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Secretary:
G. J. van Alphen
Stevinweg 1
P.O. Box 5048
2600 GA Delft, The Netherlands
Tel. 0031-15-785919
Telex 38070 BITHD

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PLASTIC DESIGN OF BRACED FRAMES ALLOWING PLASTIC HINGES IN THE COLUMNS

S. E. van Manen
IBBC-TNO
Institute for Building Materials and Building Structures
Lange Kleiweg 5, Rijswijk, The Netherlands

and

INVESTIGATION OF THE STRESSES IN RAILS

Secondary effects associated with the bending of a centrically loaded rail

A. J. Kolvoort[1]
and
T. Woestenburg[2]

[1] Jol 18-08, 8243 GG Lelystad
[2] Isebrandtsheerd 53, 9737 LJ Groningen

Abstract

The rotational capacity of steel columns in braced frames is the subject of this paper. The behaviour of a braced portal frame up to collapse is examined. Requirements for rotational capacity of the columns according to some existing standards are scrutinized with the aid of calculated relations between bending moment, normal force and angular rotation of columns. An alternative approach is also presented, enabling a simple check of rotational capacity to be performed with the aid of a table giving values to the rotational capacity of a column, depending on slenderness, axial load and ratio of end moments. The application of this new method of analysis is illustrated with two examples.

Plastic design of braced frames allowing plastic hinges in the columns

1 Introduction

According to the current ECCS Recommendations [1], braced frames may be designed on the basis of elastic or plastic theory. Plastic design is advantageous in two respects. First, plastic theory provides better insight into the structural behaviour up to failure. Secondly, the design will in general turn out to be more economical thanks to the free choise of the distribution of forces. This latter consideration is especially relevant when beams or columns have to fulfil additional requirements, such as resticted availability of various rolled steel sections, headroom or construction depth restrictions, or the need to maintain column dimensions unchanged through a number of storeys. Particularly when having to cope with requirements of this kind, elastic design is liable to result in a less economic design of structural components.

The plastic design of braced frames can most simply be carried out with the aid of the lower bound theorem. This theorem states that a lower bound for the collapse load is obtained if that load is determined from an equilibrium analysis in which the yield moment or the yield force is not exceeded anywhere in the structure. However, it is only permissible to apply plastic theory if a number of conditions are satisfied. These conditions are necessary to ensure a plastic behaviour of the structure. This is indeed important because the theoretical ultimate load will be attained only if the structure fails as a "mechanism" in accordance with the envisaged collapse mode. Some of these conditions are:

- adequate thoughness of the material;
- adequate ductility of the structural connections;
- premature local buckling is ruled out;
- premature overall instability is ruled out (buckling of columns, lateral-torsional buckling).

This paper investigates the requirements that columns in braced frames have to satisfy, in order to ensure the valid application of plastic design. Local buckling or overall instability of the columns may prevent attainment of the calculated ultimate load in such structures. This will be particularly important when plastic hinge formation in the columns occur [2] and, in view of the above mentioned instability phenomenae, this is often ruled out as not permissible.

2 Statement of the problem

To elucidate the nature of the problem, the following case will be considered. Suppose that a portal frame as shown in Fig. 1 is to be designed. It has a height h and a span l. Point loads F_c act on the columns and a distributed load λq on the beam. The point loads

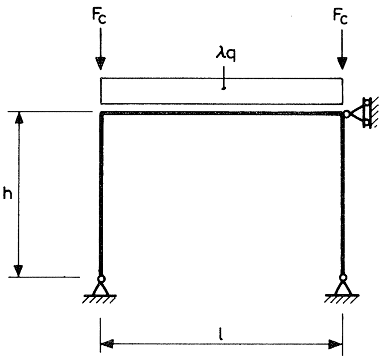

Fig. 1. Braced portal frame.

can be conceived as transmitted from the superstructure.

For design based on the lower bound theorem the first step consists in choosing an equilibrium system. More particularly, three possibilities as shown in Fig. 2 will be considered. All three alternatives satisfy the equilibrium conditions. For the beams the requirement is that their plastic moment must be at least equal to the assumed maximum bending moment. Hence it follows that in case (a) the plastic moment of the beam has to be not less than $\lambda ql^2/16$, while in case (c) it has to be not less than $\lambda ql^2/8$. In the intermediate case (b) the plastic moment of the beam will depend on the moment that the columns can develop. A design according to case (c) gives the heaviest beam section, while a design according to case (a) will result in the lightest beam section.

