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SAFETY, RELIABILITY AND SERVICE LIFE OF STRUCTURES

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Abstract

This report is concerned with the integration of various disciplines of engineering science in their approach to the safety and service life (or lifetime) of structures. A general "basic philosophy" is formulated and attention is focused on the mathematical models that can be applied in connection with it. The computational techniques for the probabilistic approach presented here are discussed, as are also economic aspects and the interpretation of technical uncertainties. A "check-list" is given in order to enable potentially hazardous factors to be detected at an early stage in a safety/service life analysis. Finally, problems associated with the simultaneous occurrence of several failure mechanisms are considered, and methods of dealing with them are described.

SAFETY, RELIABILITY AND SERVICE LIFE OF STRUCTURES

Preface

This report describes the results of a study conducted under the responsibility of the Working Group "Mechanics" of the TNO Division for Building and Metal Research (TNO = Netherlands Organization for Applied Scientific Research).

At the time of compilation of the report this Working Group was constituted as follows*:

Ir. F. K. Ligtenberg (IBBC), Chairman
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Ir. C. F. Etienne (MI)
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This study was financed mainly from a subsidy made available by TNO. Ir. J. W. Tichler was the project leader.

The present report has been compiled from the results of the activities of a Sub-Working Group consisting of the Authors, supplemented at a number of points by contributions resulting from discussions within the Working Group as a whole.

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* IBBC: Institute for Building Materials and Building Structures
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IWECO: Institute for Mechanical Constructions

Safety, reliability and service life of structures

1 Introduction

Some time after the TNO Division for Building and Metal Research had been formed, the Working Group "Mechanics" was set up for co-ordinating the activities comprised in the "Mechanics" discipline within this Division. It became apparent to this Working Group that in the Division there existed widespread interest in such subjects as service life, service life prediction and structural safety. Combination and comprehensive treatment of the various approaches to these matters was therefore very desirable.*

While research relating to service life, service life prediction and "safety" or reliability of structures was being conducted under various projects, it emerged that these concepts were being approached in different ways by the various disciplines and specialized technical fields concerned. On the one hand, emphasis is often laid on the considerable scatter in the results of measurements and on the high degree of unpredictability of all kinds of external influences. This state of affairs prompts investigators to seek stochastic relationships, while less attention is devoted to achieving greater accuracy of the data. Quite often, on the other hand, attention is focused on materials testing and investigation, while statistical analysis of the scatter in the results and of the uncertainty of external influences is not carried out. Harmonization of the various approaches to problems will enable the various disciplines to influence one another to their mutual benefit.

In general, three aspects call for mention in motivation of carrying out research on service life and reliability of structures, namely, the safety aspect, the economic aspect and the social aspect.

1. Safety aspect.
   The safety aspect relates to the damage that occurs in the event of uncontrolled and premature failure of the structure. In the case of many structures such failure, or collapse, may result in a major disaster involving possibly fatal injuries and/or environmental harm and material damage.
2. Economic aspect.
   From considerations of accident prevention there is of course a tendency to aim at providing as much structural safety as possible. Safety and reliability cost money, however, and therefore limits have to be imposed from the point of view of economy. Besides, this problem exists even if structural failure does not result in a disaster, but merely causes interruption of a production process, for example. In both cases an economic advantage can be gained if the behaviour of the structure, and more particularly the stochastic behaviour of its life or its residual life, can be predicted.
3. Social aspect.
   The safety aspect and the economic aspect are, of course, also social aspects. In addi-

* In this report the term "life" or "service life" is used in the normal sense of the length of time for which a structure or device functions or performs in a satisfactory manner. Synonymous terms are "working life", "lifetime", etc.

tion to this, however, the following considerations call for mention:

- when a structure reaches the end of its service life, the environment is burdened with the structural remains;
- the existing raw material reserves are put under strain in building a new structure;
- building a new structure offers employment opportunities; on the other hand, conservation of existing structures can also provide employment.

With regard to these considerations, too, it is possible to achieve an optimum only if a proper stochastic description of the service life is available.

Obviously, optimalizations as envisaged above are far from simple. Even if sufficient technical and statistical data for that purpose are available, the set of norms on which the optimalization is to be based will depend very much on who is more particularly interested in such an optimalization. Furthermore, the result may depend on the assumptions, omissions, schematizations and approximations that inevitably have to be made, i.e., it will depend on who is responsible for applying the procedure.

To promote a wider measure of agreement in the views of various persons responsible for such procedures was, accordingly, an important aspect in the objectives of this report, which can be defined as follows:

Objectives
To establish a general "philosophy" by harmonization of the various relevant approaches specific to the respective disciplines, and to derive therefrom a procedure (or procedures) for estimating the (residual) service life of structures and/or the reliability of structures during a certain future period.

First among these objectives is to establish a *general philosophy;* this is developed in Chapter 2. In view of the many hazards to which structures are exposed, there was found to be a need for a review of general aspects within the context of service life and safety considerations, called a *"check-list"* for convenience, it is presented in Chapter 3. Next, Chapter 4 deals with the problems associated with the fact that structures are in general subject to several *causes of failure* acting simultaneously. Chapter 5, finally, gives a *summary, conclusions and suggestions for further investigations.*

2 Philosophy relating to service life and to reliability

2.1 *Introduction*

Designing a mechanical or civil engineering structure in the expectation that it will continue to function safely, economically and efficiently over a certain anticipated period of time - often a considerable number of years - is based mainly on assumptions as to loads, material behaviour, stress concentrations, etc., which in turn are based on knowledge and experience previously gained with more or less similar structures.

The design methods derived from such expectation, knowledge and experience functioned very well for a long time. The magnitude of the uncertainties in the load and strength quantities was often too imperfectly reflected in the design procedure, however. The existence of these uncertainties was indeed recognized, and they were introduced into the procedure by the application of a set of safety factors and load factors in order to obtain what was considered to be a justified margin between "strength" and "load". The great drawback of this method is the unavailability of accurate information concerning the actual service life or the reliability of structures, while there is moreover only little insight into the constitution of the overall safety of life, i.e., insight into those factors that are of dominant importance with regard to these quantities.

Factors that tell strongly in favour of developing a more sophisticated approach to the questions of safety and service life are:

- The increasing cost of development and construction of modern structures; furthermore this very high cost which makes it essential to achieve an optimal economic service life.
- The increasing complexity of structures, so that the latitude to compensate for errors or uncertainties of design becomes less.
- The increasing number and range of new demands that are being made upon structures. This often necessitates, for example, the use of new materials concerning with there is as yet relatively little knowledge available.
- The increasingly far-reaching interests associated with the failure of a structure or with its inability to go on performing its function - more particularly from the economic and safety points of view (e.g., economic consequences of interruptions of production, accident prevention legislation, etc.).

For civil engineering and for mechanical engineering structures these matters are at present rather in a state of development. In recent years there has been a growing realization in an ever wider circle that a purely deterministic approach to the safety or the service life problem does not offer sufficient backing for soundly based design or management, of structures. This implies, much more than used to be the case, that it is necessary to aim at quantification of the uncertainties playing a part in the process. In consequence of this, probability theory will have an increasingly important part to play in service life or safety analysis. However, it still involves quite a number of difficulties, so that this approach is only slowly gaining ground. The fact that statistical information for the formulation of probability density functions for load and strength quantities is often lacking is also a deciding factor in this context. Most structures are still designed in accordance with well-tried deterministic concepts. It is to be noted, however, that sophisticated probabilistic procedures are increasingly being used for establishing a better basis for existing design rules or codes or for carrying out parameter studies for analysing the influence of quantities playing a part in the process.

Actually, there is no fundamental difference between a service life prediction (life expectancy estimate) and a reliability analysis for a structure. This is the reason why these two aspects are treated together in this report. These matters will be further examined and explained in the subsequent sections of this chapter. It is, of course, true that in the design philosophies hitherto commonly applied to mechanical and/or civil engineering structures the emphasis in some cases was indeed definitely on the reliability and the safety aspect, whereas in other cases the service life was more in the focus of attention. The similarity between the two concepts is that both the service life and the reliability of structure are closely bound up with a (sudden) transition of a structure from functioning to not (properly) functioning, this being due to a great variety of factors such as fracture, wear, inadequate efficiency, unsightliness, buckling, etc. (Figs. 1 to 4).

For many mechanical and electrical engineering structures the service life will play an important part already in the design stage, more particularly, the need to predict this life as accurately as possible. In this context the many structures destined for production processes, power supply, household appliances, etc. come to mind. With raw materials becoming steadily scarcer and the cost of energy increasing, prediction of service life as accurately possible - both its upper and its lower limit - will be central to the present treatment of the subject and constitute an important factor in introducing probabilistic considerations into this prediction, as illustrated in [1], for example.

In civil engineering, however, the emphasis until recently was on reliability, while the service life aspect received less attention, one reason for this attitude being that a civil engineering structure is normally intended to have a long life - 60 to 100 years, say. But this does not mean that the service life aspect plays no part at all. The civil engineering designer does indeed have a conception of his structure's life expectancy (or at least the order of magnitude thereof) at the back of his mind and adapts his design to that conception. In general, however, this is hardly something he does by calculation.

Furthermore, it quite often happens that the part requiring most calculation effort in the design of a building structure - namely, the loadbearing sytem - is least decisive

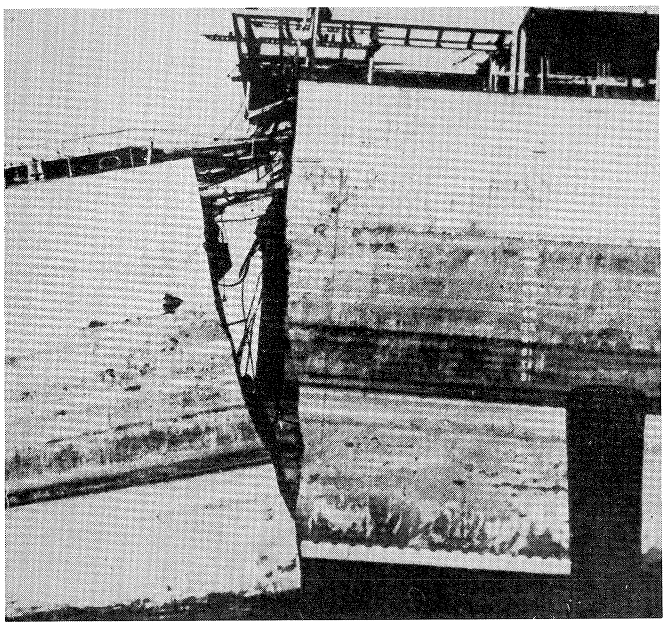

Fig. 1. Ship, broken in the harbour (Boston).

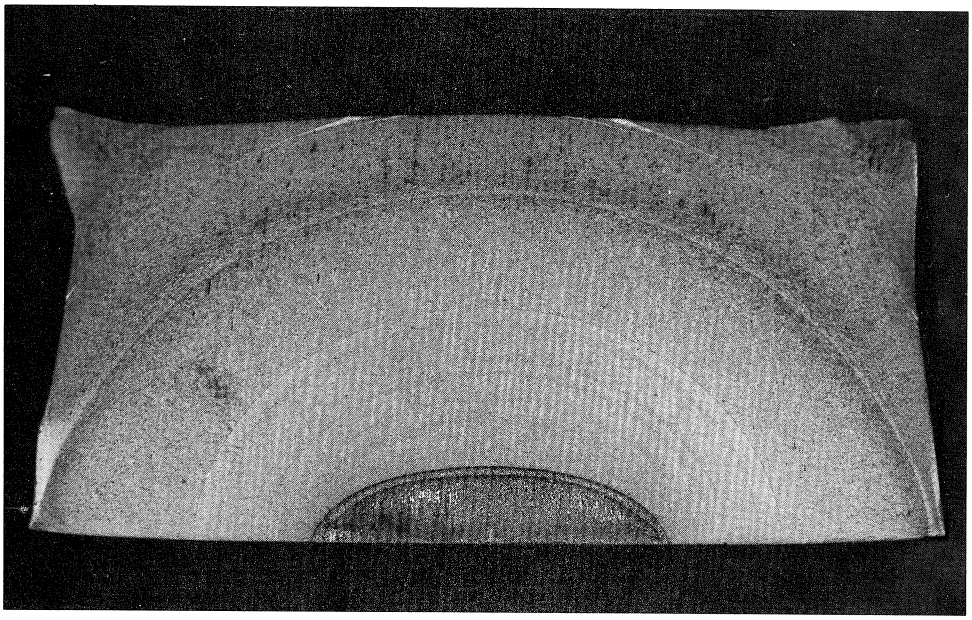

Fig. 2. Surface defect, elliptically extended under fatigue conditions (test bar broken open).

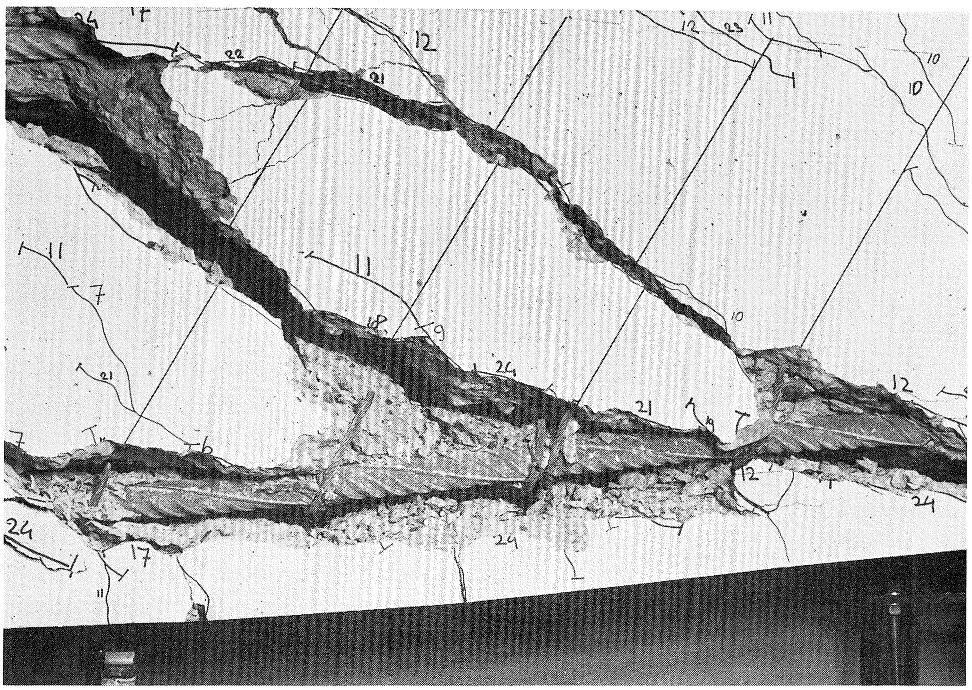

Fig. 3. Failure of reinforced concrete beam due to bending moment and shear force.

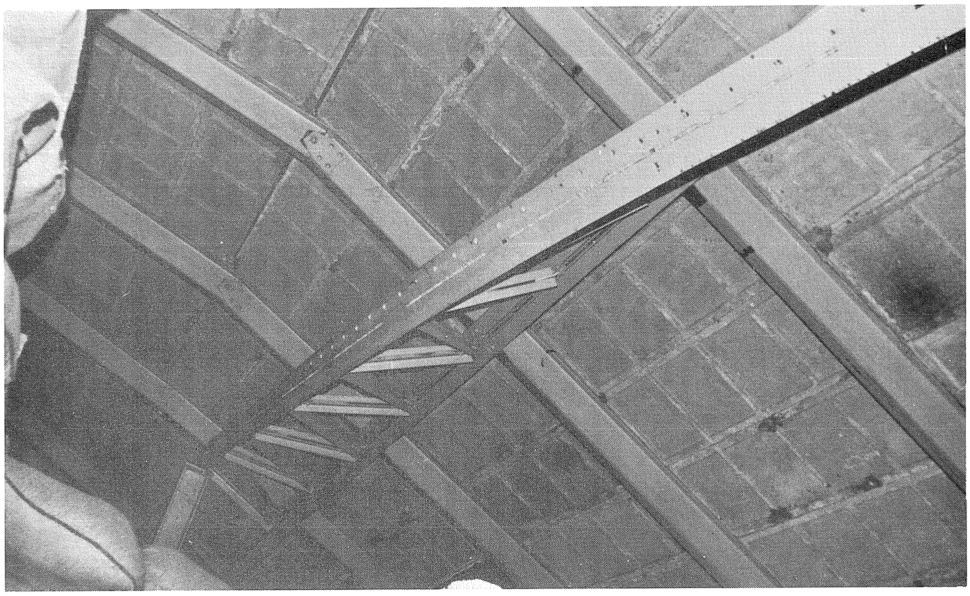

Fig. 4. Failure of lattice girder in roof framework due to buckling.

with regard to service life. Much more important in this context are factors such as the condition of the finishings and fittings or a functional deficiency in comfort, capacity or suitability. In most cases, however, the service life is governed by external factors, but without being precisely specified or determined. In fact, this resolves itself into applying a technical life and an economic life more or less independent of that. It was up to the designer to keep the risk of failure (associated with the technical life) sufficiently low throughout that given economic life. However, in these matters, too, there is emerging an increasing need to arrive at a more quantitative approach to the reliability and service life of structures, making use of probabilistic principles as set forth, for example, in [2].

In analysing methods of determining the service life and/or reliability on a probabilistic basis the technological knowledge of the materials employed plays a special part. As has already been noted, both a service life prediction and a reliability analysis are based on the description of a "change of state" of the structure. With regard to this a knowledge of the laws governing the time-dependent behaviour of the relevant material properties under the influence of the load is important because in many cases the reliability and/or service life of technical structures will be closely associated with physical processes such as (fatigue) fracture, excessive deformation, wear, damage due to creep, etc. Laboratory experiments play an important part in connection with the development of mathematical models for these physical processes. An example of this approach is given in *Appendix A*, where a model for the growth of defects on the basis of a damage model for an austenitic steel under creep conditions is described. The parameters on which this model is based are conceived as stochastic quantities.

The fundamentals of this philosophy will be elaborated in the following sections of this chapter. Section 2.2 goes more thoroughly into the basic philosophy for the above-mentioned reliability/service life considerations, while mathematical models with connect to structures and structural materials are dealt with in Section 2.3. Section 2.4 gives some more background information on the use of statistical calculation methods and Section 2.5, finally, is devoted to the interpretation of calculated probabilities and the associated economic consequences.

2.2 *Basic philosophy*

In principle, a reliability analysis is characterized by three integrated aspects, namely:

- The loads acting on a system; these are conceived as non-stationary stochastic functions (input functions).
- The manner in which the structure responds to these loads, depending on geometry and material behaviour (transfer functions).
- The stochastic response of the structure to one or more critical locations in the structure (output functions).

The last-mentioned data are used for evaluating the failure probability, the service life or the reliability of a structure by the introduction of a suitable criterion such as the exceeding of a yield point, attainment of maximum play, permissible stress, etc. Obviously, because of the considerable freedom in the choice of this criterion, a large number of failures modes, reason for termination of service life, causes of failure, etc. may occur (see also Chapter 3). A simplification (often permissible) of the analysis process is obtained it the "load acting on the structure" can be decoupled from the "failure analysis".

This leads to distinguishing between two independent general concepts, namely, the so-called load function S and the strength function R, for a certain chosen failure criterion. These two concepts in fact represent, firstly, all the causes that may give rise to failure of the structure (S) and, secondly, all the factors that determine the resistance to this failure (R).

The central criterion for these reliability and/or service life considerations can be written as:

$$\text{“FAILURE”} = \text{“}R < S\text{”} \quad \text{or} \quad \text{“FAILURE”} = \text{“}R - S < 0\text{”} \tag{1}$$

A number of examples of this approach are given in the following table:

designation	R	S	failure if:
conventional statics	yield point	actual stress	$\sigma_V < \sigma$
fracture mechanics	critical flaw length a_{crit}	flaw existing at instant t	$a_{crit} < a$
efficiency	time-dependent efficiency η	critical efficiency η_{crit}	$\eta < \eta_{crit}$
etc.			

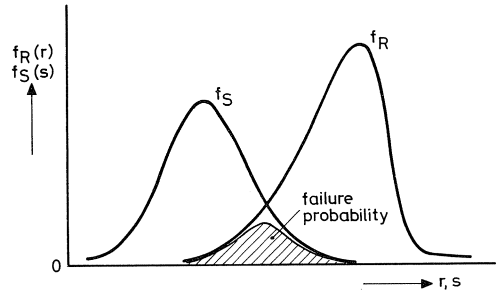

Fig. 5. Probability density function for strength and load, respectively, and the failure probability (represented by the shaded area). It is to be noted that the shaded area is merely a visual aid for indicating the failure probability and has no physical .or mathematical significance.

In general, both R and S are functions of a large number of stochastic variables. If the probability density functions of all these variables are known, the probability density functions $f_R(r)$ for the strength and $f_S(s)$ for the load can be derived. Next, a probability of failure P_f can be calculated, defined as:

$$\text{“FAILURE PROBABILITY”}\ P_f = P(R < S) \qquad (2)$$

as shown schematically in Fig. 5.

The strength R and load S will in general be functions of time. Therefore the failure probability is also a function of time, and there is usually not much point in applying the concept of failure probability without also stating the period of time to which it refers.

In the main, there are two methods of incorporating the time aspect into the failure probability analysis. These methods are bound up with the way in which R and S are defined. For further treatment of this subject we shall confine ourselves to the time-dependence in the load S. The two definitions to be distinguished are accordingly as follows:

a. $S(t)$ is the *instantaneous* load at the *point of time* t;
b. $S(t)$ is the *governing* load for the *period* $(0, t)$.

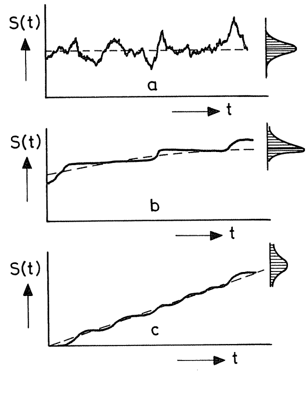

Fig. 6. Load on a structure.
a. instantaneous value
b. extreme value
c. cumulative value

By $t = 0$ is denoted the starting point of time of a period under consideration; in many instances this will coincide with the point of time when the structure is put (or put back) into service.

