Finally, it must be checked that if $\bar{\omega}_r = 0,003$ is adopted, the crack spacing y_{vmin} (= strip width h_{wo}) is not less than twice the transmission length z, or according to formula (13-20):

$$y_{v\,min} \geqq \sqrt{\frac{14 \cdot 2,1 \cdot 10^5 \cdot 0,14}{2,5 \cdot 1,5}} = 331 \text{ mm}$$

Since $y_{vmin} = 295$ mm, this condition is not satisfied. However in view of the smallness of the difference, a value of 295 mm can acceptably be adopted for h_{wo}.

To summarize, the arrangement adopted for restricting the crack width in the wall consists in installing the reinforcement in three strips, namely:

- close to the floor: a strip about 0,30 m wide, designated as h_{wo} and provided with nominal reinforcement;
- along the upper edge: a strip about 1,20 m wide, designated as h_{wb} and provided with nominal reinforcement;
- between these two strips: an intermediate zone about 5,80 m wide in which the calculated reinforcement $\bar{\omega}_{0w} = 1,01\%$ is provided.

APPENDIX A

Formulas for the „exact" calculation of curvature

In Chapter 7 a simple method of determining the curvature of structures subject to restraint of deformation, such as tunnels, shipping locks and basements, has been presented. It yields sufficiently accurate results in the case of structures of normal type, i.e., with "normal" dimensions. For structures with dimensions outside this range, however, the curvature will have to be calculated with the aid of the formulas given in this appendix. Before applying these formulas it is advisable to read Chapter 7.

A1 Formulas for the wall

Normal force:

$$H'_{w} = \frac{A_{bw}E'_{b}}{2}\left\{n\bar{\omega}_{w}(\varepsilon_{w}+\varepsilon'_{b}) + \frac{a}{h_{w}}(\varepsilon_{u}+\varepsilon'_{b})(1-\bar{\omega}_{w})\right\} \qquad \text{(A-1)}$$

Bending moment (with respect to the underside of the wall at the junction with the floor):

$$M_{w} = \frac{A_{bw}E'_{b}}{6}h_{w}\left\{n\bar{\omega}_{w}(\varepsilon_{w}+2\varepsilon'_{b})+3\varepsilon'_{b}\frac{a}{h_{w}}\left(2-\frac{a}{h_{w}}\right)(1-\bar{\omega}_{w})+\right.$$

$$\left.+\frac{a}{h_{w}}\left(3-2\frac{a}{h_{w}}\right)(\varepsilon_{u}-\varepsilon'_{b})(1-\bar{\omega}_{w})\right\} \qquad \text{(A-2)}$$

In both formulas:

$$n = \frac{E_{a}}{E'_{b}} \quad \text{and} \quad \frac{a}{h_{w}} = \frac{\varepsilon_{u}-\varepsilon'_{b}}{\varepsilon_{w}-\varepsilon'_{b}}$$

if $\varepsilon'_{b} < \varepsilon_{v} \leqq \varepsilon_{w}$, then $a = h_{w}$

if $\varepsilon_{w} > \varepsilon_{u}$, then $a = 0$

A2 Formulas for the roof

Normal force:

$$N'_{d} = \frac{b_{d}E'_{b}}{2}\left[a(1-\bar{\omega}_{d})\left\{\frac{a}{h_{w}}(\varepsilon_{w}-\varepsilon'_{b})+2\varepsilon'_{b}\right\} + d_{d}n\bar{\omega}_{d}\left\{\frac{d_{d}}{h_{w}}(\varepsilon_{w}-\varepsilon'_{b})+2\varepsilon'_{b}\right\}\right] \qquad \text{(A-3)}$$

Bending moment (with respect to the underside of the wall at the junction with the floor):

$$M_d = \frac{b_d E_b'}{6}\left[a(1-\overline{\omega}_d)\left\{ a(\varepsilon_w - \varepsilon_b')\left(3-2\frac{a}{h_w}\right) + h_w \varepsilon_b' \left(6-3\frac{a}{h_w}\right)\right\} + \right.$$

$$\left. + d_d n \overline{\omega}_d \left\{ d_d(\varepsilon_w - \varepsilon_b')\left(3-2\frac{d_d}{h_w}\right) + h_w \varepsilon_b' \left(6-3\frac{d_d}{h_w}\right)\right\} \right] \qquad \text{(A-4)}$$

In both formulas: if $a > d_d$, then $a = d_d$

A3 Formulas for the floor

Normal force:

$$N_v' = \frac{A_{bv} E_b'}{2}\left\{ 2(\varepsilon_w + \Delta\varepsilon_v) + \frac{d_v}{h_w}(\varepsilon_w - \varepsilon_b')\right\}\{1+(n-1)\omega_v\} \qquad \text{(A-5)}$$

Bending moment (with respect to the top of the floor at the junction with the wall):

$$M_v = \frac{A_{bv} E_b'}{6} d_v \left\{ 3(\varepsilon_w + \Delta\varepsilon_v) + 2\frac{d_v}{h_w}(\varepsilon_w - \varepsilon_b')\right\}\{1+(n-1)\omega_v\} \qquad \text{(A-6)}$$

In both formulas a negative value must be introduced for $\Delta\varepsilon_v$.

On the basis of the conditions stated in Chapter 7 the following equations are valid for the equilibrium of the forces and bending moments respectively:

$$N_w' + N_d' - N_v' = 0 \qquad \text{(A-7)}$$

$$M_w + M_d - M_v = 0 \qquad \text{(A-8)}$$

The two unknowns ε_w and ε_b' can be solved from the formulas given here.

In the case of "roofless" structures, such as locks, the formulas for N_d' and M_d should be equated to zero.

Cantilevered structures such as, for example, balconies and footways on bridges, cannot curve. In such cases no curvature calculation is therefore required.