In case (a) the column must be capable of resisting a moment $\lambda ql^2/16$ at its top end, while in case (c) the column is mere axially loaded. Hence the columns in case (c) can be dimensioned more economically than in case (a). Case (b) is an intermediate solution in which the full plastic moment of the beam can be adjusted to suit the moment-resisting capacity of the columns, or vice versa. By "shifting" the bending moment distribution the designer acquires the freedom to choose lighter columns and a heavier beam or a lighter beam with heavier coumns.

If, in case (c), the beam-to-column connections are not actual pin joints, plastic hinges will be formed in the columns. It will then be necessary to ensure that these plastic hinges possess sufficient rotational capacity to enable the plastic moment in the beam to be attained and to prevent premature failure of the portal frame.

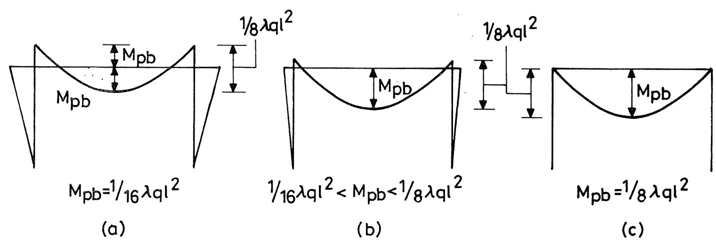

Fig. 2. Possible equilibrium systems of a portal frame, loaded as indicated in Fig. 1.

If the portal frame, designed in this way, is subjected to a distributed load λq, as shown in Fig. 1, with λ increasing from zero to failure, the behaviour of the frame can be studied step by step. There are several possibilities, depending on the stiffness ratio of the column and the beam, on the span, on the normal force, etc.

Case 1: $M_k > M_{pb}$

First, a design will be considered in which the bending moment that the column can develop (M_k) is greater than the plastic moment of the beam (M_{pb}).

The increasing load λq will cause failure of the frame by a beam mechanism in which all three plastic hinges develop in the beam. The distribution of the bending moments just before collapse and the maximum load $\lambda q = 16M_{pb}/l^2$ is shown in Fig. 2a, while Fig. 3 shows the mechanism.

Fig. 3. Collapse mechanism when $M_k > M_{pb}$.

In this case the sequence in which the plastic hinges are formed is unimportant. The first hinge to form will have sufficient rotational capacity to provide the angular rotation needed to enable the last hinge to develop. This is so because the rotational capacity of a plastic hinge in a beam (without - or with only a small - normal force acting in it) is always sufficient if the steel sections employed have width to thickness ratios conforming to the requirements stated in the ECCS Recommendations.

Case 2: $M_k \leqslant M_{pb}$

In this case the maximum bending moment that the column can develop (M_k) is smaller than, or equal to, the plastic moment of the beam (M_{pb}).

Two subcases have to be distinguished now, namely:

Case 2a: the mid-span plastic hinge will be formed first.

Case 2b: the plastic hinges at the ends of the beam will be formed first (or, rather, the maximum column moment is attained first).

In *case 2a* the angular rotations at the ends of the beam are, when the plastic hinge at mid-span is formed, not yet large enough that the columns can develop their maximum resisting moment. In that case, for a certain value of λ ($0 < \lambda < 1$) the bending moment distribution may have the appearance shown in Fig. 4a. The moments at the ends of the beam have not yet attained the design moment ($\lambda ql^2/8 - M_{pb}$). With further load increase a plastic angular rotation will occur at mid-span, while the mid-span moment (plastic moment) remains constant. As a result, the angular rotations at the ends of the

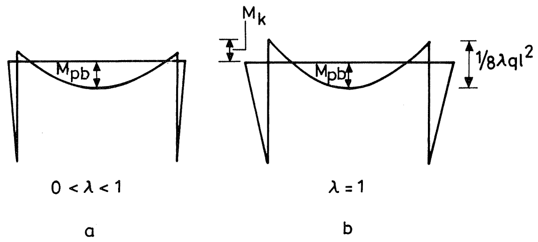

Fig. 4. Bending moment distribution when the plastic hinge at mid-span is formed, but the maximum resisting moment of the columns is not yet attained (a), and just before collapse (b).

beam will increase, and so can the resisting moments developed by the columns. Eventually these moments attain the value $M_k = \lambda q l^2/8 - M_{pb}$, and the portal frame then collapses. Just before collapse the equilibrium system shown in Fig. 4b will have developed.

In *case 2b* the sequence in which the plastic hinges are formed is just the reverse: the maximum moment is attained first in the columns. The plastic hinge at mid-span in the beam has then not yet been formed. This situation is shown in Fig. 5a and will occur when a certain value of λ has been reached.