According to the first definition, $S(t)$ corresponds directly to the external physical quantity, e.g., the wind pressure on a building or the temperature in a boiler. A load defined in this way will in general display a time-related behaviour as shown in Fig. 6a. An essential requirement in method (a) is that the time aspect - or, rather, the time duration aspect - is absent in the definition of $S(t)$. Consequently, the probability $P\{R(t) < S(t)\}$ relates only to a particular point of time and not to a period of time. The interval failure probability for the period $(0, t)$ is therefore not given by $P\{R(t) < S(t)\}$ but should instead be written, for example, as:

$$P\{\text{failure in } (0,t)\} = 1 - P\{\text{no failure in } (0,t)\}$$
$$= 1 - P\{R(t') > S(t') \text{ for all } t' \text{ in } (0,t)\} \quad (3)$$

Arithmetically the step from $P\{R(t) < S(t)\}$ to the failure probability for a period is often very difficult. For this reason it is advantageous to employ definition (b). As that definition uses a governing load S for the whole period $(0, t)$, we have directly:

$$P\{\text{failure in } (0,t)\} = P\{R(t) < S(t)\} \quad (4)$$

Of course, in this way the problems are not entirely eliminated: the question of dealing with the time aspect is, as it were, shifted to the determination of the probability density function for $S(t)$. Experience shows, however, that this does make things easier. Against this there is the disadvantage that it means committing oneself in advance to a particular mechanism. In the statical approach the governing load is the extreme load in a particular period, whereas in the case of fatigue some cumulative measure is just what matters. By way of illustration these two cases are shown in Figs. 6b and 6c, each based on the instantaneous S-values represented in Fig. 6.

It may of course occur that, besides the load, also the strength or the resistance of a system is subject to time-dependent variation. Some systems become "stronger" in course of time (e.g., concrete as a result of continuing hardening) whereas others become "weaker" (e.g., ageing of metals). If R and S are both time-dependent, it is necessary to exercise caution in defining the governing load and strength: a minimum value of R will not necessarily coincide with a maximum value of S; see Fig. 7, for example. In principle, however, in all cases it is possible so to choose the definitions

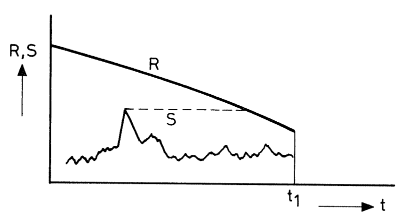

Fig. 7. After the period considered, the maximum of S is larger than the minimum of R; yet no failure occurs.

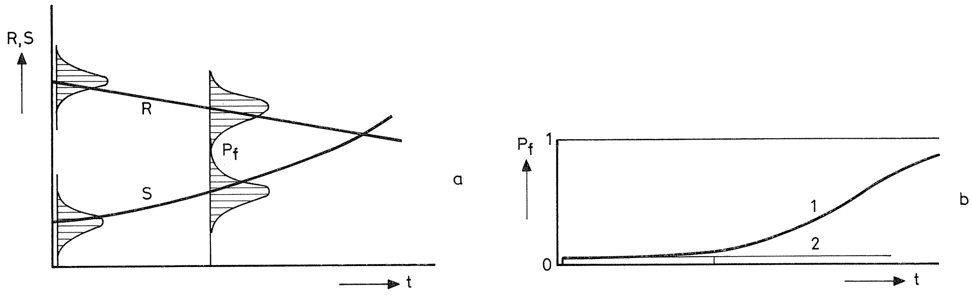

Fig. 8. a. Time dependent R and S
b. Possible corresponding failure probability (P_f) as a function of time (t) (curve 1) and failure probability according to a possible time-independent model (curve 2).

that the general configuration shown in Fig. 8a is obtained: a monotonically descending line for the governing R, a monotonically rising (or a horizontal) line for the governing S, and for both lines an average around which there occurs a scatter range due to uncertainties and fluctuations.

Thus, for each point of time t, a failure probability for a particular period can be calculated. A possible result of this probability as a function of time is represented in Fig. 8b. At $t = 0$ this function begins with a certain initial value, this being the probability that failure will occur at the very outset, i.e., when the structure is put into service. Subsequently, for $t > 0$, the failure probability will gradually increase and will in principle approach the limiting value of 1, because in the long run every structure has a finite life. Mathematically, within a particular theoretical model, a different limiting value is alternatively conceivable, e.g., a time-independent model in which P_f for all values of $t > 0$ is equal to the initial value $P_f(0)$.

In general, however, the failure probability is a monotonically rising function of time, starting at 0 for $t = 0$ and increasing to 1 for $t = \infty$. The function $P_f(t)$ therefore has the character of a distribution function. The stochastic variable associated with this is the service life L of the structure, since the event "$L < t$" is identical with the event "failure in $(0, t)$", or:

$$F_L(t) = P\{L < t\} = P_f(t) \tag{5}$$

If the distribution function for the service life is given, the probability density function can readily be determined:

$$f_L(t) = \frac{\mathrm{d}}{\mathrm{d}t} F_L(t) \tag{6}$$

On multiplying the probability density function by $\mathrm{d}t$ we obain the probability that the service life will terminate between the times t and $t + \mathrm{d}t$:

$$f(t)\ \mathrm{d}t = P\{t < t_L < t + \mathrm{d}t\}$$

Termination of the service life in $(t, t+\mathrm{d}t)$ implies that $R < S$ in $(t, t+\mathrm{d}t)$, but also that $R > S$ in $(0, t)$, for if the load had exceeded the strength already earlier than at the time t, termination of the service life would have occurred at that instant. Therefore:

$$f(t)\ \mathrm{d}t = P\{R < S \text{ in } (t, t+\mathrm{d}t) \text{ and } R > S \text{ in } (0, t)\} \tag{7}$$

The formulation (7) is correct, irrespective of whether R and S are defined as instantaneous or as governing values.

The probability density function for the service life can alternatively be conceived as the probability of failure per unit time or a failure rate. In this context the term *unconditional failure rate* is employed, as contrasted with the so-called *conditional failure rate* or hazard function $r(t)$. With the function $r(t)$ the value of $r(t)\ \mathrm{d}t$ also indicates the probability that failure will occur in $(t, t+\mathrm{d}t)$, but then on condition that no failure has occurred in $(0, t)$:

$$r(t)\ \mathrm{d}t = P\{R < S \text{ in } (t, t+\mathrm{d}t) | R > S \text{ in } (0, t)\} \tag{8}$$

Naturally, the functions F, f and r are linked by a direct relation, which can most simply be derived by basing oneself on the meaning of the probability density function according to (7):

$$\begin{aligned} f(t)\ \mathrm{d}t &= P\{R > S \text{ in } (0, t) \text{ and } R < S \text{ in } (t, t+\mathrm{d}t)\} = \\ &= P\{R > S \text{ in } (0, t)\} \cdot P\{R < S \text{ in } (t, t+\mathrm{d}t) | R > S \text{ in } (0, t)\} = \\ &= \{1 - F(t)\} \cdot r(t)\ \mathrm{d}t \end{aligned}$$

or

$$r(t) = \frac{f(t)}{1 - F(t)} \tag{9}$$

It clearly emerges that the conditional and unconditional failure rates for low values of F are almost equal. Hence, for structures for which F is always low, the two functions can quite justifiably be equated to each other.

For given $F(t)$, the functions $f(t)$ and $r(t)$ can easily be derived via (6) and (9). Conversely, $F(t)$ can be determined from $r(t)$, as follows (the formula is given here without its derivation):

$$F(t) = 1 - \exp \int_0^t - r(\tau)\ \mathrm{d}\tau \tag{10}$$

In Fig. 9 the functions F, f and r are shown for a number of cases. Fig. 9a represents a constant $r(t)$. The given circumstance that a structure has functioned for some time without having failed is evidently without effect on the failure probability for the period ahead. A situation of this kind may occur, for example, when the cause of failure is a disaster such as an outbreak fire in a building. A descending function $r(t)$ is shown in Fig. 9b. Here the failure probability for the period ahead is favourably affected by the structure having functioned satisfactorily for a time. The fact that it has already been serving its purpose without problems for some length of time and has withstood a num-

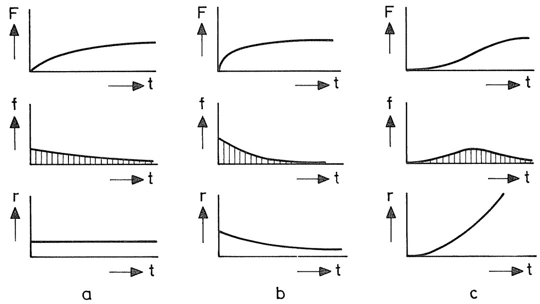

Fig. 9. Three types of service life distributions.
a. with constant r-function
b. with descending r-function
c. with rising r-function

ber of loading situations justifies the supposition that there are no "weaks spots" in the structure. On simultaneously considering a large number of structures it could be stated that the bad ones are slowly but surely filtered out, so that the good ones are left. Finally, Fig. 9c shows a rising function $r(t)$. A function of this type occurs when a structure is subject to "ageing" or "wear out". The fact that the structure has already been in service for some time leads to the conclusion that it has suffered "damage", so that its probability of failure has increased.

In an actual structure, of course, combinations of these respective cases are possible, giving a mixture whose proportions are liable to vary from one structure to another. In general, however, the resulting r-function will be as shown in Fig. 10, sometimes called the "bathtub curve". Three regions can be distinguished: an initial phase in which failure occurs as a result of faults of construction or error of design, a middle phase in which disasters and particularly extreme conditions play a part, and a final phase with failure due to ageing or wear out.

In the foregoing discussion, three functions for describing the service life have been presented, all of which in principle contain the same information: the distribution function of the service life, the probability density function of the service life or the unconditional failure rate, and the conditional failure rate. Which function is most suitable for use in a particular case will depend on what is required.

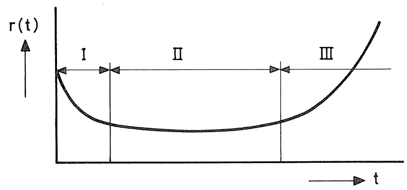

Fig. 10. General form of the conditional failure rate function (r-function; bathtub curve).
I failure due to low strength
II failure due to extreme conditions
III failure due to cumulative effects

First, consider a structure which must attain a certain specified effective service life imposed by external factors. During that life period it must function without problems; hence the probability of failure must be low, e.g., because the damage or loss resulting from failure would be very serious. Quite often, in designing or assessing a structure in this kind of situation, only the failure probability for the whole period (P_f or $F(t)$) is considered, i.e., the question of how this probability is distributed over the period is then of no interest and the probability density function $f(t)$ plays no part. Actually, of course, it does make difference: thus, if the structure fails at the beginning, rebuilding may be necessary, whereas if it fails at the end of the period, there will at most be some disadvantage due to reduced residual value. The effect of this kind of differences is often neglected, however.

A different situation exists where the effective service life of a given structure has to be ascertained. In other words: to determine the length of time for which it can function before its further utilization becomes undesirable for economic or safety reasons. In that case the conditional failure rate is the appropriate function to apply, because the problem consists in deciding to terminate the service life of the structure at the time t if is estimated that it will no longer be profitable or safe in the time interval $(t, t + dt)$. Obviously, the decision to terminate will have to be made only if the structure is in fact still in existence at the time t.

A conditional failure rate function is shown also in Fig. 11, which moreover indicates the limiting value which, on being exceeded,signifies that the structure has reached its effective service life. As represented in this diagram, it is apparent that at $t = 0$ the conditional failure rate is above the limit. This need not, however, be regarded as a reason for immediately terminating the life of the structure. In fact, the r-function descends fairly rapidly and thus soon goes below the limiting value, where it then remains for a considerable time. The term "invested risk" could be employed in this connection. A risk of that kind may be acceptable if its total extent remains within tolerable limits, if its duration is not too long, and if it is followed by a period of diminished risk.

Basing oneself on the above line of reasoning, it might be considered that the curve could be allowed to rise above the limiting value also at the end of the service life. That would be incorrect, however. It is never justifiably possible to say: "let us accept a bit more risk next year, seeing that it was so low in the past year". Decisions are always aimed at the future. As soon as a period has ended without the occurrence of damage or loss, it does not matter whether the risk was high or low.

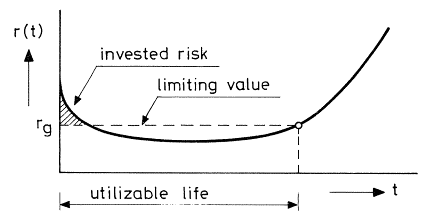

Fig. 11. The utilizable service life is determined by the intersection of the conditional failure rate with the limiting value r_g.

Summary of basic philosophy

For assessing a structure it is often convenient to introduce two parameters R and S. The parameter R (the strength) comprises those quantities which have a favourable effect on the behaviour of the structure, while the parameter S (the load) comprises those which have an unfavourable effect. The conception is that the structure will function as long as $R > S$, but will fail as soon as $R < S$.

In general, the functions R and S are also functions of time. This means that a distinction must be made between instantaneous values of R and S (at a certain point of time) and governing values (in a certain interval of time). Depending on the case considered, it is simpler to use one or the other of these two.

If the probability of failure for a period $(0, t)$ is plotted as a function of t, a monotonically rising function is obtained, increasing from 0 to 1 for t increasing from 0 to ∞. This function is found to be identical with the distribution function of the service life $F_L(t)$. If this distribution function is differentiated once, the probability density function of the service life $f_L(t)$ is obtained. This latter function is known as the unconditional failure rate, as contrasted with the conditional failure rate $r(t)$, which gives the probability of failure in the interval $(t, t + \mathrm{d}t)$ on condition that no failure has occurred up to the time t. In principle, these three functions contain the same information and can be derived from one another. Which function is most suitable for application in any particular case will depend on circumstances. The distribution function $F(t)$ and the probability density function $f(t)$ are often considered important for structures whose effective service life is determined by factors other than those inherent in the structure itself. But if the effective service life has to be determined partly on the basis of the risk of failure, it will be necessary to apply the conditional failure rate $r(t)$, except for the assessment of the initial risk for a descending course of this function r. Incidentally, it is to be noted that $f_L(t)$ and $r(t)$ quite often do not exhibit any substantial differences from the numerical point of view.

2.3 *Mathematical models*

2.3.1 Schematic set-up of structural analysis

The response of a structure is determined by the external conditions in which it is placed and by its own properties. For assessing the response and the reliability of a structure it is necessary to have formulae and equations which describe the properties of the structure and its environment, a so-called mathematical model. In this respect there is no difference between a deterministic and a probabilistic approach: in principle, the same mathematical models are employed.

For the analysis of a structure several mathematical models are generally used – side by side or consecutively. The result yielded by one model often constitutes an input for the next one. By way of example an offshore structure will be considered which has to be assessed with regard to fatigue. The fatigue is caused by the waves of the sea, and the following mathematical models are therefore required:

1. wave motion model for determining wave velocities;
2. wave forces model for determining wave forces;
3. dynamic structural model for determining stresses;
4. material model for determining fatigue.

This example thus comprises four mathematical models "in series". The first two serve to determine the load from the external influence (sea waves); the last two serve to establish the response of the structure, first at macro level, then at micro level. This is shown schematically in Fig. 12. From the external influence we calculate the load need-

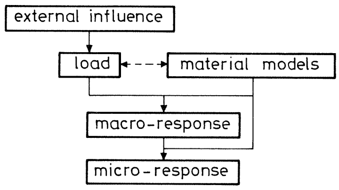

Fig. 12. Diagram of a structural analysis.

ed for choosing the appropriate material model(s), e.g., a linear-elastic or an elastic-plastic model. The macro response (stresses and displacements, dynamic or otherwise) are then calculated from the load and the material model. Micro-response comprises, for example, fatigue crack growth. To predict this it is, in general, necessary to make use of macro-response as well as material models and the load. In reality the situation is of course more complex, if only because there are many different forms of interaction. For example, in the case of the offshore structure under consideration, there may be an interaction between the macro-response and the wave forces. With structures of low rigidity this interaction will be strong, whereas with rigid structures it will be weak. Also, interaction may occur between the micro-response and the macro-response, e.g., because a fatigue crack affects the rigidity properties of a structural connection or joint. Despite these complications the schematic representation does of course indicate the main features involved. In the following discussion of a number of commonly employed mathematical models this scheme will therefore be adopted as the basis. The discussion of these models will be very succinct, however, and be intended merely to give a general overall picture.

2.3.2 Mathematical models for load determination

Mechanical loads on structures may arise from a great many different sources. A possible classification (but by no means intended as an exhaustive one) is as follows:

1. loads due to weight;
2. dynamic loads due to movements or collisions;
3. loads due to liquids;
4. loads due to wind and gas pressures;
5. temperature.

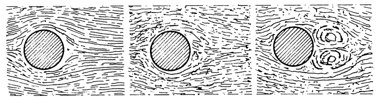

Fig. 13. Development of flow round a circular cylinder. a. laminar; b. turbulent; c. vortices. (from a publication by Prandtl, 1927).

The mathematical models which are needed for passing from the description of the various external conditions to loads on the structure vary quite considerably. The loads due to weight are of course the simplest. Starting from the mass m or the mass density ϱ, the forces or volume loads are obtained on multiplying by the acceleration of gravity g.

The determination of dynamic loads, e.g., those produced by rotating machinery, collisions, etc., often requires elaborate calculations in accordance with the theory of engineering mechanics.

Liquid loads [3] may occur in many different ways. The hydrostatic forces in tanks and reservoirs are simple to determine. The forces associated with the flow of liquid through a pipe or channel, involving the determination of frictional and impact forces, present a more complex problem. Then there are forces acting on bodies in a liquid stream (permanent or impermanent), e.g., offshore structures and ships. In that case there are not just the forces due to irrotational flow to consider, but also the consequences of turbulence and boundary layer phenomena (see example in Fig. 13).

Similar calculations can be done for wind and gas pressures. In some instances it may be sufficient to determine static gas pressures (storage tanks); in others, more or less elaborate aerodynamic calculations are needed (wind loads [4, 5] and explosion pressures on buildings, chimneys, masts; aircraft design calculations). In those cases where complicated hydrodynamic or aerodynamic calculations have to be carried out, computers are now employed, while - apart from a great many ad hoc procedures - the finite element method is extensively applied.

The fifth category in the classification given above comprises temperature influences. Determining the temperature behaviour in a structure, both as a function of time and as a function of place, often requires elaborate calculations. Just as in the previously mentioned cases, the conditions may involve steady heat flow (processors, motors, solar radiation) or unsteady flow (fire, switching-on and switching-off effects, chemical hardening process of concrete, etc.). In complex cases of this kind, too, a finite element analysis with the aid of a computer may be required.

2.3.3 Mathematical models for the determination of macro behaviour

The determination of the macro behaviour of a structure, starting from given loads, is largely the subject of structural analysis or engineering mechanics. In general, the analysis for this purpose is based on three kinds of equations:

1. kinematic equations;
2. constitutive equations;
3. equations of motion.

The *kinematic equations* establish a relation between the displacements of the structure and the internal deformations. The displacement field is usually given as a function of time in the original co-ordinates (Lagrange description). The kinematic equations may be linear or non-linear. The linear equations hold for small displacements; the non-linear ones are required for the analysis of geometrically non-linear effects such as buckling and membrane action.

The *constitutive equations* comprise material properties and establish the relation between the strains, the rates of strain and the stresses, possibly as a function of temperature. The simplest material behaviour is linear-elastic behaviour. More sophisticated mathematical models take account of plasticity, viscosity, cracking, etc. In this category of equations there is distinctly an interaction with the mathematical models for micro behaviour.

Finally, there is the third category, the *equations of motion*, based on Newton's law: $F = m \cdot a$, where F is the sum of the external load and the internal stresses. In many cases with predominantly static loads the equations of motion can be reduced to equations of equilibrium. Just as with the kinematic equations, linear or non-linear equations may be chosen, depending on whether or not geometric non-linear effects are to be taken into account.

With the aid of the above-mentioned mathematical models it is possible already to find out quite a lot about the behaviour of the structure. Limit states such as maximum loadbearing capacity governed by elastic or elasto-plastic instability can be investigated, as well as the occurrence of unacceptable deformations or vibrations. For dealing with complex problems an extensively used and successful approach consists in using computer programs based on the finite element method [6]. Such problems include, more particularly, the buckling and collapse load calculations for shells and stiffened plate panels such as those in bridges, offshore structures, aircrafts, etc. Apart from these sophisticated mathematical models there are of course many empirical design and analysis methods. Much of the research conducted with advanced computer programs aims at verifying and, where necessary, improving these empirical methods.

2.3.4 Material models

These often comprise mathematical models in which time does not explicity play any part and models in which it does. The procedures to be applied for these two classes of models if they are employed in a reliability assessment, have already been explained in general terms in Section 2.2. In material models that have been designed for the purpose of service life estimates the time-dependence of the material of course usually plays an important part. Therefore this section reviews various mathematical models for time-dependent material behaviour. Only a number of the most important limit states will be considered (see also Chapter 4) and some characteristics of the material models to be used in connection with these will be outlined.

Fracture

From the viewpoint of materials technology it can be said that fracture occurs at the instant when a hitherto subcritical defect or flaw becomes critical. In this conception, therefore, the actual size of a defect in the structure is compared with a size of defect that would produce fracture in this structure. This may occur because the defect in question has grown in course of time or because the critical size has decreased. The latter condition may occur as a result of material degradation or of an increase in (local) stress due to varying external loads.

Not much is yet known concerning material degradation under the influence of load and temperature. It can only be stated that a number of quantities defined in fracture mechanics are available for characterizing the critical sizes of defects, such as the critical stress intensity factor (K_{Ic}), the critical strain energy release rate (G_{Ic}), the crack growth resistance curve (R curve), the crack opening displacement (COD), the critical value of Rice's path-independent contour integral (J_{Ic}), etc. [7].

For describing the growth of actually existing defects under alternating load, Paris's relation [8] is often used:

$$\mathrm{d}a/\mathrm{d}N = C(\Delta K)^m \qquad (11)$$

where:

a = characteristic crack dimension
N = number of stress cycles
ΔK = variations in the stress intensity factor which occur in consequence of the stress variations

C and m can be regarded as constant within not too large a range; see also Fig. 14.

If it is assumed that always the same initial defect occurs, the familiar stress-number curve or "S-N" diagram ($\log \Delta\sigma$ versus $\log N$) can be derived from Paris's relation. If the stress amplitude is not constant, the effect of the various stress amplitudes can be taken into account by applying the Palmgren-Miner rule [9, 10] which, on the assumption of constant C and m, can likewise be derived from Paris's equation. Of course, the actual historical sequence as just the reverse: the stress-number curve and the Palmgren-

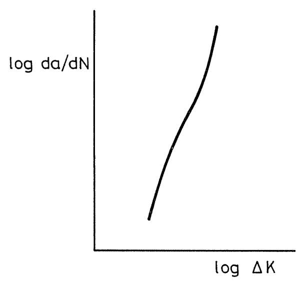

Fig. 14. Crack growth rate ($\mathrm{d}a/\mathrm{d}N$) as a function of the stress intensity factor range (ΔK).

Miner rule are older than Paris's equation. The concepts developed by this last-mentioned author and others can, however, contribute to obtaining better insight into the applicability of the stress-number curve and the Palmgren-Miner rule.

