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THE DESIGN OF TRANSVERSE AND LONGITUDINAL STIFFENERS FOR STIFFENED PLATE PANELS

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Abstract

A number of instability problems are encountered in the design of stiffened plate panels loaded within their own plane. Two such problems specifically associated with the stiffeners are treated here:

a. The instability of transverse stiffeners. In consequence of imperfections these stiffeners are subjected to a load which increases with the deformations and is produced by the primary load acting within the plane of the plate.
b. The torsional buckling instability of longitudinal stiffener of open cross-sectional shape which are primarily subjected to compressive load.

Rules for the analysis of both forms of instability are derived.

The design of transverse and longitudinal stiffeners for stiffened plate panels

1 Introduction

In connection with the revision of the Dutch Code VOSB '63 (Code of Practice for the Design of Steel Bridges) [9] it emerged that, till then, little attention had been paid to some particular forms of instability which may occur in the stiffeners of plate girders. There was a design rule only for transverse stiffeners by means of which the required flexural stiffness could be calculated. The background to the rule was, however, based on the development of instability in an initially perfect structure.

For the revised version of VOSB '63 some rules for analysing stiffened and unstiffened plate panels, loaded in their own plane, were established [8]. Two specific instability problems for stiffeners are dealt with in the present article. The necessary analysis rules are also derived.

a. *Instability of transverse stiffeners* [1, 4]

If a plate panel stiffened with transverse stiffeners is loaded in two directions within its plane and is moreover loaded perpendicularly to its plane, such stiffeners will be subjected to the following loads:

1. A directly-acting transverse load of constant magnitude.
2. A deflection-dependent transverse load arising from the loading within the plane of the plate and attributable to geometric imperfections of the transverse stiffeners.
3. The normal force and moment acting at the ends of the transverse stiffeners and caused by various possible circumstances, e.g., loading in the plane of the plate in a direction parallel to these stiffeners, cantilevering of the stiffeners, etc.

The stability of a transverse stiffener loaded in this way can be assessed with reference to the non-linear load-deformation behaviour, which can be described by means of a fourth-order differential equation with non-constant coefficients. Alternatively, this behaviour can be described by the repeated solving of a series of fourth-order differential equations with constant coefficients.

From the solution obtained by either of these methods, which is in fact a representation of the state of deformation of the transverse stiffener, it is possible to calculate the maximum moment, or the maximum stresses, acting in the stiffener. It can then be stipulated as a condition that the maximum moment must not exceed a particular value.

b. *Torsional buckling of longitudinal stiffeners of open cross-section* [3]

Longitudinal stiffeners of open cross-section which are loaded in compression may undergo torsional buckling as an instability mode. A check for the possible occurrence in this instability mode can be performed in analogy with that for flexural buckling. The procedure is as follows.

The specific slenderness ration of the longitudinal stiffener is first determined. It depends on the stress at which Euler torsional buckling occurs. Formulae for determining the critical elastic torsional buckling stress have been derived for flat bar-on-edge, *T*-section and *L*-section stiffeners in accordance with [5].

On the basis of the calculated specific slenderness ratio the stress is determined at which the longitudinal stiffener becomes unstable, taking account of initial imperfections, residual stresses and plastic behaviour [6].

2 Transverse stiffeners

2.1 *General*

As a result of geometric imperfections of the transverse stiffeners, measured perpendicularly to the plane of the plate, a load will act upon the stiffeners when the plate is loaded in its own plane.

On the assumption that the pre-existing deflection varies in sign from one transverse stiffener to the next, loading of the plane panel in its own plane will produce a so-called "concertina" action, as shown in Fig. 1.

With this schematization a requirement for the stiffness of the transverse stiffeners can be derived, based on geometrically non-linear and physically linear elastic behaviour.

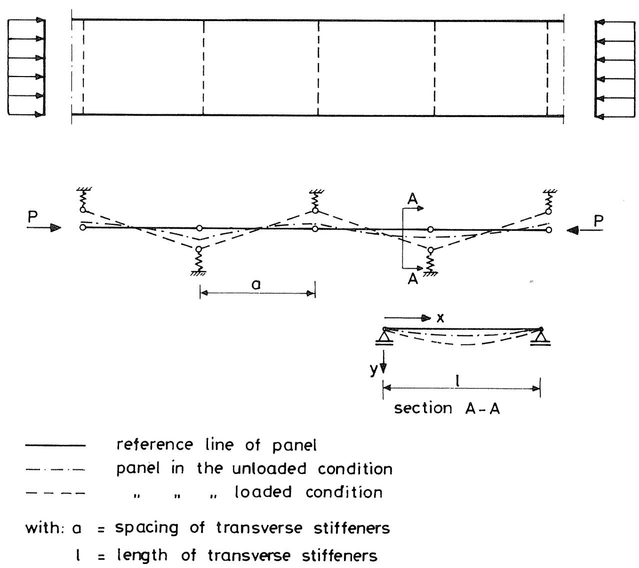


Fig. 1. Schematization of a plate panel stiffened with transverse stiffeners.