Further increase of the load will cause increasing angular rotation of the column top ends, so that the mid-span moment can likewise increase. The moments at the top ends of the columns will not become any greater, having already reached their maximum. It is sufficient if the column top end moments do not decrease due to the increasing angular rotation. The mid-span moment can then become M_{pb}, so that finally the design bending moment distribution is attained as shown in Fig. 5b with the associated design load.

In all cases 1, 2a and 2b the collapse load for which the structure has been designed will be attained, provided that the plastic angular rotations can increase without the associated bending moment decreasing. In case 1 the plastic hinges formed in the beam must be able to develop a certain rotation, on conditions that the plastic moment is maintained. In case 2a it is necessary only that the plastic hinge at mid-span in the beam has enough rotational capacity, since the collapse load is attained when the columns attain the maximum moment that they can resist.

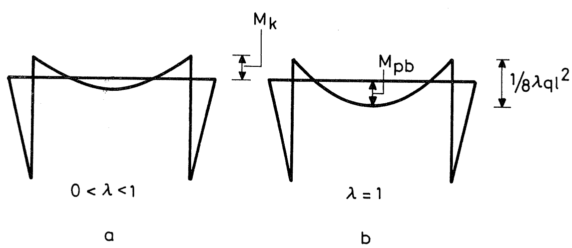

Fig. 5. Bending moment distribution when the maximum resisting moment of the columns is attained, but the plastic hinge at mid-span is not yet formed (a), and just before collapse (b).

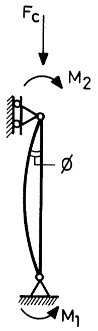

Fig. 6. Isolated column.

In case 2b the column top end should develop a certain rotation while the column top end moment remains constant. The rotational capacity of the beam is normally always sufficient, whereas the columns do not always possess sufficient rotational capacity. As a result of the instability phenomena mentioned in the Introduction the column top end moment may undergo a considerable decrease with increasing rotation. Therefore it is necessary to check that the column does indeed have the necessary rotational capacity.

It should be noted that in this case there is no question of the rotational capacity of a plastic hinge conceived as concentrated at a particular cross-section, or deformation capacity as a material property. Instead, the rotational capacity of the column end as part of the whole column is considered here.

The key to a check of this kind is provided by so-called M-N-ϕ curves for the columns. In these curves the column top end moment M is determined as a function of the angular rotation ϕ of the column end for various values of the normal force N (Fig. 6). This will be further considered in the next chapter.

3 M-N-ϕ curves

An obvious approach to the determination of M-N-ϕ curves is to use a finite element method. A number of aspects that affect these curves can be fairly simply taken into account with such a method. Imperfections of the column can be expressed in an initial eccentricity, and the residual stresses in the column can be simulated by assigning different stress-strain relations to the various parts of the column section.

For local buckling, which may (as test results have shown, e.g. [3]) affect the M-N-ϕ curves, more particularly of stocky columns, there is no satisfactory criterion available. It is also known from tests that the column top end moment in columns consisting of certain rolled steel sections undergoes very little or no decrease with increasing rotation after the flange has buckled. Hence it is not enough to know just when plastic buckling occurs; it is also necessary to be able to describe the post-buckling behaviour.

Lay [4] has developed a theory with which the occurrence of plastic buckling can be predicted, depending on the (local) strain distribution. This strain distribution can be established in a simple way with a finite element method. Lay's theory is based on the elastic buckling force of the flange of the I-section. His formula contains three material constants, namely, two moduli of elasticity (in lateral and longitudinal direction) and a shear modulus. By assuming a shear plane it is possible to estimate these constants when the material undergoes strain-hardening. On substitution of these values the buckling force is minimalized with respect to the wavelength of the buckled shape, whence the following expression is obtained for the wavelength l_b itself:

$$l_b = 0.7 \frac{bt_f}{t_w} \sqrt[4]{\frac{A_w}{A_f}} \approx 0.7b \tag{1}$$

where:

b = width of the flange
t_f = thickness of the flange
t_w = thickness of the web
A_w = cross-sectional area of the web
A_f = cross-sectional area of the flange

Because the wavelength has been determined on the assumption that the material undergoes strain-hardening, the buckling criterion is as follows:

The flange of the structural section will buckle when the strain at which strain-hardening occurs has been attained over a length l_b.