If crack growth occurs in consequence of a constant load, it is known as creep crack growth, a phenomenon which investigators try to describe with relations of the same type [11]. Crack growth can also occur in consequence of mechanisms other than fatigue and creep. For example, the combined action of stress and corrosive influences may give rise to stress corrosion. This occurs if the stress intensity factor of existing defects exceeds the critical value defined for that phenomenon (K_{Iscc}). A quantity of this kind must essentially also be embodied in the above-mentioned crack growth models, such as Paris's equation and its modifications, in the case where alternating load and environmental influences act in conjunction with one another [12]. The resulting phenomenon is known as corrosion fatigue.

Loadbearing capacity

When the loadbearing capacity of a structure is exceeded, this is associated with elastic or with plastic instability. The material properties that are important in connection with this are, inter alia, the elastic properties, the yield stress and the tensile strength. The cases where the possible time-dependence of these quantities is not taken into account have, in substance, already been discussed in Section 2.3.3.

In addition, it is sometimes necessary to take acount of dimensional changes in the structure, e.g., due to wear or corrosion. Wear will be considered below. As for corrosion (i.e., chemical attack brought about by environmental influences), this subject is of such wide scope that it can hardly be dealt with in the present brief review. In many instances the loss of material due to corrosion can be assumed to proceed linearly with time. And, anyway, it would seem that corrosion specialists are generally less interested in this aspect than in providing protection against corrosion, e.g., by means of coatings, cathodic protection, etc. The service life of the protection system then becomes the important consideration.

Deformation

Material deformation may, under the influence of stress, occur suddenly or it may proceed slowly. In the latter case it is called creep. The time-dependence of deformation due to creep is determined by the familiar creep curves; see Fig. 15. In a typical curve as shown here there are three regions, as indicated. In the past the secondary creep, defined as the region of constant rate of deformation or strain, has received most attention. The strain rate in this secondary creep region is considered to satisfy an Arrhenius equation with an activation energy that is dependent on the stress. On this assumption is based, for example, the Larson-Miller parameter for the extrapolation of the results of creep tests performed at elevated temperature and/or stress. Thus applying these results to practical conditions; see Fig. 16.

Many other parameters of this kind have been proposed over the years [13]. Besides these parameter approximations, other methods of extrapolation have also been

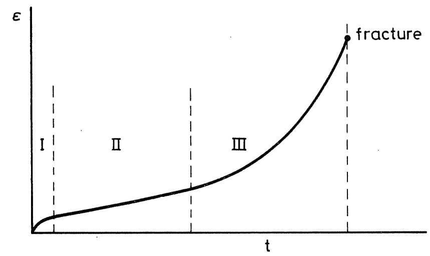

Fig. 15. Creep curve: deformation (strain) (ε) versus time (t).
I primary creep
II secondary creep
III tertiary creep

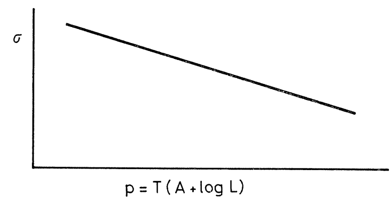

Fig. 16. Parameter approximation for service life (L) under creep conditions.
σ = stress
p = parameter with: T = absolute temperature, A = constant

proposed; for these the reader is referred to the relevant literature. Also, mathematical models have been proposed with which the primary creep region is described as well [14]. It has been found, however, that the structure of the material is liable to undergo changes during the creep process. Obviously, in such cases each material structure will involve a number of material parameters, so that a creep model that is based not merely on mathematical concepts, but also on available material-technological knowledge, will be of a complex character. Furthermore, it must be very emphatically stated that any model can at best be valid only for a limited group of materials. Finally it is pointed out that, according to a recently developed approach, the creep phenomenon can advantageously be described as the evolution of a collective of micro defects (cf., e.g., Appendix A).

With regard to creep and fatigue it is moreover to be noted that these forms of material damage often occur under successively more than one set of conditions (namely temperature, state of stress or stress amplitude). Besides, they may occur in conjunction with each other. In such cases so-called life fraction rules are employed (see also Chapter 4), such as the Palmgren-Miner rule already mentioned [9, 10]. The strain-range partitioning method is used for dealing with creep-fatigue interaction.

Cracking

The cracks envisaged here may be those in building structures or, for example, in walls of tanks or other such containers. In general, time-dependent models are not very relevant to cracking in ceramic materials (brick) or concrete. For cracking in metals the same time-dependent relations as mentioned above are valid (Fig. 14). However, crack propagation under such circumstances often takes place in more than one direction, whereas the relations in question have reference to propagation in one direction. Insight into the application of such relations to multidirectional crack propagation has been deepened in recent years, however. In such structural features as the walls of tanks or similar vessels (e.g., pressure vessels) the limit state is reached either because the critical crack size is attained or because complete penetration of the wall occurs. In the former case, fracture will occur before leakage (guillotine fracture), whereas in the latter it will be preceded by leakage. Obviously, this second alternative is far preferable to the first, which is why many "leak-before-break" criteria have been developed.

Wear

Tribology is concerned, inter alia, with the investigation of the time-dependence of wear processes. Failure of tribological systems very often occurs as a result of failure of the lubrication system, however. This being so, a considerable amount of attention in tribology is directed at the development of better lubrication systems. If loss of material is permitted, it can often be taken to proceed linearly with time (expressed as a decrease in volume or a decrease in height). More particularly, this is commonly the case both with adhesive and with abrasive wear mechanisms, including erosion by solid particles. In connection with erosive wear the angle of impingement is of major importance. The time-dependence of cavitation and droplet impact erosion is fairly complex, on the other hand; see Fig. 17.

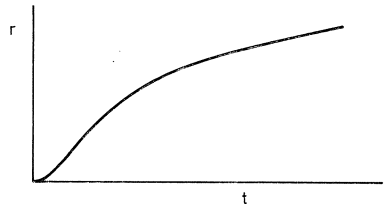

Fig. 17. Mean depth of erosion (r) as a function of time (t) in the case of cavitation and droplet impact erosion.

Others

In cases where corrosion plays an important part, or cases where large deformations are liable to occur which are not in themselves dangerous, termination of service life may nevertheless ensue in consequence of unsightliness. So far as the authors of the present report are aware, no time-dependent models for this type of behaviour are available.

In process engineering the question of efficiency is important. This technical field will not be further considered here, however.

Furthermore, a structure may constitute a hazard to its surroundings. Often there is time-dependence involved, or otherwise this dependence, if it exists, can be approximated by means of one of the models considered above. Also, for example, accumulative action of ionizing radiation may occur, the time-dependence of which can be described in nuclear physical and/or process engineering terms. This, too, is a field that goes beyond the scope of the present report.

2.3.5 Special effects

Although in connection with all the above-mentioned aspects considerable simplifications will often have to be accepted in order to arrive at a solution for the reliability problem, it is found that actually in many instances of failure or premature termination of service life the human factor has been of major influence. Also, it is now more and more clearly realized that this human influence should somehow be represented in the overall model. This realization, too, strengthens the tendency for a design to result, not in a single (uniquely determined) structure, but in a range of possible alternatives.

Various models have been developed - e.g., by Rackwitz at Munich and by Bjerager and Karlsson in Denmark - for taking account of human errors. One of the problems encountered in developing such models is the contrast between the large number of human errors committed in actual practice and the small number of serious mishaps arising from them.

Some investigators [15, 16] propose adopting an empirical approach, using the correlations between incidents, for the detection of error-sensitive structures. Nowak [17] adopts the principle that the various error-introducing processes should be considered independently of one another - thus establishing a sound basis of insight into the mechanisms underlying these errors.

In conclusion it can be stated that the human failure probability can be taken into account by the introduction of an extra limit-state-dependent random variable for each individual limit state.

2.4 *Techniques for performing the calculations*

2.4.1 Levels

In general, the work involved in doing a probabilistic safety analysis comprises a good deal of computational effort. Even if the calculations are performed entirely by computer, the sheer quantity of computation to be handled remains a restricting factor with regard to practicability. For this reason a considerable amount of research has been undertaken, over the years, with a view to developing suitable approximate methods. In order to co-ordinate the various approaches to these matters, the Joint Committee on Structural Safety has defined three levels at which reliability analyses can be performed, intended in principle for application in civil and structural engineering [18]. This classification has proved very convenient. It will therefore be outlined here and illustrated with the aid of a simple example.

Characterization of levels I, II and III

Level I: This level is intended for routine design use. For each of the stochastic variables a certain unfavourable value is chosen, the so-called characteristic value, usually based on a 5% confidence limit. In addition, a set of partial safety factors is applied; these establish the margin between load and strength for the various cases to be considered.

Level II: The analysis at level II is an approximative analysis based on the so-called "first order/second moment" principle. Briefly, what it amounts to is that only the mean value and the standard deviation of each stochastic variable are taken into account, linearization being applied if necessary. There are several variants as to how the approx-

imations are to be applied in detail. In principle, level II approximations are used for two purposes, the more important of which is to provide a basis for level I procedures. More particularly, by means of studies at level II, it is endeavoured, as it were, to underpin the partial safety factors of level I as soundly as possible. The second purpose consists in applying the level II analysis to special or important structures for which an above-normal standard of reliability analysis is considered essential, e.g., for offshore structures, nuclear power plants, storage installations for liquefied natural gas, etc.
Level III: This level comprises complete and exact analyses performed with the aid of analytical or numerical integration procedures or Monte Carlo simulations. As already mentioned, level III analyses generally demand too much computational effort to be practicable for dealing with major problems. The scope for applying them exists mainly in verifying and complementing level II approximations.

Example I
To illustrate and explain the reliability analysis levels introduced here, a simple example - a so-called basic case - will now be presented. Consider the *limit state of failure* of a simply-supported beam subjected to a point load at mid-span (see Fig. 18). Both the load and the strength are to be regarded as constant, i.e., not varying with time. It is further assumed that the beam will fail when the mid-span bending moment exceeds the maximum moment that the beam can resist. In order to formalize this failure criterion we may introduce a so-called *reliability function* Z, defined as follows:

$$Z = R - S$$

where

$R =$ is the moment that the beam can resist
$S = Fl/4$ is the moment produced by the load

The event "failure" is thus identical with the event "$Z < 0$".

In this case the reliability analysis at *level III* consists in working out the well known convolution integral:

$$P(Z<0) = \int_0^{\infty} f_S(\phi) F_R(\phi)\, \mathrm{d}\phi \tag{12}$$

where $f_S(\phi)$ is the probability density function of the load F and $F_R(\phi)$ is the cumulative probability distribution function of the strength R.

In a *level II* analysis the mean value and the variances of the reliability function Z are

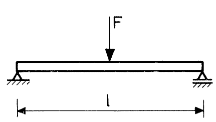

Fig. 18. Simply-supported beam supporting a point load acting at mid-span; the beam fails when the maximum mid-span moment $Fl/4$ exceeds the resisting moment R.

calculated with the aid of:

$$\mu(Z) = \mu(R) - \mu(S) \tag{13}$$

$$\sigma^2(Z) = \sigma^2(R) + \sigma^2(S) \tag{14}$$

From this mean value and the standard deviation the *reliability index* β is then determined:

$$\beta = \mu(Z)/\sigma(Z) \tag{15}$$

Finally, on the assumption that Z conforms to a normal distribution, the failure probability is obtained as:

$$P(Z < 0) = \Phi_N(-\beta) \tag{16}$$

where $\Phi_N(-\beta)$ is the distribution function of the standard normal distribution. It is evident that the result will be exact if R and S are both normally distributed; in all other cases it will be an approximation.

Finally, treatment of this problem at *level I* will be considered. For this purpose the characteristic values S_k and R_k are determined for which S_k has a 5% probability of being exceeded and R_k a 5% probability of not being exceeded:

$$P(S < S_k) = F_S(S_k) = 0{,}95 \tag{17}$$

$$P(R < R_k) = F_R(R_k) = 0{,}05 \tag{18}$$

The designer's requirement could then be, for example:

$$S_k \cdot \gamma_S < R_k/\gamma_R \tag{19}$$

where γ_R and γ_S are the partial safety factors. For guidance: if $\gamma_S = \gamma_R = 1.0$, the probability of failure for a wide range of problems is approximately 10^{-2}. For structural steelwork the factors commonly adopted are $\gamma_S = 1.5$ and $\gamma_R = 1.0$. Depending on the nature of the load, this assumption involves a failure probability of between 10^{-5} and 10^{-7}.

2.4.2 Several causes of failure

In the foregoing example there was assumed to be just one cause of failure, namely, the situation where the structural loadbearing capacity of one cross-section is exceeded under the influence of a single load. In reality, of course, matters will often be much more complex. As already explained in Section 2.2, for example, R and S may vary with time (this case will be further considered below); also, several causes of failure may play a part. For a certain structure the occurrence of failure may result from its maximum loadbearing capacity being exceeded, from fire, from fatigue, etc. Furthermore, with structures of greater complexity there may be several failure mechanisms involved in each main cause. Combining the probabilities associated with separate mechanisms into the overall failure probability for the structure is still a major problem. It is not so much a theoretical problem (indeed at level III there is no problem); the difficulty lies in finding suitable approximations. These matters will receive further attention in Chapter 4.

2.4.3 Service life and time dependence of strength and load

If R or S varies with time, or if failure is bound up with a cumulative load effect, the probability of failure will depend on the period of time considered. By calculating the probability of failure for various period durations we find, as already explained in Section 2.2, the distribution function of the service life (Fig. 19). However, in some cases this function can be obtained in a different and possibly more convenient way. To do that it is necessary to be able to write the service life L directly as a function of the basic variables:

$$L = f(x_1, \ldots, x_n) \tag{20}$$

The variables $x_1, \ldots, x_n$ are the same basic variables that were adopted for R and S in the foregoing treatment of these matters. Starting from (20) we can directly determine the distribution function for L. This analysis can of course be performed at all three levels I, II and III. For the analysis at level II we must then determine the mean value $\mu(L)$ and the standard deviation $\sigma(L)$ on the basis of the parameters $\mu(x_i)$ and $\sigma(x_i)$ and a linearization of (20). Next we can assume for L a normal distribution; with this the whole distribution function is established. If desired, we may alternatively assume a log-normal or a Weibull distribution and/or introduce other variants into the analysis. The following example will serve to illustrate this.

Example 2
Consider again a simply-supported beam, but now under fluctuating load (F); see Fig. 20. Suppose that at a certain point of time this beam - length l, width W, thickness B - is found to contain a fatigue crack of depth a. We wish to know what the reliability of the beam now is. First, we must realize that it will fail if the crack exceeds a certain critical

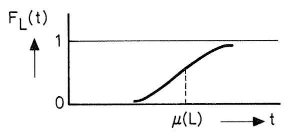

Fig. 19. The distribution function for the service life is equal to the failure probability as a function of the period considered.

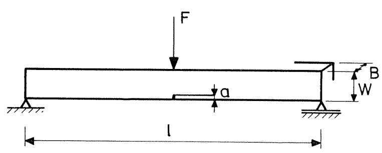

Fig. 20. Simply-supported beam with fluctuating point load F acting at mid-span; the beam fails when the crack depth (a) exceeds the critical value (a_c).

size (a_c). To calculate this critical crack size we must know whether the beam will fail as a result of unstable cracking that spreads through it or as a result of plastic instability. In the former case the magnitude of a_c is determined by the maximum load and a suitable criterion of fracture mechanics, e.g., the critical crack opening displacement (δ_c) or the critical value of the path-independent contour integral (J_{Ic}). In the latter case a_c is determined by the maximum load and a criterion that describes the plastic instability, e.g., the yield stress (σ_y) or the ultimate tensile stress (σ_u). Actually (even in this simple example) both failure modes must be taken into account, as described in Chapter 4. In this example we shall, however, confine ourself to the case of plastic instability. For this we have, in accordance with the stress model indicated in Fig. 21:

$$F_c = (B\sigma_u/L)(W - a)^2 \tag{21}$$

or:

$$a_c = W - (F_{\max} L/B\sigma_u)^{1/2} \tag{22}$$

where F_c denotes the critical load, i.e., the load at which failure takes place, and $F_{\max}$ the maximum load during the time interval between t and $t + \Delta t_s$, the term Δt_s being a suitably chosen "sample time" (the average cycle duration, for example).

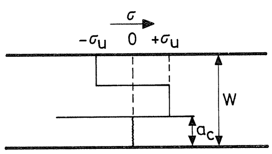

Fig. 21. Stress model for failure due to plastic instability.

In (21) and (22) the ultimate tensile stress σ_u may (for example) moreover be time-dependent. This possibility will not be considered in the present example, however.

A prediction with regard to the crack size (a) in the same time interval can be made with, for example, the aid of Paris's relation (11) (or some other relevant crack growth relation).

In this case the value for ΔK to be substituted into this relation can be calculated from:

$$\Delta K = (L\Delta F/BW^{3/2})\,\Phi(a/W) \tag{23}$$

if the geometry-dependent function $\Phi(a/W)$ is known; this function is often given as a polynomial approximation.

With the aid of (11), (21), (22) and (23) we can calculate a_c and a or F_c for the time interval ($t, t + \Delta t_s$). On the basis of this the probabilistic analysis can be carried out by three methods.

First method

We define:

$$R = a_c \text{ and } S = a, \text{ therefore } R - S = a_c - a \tag{24}$$

This is, for example, in agreement with rule 2 in the table in 2.2.

From (11), (22), (23) and (24) follows:

$$R = R(W, F_{\max}, L, B, \sigma_u) \tag{25}$$

and

$$S = S(C, m, L, \Delta F, B, W, a_0, t, \Delta t_s) \tag{26}$$

where a_0 denotes the size of the defect at time $t = 0$ and where it has furthermore been assumed that the number of load cycles (N) has been converted via the (mean) frequency into a time (t).

The functional relationships (25) and (26) can in principle be determined from (11), (22) and (23). Thus the distribution function of R and S can likewise be determined if the distribution functions of the other parameters are known. Therefore it is also possible to determine $P(R - S < 0)$. By using the quantity $F_{\max}$, which has been introduced in (22) and is defined with respect to the time interval $(t, t + \Delta t_s)$, we ensure that R and S are valid for this interval. Therefore (approximately, for not too large a value of Δt_s):

$$P(R - S < 0) = r(t)\Delta t_s \tag{27}$$

By performing this calculation for various times (t) we can find the conditional failure rate as a function of $t, r(t)$. F_L and f_L can be calculated from this.

Second method

We define:

$$R = F_c \text{ and } S = F_{\max}, \text{ therefore } R - S = F_c - F_{\max} \tag{28}$$

Now, in analogy with (26), we obtain from (11), (21), (23) and (28):

$$R = R(B, \sigma_u, L, W, a_0, t, \Delta t_s, \Delta F, C, m) \tag{29}$$

This functional relationship can in principle be determined from (11), (21) and (23), so that again the distribution function of R can be determined if the distribution functions of the other parameters are known. The distribution function of $S = F_{\max}$ is dependent on Δt_s (maximum value distribution) and is assumed to be known. Thus $P(R - S < 0)$ can again be determined, and the procedure is the same as in the first method.

Third method

Let L again denote the service life and t the functioning time. We then define:

$$R = L \text{ and } S = t \tag{30}$$

Hence we see that, in accordance with (20):

$$R = L = f(W, F_{\max}, L, B, \sigma_u, C, m, \Delta F, a_0, \Delta t_s) \tag{31}$$

Now $R = L$ is calculated by integrating (11) between the limits a_0 and a_c while making use of (22) and (23). We can again calculate $P(R-S<0)$ and proceed as in the other two methods. The present method corresponds to the procedure outlined at the beginning of this section. It might be supposed that $F_L(t)$ is directly calculated in this way. This is not so, however: comparison of this method with the two preceding ones shows that here too:

$$P(R-S<0)=r(t)\ \mathrm{d}t \tag{32}$$

By working out the three methods it can be shown that they are not substantially different from one another and the results they yield must be the same.

2.5 *Probability interpretation and economy*

Several methods and techniques that can be used in carrying out probabilistic analyses have been indicated in Section 2.4. The central problem is to calculate the probability of failure in a certain period, basing oneself on a given mathematical model and given properties of the stochastic variables. Of course, this is only part of the overall design or advisory assessment problem. At least as important are such questions as "How are the necessary data to be obtained?" and "What next to do when a probability of failure has been calculated?"

The second question will be discussed first. Obviously, determining the probability of failure is not an end in itself. The failure probabilities or service life distributions that are thus found must be further utilized in a deliberative process that must ultimately lead to the best possible design or to optimum advice as to the use and maintenance of the structure. The most obvious approach is to conceive to concepts of "best" and "optimum" in the economic sense. The design or the advice should be so devised that the sum of all the cost expectations is as low as possible. By way of example consider again the simply-supported beam loaded in bending. Suppose that its depth W has to be determined. If a low depth is chosen for the beam, the construction cost c_B will in principle likewise be low. But in that case there is a high expectancy of damage (product of damage cost c_F and failure probability P_f) because with a shallow beam (low W) the probability of failure is high. If W is increased, the cost of construction will increase and the damage expectancy will decrease (Fig. 22). An optimum will exist for a particular value of W. This optimum will depend to a great extent on the ratio between damage cost and construction cost $r=c_F/c_B$. If the cost associated with damage is relatively low,

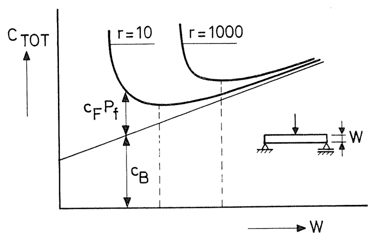

Fig. 22. Total cost as a function of the design parameter W. The optimum depends on the ratio $r=c_F/c_B$.

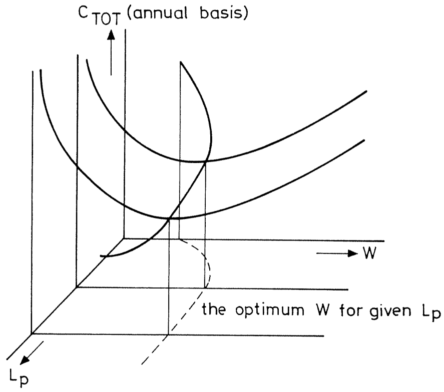

Fig. 23. Optimalization if the planned service life (L_p) is also a design parameter.

it will be advantageous to choose a low depth for the beam. On the other hand, if the cost is relatively high, a design offering greater reliability is to be preferred. Hence it follows that - as contrasted with what is commonly supposed - it does not constitute an optimum solution to make all the parts or components of a structure exactly equal as regards their functional reliability. Relatively cheap components have a high value of r and should therefore be designed to a higher degree of reliability than relatively expensive ones with a low r.

In preparing a diagram as shown in Fig. 22 it is necessary in principle to start from a certain planned service life (design life). Of course, an analysis of this kind can be carried out for more than one value of the design life. In that case a diagram as in Fig. 23 is obtained. With each value of the design life L_p is associated an optimally proportioned design, characterized by a set of optimum structural dimensions. In general, for a short design life the optimum will be obtained with correspondingly small dimensions; the structure will have to be more stoutly proportioned according as L_p increases. This is more particularly relevant if ageing and wear play a part; it is less so in the case of structures subjected to predominantly static load and functioning at normal temperatures.