2.2 *Plate panel, subjected to linear stress distribution, supported by transverse stiffeners with an initial imperfection*

Starting points:

a. The panel is loaded with stresses acting in its plane in accordance with the following distribution (see Fig. 2):

$$\sigma(x) = \frac{\sigma_2 - \sigma_1}{l} \cdot x + \sigma_1 \tag{1}$$

Putting $\sigma_2 = \psi\sigma_1$ this may alternatively be written as:

$$\sigma(x) = \sigma_1 \cdot \left(\frac{\psi - 1}{l} \cdot x + 1\right) \tag{2}$$

where:

σ_1 = largest compressive stress in the panel
σ_2 = smallest compressive or largest tensile stress in the panel
l = length of the transverse stiffener

Note: compression = positive
tension = negative

b. The transverse stiffeners have an imperfection which is affine with respect to the deflected shape which occurs if the loading $\sigma(x)\delta$ acting in the plane of the plate would act perpendicularly to the transverse stiffeners. This deflected shape is a good approximation for the buckled shape for the transverse stiffener without imperfection.
The deflected shape obtained in this way is so scaled that the maximum displacement of the transverse stiffener is equal to a chosen initial imperfection e, for example: $e = \frac{1}{400}l$.

c. The plate panels are connected by hinged joints to the transverse stiffeners (so-called piano hinges).

d. The transverse stiffener is conceived as a beam on two hinged supports.

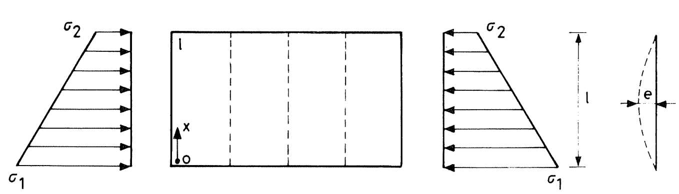


Fig. 2. Stress distribution in the plane of the plate; all edges freely supported.

2.3 *Analysis with the aid of a fourth-order differential equation with non-constant coefficients*

2.3.1 Transverse loading on the transverse stiffener

If the transverse stiffeners deflect alternately, the transverse loading on these stiffeners is:

$$q(x) = \frac{4p(x) \cdot (y(x) + y_i(x))}{a} \tag{3}$$

where:

a = centre-to-centre spacing of the transverse stiffeners

$$p(x) = \sigma_1 \cdot \left\{(\psi - 1) \cdot \frac{x}{l} + 1\right\} \cdot \delta' \tag{4}$$

where δ' is the average plate thickness:

$$\delta' = \frac{\delta \cdot l + \Sigma A_s}{l}$$

and:

l = width of plate panel
A_s = cross-sectional area of longitudinal stiffener
δ = plate thickness

Substitute (4) into (3):

$$q(x) = \frac{4\delta'\sigma_1}{a} \cdot \left\{(\psi - 1) \cdot \frac{x}{l} + 1\right\} \cdot (y(x) + y_i(x)) \tag{5}$$

2.3.2 Determination of the initial deflected shape $y_i(x)$ of the transverse stiffeners

The initial imperfection of the transverse stiffeners scaled to a chosen imperfection e can be written as:

$$y_i(x) = y_p(x) \cdot \frac{e}{y_p(x)_{\max}} \tag{6}$$

The following fourth-order differential equation can be established for the deflection $y_p(x)$ of the transverse stiffener under a load $p(x)$:

$$EIy_p''''(x) = p(x) \tag{7}$$

Substitute (4) in (7):

$$EIy_p''''(x) = \sigma_1 \cdot \left\{(\psi - 1) \cdot \frac{x}{l} + 1\right\} \cdot \delta' \tag{8}$$

Equation (8) can be written as:

$$y_p''''(x) = Mx + N \tag{9}$$

where

$$M = \frac{\delta'\sigma_1}{EI} \cdot \frac{(\psi - 1)}{l}$$

$$N = \frac{\delta'\sigma_1}{EI}$$

General solution of the homogeneous differential equation:

$$y_p''''(x) = 0 \tag{10}$$

Put

$$y_p(x) = Ax^3 + Bx^2 + Cx + D \tag{11}$$

$$y_p'''' = 0 \tag{12}$$

Conclusion (11) satisfies the homogeneous differential equation.