Although a number of questionable assumptions have been made, this buckling criterion is in reasonable good agreement with test results [4]. There are, however, indications that the rotation at which buckling occurs is underestimated in certain cases. Some sections do not buckle at all, not even when very large curvatures and therefore very large strains occur. Not taking account of post-buckling behaviour means that this criterion remains on the safe side. It will be shown that the number of cases where the rotational capacity is restricted by this criterion is not very large, however, and these cases can readily be accommodated when in due course a more suitable criterion becomes available.

Parameters to be varied

There are three parameters that have to be varied in establishing the M-N-ϕ curves. These are: the normal force, the slenderness and the end moment ratio. A practical quantity for expressing the normal force is F/F_k where F denotes the externally applied load and F_k the collapse load of the axially compressed column. This parameter varies between 0 and 1. The ordinary slenderness ratio λ_z has been chosen as slenderness parameter. Furthermore the symbol β denotes the end moment ratio in the elastic range ($\beta = M_1/M_2$ in Fig. 6). If the bottom end moment is greater than the top end moment, the reciprocal value of β is employed, in which case β varies from -1 to $+1$. As the angular rotation is controlled, β is accurately constant only in the elastic range but the end moment ratio was found to undergo little change in the plastic range as well.

Strictly speaking, there is a fourth parameter, namely, the shape factor η. However, since η ranges from 1.11 to 1.24 for the rolled I-sections under consideration here, it was assumed, as a preliminary approximation, that η could be taken as constant. This subsequently turned out to be a sufficiently accurate assumption.

The steel grade adopted in the various calculations is Fe 360, with $\sigma_r = 235$ N/mm^2.

Results

Calculations were carried out with the following numerical values of the parameters. Parameter F/F_k was given the values, 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1, the parameter λ_z the values 40, 80, 120, 160 and 200, and the parameter β the values -1, 0 and $+1$. Calculations with intermediate values of these parameters were also carried out in order to check whether linear interpolation is permissible.

One of these results is represented in Fig. 7. It relates to a pin-jointed column consisting of an HE 200 A rolled steel section and subjected to a normal force characterized by $F/F_k = 0.725$. Furthermore: $\lambda_z = 100$ and $\beta = 0$. Using the finite difference method, Ojalvo [5] performed a similar analysis for an American rolled steel section 8WF31. This result is also shown in Fig. 7. Both calculations provide an accurate description of the elastic branch of the curve. In the plastic range the results diverge, but the difference can readily be explained by the fact that the two sections analysed were different, resulting in different plastic moments.

Obviously, it is not possible to include all the curves in this publication, which will therefore have to confine itself to present a few significant aspects. The complete set of curves, together with a detailed treatment of the problem with which the present paper is concerned, is given in [6].

The following emerges from the research that has been conducted:

- Columns with a high slenderness ratio remain elastic even when large angular rotations occur at the column top end. They respond as an elastic spring, with low stiffness caused by geometric non-linearity.

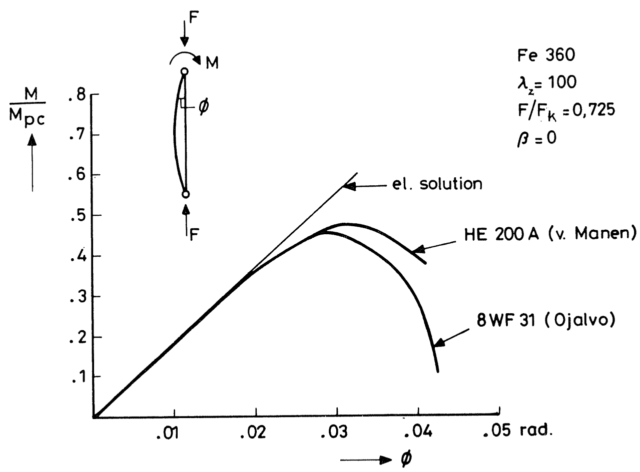

Fig. 7. *M-N-ϕ* curve.

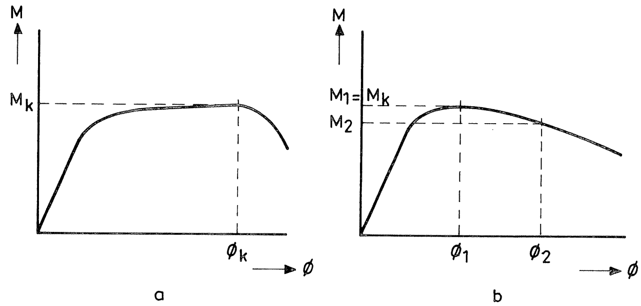

Fig. 8. Two different shapes of *M-N-ϕ* curves.