Of course, the next step in the chain of reasoning is to conceive L_p as a design parameter. Then we can seek that value for L_p at which the lowest minimum for the overall cost is obtained. In that case it is necessary to base ourselves on annual cost, certainly not on cost for the service life. Furthermore, an analysis of this kind is not meaningful for just one individual part or component; the structure as a whole must be considered. Finally, the purpose and the use of the structure must be taken into account. There is no point in designing a structure to last for thirty years, however advantageous that may be on a "per year" basis, if in all probability the need for that structure will no longer exist in, say, ten years' time.

Apart from practical problems associated with the assessment of the various cost factors, the economic criterion does not present many problems so long as the damage (and monetary loss) is confined to strictly economic objects. The situation becomes different when human life or other values that are difficult to valuate in monetary terms

– such as environment, art treasures or scenic beauty – are involved. Of course, it can be endeavoured nevertheless to express all these things in money and thus bring all the factors under one common denominator. There are, however, many people who reject such an approach and who wish to differentiate between economic factors and social/ ethical factors. The risk that a human being can acceptably run in relation to a particular structure is in that case referred to risks that people run in other sectors, e.g., from road traffic accidents, accidents in the home, sports activities, illness, etc. The design problem is thus reduced to finding the most inexpensive design that does not exceed a certain maximum acceptable failure probability.

We now come to the other question posed at the beginning of this section, namely: how are we to determine the properties of the respective stochastic variables? The core of the problem is that in principle there is always a shortage of data for objectively establishing various types of distribution. In order to arrive at results, we are compelled to proceed intuitively in making a number of subjective assumptions. This state of affairs leads some people to the assertion that a probabilistic reliability analysis is impossible or, to say the least, rather pointless. Yet this is hardly a logical argument. What it amounts to is this: They have recourse to probability theory because there are uncertainties involved, but then they reject that same probability theory when the uncertainties turn out to be even greater than anticipated. Besides, it is represented as if, with the rejection of the probabilistic element, the subjective element would disappear too. Of course, that is not so. The subjectivity is inherent in the problem, and an objective solution simply does not exist. No matter how the problem is tackled, there always comes a time when we have to rely on intuitive statements – "engineering judgement". It is precisely by bringing probability theory into the picture that we can help to ensure that we do not have to make any greater appeal to intuition than is necessary.

The branch of probability theory offering the possibility of assigning probabilities on subjective grounds is known as "Bayesian probability theory" [19]. Briefly, this theory comes to the following. So long as data are not available, we start from so-called a priori distributions which are based entirely on an intuitive assessment of the situation. If objective statistical information subsequently becomes available, we convert the a priori distributions by formal procedures into the a posteriori distributions. Then, if certain requirements are fulfilled, it can be shown that, according as more statistical informa-

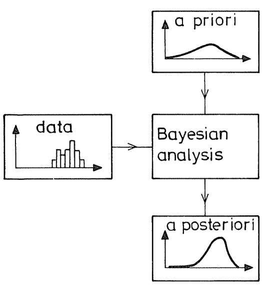

Fig. 24. Principle of a Bayesian analysis.

tion becomes available, the a posteriori distribution will depend less and less on the a priori distribution. In other words: when abundant statistical information has been acquired we finally come back to "ordinary probability theory" (see Fig. 24).

The fact that the stochastic characteristics of the problem variables are interpreted "in a Bayesian way" does of course have consequences with regard to the interpretation of the result. The calculated probability of failure should not be regareded as a frequency of cases of damage which will in fact be observable in the future. It is more difficult to indicate how it should be regareded. Actually, the failure probability is a number which, in a scale extending from 0 to 1, reflects the engineer's confidence (or lack of confidence) in the structure concerned. This number, and also the manner in which it is constituted, can help in making the best possible decisions in the light of the available information and assessments. The advantage over a deterministic design lies in the quantification of the uncertain factors and their effect on the final result. Major uncertainties at the level of the problem variables are often found not to work their way through into the probability failure, whereas other and relatively minor uncertainties may well do so. With the probabilistic approach the designer becomes more clearly aware of where the critical point are located an how these may be taken into account in as balanced a way as possible both in the design and in the subsequent utilization of the structure.

3 Review of general aspects associated with service life and safety considerations

3.1 *Reasons for this review of general aspects*

In general, depending on the type of structure and the environment within which it functions, it often occurs that widely different sets of factors play an important part in the assessment of the safety and service life of engineering structures. With regard to buildings, for example, the structural strength and the climatic influences (wind load, etc.) are important, while in the case of, say, production machinery and technical installations it is often necessary to take account of thermal fatigue, wear, importance of avoiding interruptions in production, etc.

Despite the above-mentioned differences in the aspects included in the service life and safety considerations, these considerations do have a number of features in common for a wide range of different structures. In principle it is therefore possible to set up a more or less general scheme in which all conceivable aspects can be more or less logically accommodated. The use of such a review consists more particularly in the fact that more and more structures are being designed in which a wide range of aspects is simultaneously involved. In such cases the scheme can serve as a check-list with the aid of which it can be verified which elements may play an important part in the safety considerations in a given case.

A possible set-up for a scheme of this kind is given in Section 3.2 of this chapter. The basic idea is that a structure has to endure a number of external influences, develops a response to these, and perhaps attains a limit state.

On the basis of this approach five main inputs have been chosen:
1. external influences;
2. properties of the structure (material and geometry);
3. limit states;
4. special effects;
5. safety criteria.

The external influences and the various forms of response in most cases exhibit a distinctly stochastic character. In every case where service life is a consideration, it will have to be indicated whether or not this stochastic character is to be explicitly taken into account. Limit states should in principle be defined in a deterministic manner.

Errors and certainties of design, execution and operation constitute a special category of aspects in connection with service life considerations. As these aspects cannot readily be accommodated in the first three of the above-mentioned categories, they have been assigned a separate fourth category. Determining the reliability of a structure or estimating its service life is in fact not an end in itself. The results of this analysis will have to be verified with reference to particular safety criteria in order to ascertain whether the design, or the further functioning of the structure, is to be regarded as meaningful. Since it is advisable to adopt a more systematic approach also in imposing these initial requirements, the category "safety criteria" has been included in the check-list.

Each of these five categories will now be briefly discussed.

3.2 *Basis for the set-up of the check-list* (see folding-out page)

Category 1: external influences

Any influence that is able to shorten the service life of a structure or part of a structure must possess energy in one form or another. For the purpose of further subdividing the various influences it is therefore helpful to base oneself on the well-known classification of energy in the forms of mechanical, thermal, chemical energy, etc. A similar approach is adopted, for example, also in [20] and [21].

The *mechanical* influences are, in the first place, the normal static and variable loads. The latter may be further subdivided into rapidly varying loads (with frequencies of the order of the natural frequency of the structure) and slowly varying loads. The static and variable loads will for the most part be due to the usual causes such as dead weight, live load, wind, machinery driving forces, etc. Quite a different type of load is that consisting of impulse loads, separately listed in the review of aspects, which are mostly of a more "disastrous" character: collisions, crashes, explosions, etc. Yet another possible mechanical influence is constituted by imposed displacements of supports, exemplified by earthquake effects and the settlement of foundations.

The *physical* influences include temperature and radiation. Temperature is important for several reasons. For one thing, rises or falls in temperature may cause non-uniform expansion or contraction and thus produce stresses in a structure. Furthermore, temperature has considerable effect on the mechanical properties of the structure. At low temperatures the material may become brittle; at high temperatures there is likely to be a loss of rigidity, strength and creep resistance. Just as with mechanical influences, temperature fluctuations may exhibit a more or less normal character of a "disastrous" character. The rise in temperature of the stay cables of a cable-stayed bridge or the heating-up of an internal combustion engine are examples of normal temperature changes, whereas an outbreak of fire in a building or the boiling-over of a nuclear reactor is to be classed as disastrous.

Chemical influences comprise the action of acids, salts, catalyst poisons, etc. or simply the consequences of rusting due to the presence of air and water.

Finally, there are the *biological* influences, more particularly those due to microorganisms, which are of importance in connection with the fouling of ships' hulls due to growth of organisms attaching themselves.

Category 2: response of the structure

Every influence to which a structure is subjected will call forth a particular response, depending on the nature of the influence concerned. Static forces mostly bring about elastic deformations and stresses; elevated temperatures cause a reduction of strength and rigidity; a varying load causes fatigue; etc.

The various forms of response are not easy to classify, the more so as all sorts of interactions may occur. In [21] a distinction is drawn between two groups, namely, the reversible and the irreversible processes. The purely elastic response is an example of a reversible process; fatigue and wear are irreversible. With some processes the distinction is less clear-cut: creep, for example, may be partly reversible and partly irreversible. Another possibility is to distinguish between responses that are mainly spontaneous and those where the time and ageing effect play a part. This subdivision partly coincides with the preceding one, as in the case of elastic response (reversible, instantaneous) and fatigue and wear (irreversible, cumulative). On the other hand, plastic deformations are irreversible, but mostly the instantaneous plastic behaviour will be important, whereas with reversible creep the time factor is of importance. We must conclude that a clear-cut and fully satisfactory subdivision is not possible. In the review presented here such a subdivision has indeed not been aimed at. The various conceivable forms of response have merely been so arranged that the reversible and/or instantaneous character predominates as we move further up in the list and that the irreversible and/or cumulative time character predominantes as we move further down.

Precisely what forms of response should be included, and where they should be fitted into the scheme, are questions which are of course susceptible of various interpretations. Much will depend on the degree of refinement sought, and on where it is desired to set the limits of the systems to be considered. If we confine ourselves to the purely structural aspects, we arrive at a different list than the one obtained if functional aspects are also included. By way of example, consider the sludging-up of a pipeline. In the purely structural sense this will not necessarily have serious consequences. From the functional point of view, however, it is a form of response that shortens the service life.

Category 3: limit states

In the context of the service life and/or reliability of a structure the so-called limit states play an important part. By limit state is understood a state of the whole system, characterized by a change - whether or not of a distinctly instantaneous kind - which so affects the structure that its further functioning or its continuation in existence will have to be reconsidered. In general, a certain safety margin with respect to the attainment of the limit state is often required in order to avoid the consequences that could result from attaining that state.

The actual limit states of systems can be subdivided into:
- ultimate limit states;
- limit states of serviceability (or fitness for use);
- progressive limit states.

Quite often, too, limit states of fatigue are distinguished, which are related to the development of micro-defects into unacceptable cracks under the influence of a large number of load variations. These do not in fact constitute a separate category, because they eventually merge into one of the above-mentioned categories.

The ultimate limit states are associated with actual cessation of existence of the structure, i.e., complete failure. This may involve fracture, plastic deformation on a large scale, or local or overall loss of stability. The limit states of serviceability are associated with less-than-optimal functioning of the structure, though without loss of mechanical equilibrium. In the latter case it is necessary to give close consideration to the question as to how far it is desired to refine the system and where its limits are precisely to be set. Without as yet committing itself to an explicit choice the "review of aspects" lists a number of limit states of this type.

These matters call for a few explanatory comments. Thus, "inadmissible deformations", "inadmissible cracking" (e.g., in the walls of a liquid storage tank) and "excessive wear" (e.g., in the engine of a motor vehicle) are examples of limit states with a typically technical background. On the other hand, with "unsightliness" the aesthetic aspect plays the dominant part, while with "inadequate efficiency" the economic aspects are more particularly relevant. With regard to the limit state "danger to the environment" we may think of, for example, cumulative radioactivity which must not exceed a particular permissible maximum level. Against this, danger to the environment is sometimes conceived as a second-order limit state: first a particular limit state is considered, e.g., fracture, and the probability of its occurrence is calculated; then it is checked whether or not this probability exceeds a particular limit value.

Besides the above-mentioned limit states, in present-day design methods a specific distinction is sometimes made with regard to the so-called progressive limit states in which the capacity of the structure to withstand and survive harmful influences plays an important part.

Category 4: special effects

It is a well known fact that substantial proportion of structures that have been (or are to be) put into service do not comply with requirements applicable to them, this being due to unforeseen human errors. Although there is generally little statistical information available concerning this type of errors, it is nevertheless necessary to take them into account as well as possible in an overall analysis of service life or reliability. If necessary, missing statistical data should be supplemented with subjective assessments as conceived in Bayesian probability theory. In this category of influences the following groups are to be distinguished:

Uncertainties of design

These uncertainties in the design, which are separate from those in the modelling of the structure, may be associated with anything ranging from relatively unimportant errors of arithmetic to major fundamental errors. Experience with matters of this kind has shown, inter alia, that a substantial proportion of causes of failure is due to possibly critical steps in the design process having been overlooked. The effect of arithmetical errors committed in design calculations has hitherto not been the subject of any appreciable scientific investigation, though examples could be given to show the increase and decrease of such errors. In this context it is very important that the problem should be tackled by independent scrutiny and by the application of many different checking methods, more particularly in relation to the application of increasingly sophisticated analysis procedures and complete numerical computation techniques.

Errors of communication

These may comprise errors and uncertainties due to gaps in communication between the owner and the designer of a structure with regard to the requirements that it has to fulfill, or between the designer and the builder with regard to materials and manufacturing processes, or between the designer and the user with regard to installation procedures, or between the owner and the user with regard to safety regulations, etc.

Uncertainties of manufacture and/or execution

The functional reliability of a structure depends on the quality of the materials and the workmanship actually applied, as compared with the assumptions with regard to these factors adopted in the design. Properly organized quality control and an adequate level of inspection can do much to limit these uncertainties. However, with regard to this class of uncertainties there is likewise an almost complete lack of statistical information.

Commercial and political subsidiary influences

Political or economic factors may cause departures from the original design, the technical installations provided, or the management of a structure, and they may thus adversely affect its reliability and safety. This risk will also increase if insufficient time or capacity (human as well as financial) is available for attaining the required level of quality and specified degree of care in building and commmissioning the structure.

Mathematical models and statistical models
This set of influences which is liable to introduce errors and uncertainties is bound up with the mathematical models chosen to describe the physical processes and with the statistical parameters that have been introduced for the stochastic variables. In their nature these uncertainties resemble human errors.

Category 5: criteria for the assessment of safety/reliability

In considering the safety aspects or the service life aspects of a structure it is relevant to define the objective that is aimed at for the structure. In other words: what is the probability of a desired service life not being attained? For determining this level a number of criteria have to be considered, such as:

Social criteria
In general, what constitutes an acceptable level of risk is determined by the degree in which a particular risk is voluntarily accepted, the reasons why a certain risk is taken, difficulties in verifying a particular level of risk, the number of human lives at hazard, and also the historical background of the risk.

Economic criteria
The reliability or the specified service life is also linked to the economic value, which is determined by continuation of production and by the cost entailed by unserviceability or failure. In this respect it is endeavoured to establish a relation between failure risk, failure cost, cost of construction and level of inflation in order thereby to arrive at an economically justified choice for the desired failure probability or service life prediction, as has already been discussed earlier (Section 2.5).

Environmental criteria
The effect of environmental requirements upon the desired safety levels for present-day structures has greatly increased in recent years, partly as a result of a number of very serious disasters involving considerable danger to the environment of, in some cases, large numbers of people. Offshore activities, transport by sea, etc. are instances in point. It will be obvious that any attempt to refer the above-mentioned factors to one common denominator and to translate the result into a desired safety level must as yet encounter major problems. For the time being, therefore, we must content ourselves with applying some fairly general guidelines.

3.3 *The check-list applied to a number of studies performed*

3.3.1 General remarks with regard to the review of aspects

The check-list presented in the preceding section of this chapter gives a preliminary review of factors which may be of importance in carrying out reliability and/or literature studies. Obviously, a check-list of this kind can never be really complete and thus offer a ready-made solution to every problem. Nevertheless, this check-list does provide a survey of importance influencing factors affecting reliability/service life over a fairly wide range.

Day-to-day experience in actual practice shows that, in carrying out a reliability analysis, it is essential to restrict the number of factors involved in the analysis in order to arrive at a solution and that quite often such restrictions are indeed justified. In connection with problems liable to arise in practice, factors that are really outside the investigator's specific discipline may also play a part. In such cases the check-list enables him to review, in an early stage, all the relevant factors that may be involved and to initiate a soundly based analysis.

For a number of studies in the field of the reliability/service life of structures (carried out, at least partly, at the TNO Division for Building and Metal Research), whether on a probabilistic basis or otherwise, it has been attempted to classify these with the aid of the check-list (see summary of results on page 44).

First in column 0 it is indicated which factors are involved, generally speaking, in an investigation of this kind when 90% of the cases encountered in everyday - mechanical as well as civil or structural engineering - practice are considered. The variables taken into account for a number of studies, briefly described below, are indicated in columns 1 to 12. With regard to the variables belonging to categories "external influence" and "properties of the structure" a distinction is made between a deterministic approach to the variable concerned (designated by D) and a statistical approach (designated by S).

What is especially notable in this table is that in "everyday practice" (column 0) in general only a very limited number of factors is brought into the analysis. The same is true, though to a less extent, of the studies 1 to 12, although the investigators who undertook these studies found then nevertheless to be fairly complex. This fact illustrates the complexity of the whole set of problems with which we are concerned here. A second notable feature is that in some of these studies (more particularly 9 to 12) a large number of factors is indeed taken into account, but that all these factors are regarded as known and given quantities (deterministic). Also, a number of apparent or real discrepancies can be found. To mention just one example: in some of the studies, fracture was particularized as a limit state, although no fracture mechanics analysis was performed. It emerges, too, that - in the studies presented here, anyway - the reliability analysis is often concerned with "technical factors", whereas such concepts as unsightliness, lubrication, chemical influences and non-mechnical thermal loads are only exceptionally taken into consideration. Finally, it appears that only in a few of the studies the reliability that has been determined is in fact compared with the reliability required on the basis of economic factors.

3.3.2 Short description of the studies performed

(The numbering corresponds to the numbers 1 to 12 of the columns in the summarizing table on page 44):

1. Stupoc safety study of offshore structures [22]
   With the aid of level II reliability analyses combined with spectral calculation techniques for wave loading, a simple offshore structure consisting of a single column was investigated. Its safety with regard to maximum loadbearing capacity, fatigue and the foundation was considered. See also *Appendix B.*
2. Lighting columns subjected to wind load [23]
   Lighting columns vibrate under the action of wind load and are thus liable to fatigue. This was analysed with the aid of spectral calculations. Then a procedure for optimum design on the basis of economic criteria was outlined.
3. Reinforced concrete slabs exposed to fire [24]
   With the aid of Monte Carlo simulations it was shown that the reinforcement should preferably be installed higher up in the slabs than has hitherto been normal practice. Although the risk of overloading is thereby increased (as the internal lever arm is reduced), the greater depth of concrete cover to the bars reduced the risk of failure in the event of fire.
4. Surge tide barrier in Oosterschelde [25]
   In connection with the design and construction of the surge tide barrier in the Oosterschelde (Eastern Scheldt) probabilistic reliability analyses were performed for a number of structural members. The results were further elaborated in probabilistic fault tree analysis in which other failure causes were also considered, such as collisions, mistakes of management or failure of hydraulic and electric systems. The resultant failure probability was verified with respect to a previously accepted social norm.
5. Low-cycle fatigue of thin-walled tubes [26]
   Large plastic deformation may be caused by the sometimes quite considerable thermal gradients that are liable to occur in cylindrical members (e.g., of nuclear power stations) when expected and/or unexpected power variations occur. In combination with creep this phenomenon can result in fatigue damage. With the aid of Monte Carlo simulation this damage was analysed as a function of the transients acting on the member and of the material data. The central theme of this study was to investigate the use of probabilistic methods for obtaining a better basis for reliability or safety predictions. See also *Appendix C.*
6. Plastic collapse of a portal frame [27]
   The effect of combined interdependent failure mechanisms was investigated for a simple portal frame.
7. Stress analysis of a mould for die-casting [28]
   By means of the finite element method the thermal stresses occurring in a mould for the die-casting of aluminium parts for coffee-making machines were calculated. On the basis of these stresses and their dynamic character an estimate of the service life expectancy was made, and ways of so improving the design were shought that this life could be increased.
8. Service life predictions for insertion vibrators of a soil compaction barge [29]
   Measurements were performed with a view to determining the vibration ampli-

tudes and stresses in the poker-type soil compacting vibrators for insertion into the ground. Furthermore, these amplitudes and stresses were determined also by calculation. The measured and the calculated values were compared and interpreted. A service life prediction was based on the stresses thus found to occur, and it was investigated how this life could be increased.

9. Gas pipes [30]
   Rapid crack propagation in gas pipelines presents a serious hazard. This is due to the large amount of energy stored up in gas under pressure (as distinct from high-pressure liquids). There is sufficient energy even to cause ductile cracking to proceed in an unstable manner. As a result, cracks hundreds of metres in length can be formed quite suddenly, crack growth rates of up to 300 m/s having been observed. In this project, the resisance to (or energy absorption associated with) unstable crack propagation was studied, and measuring methods were (further) developed.
10. BROS fracture research [31]
    This program was concerned with the evaluation of the risk of small cracks occurring in nozzles and walls of nuclear reactor pressure vessels as defined in the ASME Boiler and Pressure Vessel Code, Section III, Appendix G, where reference is moreover made to WRC Bulletin 175. The first part of the program (BROS I) ran from 1973 to the end of 1979. In the main, it comprised the following investigations:
    - elastic-numerical analyses of the stress field at the tips of nozzle corner cracks;
    - experimental verification of the applicability of those analyses;
    - experimental determination of critical crack size and crack growth rates under fatigue conditions on the basis of elastic criteria ($K_{Ic}, \Delta K$);
    - evaluation of methods for the determination of critical crack sizes on the basis of elastic-plastic criteria (COD, J_{Ic});
    - study of the suitability of acoustic emission for the detection and location of defects.
11. Evaluation of two-dimensional defects [32]
    If inspection of offshore structures reveals small fatigue cracks, these will generally be surface cracks or embedded cracks, i.e., cracks wich extend in several directions at once under fatigue conditions (two-dimensional defects). Much less is known about this phenomenon than about fatigue crack propagation in one direction (one-dimensional defects). Fatigue crack propagation in several directions was studied in this research. The results lent support to the recent hypothesis that this crack propagation takes place in such a manner that maximum relaxation of the material (maximum increase in compliance) takes place.
12. Service life estimation by NDT (non-destructive testing)
    The purpose of this project is to develop a non-destructive testing method for estimating the service life of structures which operate under mechanical load at high temperature. The development process will be backed by destructive testing for evaluating the non-destructive testing method. It will be endeavoured to measure the growth of a defect, or a collective of micro-defects, by means of fatigue tests and metallographic examination, while furthermore the decrease in critical defect size

will be determined by fracture mechanics investigation.

In the studies 10, 11 and 12 the phenomenon of crack growth under fatigue or creep conditions is very much to the fore, while in 12 attention is moreover focused on structural changes and viscous behaviour of the material during service. All these phenomena imply that the material behaviour is time-dependent. This may have an important effect on the nature of a stochastic treatment when that is added to these subjects that have hitherto merely been treated deterministically.

Review of aspects relating to service life considerations

	see page 40 et seq. →	0	1	2	3	4	5	6	7	8	9	10	11	12
1. *External influences*														
mechanical	static force	•			S	S	D	S		S	D	D		D
	variable forces	•	S	S		S	S		D	D		D	D	
	frictional forces													
	impulses										D			
	imposed displacements						S				D			
	imposed accelerations													
physical	temperature				S		D		D		D	D		D
	radiation													
	moisture													
chemical	oxygen + water					D					D			
	acids										D			
	salts		D			D					D			
	poisons													
physicochemical	lubricants or the absence thereof													
biological	micro-organisms													
2. *Properties of the structure (material and geometry)*														
mechanical	elastic	•	S	D		S	S		D	D	D	D	D	D
	plastic				D	S	S	S			D	D		D
	dynamic	•	S	D						D	D	D	D	
mechanical/physical	viscous: creep						S							D
	viscous: damping		S	D						D				
	fatigue	•	S	D			S		D			D	D	
	fracture mechanics (defects)										D	D	D	D
	thermodynamic				D									
	effect of temp. on material properties													
physical	thermal				D		D				D	D		D
	absorption/emission													
	ageing													D
	electrical													
mechanical/chemical/physical	wear resistance													
chemical	corrosion resistance									S	D			
	susceptibility to accretion													
	internal chemical activity													
geometric	shape design		D	D		D	D	D	D	D	D	D		
	slenderness	•												
	dimensional accuracy		S		S	S	S			D	D	D		
	initial imperfections	•				S					D		D	
	initial micro-defects										D			D
	inhomogeneties										D			D

(continued)

	see page 40 et seq. →	0	1	2	3	4	5	6	7	8	9	10	11	12
3. *limit states*														
ultimate limit states	fracture		x	x			x			x	x	x	x	x
	maximum loeadbearing capacity	•			x	x		x						
limit states of serviceability	inadmissible deformation	•				x	x							x
	inadmissible cracking								x			x	x	
	excessive wear													
	unsightliness													
	inadequate efficiency									x				
	danger to environment										x	x		
progressive limit states	insufficient redundancy													
	stress redistribution not possible													
4. *special effects*														
errors and uncertainties in	design									x		x		
	execution/manufacture												x	
	operation/management					x			x					
uncertainties associated with	mathematical models					x			x	x			x	
	statistical models													
general	maintenance													
	inspection										x	x	x	x
	repairs													
5. *damage criteria*														
social criteria	voluntary risk													
	risk modifiable					x								
	injuries/deaths involved					x								
economic criteria	importance to maintain production	•							x	x				
	cost of repairs			x		x								
	cost of inspection													
environmental factors	effects on wildlife					x								
	effect of people and animals etc.					x								

4 Various causes for attaining a limit state

4.1 *Introduction*

If a structure fails, this is due to a particular limit state being exceeded through the action of a particular mechanism. In general, however, that limit state is just one of the many that could have been exceeded, and the mechanism, too, was just one among many possibilities. In designing or assessing a structure it is necessary to take account of this overall set of possibilities. The check-list presented in the preceding chapter serves in preparing an inventory of the factors that could be relevant in that context. It involves:

1. giving due consideration to all the failure mechanisms (limit states) that may play a part, and describing the extent to which these influence one another (taking account of the so-called special effects, where relevant);
2. investigating the statistical behaviour of the parameters (loads, temperature, yield point, etc.) that play a part with regard to the various limit states.

With the aid of the check-list it is possible to obtain a qualitative conception as to which limit states could be so important as to call for a more quantitative analysis. Broadly speaking, the probability of the limit state of structure being attained increases according as there are a greater number of possible causes. These matters will be considered in more detail in the present chapter.

4.2 *A structure conceived as an assembly of series and parallel systems*

Fig. 25a shows an example of an elementary series system. The force S has to be transmitted in its full magnitude by each of the individual elements. If just one element does not succeed in doing this, the structure fails. A series system is therefore comparable to a chain whose strength is equal to that of the proverbial weakest link.

Its counterpart is the parallel system, as shown in Fig. 25b, of which, incidentally, there exist three variants. These variants are (see Fig. 26):

1. The strength of the system is equal to that of its best element.
2. The strength of the system is equal to the sum of the strengths of the individual elements.
3. The strength of the system is equal to the maximum in the force-deformation diagram obtained by summation of the forces in the force-deformation diagrams of the individual elements.

Variant 1 is the extreme opposite of the series system in that now it is the strongest, not the weakest, element that governs the strength of the system. In electronics a system of this type is known as the parallel system with hot reserve. The second variant is called the parallel system with cold reserve. For purely mechanical systems it is more important than the first. For a system like the one shown in Fig. 25b to be behave indeed as envisaged here, the material must possess adequate ductility (toughness). If this condition is not satisfied, it is not permissible simply to add together the respective individual strengths; the overall force-deformation pattern will then have to be considered instead. This brings us to the third variant. An instructive publication dealing with it originated in the fibres industry of the 1940s [34]. A recent review of the present position is given in [35].

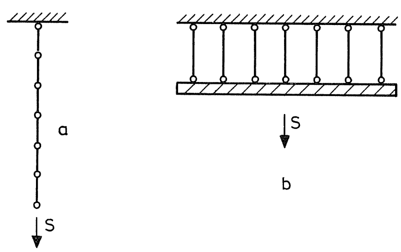

Fig. 25. Series system and parallel system.

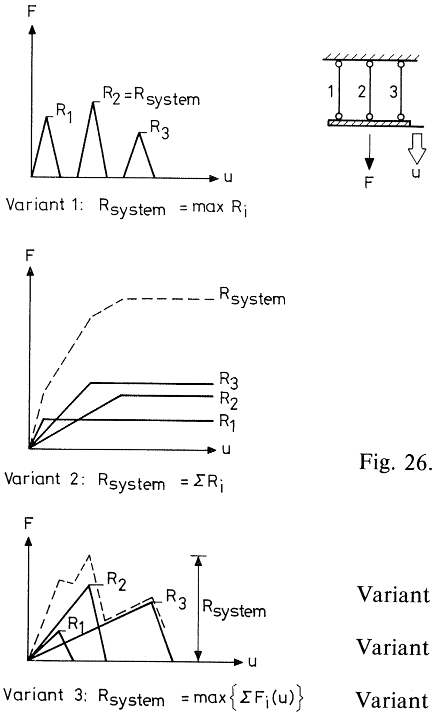

Fig. 26. Variants of parallel systems.
1. strict parallel system
2. ductile parallel system
3. brittle parallel system

Variant 1: $R_{\text{system}} = \max R_i$

Variant 2: $R_{\text{system}} = \Sigma R_i$

Variant 3: $R_{\text{system}} = \max \{\Sigma F_i(u)\}$

From the mechanics point of view the series system corresponds to a fully statistically determinate structure, while the parallel system corresponds to a kinematically determinate structure. Actually, pure forms of these systems are comparatively seldom encountered in practice. Some examples do exist, however (Fig. 27). In order to avoid temperature stresses and residual stresses, many lattice girder bridges are built as statically determinate structures. A telecommunication transmission tower on a piled foundation comprising a large number of piles is an example of an almost ideal parallel system. In most cases, however, both types of system are presented in one and the same structure. Consider the portal frame shown in Fig. 28, for example. According to elementary plastic theory for frames this structure has three possible collapse mech-

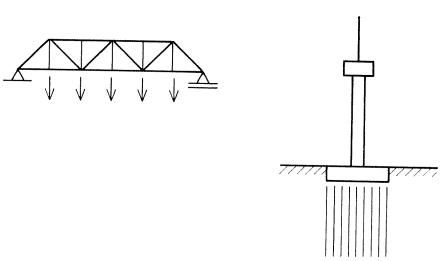

Fig. 27. Examples of a series and a parallel system in practice.

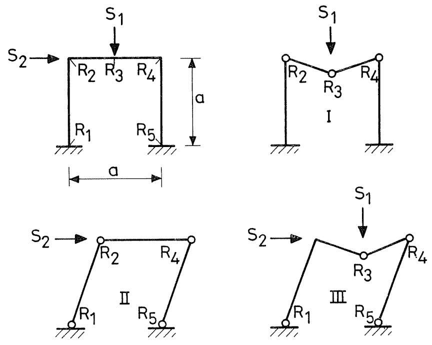

Fig. 28. Portal frame with three collapse mechanisms.
I beam mechanism
II sidesway mechanism
III combined mechanism

anisms, namely, a beam mechanism, a sidesway mechanism, and a combined mechanism. For each of these mechanisms (collapse modes) a reliability function can be defined:

$$Z_1 = R_2 + 2R_3 + R_4 - (\tfrac{1}{2})S_1 a \tag{32a}$$

$$Z_2 = R_1 + R_2 + R_4 + R_5 - S_2 a \tag{32b}$$

$$Z_3 = R_1 + 2R_3 + 2R_4 + R_5 - (\tfrac{1}{2})S_1 a - S_2 a \tag{32c}$$

where S_1 and S_2 are the loads and R_1 to R_5 are the moments that can be resisted at the respective points in the structure, while a denotes a deterministic length. The parallel character of the structure is manifested in the adding together of several R_i values within one and the same mechanism. The series character is bound up with the presence of several mechanisms, each with its own Z_j. Thus there are indeed various mechanisms involved, but each of these in itself forms a parallel system.

In the above-mentioned example there were various causes of failure, but they all related to the plastic limit-loadbearing capacity being exceeded. Of course, failure may be caused by much more widely divergent mechanisms such as plasticity, buckling, creep, fatigue, fire, etc. The various mechanisms may moreover affect one another, though that does not make very much difference with regard to the essence of the reliability problem. Therefore, in the next sections of this chapter, the elementary series system envisaged in Fig. 25 will be dealt with in detail. In Section 4.3 this will be done in the R and S domain, while the time aspect will largely be left out of account. The analysis in the time domain will be considered in Section 4.4.

4.3 *The series system; R and S domain*

Consider a series system comprising n elements. For each element a reliability function Z_i can be established:

$$Z_i = R_i - S_i \tag{33}$$

Per element an associated failure probability can be determined, as explained in Chapter 2:

$$P_{fi} = P\{Z_i < 0\} \tag{34}$$

In this way we find n separate failure probabilities. This information is not sufficient to enable the failure probability of the whole system to be determined, however. But it is possible to indicate lower and upper bounds [36], namely:

$$\max\{P_{fi}\} \leqslant P_f \leqslant \Sigma P_{fi} \tag{35}$$

In Fig. 29 the lower and upper bounds for P_f, as well as a possible curve for P_f itself, are indicated as a function of n for the case where all the probabilities P_{fi} are equal. Clearly, the lower and upper bounds diverge rapidly for large values of n. Indeed, these bounds have significance only if one or a few probabilities P_{fi} dominate or if there are indications that the system is close to one or the other bound, In order to investigate this latter possibility, we shall now proceed to prove (35) and to ascertain precisely when the bounds are reached. For the sake of simplicity we shall do this for the case $n = 2$. The relationships is fundamentally no differnt for any value of n.

If $n = 2$, the exact probability of failure can be written as:

$$\begin{aligned} P_f &= P\{Z_1 < 0 \quad \text{of} \quad Z_2 < 0\} = \\ &= P\{Z_1 < 0\} + P\{Z_2 < 0\} - P\{Z_1 < 0 \quad \text{and} \quad Z_2 < 0\} \end{aligned} \tag{36}$$

Since a probability is, by definition, positive, the last term of (36) is positive. Hence we may write:

$$P_f \leqslant P\{Z_1 < 0\} + P\{Z_2 < 0\} \tag{37}$$

Thus the right-hand inequality in (35) has been proved for the case $n = 2$. It follows, too, that the upper bound is exactly satisfied if the last term of (36) is zero. This is the case if the mechanisms associated with $Z_1 < 0$ and $Z_2 < 0$ are mutually exclusive, i.e., if the strength of the first element is less than the load, it is impossible for the second element

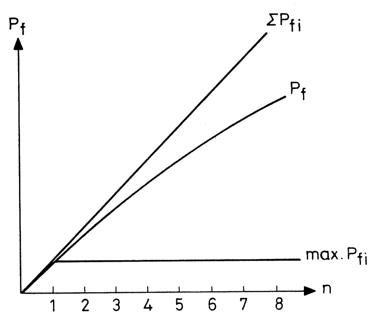

Fig. 29. Lower and upper bounds for the failure probability P_f of a series system comprising n elements with equal individual failure probabilities P_{fi}.

to have a strength that is also less than the load. A situation of this kind does not, of course, occur in actual practice and therefore the upper bound is hardly ever entirely exact. If the events $Z_1 < 0$ and $Z_2 < 0$ are not mutually exclusive, but are independent of each other, the difference between P_f and the upper bound is often very small, however. For then, basing ourselves on (36), we have:

$$P_f = P\{Z_1 < 0\} + P\{Z_2 < 0\} - P\{Z_1 < 0\}P\{Z_2 < 0\} \tag{38}$$

The product term can usually be neglected in relation to the linear terms. The condition is that the probabilities $P\{Z_i < 0\}$ are low. Besides, equation (44) in Section 4.4 is an expression for the failure rate which is exact if all the events $Z_i < 0$ are independent of one another,, from which the failure probability P_f can then be calculated, also for large values of n.

For extending the proof to the lower bound it is necessary to work out (35) further:

$$P_f = P\{Z_1 < 0\} + P\{Z_2 < 0\} - P\{Z_2 < 0\}P\{Z_1 < 0|Z_2 < 0\} \tag{39}$$

which can be rearranged to:

$$P_f = P\{Z_1 < 0\} + P\{Z_2 < 0\}[1 - P\{Z_1 < 0|Z_2 < 0\}] \tag{40}$$

By definition a probability is always less than 1, so that the term in square brackets is positive. Therefore:

$$P_f \geqslant P\{Z_1 < 0\} \tag{41}$$

It can of course similarly be shown that also $P_f \geqslant P\{Z_2 < 0\}$, thus proving the left-hand inequality in (35). The lower bound is exactly attained if $P\{Z_1 < 0|Z_2 < 0\} = 1$, i.e., the one mechanism implies the other. In other words, there are now no random differences between the various elements; instead, all the elements have precisely the same - though stochastic - strength. In statistical terms this means that the various mechanisms are completely (positively) correlated or are completely interdependent.

Therefore, as a good approximation, it can be stated that the upper bound occurs in the case of independence, the lower bound in the case of complete positive dependence. In reality there mostly occurs partial dependence. Calculations shown that the upper bound then in many instances does nevertheless provide a serviceable approximation [37]. For not too large values of n and not too high a correlation, P_f is fairly close to the upper bound. If greater accuracy of the results is required, there are methods providing closer approximations, as described in [38, 39, 40, 41], for example. Despite the fact that those methods demand less arithmetical work than a level III procedure, there is as yet no satisfactory solution available for really large values of n (in excess of 100, say). At present, efforts are being made to find procedures that link up as far as possible with level II analysis of the individual mechanisms. So far, however, these efforts have not produced a satisfactory result. Perhaps the best available possibility at present is a combination of Monte Carlo with level II, i.e., a Monte Carlo simulation is performed, not for the purpose of directly determining P_f, but in order to determine a mean and a standard deviation for $Z_{\min} = \min(Z_i)$. This procedure requires far fewer simulations, and the failure probability is ultimately obtained via $P_f = \Phi_N(-\beta)$ - where $\beta = \mu(Z_{\min})/\sigma(Z_{\min})$ - or possibly other distribution function.

4.4 *The series system; time domain*

4.4.1 Some comments on one possible limit state

The methodology described in the foregoing resolves itself into the following: for a particular time interval we suitably define R and S; then we determine, from the distribution functions of R and S, the probability that the event $Z = R - S < 0$ will occur during that interval. A special choice for R and S is that the "consumed" operating time or service time (t) is defined as S, while the total service life (L) is defined as R. In that case therefore the distribution function of R is immediately equal to F_L. With this choice the service life (L) is expressed as a function of a number of other quantities (x_1, x_2, etc.):

$$L = t(x_1, x_2, \ldots, x_p);\ p \geqslant 1 \tag{42}$$

The distribution function of L is then calculated from this.

Because of the (apparent) simplicity of this choice, it will, in the following treatment of the subject, be assumed that it is indeed always a possible choice. The same treatment is, mutatis mutandis, applicable also in conjunction with other definitions of R and S, however (see also Section 2.4.3).

A worked example for one failure mechanism, namely, creep crack growth, is given in *Appendix A*. In that example a Weibull distribution is found for F_L. This is not very surprising, the reason being that a service life distribution often has to be a smallest-value distribution. The Weibull distribution is the only such distribution which is directly - i.e., without transformation of the parameters or truncation - bounded on the left. In many other cases of fatigue and creep, if only one possible limit state is considered, an approximation based on the Weibull distribution is likewise satisfactory [42, 43, 44, 45].

4.4.2 Mutually independent causes

If several possible limit states have to be considered simultaneously, an exact expression can be found (as already noted in Section 4.3) for the case where the causes of attaining the limit states are statistically independent of one another. In that case a functional relationship as in (42) must be found for each possible limit state. In view of the independence these relationships can be represented as follows:

$$\begin{aligned} L_1 &= t_1(x_1, \ldots, x_p);\ p \geqslant 1 \\ L_2 &= t_2(x_{p+1}, \ldots, x_q);\ q \geqslant p+1 \\ L_3 &= t_3(x_{q+1}, \ldots, x_r);\ r \geqslant q+1 \end{aligned} \tag{43}$$

For each L a distribution function (F_1, F_2, F_3, etc., respectively) can be calculated. From these, with the aid of (6) and (9), we can calculate for each limit state an (instantaneous conditional) failure rate r_1, r_2, r_3, etc. The failure rate due to all the failure mechanisms collectively is then obtained by adding together the failure rates of the individual failure mechanisms. Proofs for the permissibility of this summation are given in *Appendix D* (see also the example on p. 217 of [46]). These proofs are conducted on similar lines to those given in the preceding section and make use of the statistical definition of independence: the events A_i and A_{i+1} are mutually independent if $P(A_i \text{ and } A_{i+1}) = P(A_i)P(A_{i+1})$. Hence we can write:

$$r = r_1 + r_2 + \dots \tag{44}$$

With the aid of (10) and (6) we can determine from r the distribution function and the probability density function for the attainment of a limit state due to all the causes together.

For applying (10) it is convenient to have for $r(t)$ an integrable closed-form expression. In the literature some expressions of very general applicability are indeed to be found, but quite often they are very complicated, e.g., see [47]. A simpler expression is obtained by basing oneself on an exponentially increasing failure rate:

$$r = r_0 \exp\,(t/t_c);\; t > 0, \tag{45}$$

where r_0 is the failure rate at time $t = 0$ and t_c is a characteristic time constant. Substitution of (45) into (10) and integration between the limits 0 and t yields a Gumbel distribution truncated on the left at $t = 0$.

An example of the applicability of this formulation is given in *Appendix E,* where the form of the associated distribution function and probability density function is presented, while moreover an acceptable explanation is given to show that this expression will give satisfactory results for the failure rate of structures, or parts of structures, that are subject to many possible causes of attaining a limit state.

4.4.3 Mutually dependent causes

Two different reasons why failure mechanisms may be interdependent are to be distinguished:

1. there is only statitical interdependence: the mechanisms do not affect one another physically;
2. the mechanisms do affect one another physically.

First, case 1 will be considered, i.e., the case where the various failure mechanisms do not accelerate or retard one another, but where the various expressions for L share a number of parameters in common, for example:

$$\begin{aligned} L_1 &= t_1(x_1, \dots, x_p);\; p \geqslant 1 \\ L_2 &= t_2(x_m, \dots, x_q);\; q \geqslant m,\; m \leqslant p \\ L_3 &= t_3(x_n, \dots, x_r);\; r \geqslant n,\; n \leqslant q \end{aligned} \tag{46}$$

where the expressions for L_1 and L_2 share the parameters x_m to x_p, the expressions for L_2 and L_3 share the parameters x_n to x_q, etc.

This case can in principle be solved at level III, e.g., by means of Monte Carlo simulation. With that procedure, at each simulation step a value is assigned to all the parameters x_i, and then the smallest of the values of L_i calculated with those parameters is chosen. In this way, after the requisite number of simulation steps have been performed, a corresponding number of values of L will have been obtained, to which a distribution function can be adapted. If desired, the values of L can be designated as being L_1, or L_2, or L_3, etc., so that an insight into the dominance of one or more failure mechanisms is obtained. However, because of the elaborate nature of level III calculations, the approximate methods mentioned in Section 4.3 deserve consideration.

The case where the failure mechanisms affect one another in the physical sense, as envisaged in case 2 above, calls for some comment, too. What characterizes these instances of failure is that, if it is attempted to take account of the various failure mechanisms collectively by means of a life fraction rule (e.g., the Palmgren-Miner rule), the right-hand member of this rule may be larger or smaller than 1. An example is provided by the interaction between creep and fatigue. For instance, it may be attempted to describe the life of a specimen, exposed to one set of creep conditions and one set of fatigue conditions, by means of the following life fraction rule:

$$t/t_R + n/n_R = D \tag{47}$$

where:

- t = exposure time of the specimen to the creep conditions
- t_R = length of time that results in failure under these conditions
- n = number of cycles for which the specimen is exposed to the fatigue conditions
- n_R = number of cycles that results in failure under these conditions
- D = criterion for failure: in this case fracture

Therefore the first term of (47) representes the fraction on the service life under creep conditions, and the second term represents the fraction under fatigue (see Fig. 30) con-

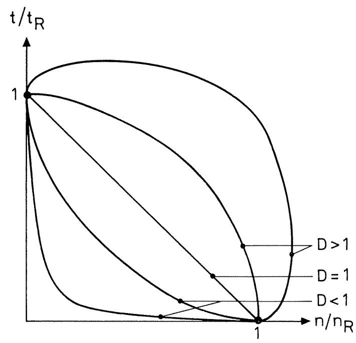

Fig. 30. t/t_R (creep) vs. n/n_R (fatigue) for cases where creep and fatigue accelerate each other ($D < 1$), can simply be added ($D = 1$), or retard each other ($D > 1$).

ditions. Now if, for example, D is smaller than 1, this means that D is dependent on the ratio of the two terms in the left-hand member of (47), for if one of these terms is zero, then (by definition) $D = 1$. Creep and fatigue therefore accelerate each other (synergistic effect). So in this case it is evidently not possible to speak of separate damage mechanisms; instead, the two phenomena must be considered together in order to arrive at one formulation (42), or a valid life fraction rule must be found in which the right-hand member is equal to 1 (see [48], for example).

4.5 *Some examples*

Numerous examples of the simultaneous occurrence of various causes for the attainment of a limit state, inclusive of special effects, can be obtained from the check-list given in Chapter 3. In the present section a few comments on two simple examples of the combined occurrence of two failure causes will merely be offered.