- Stocky columns, on the other hand, respond in a very rigid manner and are hardly sensitive to geometric non-linearity. Zones with highly concentrated plastic deformations soon develop and create the conditions under which plastic buckling occurs. This type of buckling is then often the criterion for the rotational capacity.
- If the moment decreases with increasing rotation (the descending branch), the moment decreases very rapidly in many cases. This phenomenon is shown schematically in Fig. 8a as contrasted with Fig. 8b.

There are two practical consequences due to this last finding:

1. The rotational capacity ϕ_k of the column is in most cases fairly uniquely determined while the column has attained the maximum resisting end moment M_k.

Table 1. Rotational capacity of a column consisting of a rolled steel I-section (steel grade Fe 360) for various values of λ_z, F/F_k and β in 10^{-2} rad. The cases marked by an asterisk denote those where local flange buckling determines the rotational capacity. The values in parentheses indicate the rotational capacity neglecting the local flange buckling criterion

	F/F_k	$\lambda_z = 40$	$\lambda_z = 80$	$\lambda_z = 120$	$\lambda_z = 160$	$\lambda_z = 200$
$\beta = +1$	1	0	1	2,5	4	5,5
	3/4	1	1,5	3	4,5	7
	1/2	2	3	4,5	6,5	9
	1/4	4	5	6,5	8	10
	0	10	10	10	10	10
$\beta = 0$	1	0	1	2	3	5
	3/4	1,5	2,5	3,5	5,5	7,5
	1/2	2,5* (6)	3,5	5	7	10
	1/4	3,5* (8)	8	9	10	10
	0	6* (10)	9	10	10	10
$\beta = -1$	1	0	1,5	3	4,5	6
	3/4	2* (5)	3	5	6,5	9
	1/2	3* (10)	5* (10)	8* (10)	10	10
	1/4	3* (10)	6* (10)	8,5* (10)	10	10
	0	5,5* (10)	7* (10)	9* (10)	10	10

2. It will not be possible to design the portal frame with a rotation larger than ϕ_k. It is not practicable to calculate the associated moment, which is smaller than M_k, with any accuracy. This means that the equilibrium of the beam ($M_{pb} + M_k \geqslant ql^2/8$) can not be verified then.

The rotations ϕ_k determined in this way are presented in Table 1. For practical purposes the rotational capacity can be taken to vary linearly between the values given there. How these data can be used for checking the availability of sufficient rotational capacity will be explained in the next chapter.

4 Application of the M-N-ϕ curves

A possible way to tackle the problem of whether or not there is sufficient rotational capacity consists in introducing the required rotational capacity into the calculations.

For braced structures this capacity can be calculated quite simply. In the case of braced portal frames the required rotation follows from the condition that the plastic moment has to be attained at mid-span in the beam. The column develops the end moment M_k, and, with a uniformly distributed load as shown in Fig. 9a the bending moment distribution at the instant of collapse will be as shown in Fig. 9b. The angular rotations that occur at the end of the beam and correspond to this bending moment distribution can be calculated as follows.

The distributed load and the end moments will cause the simply-supported beam to undergo an angular rotation expressed by:

$$\phi_b = \frac{\lambda q l^3}{24(EI)_b} - \frac{M_k l}{2(EI)_b} \tag{2}$$

while:

$$M_{pb} + M_k = \lambda q l^2/8 \tag{3}$$

$$\rightarrow \lambda q = \frac{8(M_{pb} + M_k)}{l^2} \tag{4}$$

(2) and (3)

$$\rightarrow \phi_b = \frac{(M_{pb} + M_k)l}{3(EI)_b} - \frac{M_k l}{2(EI)_b}$$

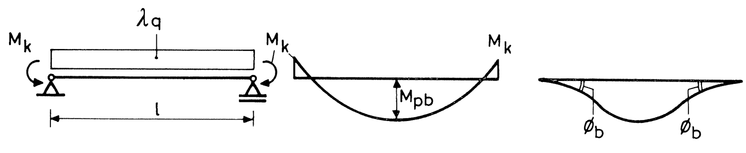

Fig. 9. The angular rotation of a simply-supported beam, loaded by a distributed load and two equal, opposite moments on both ends.

or:

$$\phi_b = \frac{(2M_{pb} - M_k)l}{6(EI)_b} \tag{5}$$

where:

M_{pb} = plastic moment of the beam
M_k = column top end moment
l = length of the beam
$(EI)_b$ = flexural stiffness of the beam

The maximum rotation that the column can develop is then obtained from Table 1, applying linear interpolation if necessary.

If the rotation *that can be developed* is larger than, or equal to, the *required* rotation, it means that the rotational capacity of the column is sufficient.