The first example is that of *leakage and fracture.* Under the heading "cracking" in Section 2.3.4 it has already been noted that, for example, in pressure vessels "leak before break" is much preferable to the reverse, "break before leak". For this reason several criteria to ensure the first of these two alternatives have been developed. It can be stated that checking a "leak befor break" criterion is not sufficient; the probabilities of the occurrence of leakage and of fracture separately must be calculated, from which in turn the overall failure probability and the probability of "leak before break" can be calculated. Alternatively, an existing "leak before break" criterion may be used and a probabilistic analysis be performed on it.

The second example to be considered here is that of *fire and overloading.* As a rule, fire is a typical instance of a failure cause with a constant failure rate. On the other hand, with overloading it is necessary to take account of possible reduction in strength of the structural material as time passes, resulting in increased failure rate.

As already stated, these are merely two examples of cases where only two causes of failure occur in conjunction with each other. Of course, a formulation such as (44) is suitable for simultaneously taking account of many more mechanisms, but it will, for practical reasons, often be necessary to apply restrictions.

4.6 *Summary*

In general, a structure has a large number of limit states. Whenever one of these is exceeded, the structure can be said to have failed. Besides, there is an even larger number of mechanisms through which those limit states can be attained. In the terminology of reliability analysis this means that we are concerned with a series system: failure occurs as soon as one of the mechanisms results in one of the limit states being exceeded. So long as the various mechanisms are stochastically independent of one another, the analysis generally presents no problems. In the R and S domain the summation of the failure probabilities of the individual mechanisms gives an excellent approximation, and in the time domain the procedure of simply adding together the conditional failure rates even provides an exact solution. Also in the case of complete interdependence there are few problems. There is then always a governing mechanism, namely, that which involves the greatest failure probability in a particular time interval. Of course, the real problem is that in most cases there is partial dependence. If the correlations are not too high and the number of mechanisms is not too large, the solutions for mutually independent mechanisms provide serviceable approximations. If there is a large number of closely correlated mechanisms, however, a more accurate analysis is necessary. Various possibilities have been described in the literature. A choice will have to be made from among these in each individual case considered. At present a judicious combination of level II and Monte Carlo would appear to be the most appropriate method.

5 Summary, conclusions and possible continuation

In this report it has been attempted to present a *general philosophy* for the assessment of the safety, reliability and service life of structures. There exists a relationship between the reliability of structures and the accuracy or certainty in the prediction of service life. Existing uncertainties can often be merely reduced by further research, not eliminated. A correct approach to dealing with those uncertainties is possible only within the framework of probability theory.

Assessing the safety of a structure comes down, in principle, to the comparing of two quantities: *the load and the strength.* Failure occurs as soon as the load exceeds the strength. Taking this conception as the starting point, a basic philosophy was developed in Section 2.2. The simplest case occurs when the strength and the load are both *independent of time.* A situation of this kind is encountered when, for example, the load is due to the dead weight of a structure which is not appreciably subject to ageing. For those cases the probability of failure is obtained through convolution of the probability density function of the load and the distribution function of the strength; see Fig. 31.

In most cases, however, *time effects* (Fig. 32) cannot be neglected. Loads may vary with time; material ages or is susceptible to cumulative effects. The concepts of load and strength may, in cases of this kind, moreover usually be defined in more than one way. For example, in the case of fatigue the load may be defined as the instantaneous value, and the strength be allowed to decrease with time as a result of fatigue. Alter-

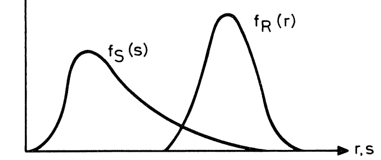

Fig. 31. Probability density functions for load S and strength R.

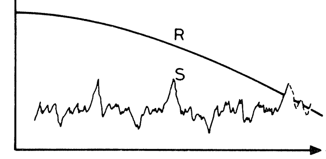

Fig. 32. Strength and load plotted as functins of time.

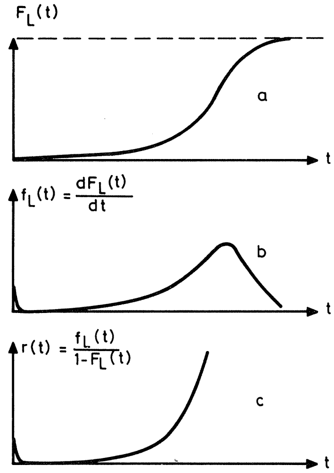

Fig. 33. a. Distribution function for service life L.
b. Probability density function for service life L (unconditional failure rate).
c. Conditional failure rate; the curve is known as a bathtub curve because of its shape.

natively, however, a constant strength may be adopted and the load be defined as a cumulative effect. The first of these two approaches is perhaps more closely in line with physical reality; in other respects, however, the two are equivalent. Which of them is to be preferred can be made to depend entirely on the computational amenability of the problem. If possible, the available freedom can best be so utilized that strength and load are independent, while one of the two is time-independent.

In the case of time-dependence the probability of failure is always calculated for a particular period, e.g., the interval $(0, T)$, where T may be an intended service life or inspection period. By calculating the failure probability for several values of T we obtain the *probability distribution for the service life* (see Fig. 33a). Differentiation of this yields the probability density function of the service life, known also as the *unconditional failure rate* (see Fig. 33b). On dividing this failure rate by the completement of the failure probability, we obtain the *conditional failure rate* (see Fig. 33c) or “hazard function”.

Both the unconditional and the conditional failure rate indicate the probability that the load will exceed the strength in the time interval $(t, t + \mathrm{d}t)$, divided by the interval duration $\mathrm{d}t$. The unconditional failure rate relates to the probability of the strength being exceeded in the interval considered *and* not earlier. The conditional failure rate likewise relates to the probability of the strength being exceeded in the interval considered, *given the condition* that this does not occur earlier. In both cases there is therefore a tie-up with the period preceding the interval considered, and it is this fact that is at the centre of the difficulties encountered in seeking to calculate these functions. In many instances these relationships with the past are accordingly neglected or simplified. What consequences this is liable to have in various cases has not yet been properly clarified and will have to be a subject for further research.

Which of the three above-mentioned functions will have to be used in a particular case will moreover depend on circumstances. The distribution function and the unconditional failure rate are often considered to be of importance for structures whose effective service life is determined by factors other than within the structure itself. But if the effective service life is to be determined partly on the basis of the failure risk, it becomes necessary to base oneself on the conditional failure rate (e.g., the "bathtub curve" in Fig. 33c). If the curve is decending, no replacement for technical reasons must be done. If it is ascending, the conditional failure rate may be just as important as the reliability for the whole service life. It is to be noted, incidentally, that the unconditional and the conditional failure rate do not show any major numerical differences in the range that is usually of importance.

In general, the calculation of a failure probability is a computationally intensive operation, especially if an exact analysis is aimed at. This being so, in many cases we must content ourselves with *approximations.* An extensively used approximate method is the so-called level II analysis, in which approximations are applied by means of linearizations and substitutive normal distributions. This level II occupies the middle position in a subdivision into three levels, in which level III denotes the exact probabilistic analysis, and level I the so-called semiprobabilistic practical analysis. The three levels defined in this way should not be regarded as unconnected with one another. The object is to use the same concepts, as far as possible, on all three levels and to compare the results obtained. However, it cannot be said that satisfactory approximate procedures are available for all problems in his field. More particularly the case where there are several correlated causes of malfunction still presents unsolved problems. If level III calculations become too unwieldy for the purpose, an approach based on Monte Carlo simulation would at present appear to be one of the most appropriate methods. In order to keep the number of simulations within acceptable limits, we would content ourselves with a level II-type analysis on which only a mean value and standard deviation of a failure-relevant quantity are determined. It will then be necessary to make an assumption as to the nature of the distribution (normal, Weibull, etc.).

Besides the above-mentioned computational problems associated with probability and service life analyses, there are problems relating to the availability of the necessary data. In general, there will not be enough data available to enable statistical distributions to be estimated with sufficient reliability. From this state of affairs it is often concluded that probabilistic analysis is obviously an impossibility. Such a conclusions is indeed inescapable if we completely hold on to the conventional frequentistic probability concept. An alternative possibility, however, is to change over to the ***Bayesian** probability concept*, which offers the possibility of adding the missing information on an intuitive basis (see Fig. 34). The objections attaching to the introduction of subjective elements into the analysis can be countered by pointing out that otherwise a subjective assessment will have to be made at some other point in the analysis anyway. Thus the Bayesian analysis is no more than a tidy method of dealing with the intuitive aspects and moreover of keeping the influence thereof to a minimum.

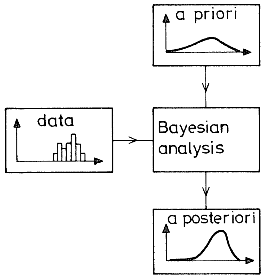

Fig. 34. Principle of a Bayesian analysis: an intuitive a priori distribution is combined with objective data to give a so-called a posteriori distribution, with the aid of Bayes's theorem.

The failure of a structure is caused by a number of external influences that act upon the structure from the environment. Generally speaking, these influences may be of a mechanical, physical, chemical or biological nature. To translate an external influence of this kind into an actual load acting on a structure will in principle require a *mathematical model*, as will also the determination of the response from the load (see Fig. 35). Mathematical models are just as essential in a probabilistic analysis as in a deterministic one. In principle, too, the same models can be used for both types of analysis. A number of commonly employed mathematical models are briefly described in Section 2.3.

It is of course very important that, for dealing with a particular structure, the correct mathematical models are set up and that the appropriate experts are consulted. A prime requirement is to have a proper understanding of the hazard to which the structure may be exposed and how it will - for given material and geometry - respond to them. To assist the designer in these matters, a *check-list* (see Fig. 36) has been included in this report (Chapter 3). This list lays no claim to completeness or to offering an ideal solution; it is to be regarded as an initial attempt in that direction. It can be enlarged or improved in the future. Also, the list does not offer any criterion of the intensity at which any particular phenomenon could be disregarded from the outset. Indeed, this appears very difficult, having regard to the unnumerable interactions that are possible. Thus, "mechanical stress" and "chemical aggression" may together constitute a much more serious hazard than the mere sum of the two actions occurring separately would lead us to suppose. Applying the check-list therefore does not offer a guarantee against unpleasant surprises. It can, however, provide a safeguard against "forgetting" to make

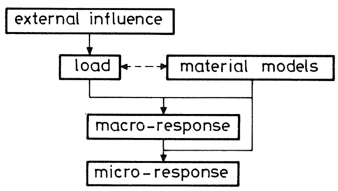

Fig. 35. Diagram of an analysis.

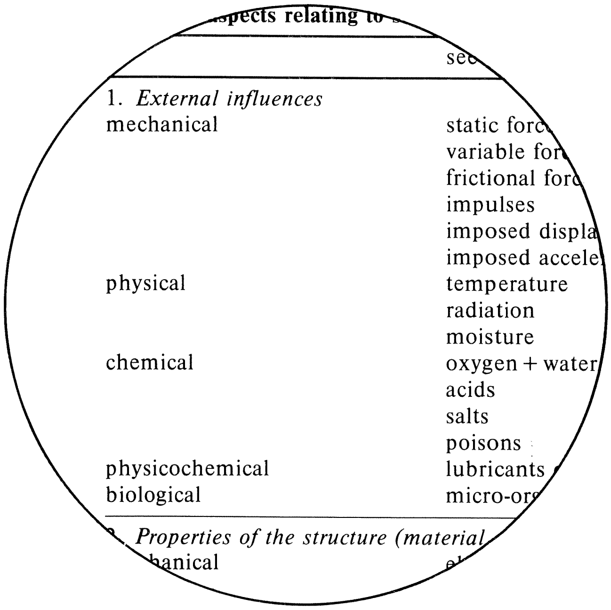

Fig. 36. Check-list (detail).

account of particular influences and furthermore serves as a suitable framework for efficient investigation in the event of unexpected difficulties nevertheless occurring in practice.

In Chapter 4 some attention is devoted to the simultaneous occurrence of several possible causes of failure (see Fig. 37). A *service life distribution function* (see Fig. 38) is proposed which, backed by theoretical considerations, can be expected to possess more general validity for structures exposed to many simultaneous causes of failure. It is considered important to seek information that will support, or refute, this expectation.

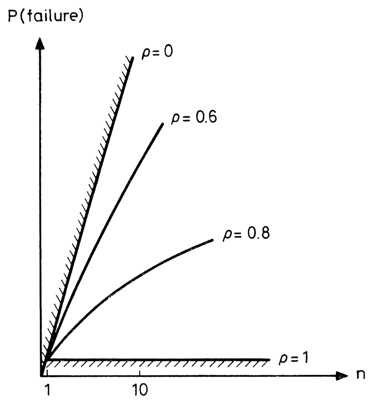

Fig. 37. Failure probability as a function of the number of possible causes of failure *n* and their mutual correlation *p*.

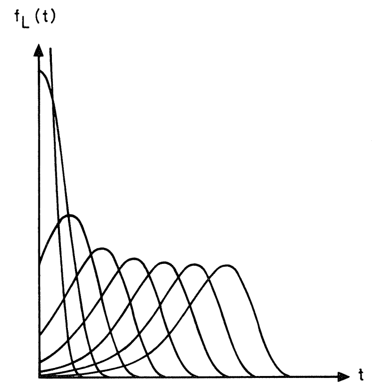

Fig. 38. Possible shapes of probability density functions of the service life when there are several possible causes of failure.

The *objective* of this research has been to investigate whether, in the sphere of safety and service life, it would be possible to achieve harmonization of conceptions between engineers of different disciplines, more particularly civil engineering, mechanical engineering and metallurgy. It clearly emerges from the research that this objective is by no means simple or easy to achieve. The problem that an engineer encounters in his day-to-day practical duties are of major influence on the way in which he approaches such matters as safety and service life. In civil (and stuctural) engineering, for example, the concept of "service life" does not attract any great interest, certainly not as a parameter to be optimalized. Yet design engineers in this field are alive to the uncertainties besetting structural analysis. This state of affairs means that while they do not, on the one hand, feel much need for accurate mathematical models, they are, on the other, very receptive to probabilistic thinking. In mechanical engineering the environmental conditions of a structure are generally of a more extreme nature than in civil engineering, but are generally also better known. Therefore the use of sophisticated mathematical models is more likely to prove advantageous (high temperature, fatigue, etc.). In the sphere of safety and service life the emphasis is on the "safety aspect", to which a probabilistic approach is increasingly being applied. The materials technologist, finally, is often confronted with problems that can primarily be solved in the laboratory. Because of this, there is a preference to obviate the uncertainties as far as possible by means of experimental research and the application of physical models. This approach is obviously associated with a more deterministic pattern of thinking. Considerable attention is focused on the "service life aspect", i.e., the service life is regarded as a design parameter or otherwise, in the case of an existing structure, is determined partly also on the basis of safety considerations.

The differences in conceptual approach outlined above – inevitably in a schematic way – should not always be regarded as undesirable. However, engineers of different disciplines have to work in collaboration with one another in complex and socially important constructional projects (nuclear power stations, tanks for liquefied natural gas, surge tide barrier in the Oosterschelde, offshore structures). They should therefore be able to establish contacts with one another's specialized skills at the appropriate stage and to communicate without difficulties. The safety of the structure is their joint concern, and it is important that each can accommodate his specialized knowledge correctly in the overall analysis. To this end, it is essential to have a shared overall concept of the safety problem with terminology, methodology and interpretation that are clear to all concerned. The present authors trust that they have made a contribution to that aim with this report. Concepts from the various technological disciplines have been collected and placed in relation to one another or accommodated in more comprehensive schemes. It has been attempted to present a generally-utilizable approach for the assessment procedure relating to a wide range of structures. It virtually goes without saying that a first-time attempt of this kind can, despite much effort, never claim to be one hundred per cent successful in achieving its stated objective. What it has merely done is to start out on a road and to ascertan that it is necessary to proceed further along this road. Besides, this will involve several lines of advance. Firstly, there are still many theoretical problems waiting to be explored in depth, more particularly in connection with the occurrence of several causes of failure. Secondly, it is important to try to solve a number of practical problems with the theory presented here. This will undoubtedly lead to modification and adaptation of the schemes and procedures given in this report and to a more precise demarcation of sets of problems which as yet have not been very clearly established. Finally, there is a third line of advance: publicity. It would indeed be rather pointless if these conceptions were recorded in a report that would be consulted and applied merely within a small circle of specialists. So it will be necessary to engender interest in these matters, both within and outside the orbit of TNO, e.g., by means of publications, courses or symposia. Besides, the reactions that can be expected from these initiatives will be essential to the conduct of the further research that has to be done.

Conclusions

1. The research has shown that there are differences in conceptual approach between the respective technological disciplines concerned. The aim to harmonize these different approaches has resulted in the *"basic philosophy"* and the *"check-list"* (as an instrument for the demarcation of reliability and/or service life problems) presented in this report.
2. Studies relating to the reliability/service life of actual structures are at present hampered mainly by the large amount of computational effort involved (computer costs) and by the lack of sufficient (statistical) information. *Schematizations* and *approximations* are therefore necessary. Although good progress has been made in recent years and serviceable approximations have become available, many problems are still unsolved or inadequately solved. Thus, notably it is necessary to gain more insight into the consequences of particular schematizations and/or the effect and scope of the approximations introduced through the mathematical models.

3. In applying probability theory to the assessment of the safety of structures it is preferable to base oneself on a *Bayesian* (subjective) interpretation of probability. In general, not enough statistical data are available to allow successful application of a frequentistic probability interpretation.

Essential lines of *further research:*
1. Applying the check-list and basic philosophy, presented in this report, in actual practice and supplementing or adjusting them on the basis of the practical experience that will be grained with them.
2. Further *elaboration* of the *sub-problems* discussed in the report and the verification of solutions for existing or future practical reliability/service life problems. In particular, this should comprise the solving of computational problems, the study of several correlated causes leading to the attainment of limit states and, finally, the evaluation (technical/economic) of the reliability/service life levels to be determined.
3. Giving wider publicity, both within and outside TNO, to the conceptions that have evolved from this research - by means of publications and/or symposia.

6 Notation

$f_X(\xi)$	probability density function of X
$F_X(\xi)$	distribution function of X
$f_L(t)$	probability density function for service life
$F_L(t)$	distribution function for service life
L	(service) life, lifetime
$P\{...\}$	probability of event $\{...\}$
P_f	probability of failure
$r(t)$	conditional failure rate
$R(t)$	strength as a function of time
$S(t)$	load as a function of time
t	time
Z	reliability function
β	reliability index
$\mu(X)$	mean value of X
$\sigma(X)$	standard deviation of X
$\Phi_N()$	distribution function of normal distribution

7 Appendices

A. *Example of procedure in the time domain with one possible cause of failure*

1 *Background and set-up of the analysis*

The serivce life of a structure or part of a structure can often be described deterministically with the aid of a formulation for the growth of a defect (or a collective of defects), e.g., in the form:

$$\mathrm{d}\bar{a}/\mathrm{d}t = f_{\mathrm{growth}}(y_1, y_2, \ldots, y_n) \quad \text{(A1)}$$

where:

$\bar{a}$ = mean characteristic magnitude of the defects
t = time
f_{growth} = a known growth function
$y_1 \ldots y_n$ = parameters which may be dependent both on $\bar{a}$ and on t

Integration of (A1) between the initial value of $\bar{a}$ (a_0) and the critical value thereof (a_c) (which may moreover be time-dependent) yields the service life of the structure considered.

An example of such a growth law is obtained by application of the "damage model" (see *Fig. A1)* to an austenitic steel under creep conditions (constant stress, high temperature). In the fracture mechanics literature various quantities are introduced, of which it can be supposed that they govern the growth of a defect (or a collective of defects) – such as K_I, J_I, C^* and σ_{net}. If we assume that this last-mentioned quantity, the ligament stress (σ_{net}), is the defect-governing quantity for this material under these conditions, we can define the growth law as follows:

$$\mathrm{d}\bar{a}/\mathrm{d}t \div \sigma_{\mathrm{net}}^{p} \quad \text{(A2)}$$

From the model we obtain the ligament stress:

$$\sigma_{\mathrm{net}} = \sigma/(1-\lambda) \quad \text{(A3)}$$

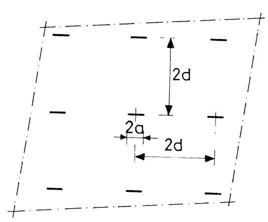

Fig. A1. Collective of defects, 2D model. Here it is assumed that: $\bar{d}_1 = \bar{d}_2 = \bar{d}$ and defined: $\lambda = \bar{a}/\bar{d}$.

where σ denotes the (constant) nominal stress.

With the definition of λ:

$$\lambda = \bar{a}/\bar{d} \tag{A4}$$

we can formulate the growth law as follows:

$$\mathrm{d}\lambda/\mathrm{d}t = C\{\sigma/(1-\lambda)\}^p/\bar{d} \tag{A5}$$

The measure for the mean spacing of the defects ($\bar{d}$) is here regarded as constant, as also are C and p. By comparison with growth laws of similar type, and from fracture mechanics considerations concerning the energy required for plastic deformation associated with the propagation of defects, it can reasonably be supposed that the magnitude of the exponent p is not less than 2. With these assumptions it is possible to integrate (A5) between the limits λ_0 (the value of λ at time $t = 0$) and λ_c (the critical value of λ at failure). We shall further assume:

$$\lambda_0 = 0 \tag{A6}$$

and:

$$\lambda_c = 1 - \frac{\sigma}{\sigma_u} \tag{A7}$$

where σ_u is the tensile strength of the material.

The result of the integration is:

$$L = (t_1/q)\{1 - (\sigma/\sigma_u)^q\}(\sigma_0/\sigma)^{(q-1)} \tag{A8}$$

where:

L = service life
$t_1 = \bar{d}/C\sigma_0^{(q-1)}$, a time constant
$q = p + 1$
σ_0 = nominal value of σ

In this formulation all the service-life-governing quantities, and therefore the service life itself, are conceived as deterministic quantities. By conceiving the service-life-governing parameters as stochastic quantities it is possible to calculate a distribution function for L. The language called SIMULA, available to the IWIS-TNO, has been designed for procedures of this kind. Such a calculation for (A8) has been carried out by that Institute on the following assumptions.

There are no indications that other than normal distributions should be chosen for the distribution functions to be applied, except for that of q. Experience has shown that in expressions like (A5) the value of p is more often just above 2 instead of being much higher (although this latter possibility certainly exists, with values ranging up to about 7 or 8). A non-skew distribution would be decidedly unacceptable. These findings, and the theoretical limits resulting from the above-mentioned fracture mechanics considerations, make an exponential distribution appear a very reasonable assumptions. Accordingly, normal distribution functions were adopted for all the quantities, except

q, for which an exponential distribution was chosen with 3 as the lower limit.