In other words, adequate rotational capacity is ensured if:

$$\phi_{\text{developable by column}} \geqslant \phi_{\text{required by beam}}$$

In the check for rotational capacity as outlined here it has been presupposed that the normal force in the columns is constant, and the *M-N-*ϕ curves have indeed been based on this assumption. Actually, however, the rotational capacity is found always to decrease with increasing normal force. Hence it follows that the method can be used also in those cases where the load on the column increases, the check then being performed in the ultimate state (i.e., at failure), when the column is under maximum load.

This method will be further explained with the aid of the following examples:

Example 1

Suppose that the portal frame shown in Fig. 1 is to be designed. The span is $l = 8$ m, the height $h = 8.12$ m, the column load $F_c = 6$ kN, and the distributed load $q = 10$ kN/m. For reasons of economy (or of availability) the columns consist of the HE 100 A rolled steel section, steel grade Fe 360. In order to be able to use these slender columns, the load transmission as shown in Fig. 2b is chosen.

The maximum mid-span moment under design load ($\gamma = 1.5$) is:

$$M = \gamma q l^2/8 = 120 \text{ kNm}$$

The section chosen for the beam of the portal frame is the IPE 300, with a plastic moment $M_{pb} = 150$ kNm and a second moment of area $I = 8356 \times 10^{-8}$ m^4.

With the chosen bending moment distribution the column is conceived as axially (i.e., centrally) loaded with normal force F:

$$F = \gamma \{F_c + \tfrac{1}{2}ql\} = 69 \text{ kN}$$

The slenderness ratio of the column is:

$$\lambda_z = \frac{h}{i} = \frac{8.12}{0.0406} = 200.$$

According to Table B1-36 of the European (ECCS) Recommendations [1] it appears that $\lambda_z = 200$ is associated with a critical stress $\sigma_k = 43.8$ N/mm^2.

The stress occurring in the column under design load is:

$$\sigma = \frac{F}{A} = 32.5 \text{ N/mm}^2$$

Being an axially loaded column this column therefore fulfils the requirements.

Finally the check for rotational capacity is carried out. The rotation that the beam requires in order to attain M_{pb} at mid-span is (see equation 5):

$$\phi_b = \frac{(2M_{pb} - M_k)l}{6(EI)_b} = 2.3 \times 10^{-2} \text{ rad.}$$

The rotation that the column can develop is as follows (see Table 1 for $\beta = 0$, $\lambda_z = 200$ and $F/F_k = \sigma/\sigma_k = 0.75$):

$$\phi_k = 7.5 \times 10^{-2} \text{ rad.}$$

In this case therefore: $\phi_{\text{developable}} > \phi_{\text{required}}$

This means that the rotational capacity of the column is sufficient.

Example 2

The portal frame shown in Fig. 1 will again be the subject of this example, but now with stocky columns and large normal force. The span in this case is 5 m and the height 4.04 m. The column load is $F_k = 740$ kN and the distributed load $q = 32.0$ kN/m. The normal force under design load is:

$$F = \gamma \{F_c + \tfrac{1}{2}ql\} = 1230 \text{ kN}$$

The section chosen for the column is the HE 240 A, steel grade Fe 360, with $i = 101$ mm and $A = 76.8 \times 10^2$ mm^2. The slenderness ratio is therefore $\lambda_z = 4040/101 = 40$.

According tot the European Recommendations, Table B1-36:

$$\sigma_k = 231.5 \text{ N/mm}^2.$$

Hence it follows that:

$$\frac{F}{F_k} = 0.69$$

The rotation that can be developed is obtained from Table 1
($\beta = 0$, $\lambda_z = 40$, $F/F_k = 0.69$):

$$\phi_k \simeq 1.5 \times 10^{-2} \text{ rad.}$$

Since the column is not loaded to its utmost axial capacity, the column is additionally able to resist a bending moment. The load transmission shown in Fig. 2b is chosen, and the column top end moment is calculated in accordance with the European Recommendations for columns loaded in compression and bending:

$$F/F_p + \frac{\mu}{\mu - 1} (\beta^* M_k + \text{Fe})/M_{pk} \leqslant 1 \tag{8}$$

The maximum bending moment is found by equating the expression on the left-hand side to 1.

The factor μ in (8) denotes:

$$\mu = \frac{F_E}{F}, \text{ where } F_E = \frac{\pi^2 EI}{l^2}, \text{ furthermore } \beta^* = 0.6 + 0.4\beta.$$

The initial eccentricity e is calculated as follows (again in accordance with the European Recommendations):

$$e = \left(\frac{\sigma_r}{\sigma_k} - 1\right)\left(1 - \frac{\sigma_k}{\sigma_{cr}}\right)\frac{Z}{A} \tag{9}$$

where σ_r is the yield stress and $\sigma_{cr} = F_E/A$ while Z is the plastic section modulus of the column. On substitution of the data, the following values are obtained:

$F/F_p = 0.667$
$\mu = 8.02$
$\beta^* = 0.6$
$\sigma_r = 235 \text{ N/mm}^2$
$\sigma_{cr} = 1284 \text{ N/mm}^2$
$e = 8.1 \text{ mm}$

and finally the maximum moment which can be developed is:

$$M_k = 75.5 \text{ kNm}$$

The moment that the beam must be able to resist is calculated from equation (3):

$$M_{pb} = \lambda q l^2/8 - M_k$$

On evaluation this becomes: $M_{pb} = 74.5$ kNm

The IPE 270 rolled steel section can be chosen for the beam, for which we thus have:

$$M_{pb} = 123 \text{ kNm}$$

Finally, the required rotational capacity is calculated (equation 5):

$$\phi_b = \frac{(2M_{pb} - M_k)l}{6(EI)_b} = 1.2 \times 10^{-2} \text{ rad.}$$

In this case, too, the required rotation is less than the rotation that can be developed. The rotational capacity of the column is therefore adequate.

5 Conclusions with regard to current codes of practice

If the largest bending moment that the columns can resist is smaller than the plastic moment of the beam - or, stated differently, if the plastic hinges will occur in the columns - it is necessary to check that the columns possess sufficient rotational capacity.

As indicated in the preceding chapter, the question whether or not a column fulfils the rotational capacity requirement will depend not only on the column itself, but also on the beam or, in a more general way, on the rest of the structure to which the column is connected. It is in fact the structural members connected to the column that determine the amount of rotation required, and this will vary from one case to another. If the required rotation is small, the column need not have a large rotational capacity. Conversely, if the required rotation is large, the rotational capacity of the column will likewise have to be large.

Yet the codes of practice relating to this subject which have come to the present author's notice consider only the column. Thus, both the Netherlands code [7] and the Australian code [8] are based on the conception that, if the column can develop a certain rotation, "all will be well", on the assumption that any larger rotational capacity is not necessary in practice.

Deciding whether or not a column has sufficient rotational capacity by applying the criterion of an arbitrarily determined rotation limit admittedly has the merit of simplicity for practical use, but suffers from the disadvantage of being rather inflexible. Thus, in a majority of cases this criterion will be too conservative in that it will rule out the use of columns which could easily have developed the desired rotation. And, conceivably, in a small number of cases the rotational capacity of a column not ruled out on the basis of this criterion, may yet prove to be inadequate.

The two codes do indeed take account of the situation - commonly encountered in design practice - that a stocky column requires only little rotational capacity. This is done by reducing the rotational capacity required of such columns according as the slenderness ratio is less. The implicit embodiment of this principle in the design rules

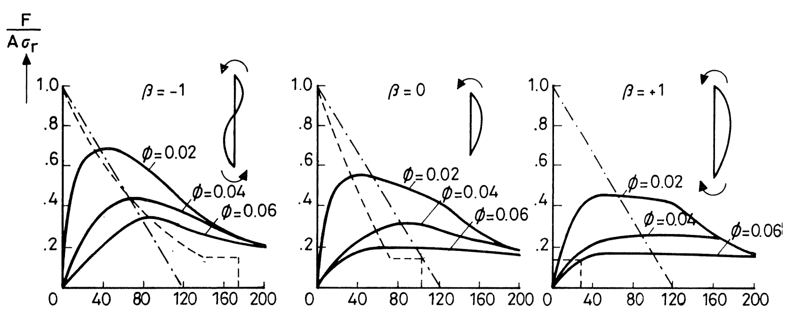

Fig. 10. Permissible combinations of normal force and slenderness with respect to the rotational capacity according to the Australian code (– – – – –), the Netherlands code (– · – · –) and the results of the present analysis (————) for $\beta = -1$, $\beta = 0$ and $\beta = +1$.

makes a direct comparison with the results obtained impossible. For this reason the design rules are indicated in Fig. 10, together with the various values of the total rotational capacity arrived at in the analysis. The curves give the limits of permissible combinations of slenderness and normal force for various values of β. The (arbitrary) limiting rotations with which the results of the analysis are presented are 0.02, 0.04 and 0.06 rad. 0.02, 0.04 and 0.06 rad.