In the program it was taken into account that perhaps the following future developments can be realized as simply as possible:

1. The substitution of a different right-hand member into (A8), perhaps containing parameters with possible other distribution functions, conceivably including some that are not available as standard ones in SIMULA.
2. The application of the procedure to a non-integrable growth law, so that a numerical integration of a differential equation like (A1) will have to be accommodated in the program.

In connection with the stochastic properties of q, already referred to, and the way in which the exponential distribution is available as a standard feature in SIMULA (namely, as a one-parameter distribution), it was necessary to recast (A8) in a somewhat different form for the purpose of the analysis:

$$L = \{t_1/(u+3)\}\{1-(\sigma/\sigma_u)^{(u+3)}\}(\sigma_0/\sigma)^{(u+2)} \qquad \text{(A9)}$$

where: $u = q - 3 = p - 2$

The numerical values employed in (A9) are indicated in *Table A1.*

Table A1. Stochastic properties introduced for the parameters; $\sigma_0 = 80$ MN/m^2, deterministic

parameter	t_1	u	σ	σ_u
dimension	(year)	(−)	(MN/m^2)	(MN/m^2)
distribution*	normal	negexp	normal	normal
a*	150	1	80	400
b*	40	not applicable	5	20

* According to IWIS-TNO's manual "SIMULA", Section 12.2, pp. 104 and 105, under "4. *real procedure* normal" and "5. *real procedure* negexp." For this analysis:
a = mean value
b = standard deviation

2 *Detailed analysis*

IWIS-TNO investigated how many simulation steps had to be performed in order to obtain a smoothly continuous curve. For each step four draws had to be made, namely, one for each parameter mentioned in *Table A1.* Calculations were carried out with 1000, 10 000 and 50 000 simulation steps respectively. The scatter in the curve obtained depends also on the number of classes into which the values obtained for L are divided. The result of each of the three above-mentioned analyses was assessed for division into 100 and into 25 classes.

3 *Results and discussion*

The result of the assessment of the scatter is presented in *Table A2.* Not surprisingly, the

Table A2. Result of scatter assessment

number of simulation steps	number of classes	result*
1 000	100	– –
1 000	25	±
10 000	100	–
10 000	25	+
50 000	100	+
50 000	25	+ +

* – – = very much scatter
– = much scatter
± = moderate scatter
\+ = little scatter
\+ + = very little scatter

best result was obtained with the largest number of simulation steps (50 000) and the smallest number of classes (25). It is found, however, that a good result is also obtained even with 10 000 steps and 25 classes. It is to be noted that division into 25 classes can claim to be a reasonable optimum. This number is small enough to be conveniently manageable, yet it is amply sufficient with regard to the attainable accuracy, especially in view of the major uncertainties that will generally exist in the stochastic properties of the initial parameters, as given in *Table A1.*

The result of the analysis comprising 50 000 simulation steps and 25 classes is presented in *Table A3.* Included in the computer program is the plotting of the values listed in the second and third columns of this table (number and cumulative number, respectively) as "printer plots" (see *Figs. A2 and A3*). These give a good idea of the scatter and of the functional behaviour of f and F respectively. The correct numerical values of f (in year^{-1}) can be calculated from the fourth column of *Table A3* (percentage) by dividing the values of this column by 100 times the class widths. By division of the values listed in the fifth column (cumulative percentage) by 100 the values of F are directly obtained. It is to be noted, however, that the values of f found in this way relate to the class midpoints, whereas the calculated values of F relate to the class boundaries. Values of F for the midpoints were obtained by linear interpolation. The values of r in

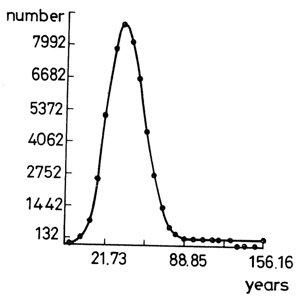

Fig. A2. "Printer plot" of the second solumn (number) of *Table A3.*

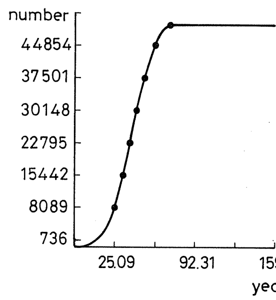

Fig. A3. "Printer plot" of the third column (cum. number) of *Table A3.*

Table A3. Results of the analysis in 50000 simulation steps and 25 classes

class (year)	number	cum. number	perc.	cum. perc. = 100F	$r=f/(1-F)$ (k year^{-1})	class midpoint (year)
< −1.79	0	0	.00	.00	-	-
−1.79- 4.93	34	34	.07	.07	0.101	1.57
4.93- 11.65	614	648	1.23	1.30	1.84	8.29
11.65- 18.37	2383	3031	4.77	6.06	7.36	15.01
18.37- 25.09	5008	8039	10.02	16.08	16.8	21.73
25.09- 31.82	7649	15688	15.30	31.38	29.8	28.46
31.82- 38.54	8908	24596	17.82	49.19	44.4	35.18
38.54- 45.26	8376	32972	16.75	65.94	58.7	41.90
45.26- 51.98	6698	39670	13.40	79.34	72.9	48.62
51.98- 58.70	4674	44344	9.35	88.69	87.0	55.34
58.70- 65.42	2832	47176	5.66	94.35	99.4	62.06
65.42- 72.14	1455	48631	2.91	97.26	103	68.78
72.14- 78.86	763	49394	1.53	98.79	115	75.50
78.85- 85.59	369	49763	.74	99.53	130	82.23
85.59- 92.31	124	49887	.25	99.77	105	88.95
92.31- 99.03	67	49954	.13	99.91	125	95.67
99.03-105.75	27	49981	.05	99.96	124	102.39
105.75-112.47	10	49991	.02	99.98	106	109.11
112.47-119.19	5	49996	.01	99.99	114	115.83
119.19-125.91	2	49998	.00	100.00	99.2	122.55
125.91-132.63	1	49999	.00	100.00		
132.63-139.35	0	49999	.00	100.00		
139.35-146.08	0	49999	.00	100.00		
146.08-152.80	0	49999	.00	100.00		
152.80-159.52	1	50000	.00	100.00		

the sixth column of *Table A3* were determined with the aid of (9) from the values f and F for the class midpoints. The last column of the table gives the values of the midpoints, these being the arithmetical mean values of the relevant class boundaries.

The shape of *Fig. A2* suggests that the distribution in question can be satisfactorily

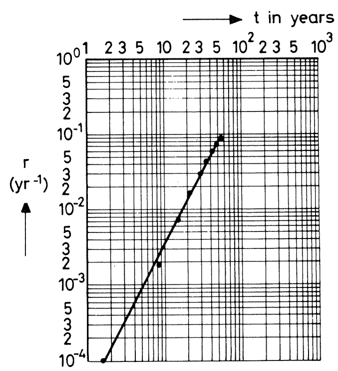

Fig. A4. r versus t (double-logarithmic): the straight line corresponds to a two-parameter Weibull distribution. Points for which $F > 0.9$ have been omitted.

approximated by a two-parameter Weibull distribution. If this is correct, the values of *r* plotted logarithmically as a function of time (i.e., the class midpoints) should be located approximately on a straight line. *Fig. A4* shows this requirement to be quite closely fulfilled for the most important part of the distribution. This is in agreement with what was found in practice (for example, see [44] and [45] for the service life functions of test specimens which were loaded under fatigue and creep conditions.

The normal distribution and the log-normal distribution are found to be less satisfactory, as is apparent from *Figs. A5 and A6.* It is to be noted that in these diagrams the deviation from the straight line (associated with the normal and the log-normal distribution respectively) is greatest in the range of the low values of *F*, i.e., where the reliability is still acceptably high, this being therefore precisely the range that is of greatest interest.

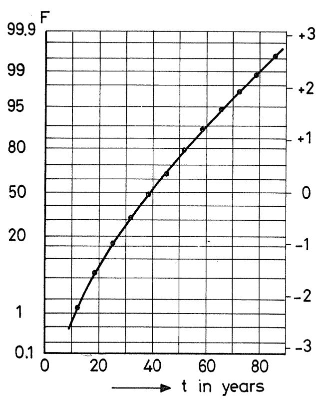

Fig. A5. *F* versus *t* on normal statistical paper: a straight line corresponds to a normal distribution.

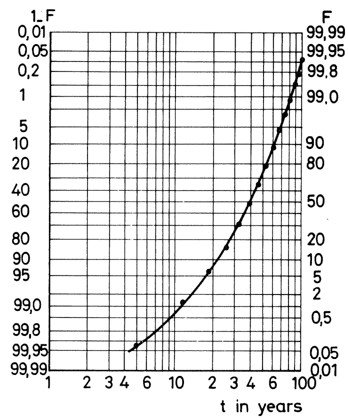

Fig. A6. *F* versus *t* on log-normal paper: a straight line corresponds to a log-normal distribution.

B. *Fatigue analysis of offshore structures*

1 *Introduction*

As part of the MaTS 28 project relating to the safety of offshore structures the probability of fatigue failure of a structure consisting of a single tube was calculated [49, 50, 51]. This probability was calculated for various values of the wall thickness of the tube and for an assumed service life of 50 years. Within the framework of the project "Service life of structures" it was considered relevant also to vary the service life. This appendix gives the results of such an analysis.

2 *The structure and the mathematical model*

For a full discription of the mathematical model and of the structure the reader is referred to the above-mentioned literature [49, 50 51]. The principal data are recapitulated here.

The structure is shown in *Fig. B1.* A hollow steel tube is installated in water 100 m in depth and supports a deck superstructure weighing 2000 tons. The base of the tube rests on a piled foundation of such high rigidity that the tube can be regarded as completely restrained (fixity). Of the various loads acting on the structure only wave load is assumed to contribute to fatigue damage.

The procedure adopted in this analysis is shown in the flow diagram of *Fig. B2.* It is assumed that the sea action throughout the service life of the structure can be subdivided into a number of stationary (steady-state) stochastic processes, called sea states. The water motion within a sea state is described statistically with the aid of a wave spectrum.

On the basis of the wave spectrum a stress spectrum for the stress variations can be calculated. The resulting fatigue damage is quantified as a Miner sum. Summation of Miner sums per sea state results in a Miner sum for the whole service life. According to the model, failure occurs when the total Miner sum is larger than 1.

The procedure outlined above was worked out probabilistically. The following were adopted as stochastic variables to be taken into account:

1. water fluctuation within a sea state;
2. significant wave height per sea state;
3. predominant wave direction per sea state;
4. wall thickness of the tube;
5. added mass resulting from the surrounding water;
6. damping constant;
7. fatigue parameter.

For quantitative data see *Table B1.*

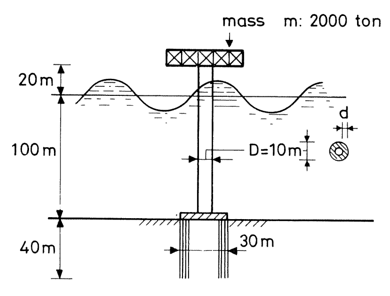

Fig. B1. The offshore structure considered.

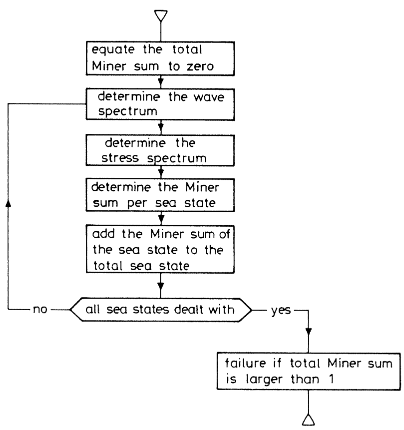

Fig. B2. Flow diagram of fatigue analysis.

Table B1. Summary of all the problem variables

var.	designation	type	mean	σ/μ
d_w	depth of water	D	100 m	-
l	length of tube	D	120 m	-
D	diameter of tube	D	10 m	-
d	wall thickness	N	0.15–0.25 m	0.05
ϱ_w	mass density of water	D	1024 kg/m^3	-
ϱ_{st}	mass density of steel	D	7800 kg/m^3	-
g	acceleration of gravity	D	9.8 m/s^2	-
C_M	wave force coefficient	D	2	-
m_d	mass of deck	D	$2 \cdot 10^6$ kg	-
γ	coefficient for added mass	N	0.9	0.10
L	service life	D	$158 \cdot 10^7$ s	-
η	displacement of free water surface	NPM	0 m	pm
ϕ	direction of individual wave train	C2	0 rad	pm
ϕ_0	predominant wave direction	U	3.14 rad	pm
H_s	significant wave height	W	2.17 m	0.60
ζ	damping constant	N	0.01	0.25
E	modulus of elasticity of steel	D	$210 \cdot 10^9$ N/m^2	-
k	slope of S–N line	D	4	-
C	fatigue constant	LN	$9 \cdot 10^{38}$ N^4/m^8	0.62

type designation:
D = deterministic
N = normal
NMP = normal with Pierson-Moskowits spectrum
LN = log-normal
U = uniform
C2 = cos^2 distribution
W = Weibull

3 *Results of the analysis*

Calculations were performed for wall thicknesses of 150, 170, 190, 210, 230 and 250 mm for planned service lives of 25, 50, 75, 100, 200, 400, 600 and 1000 years. The probability of failure was determined for each wall thickness and service life. The numerical results are presented in *Table B2 and B3.* In *Fig. B3* they are shown in graphic form, where the

Table B2. Results of the analysis (β-values)

L	$d = 0.15$	0.17	0.19	0.21	0.23	0.25
25	1.39	2.70	3.90	4.99	5.81	5.55
50	0.54	1.87	3.11	4.08	4.87	5.65
75	0.03	1.27	2.59	3.53	4.36	5.10
100	−0.33	1.16	2.22	3.14	3.98	4.71
200	−1.23	0.11	1.25	2.17	3.00	3.73
400	−2.16	−0.82	0.26	1.18	2.00	2.74
600	−2.71	−1.45	−0.32	0.59	1.40	2.14
1000	−3.42	−2.11	−1.07	−0.16	0.64	1.38

Table B3. Probability of failure

L	$d = 0.15$	0.17	0.19	0.21	0.23	0.25
25	$0.82 \cdot 10^{-1}$	$0.35 \cdot 10^{-2}$	$0.48 \cdot 10^{-4}$	$0.29 \cdot 10^{-6}$	$0.31 \cdot 10^{-8}$	$0.29 \cdot 10^{-10}$
50	0.29	$0.31 \cdot 10^{-1}$	$0.94 \cdot 10^{-3}$	$0.23 \cdot 10^{-4}$	$0.57 \cdot 10^{-6}$	$0.85 \cdot 10^{-8}$
75	0.49	$0.85 \cdot 10^{-1}$	$0.48 \cdot 10^{-2}$	$0.20 \cdot 10^{-3}$	$0.60 \cdot 10^{-5}$	$0.17 \cdot 10^{-6}$
100	0.63	0.15	$0.14 \cdot 10^{-1}$	$0.85 \cdot 10^{-3}$	$0.35 \cdot 10^{-4}$	$0.12 \cdot 10^{-5}$
200	0.891	0.46	0.11	$0.15 \cdot 10^{-1}$	$0.13 \cdot 10^{-2}$	$0.98 \cdot 10^{-4}$
400	0.9842	0.79	0.40	0.12	$0.23 \cdot 10^{-1}$	$0.31 \cdot 10^{-2}$
600	0.9966	0.9265	0.63	0.28	$0.81 \cdot 10^{-1}$	$0.16 \cdot 10^{-1}$
1000	0.9998	0.9826	0.86	0.56	0.26	$0.84 \cdot 10^{-1}$

planned service life L is marked on the horizontal axis and the failure probability on the vertical axis. For the various wall thicknesses different curves are thus obained, which are located higher up in the diagram according as the wall thickness is less.

The presentation of the results in *Fig. B3* is based on an interpretation thereof as failure probabilities associated with a planned service life. However, the same results are also susceptible of a different interpretation. The life L of the structure can be conceived as a stochastic variable, and the curves in Fig. B3 then become distribution functions $F_L(t) = P\{L < t\}$ for the stochastic variable L for various wall thicknesses. Differentiation of the distribution functions gives the probability density functions $f_L(t)$ for the service life. These have been plotted in *Figs. B4a to B4f.* For small values of the wall thickness d the service life has a low mean value and the probability density function is very narrow, indicating a not very large standard deviation. With increasing wall thickness the probability density function does not "shift along", as might be expected, but progressively flattens out. The mean as well as the standard deviation increase.

A log-normal distribution for L would appear to be a reasonable assumption. This is suggested both by the results presented in Figs. B4a to B4f and by the fact that the predominant stochastic variable (i.e., the fatigue parameter C, [50] or [51]) has a log-normal distribution. The validity of the assumtion is confirmed in Fig. B5, where the functions $F_L(t)$ have been plotted on log-normal probability paper.

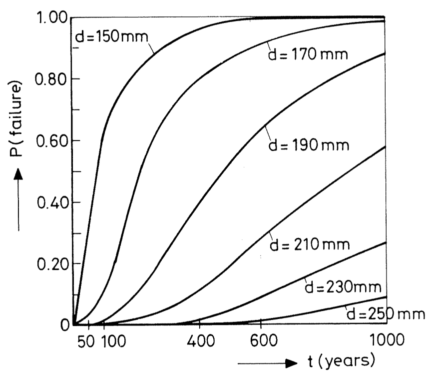

Fig. B3. The probability of failure as a function of planned service life L for various values of the wall thickness d.

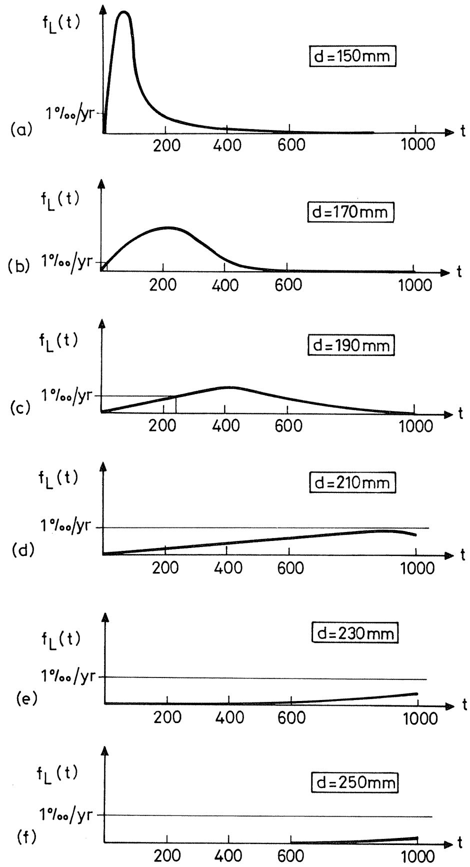

Fig. B4. Probability density functions for the service life L for various wall thicknesses.

The following rough estimates for the mean and the standard deviation of the service life L can be adopted:

d	$\mu(L)$	$\sigma(L)$
150 mm	100 years	100 years
170 mm	250 years	250 years
190 mm	600 years	600 years
210 mm	1200 years	1000 years
230 mm	2000 years	1600 years
250 mm	4000 years	3000 years

This table likewise shows that with increasing design thickness of the tube wall there is more particularly an increase in life expectancy. The relative scatter is very much the same for all values of the wall thickness d.

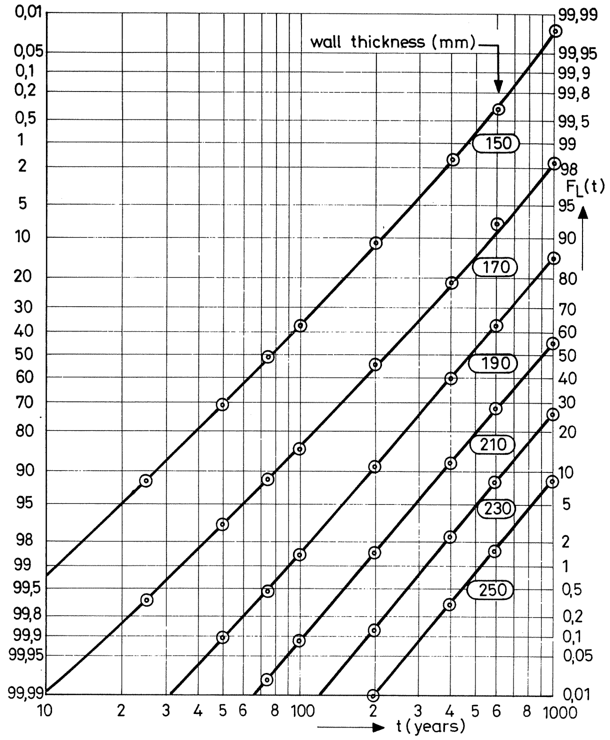

Fig. B5. Service life distribution on log-normal paper.

4 *Summary and conclusions*

The probability of failure of a simple offshore structure due to fatigue for various design dimensions (wall thicknesses) and various planned values of the service life L was calculated. The results obained can alternatively be interpreted as distribution functions for the service life L, conceived as a stochastic variable. The distribution functions arrived at are of the log-normal type. The life expectancy was found to range from 100 to 4000 years, depending on the wall thickness of the tube. The relative scatter was fairly large in all cases.

Values of between 0.8 and 1.0 were found for $\sigma(L)/\mu(L)$. Hence it appears meaningful to complement deterministic service life calculations for this type of structure by a probabilistic analysis.

C *A method of determining the reliability of structures and an extension thereof for predicting their service life*

1 *Introduction*

The starting point for the considerations presented here is formed by a study relating to the determination of the reliability of a thin-walled tube loaded by internal pressure and cyclic thermal loading [52, 53]. The method is based on Monte Carlo simulation, while the failure criterion is the occurrence of inadmissible accumulated plastic strains and creep fatigue damage. The reliability of the structure loaded in this manner was calculated on the basis of characteristics – deduced from practical data – relating to the random character of the variables that play a part on this problem. Finally, a sensitivity study was carried out in order to ascertain the effect of various parameters on the overall reliability of the thin-walled tube under the given conditions.

Within the context of the project "Service life of structures" (see, inter alia, [54]) it was investigated how the above-mentioned study could be applied to the determination of the service life of such structures by conceiving the life also as a stochastic parameter. This appendix gives some results of an analysis of this kind.

2 *The structure analysed*

First, the structure and the mathematical model will be briefly described.

The structure is shown in *Fig. C1*. The thin-walled tube is loaded by an internal pressure p which is (virtually) constant and by large cyclic temperature differences through the wall thickness, these being due to sudden shutdowns affecting the system in which this tube is accommodated. It is assumed that this occurs in accordance with the temperature variation curve at the outer face of the pipe wall shown in *Fig. C2*, resulting in a maximum temperature difference of 110 °C through the wall during a shutdown period.

The mathematical model is based on a method developed by Bree, while the criterion for failure of the structure has been chosen in accordance with the damage criteria stated in ASME Code Case 1592.

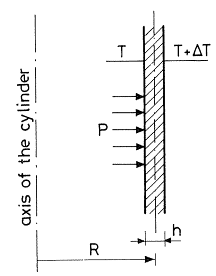

Fig. C1. Thin-walled tube, loaded by internal pressure, with temperature gradient in the wall of the tube.