In the case of braced frames the Netherlands code gives the following rule:

$$\frac{F}{A\sigma_r} + \frac{\lambda}{120} \leqslant 1 \qquad (10)$$

where σ_r is the yield stress and A the cross-sectional area. This rule is valid for steel grade Fe 360, and independent of the parameter β, whereas in the Australian code the effect of β is very important. Some notable differences are immediately evident:

1. The parameter β is found actually to have not much effect on the results. The Australian code (in which the relevant rules have been derived by a semi-analytical method) considerably overestimates this effect.
2. In contrast with what the codes envisage, the rotational capacity of columns in the "slender" range is very large, and the application of such columns is unnecessarily restricted in that range.
3. On the other hand, columns which are to be rated as "stocky" have less rotational capacity than indicated in the codes. As already stated, in this case the codes implicitly take account of the fact that such a column does not need to develop much rotation. In actual practice these columns are often so strong that the plastic hinge will always be formed in the beam, so that this part of the code is of merely secondary importance.

Despite the drawbacks of the Netherlands code with regard to the overall structural behaviour, the rule laid down in it has the advantages of simplicity and of serviceability in practice. In the "slender" range, however, the application of the lower bound theorem is unnecessarily restricted. If simplicity is nevertheless considered to be prime consideration, the existing rule could be modified as follows:

$$\frac{F}{A\sigma_r} + \frac{\lambda}{120} \leqslant 1 \quad \text{for} \quad \frac{F}{A\sigma_r} \geqslant 0.15 \qquad (11)$$

if

$$M_k \leqslant M_{pb}$$

What it amounts to is that the check can be omitted if the normal force is less than 15% of the force causing yield of the total cross-section. In the "slender" range the normal force is usually less than this value, so that the check for rotational capacity is then unnecessary.

6 Summary and conclusions

This paper gives a description of the various collapse modes of a braced portal frame. It appears that the angular rotation of the column ends play an important part in determining the collapse behaviour if the plastic hinges are formed in the columns. This situation arises when the maximum moment that the column end can resist is less than the plastic moment of the beam. A distinction is drawn between the case where a plastic hinge is formed first at mid-span of the beam, and the case where plastic hinges are formed first in the columns. In the latter case the moment developed by the column should remain constant until the plastic hinge at mid-span has also been formed and the collapse load reached. Actually, however, the column end moment may be drastically reduced due to local or overall instability effects. For this reason the Netherlands code of practice lays down requirements with regard to the columns of braced portal frames which have been designed on the basis of elementary plastic theory (lower bound theorem) and in which the plastic hinges are allowed in the columns. In this paper these requirements are compared with the results of calculations in which these instability effects have been taken into account, and it is explained why there are grounds for widening the scope of the present code.

The question whether or not sufficient angular rotation can be developed is, however, not decided as a property of the column alone, but follows from the geometry of the whole structure. The angular rotation that is needed to obtain the desired collapse mechanism is likewise important. This is demonstrated with two examples. Accordingly, an approach is proposed in which the required angular rotation is calculated. This procedure also deals with those cases where plastic hinges are indeed formed in the columns, but where the mid-span hinge is formed first and the columns need therefore not possess rotational capacity. It is shown that the normal force in the columns need not be constant and that the analysis can be based on the maximum normal force that occurs. The proposed method of analysis can be applied quite simply to braced frames of greater complexity.

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8 Notation

A	area of a cross-section
A_w	cross-sectional area of a web
A_f	cross-sectional area of a flange
b	width of a flange
e	eccentricity
E	modulus of elasticity
F	force
F_c	constant axial force on a column
F_E	Euler buckling force
F_p	yield force $(\sigma_r \cdot A)$
F_k	maximum column load $(\sigma_k \cdot A)$
h	height of a portal frame
i	radius of gyration
I	second moment of area
l	span
l_b	length of a buckling wave
M	bending moment
M_1	end moment with smallest absolute value
M_2	end moment with largest absolute value
M_{pb}	plastic moment of a beam
M_{pc}	plastic moment of a column
M_k	maximum column end moment
N	normal force at a cross-section
q	uniformly distributed load
t_f	thickness of a flange
t_w	thickness of a web
Z	plastic section modulus
β	ratio of minimum to maximum end moment of a column
β^*	factor to calculate equivalent moment when end moments are unequal
γ	factor of safety
λ	load factor
λ_z	slenderness ratio
ϕ	angular rotation
ϕ_b	angular rotation of a beam end
ϕ_k	rotation capacity of a column
η	shape factor
σ	stress
σ_k	maximum column stress (European Recommendations)
σ_r	yield stress
σ_{cr}	elastic critical stress (F_E/A)

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