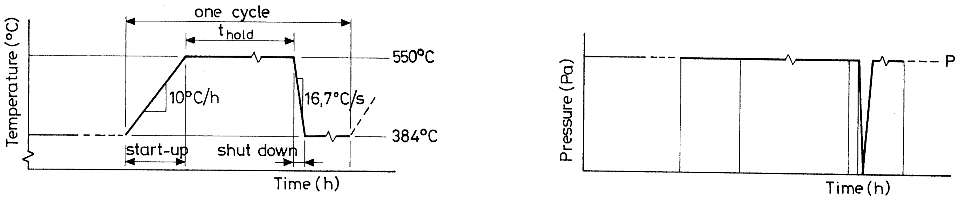

Fig. C2. Variations of temperature and pressure.

In a very general from the failure probability of these structures can be written as:

$$p_f = F(p, R, h, E, \alpha, \Delta T, \nu, \sigma_y, \sigma_{\text{ref}}, n_c, t_{\text{hold}}, T_{\text{max}}, \text{cycles}, N_d, T_d, D_{\text{crit}})$$

Of these quantities the following ones have been taken into account as stochastic variables:

- wall thickness h;
- modulus of elasticity E;
- coefficient of thermal expansion α;
- yield stress σ_y;
- creep parameters n_c and σ_{ref};
- creep fatigue parameters N_d and T_d.

Quantitative data for these parameters are given in *Table C1.*

Table C1. Summary of the problem variables adopted

quantity	designation	type	mean μ	variance σ/μ
P	internal excess pressure	D	$4.82 \cdot 10^6$ Nm^{-2}	-
R	radius	D	102.5 mm	-
ΔT	max. temperature difference	D	102 °C	-
ν	Poisson's ratio	D	0.3	-
t_{hold}	holding time at 550 °C	D	160 hr	-
T_{max}	maximum temperature	D	550 °C	-
cycles	number of cycles	D	variabel	-
D_{crit}	critical fatigue parameter	D	0.6	-
h	wall thickness	N	9.53 mm	0.004
E	modulus of elasticity	N	$1.5594 \cdot 10^{11}$ Nm^{-2}	0.04
α	coeff. of thermal expansion	LN	$15.5 \cdot 10^{-6}$ °C^{-1}	0.05
σ_y	yield-stress	N	$2.05 \cdot 10^8$ Nm^{-2}	0.058
n_c	creep parameter	N	5.52	0.04
σ_{ref}	creep parameter	LN	$1.17 \cdot 10^9$ Nm^{-2}	0.15
N_d	fatigue parameter	LN	$N_d = f(\Delta\varepsilon_p)$	0.80
T_d	fatigue parameter	LN	$T_d = f(\sigma)$	0.60

type of distribution functions: Material 1.6770
D = deterministic
N = normal
LN = log-normal

Table C2. Calculated failure probabilities as a function of the number of load cycles

	number of load cycles						
	10	100	200	1000	3000	6000	10000
$P(\varepsilon > \varepsilon_{cr})$	$0.99 \cdot 10^{-8}$	$0.14 \cdot 10^{-2}$	$0.11 \cdot 10^{-1}$	0.29	0.73	0.907	0.9678
$P(D > D_{cr})$	$0.19 \cdot 10^{-4}$	$0.11 \cdot 10^{-1}$	$0.42 \cdot 10^{-1}$	0.33	0.66	0.838	0.9187

3 *Results*

Two failure criteria will be considered separately here: maximum accumulated plastic strain ($\varepsilon > \varepsilon_{cr}$) and creep fatigue damage ($D > D_{cr}$). The reason is that in this way a better insight into the character of each of these two criteria is obtained. Since these criteria are statistically independent of each other, this approach involves no further implications.

The two criteria were analysed for 10, 100, 200, 1000, 3000, 6000 and 10000 thermal cycles, corresponding roughly to a planned service life of 0.22, 2.2, 4.4, 22, 66, 132 and 220 years. For each number of cycles the probability of not satisfying one of the two above-mentioned criteria was calculated. The numerical results are given in Table C2, while they are presented graphically in *Fig. C3.* The planned number of load cycles is marked on the horizontal axis; the probability of the two criteria being exceeded is indicated on the vertical axis. For 1500 cycles, say, the probability of the two criteria not being satisfied is, for both of them, approximately equal to 0.45. Furthermore it is notable that for $n_{\text{cycles}} > 1500$ the governing criterion is the total accumulated plastic strain and that for $n_{\text{cycles}} < 1500$ it is the creep fatigue damage.

This diagram is capable also of a different interpretation, namely, as a calculated distribution function of the stochastic service life ($F_L(t) = P\{L < t\}$). Hence it follows, for example, that the probability of the actual life L being less than a planned life (e.g. 1500 cycles) is equal to 0.45.

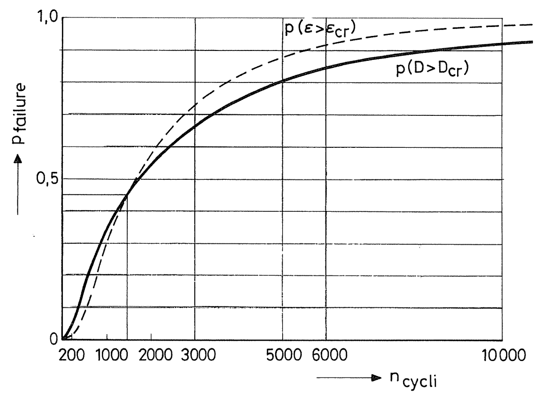

Fig. C3. Calculated failure rate as a function of assumed numbers of cycles for the two different failure criteria.

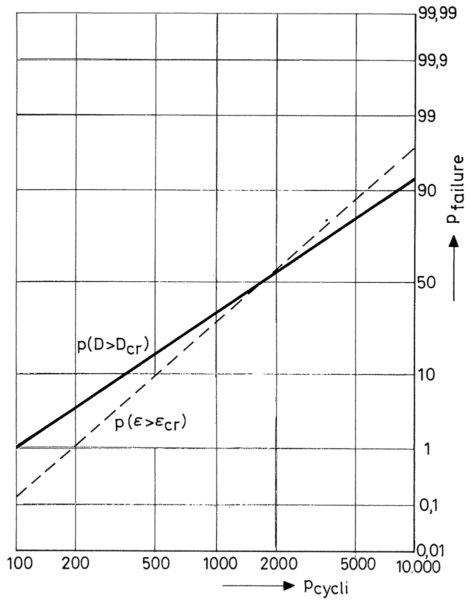

Fig. C4. Verification for log-normal distribution function.

Because the parameters most important to both damage criteria have a log-normal distribution, this distribution function can likewise be expected to have a log-normal character. In order to verify this, the curves in Fig. C3 have been plotted again in *Fig. C4*, but now in a manner devised specially for log-normal distribution functions. True log-normal distribution functions are represented in this way by straight lines. The lines in the diagram show that the calculated distribution functions are very close approximations of log-normal ones.

Differentiation of the distribution function obtained yields the probability density function $f_L(t)$ for the service life associated with the two damage criteria considered, as shown in Fig. C5. The following rough estimates for the mean and the standard deviation of the service life are obtained:

	$\mu(L)$	$\sigma(L)$
$\varepsilon > \varepsilon_r$	2452	2695 cycles
$D > D_r$	3700	7421 cycles

From these results it is apparent that the relative scatter in this "design life" is fairly large, for both criteria.

4 *Summary and conclusions*

The failure probability for a very simple structure was calculated for a range of different assumed numbers of design cycles. The results have been recast to give a distribution function for the service life L, this life being conceived as an extra stochastic parameter

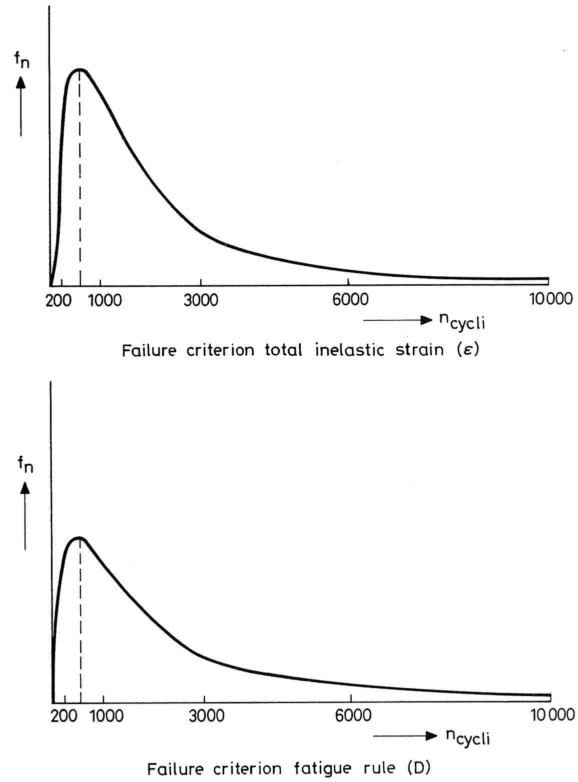

Fig. C5. Probability density functions for the service life L in the case of two failure criteria.

in the system. The distribution function for L is very satisfactorily approximated by a log-normal distribution function, which, having regard to the parameters of the problem and their probability density functions, was indeed to be expected. For the two damage criteria a mean value for this service life of about 1700 cycles is obtained, but the relative scatter (σ/μ) is found to be fairly considerable.

The important conclusion to be drawn from this study is that a service life calculated on a deterministic basis is – for this class of problems, anyway – of little value, but that certainly the statistical character of the various parameters, and therefore also the statistical character of the calculated service life, should be considered.

Of course, these results are not entirely unexpected. For example, if the random variables that determine the creep fatigue damage (N_d and T_d) are considered, it will be directly evident that a service life calculated on the basis of these quantities will assuredly also exhibit a by no means negligible scatter in the results.

D *Proofs of* $r = \Sigma r_i$

Two proofs will be given. The first is rather fundamental and indicates a possible limitation. The second makes more use of already available knowledge and is much simpler for that reason.

First proof

The precise definition of the instantaneous conditional failure rate in consequence of the i-th failure cause (r_i) is:

$$r_i \equiv \lim_{\Delta t \to 0} P\{(t < L_i < t + \Delta t)|(L_i > t)\}/\Delta t \tag{D1}$$

Define the event A_i as:

$$A_i \equiv (t < L_i < t + \Delta t)|(L_i > t) \tag{D2}$$

Then (D1) can be written in abbreviated form as follows:

$$r_i = \lim_{\Delta t \to 0} P(A_i)/\Delta t \tag{D3}$$

Furthermore the following expression can be written for the instantaneous conditional failure rate in consequence of all the failure causes together (r):

$$r \equiv \lim_{\Delta t \to 0} P\{(t < L < t + \Delta t)|(L > t)\}/\Delta t = \lim_{\Delta t \to 0} P(\bigcup_i A_i)/\Delta t \tag{D4}$$

In accordance with the relevant Venn diagram we can write:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \tag{D5}$$

As we have assumed all A_i to be mutually independent (therefore also A_1 and A_2), we have by definition:

$$P(A_1 \cap A_2) = P(A_1)P(A_2) \tag{D6}$$

Substitution of (D6) into (D5) gives:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1)P(A_2) \tag{D7}$$

Now if we define A_{12} as:

$$A_{12} \equiv A_1 \cup A_2 \tag{D8}$$

we obtain:

$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= P(A_{12} \cup A_3) = P(A_{12}) + P(A_3) - P(A_{12} \cap A_3) = \\ &= P(A_{12}) + P(A_3) - P(A_{12})P(A_3) \end{aligned} \tag{D9}$$

Substitution of (D8) into (D7) gives:

$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= P(A_1) + P(A_2) + P(A_3) + \\ &\quad - P(A_1)P(A_2) - P(A_1)P(A_3) - P(A_2)P(A_3) + \\ &\quad + P(A_1)P(A_2)P(A_3) \end{aligned} \tag{D10}$$

Proceeding in this way, we find:

$$P(\bigcup_i A_i) = \sum_i P(A_i) + \text{a number of higher-order terms} \tag{D11}$$

Now:

$$\lim_{\Delta t \to 0} P(A_i) = 0 \text{ for all } i \tag{D12}$$

so that the higher-order terms may be omitted in (D11):*

$$P(\bigcup_i A_i) = \sum_i P(A_i) \tag{D13}$$

Substitution of (D13) into (D4) gives:

$$r = \lim_{\Delta t \to 0} \left\{ \sum_i P(A_i) \right\} / \Delta t = \sum_i \lim_{\Delta t \to 0} P(A_i)/\Delta t \tag{D14}$$

which with (D3) becomes:

$$r = \sum r_i \tag{D15}$$

Second proof

If there are only two causes of failure, with instantaneous conditional failure rates r_1 and r_2 respectively, we have to prove:

$$r = r_1 + r_2 \tag{D16}$$

By complete induction we then obtain (D15).

Now:

$$r_1 = f_1/(1 - F_1) \tag{D18}$$

and:

$$r_2 = f_2/(1 - F_2) \tag{D18}$$

Therefore:

$$r_1 + r_2 = (f_1 - f_1F_2 + f_2 - f_2F_1)/(1 - F_1 - F_2 + F_1F_2) \tag{D19}$$

Furthermore:

$$r = f/(1 - F) \tag{D20}$$

From the independence of the two failure causes it follows that:

$$F = F_1 + F_2 - F_1F_2 \tag{D21}$$

---

* This is the central feature of the proof. It is possible to conceive a case where it is not correct. Thus, if it emerges that the time has been essentially quantified and that, for example, the smallest possible $\Delta t \sim 10^{-43}$ s, and if particles are found with a life not exceeding this value by more than a few orders or magnitude and these particles have various modes of decay (i.e., several possible "failure causes"), it may turn out that for these the statement (D15) is not exact.

Hence we obtain:

$$f = \mathrm{d}F/\mathrm{d}t = f_1 + f_2 - F_1 f_2 - f_1 F_2 \tag{D22}$$

Substitution of (D21) and (D22) into (D20) gives:

$$r = (f_1 + f_2 - F_1 f_2 - f_1 F_2)/(1 - F_1 - F_2 + F_1 F_2) \tag{D23}$$

From (D19) and (D23) it follows that:

$$r = r_1 + r_2 \tag{D24}$$

E *A service life distribution function for structures that may fail from many causes*

1 *An example*

With modern structures it is often very difficult to predict which cause will be the first to bring about failure. What makes it difficult is of course the great complexity of such structures, the more so as they are usually not built in large numbers, so that it is not possible to collect statistical data for service life. Besides, such data would in general become available, if at all, only at a time when they are no longer needed. So it must be virtually ruled out that it will ever be possible to make predictions on the basis of practical data of this kind. However, it appears obvious and reasonable to compare the service life of structures with, for example, the life expectancy of the human body - for which statistical data are indeed available. After all, the human body, too, can be conceived as a "structure" with a high degree of complexity and susceptible to many causes of "failure".

An example of r-values for inhabitants of the Netherlands who died in 1975 is given in *Table E1* [55]. These values, plotted linearly against time (for which the class midpoints are taken here), result in the familiar "bathtub curve", as shown in *Fig. E1*, though admittedly its shape does not always closely resemble a bathtub. The descending initial part of this curve relates to deaths in early childhood (or what are often called "teething troubles" in an engineering context) and will not be further considered here (though

Table E1. r-values deduced from Netherlands mortality table 1975 [55]

class (yr)	class midpoint (yr)	r (kyr^{-1})
0- 1	0,5	10,6
1- 5	3,0	0,7
5-10	10,0	0,3
15-30	22,5	0,6
30-45	37,5	1,3
45-65	55,0	8,0
65-75	70,0	32,3

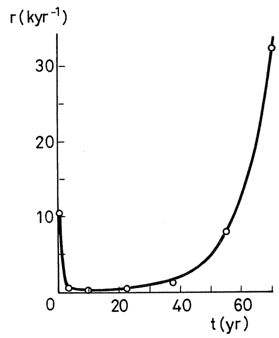

Fig. E1. r vs. t, linear: "bathtub curve".

this does not mean to say that this part is always unimportant in the case of structures). If the then following part of the curve could be approximated by a Weibull distribution, it should, when plotted as a double logarithmic graph, be represented by an approximately straight line. From *Fig. E2* it appears that this condition is not satisfactorily fulfilled: the curve remains concave upwards. However, if we adopt for r, not the expression proposed by Weibull, but the expression (45) given in Section 4.4.2 of this report, namely:

$$r = r_0 \exp\,(t/t_c); \qquad t > 0, \tag{45}$$

we obtain a straight line when we plot r versus t on single logarithmic paper: see *Fig. E3*. It turns out from this diagram that the points in the age range above about 15 years can be approximated reasonably well by a straight line (see, for example, also [56], pp. 22 and 246).

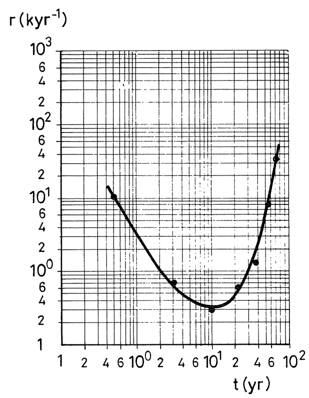

Fig. E2. Double-logarithmic representation of Table E1.

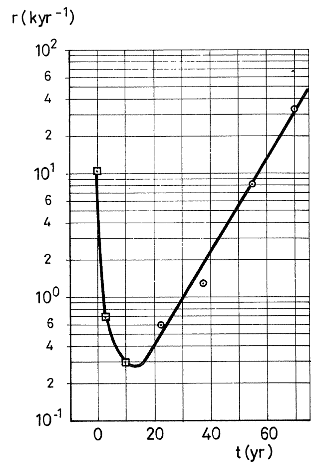

Fig. E3. Single-logarithmic representation of Table E1.

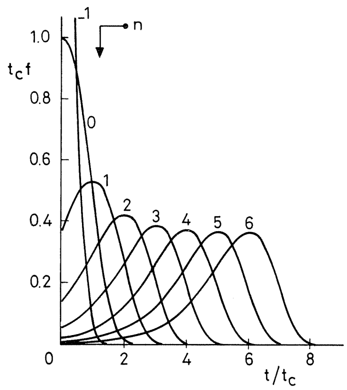

Fig. E4. A number of possible shapes of the probability density function (E2) (made non-dimensional).

2 *Distribution function and probability density function*

From (45) (see above) we can, with the aid of (10) (see Section 2.2 of this report), readily derive an expression for the associated distribution function:

$$F = 1 - \exp\{-t_c(r - r_0)\} \tag{E1}$$

From (E1), (45) and (6) the following expression for the probability density function can be obtained:

$$f = r \exp\{-t_c(r - r_0)\} \tag{E2}$$

This is a two-parameter distribution function with parameters r_0 and t_c. As in the case of the Weibull distribution, a three-parameter distribution is obtained if in (45) the condition $t > 0$ is replaced by $t > t_a$.

The shape of the two-parameter distribution is determined by the product of t_c and r_0. *Fig. E4* shows a number of possible shapes of the probability density function (made non-dimensional) with the negative natural logarithm of the product of t_c and r_0 (n) as the parameter.

3 *Justification of applicability*

Suppose that a structure or part of structure can fail in consequence of many different mutually independent causes. Suppose furthermore that each failure cause independently results in a failure rate r_i. The overall failure rate (r) can then, in accordance with (44), be written as:

$$r = \Sigma r_i \tag{E3}$$

Now suppose that the service life distribution function due to each of the failure causes separately can be approximated by a two-parameter Weibull distribution:

$$F_i = 1 - \exp\{-(t/\theta_i)^{\beta_i}\} \tag{E4}$$

The associated failure rate is:

$$r_i = (\beta_i/\theta_i^{\beta_i})t^{\beta_i - 1} \tag{E5}$$

The (independent) parameters of this distribution are θ_i and β_i. The expression (E5) for r_i can be put into non-dimensional form, while at the same time the parameters are separated, by introducing the substitution:

$$\theta_i^{\beta_i} = \beta_i t_c^{\beta_i - 1}/C_i r_0 \tag{E6}$$

This is possible because to every conceivable combination of θ_i and β_i corresponds a value of the parameter C_i introduced with this. The newly introduced quantities t_c and r_0 can be so chosen that they are equal for all Weibull distribution functions. By substitution of (E6) into (E5) we obtain:

$$r_i/r_0 = C_i(t/t_c)^{\beta_i - 1} \tag{E7}$$

where C_i and β_i are the new (independent) parameters.

Now consider the range $t > t_c$. Suppose that there are two failure causes with failure rates r_m and r_{m-1}, which both satisfy (E7) with parameters C_m and β_m, and C_{m-1} and β_{m-1}, respectively. Suppose furthermore that both failure causes contribute substantially to all cases of failure at time t considered. This means that r_m and r_{m-1} are of the same order of magnitude at that time. Now if we assume β_{m-1} to be smaller than β_m, it means that C_{m-1} is larger than C_m.

By making a number of assumptions it becomes possible to give more quantitative expression to what has been stated in the preceding paragraph. Assume that at time $t = mt_c$ failure occurs just as frequently in consequence of the m-th cause as in consequence of the $(m-1)$-th. Then:

$$r_m = r_{m-1} \tag{E8}$$

when:

$$t = mt_c \tag{E9}$$

From (E7), (E8) and (E9) follows:

$$C_m m^{\beta_m - 1} = C_{m-1} m^{\beta_{m-1} - 1} \tag{E10}$$

Now assume that:

$$\beta_m = \beta_{m-1} + 1, \tag{E11}$$

then (E10) becomes:

$$C_m = C_{m-1}/m \tag{E12}$$

Assume that, in analogy with (E11) and (E12), the following holds for every i:

$$\beta_i = \beta_{i-1} + 1 \tag{E13}$$

and:

$$C_i = C_{i-1}/i, \tag{E14}$$

Then:

$$\beta_i = \beta_0 + i \tag{E15}$$

and:

$$C_i = C_0/i! \tag{E16}$$

Furthermore, suppose that $\beta_0 = 1$ (being the lowest possible integer value that β can assume) and $C_0 = 1$ (a permissible supposition by virtue of the introduction of r_0, which can still assume all values). Then, on substitution of (E15) and (E16), we obtain from (E7):

$$r_i/r_0 = (1/i!)(t/t_c)^i \tag{E17}$$

Substitution of (E17) into (E3) gives:

$$r = r_0 \Sigma (1/i!)(t/t_c)^i \tag{E18}$$

In order to set limits to the summation we must bear in mind, on the one hand, that in the above reasoning we tacitly adopted 0 as the lowest value for i and, on the other hand, that in principle there is an infinite number of possible failure causes, so that (E18) becomes:

$$r = r_0 \sum_{i=0}^{\infty} (1/i!)(t/t_c)^i = r_0 \exp (t/t_c) \tag{E19}$$

which is identical with (45) in Section 4.4.2 of this report.

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