A particular solution of the non-reduced differential equation (9) of the following form is sought:

$$y_p(x) = Px^5 + Qx^4 \tag{13}$$

Hence the following requirement applies:

$$120Px + 24Q = Mx + N$$

This relation is satisfied if:

$$P = \tfrac{1}{120}M$$
$$Q = \tfrac{1}{24}N$$

The general solution of the non-reduced differential equation is:

$$y_p = \tfrac{1}{120} \cdot Mx^5 + \tfrac{1}{24} \cdot Nx^4 + Ax^3 + Bx^2 + Cx + D \tag{14}$$

Boundary conditions:

$$x = 0 : y_p\ (x = 0) = 0 \quad \rightarrow \quad D = 0 \tag{15}$$

$$x = 0 : y_p''(x = 0) = 0 \quad \rightarrow \quad B = 0 \tag{16}$$

$$x = l : y_p\ (x = l) = 0 \tag{17}$$

$$x = l : y_p''(x = l) = 0 \tag{18}$$

From (17) and (18):

$$\tfrac{1}{120}Ml^5 + \tfrac{1}{24}Nl^4 + Al^3 + Cl = 0 \tag{19}$$

$$\tfrac{1}{6}Ml^3 + \tfrac{1}{2}Nl^2 + 6Al = 0 \tag{20}$$

From (19) and (20):

$$A = -\tfrac{1}{36}Ml^2 - \tfrac{1}{12}Nl \tag{21}$$

$$C = \tfrac{7}{360}Ml^4 + \tfrac{1}{24}Nl^3 \tag{22}$$

The general solution of the non-reduced differential equation:

$$y_p(x) = \tfrac{1}{120}Mx^5 + \tfrac{1}{24}Nx^4 - (\tfrac{1}{36}Ml^2 + \tfrac{1}{12}Nl)x^3 + (\tfrac{7}{360}Ml^4 + \tfrac{1}{24}Nl^3)x \tag{23}$$

The location and magnitude of the maximum displacement in the interval $0 < x < l$ can be determined from:

$$y_p' = \tfrac{1}{24}Mx^4 + \tfrac{1}{6}Nx^3 - (\tfrac{1}{12}Ml^2 + \tfrac{1}{4}Nl)x^2 + \tfrac{7}{360}Ml^4 + \tfrac{1}{24}Nl^3 = 0 \tag{24}$$

Since

$$M = \frac{N(\psi - 1)}{l} = \text{(24) can be written to:}$$

$$\tfrac{1}{24} \cdot \frac{\psi - 1}{l} \cdot x^4 + \tfrac{1}{6} \cdot x^3 - \{\tfrac{1}{12} \cdot (\psi - 1) + \tfrac{1}{4}\} \cdot l \cdot x^2 + \{\tfrac{7}{360} \cdot (\psi - 1) + \tfrac{1}{24}\} \cdot l^3 = 0 \tag{25}$$

Put $\alpha = x/l$ and $l \neq 0$ and equation (25) can be written in a dimensionless form:

$$\tfrac{1}{24} \cdot (\psi - 1) \cdot \alpha^4 + \tfrac{1}{6}\alpha^3 - \{\tfrac{1}{12} \cdot (\psi - 1) + \tfrac{1}{4}\} \cdot \alpha^2 + \{\tfrac{7}{360} \cdot (\psi - 1) + \tfrac{1}{24}\} = 0 \tag{26}$$

ψ can be solved for certain values of α:

$\psi = 1.00$	$\alpha = .50$	$\psi = -1.00$	$\alpha = .76$
$\psi = 0.00$	$\alpha = .48$	$\psi = -1.25$	$\alpha = .63$
$\psi = -0.50$	$\alpha = .44$	$\psi = -1.50$	$\alpha = .59$
$\psi = -0.75$	$\alpha = .39$	$\psi = -2.00$	$\alpha = .56$
$\psi = -1.00$	$\alpha = .24$	$\psi = -3.00$	$\alpha = .54$
		$\psi = -\sim$	$\alpha = .52$

These results are represented graphically in Fig. 3.

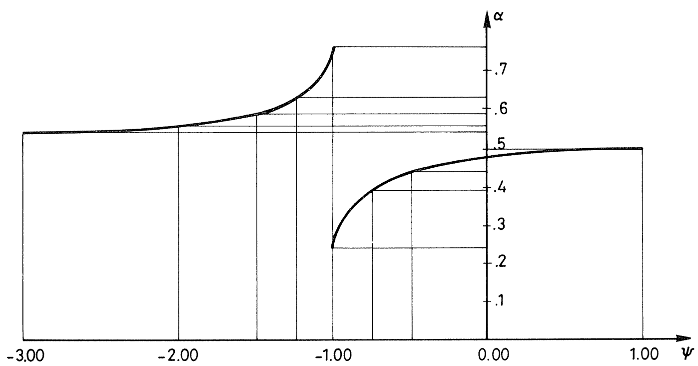


Fig. 3. Determination of the maximum deflection in the transverse stiffener.

Now $y_{p\,max}$ can be determined substituting

$$M = N\cdot\frac{\psi-1}{l},\quad N=\frac{\delta'\sigma_1}{EI}\quad \text{and}\quad \alpha=\frac{x}{l}\ \text{in (23)}$$

$$Z=\frac{y_{p\,max}}{Nl^4}=\tfrac{1}{120}\cdot(\psi-1)\cdot\alpha^5+\tfrac{1}{24}\alpha^4-\left\{\tfrac{1}{36}(\psi-1)+\tfrac{1}{12}\right\}\cdot\alpha^3+\left\{\tfrac{7}{360}\cdot(\psi-1)+\tfrac{1}{24}\right\}\cdot\alpha$$

In the following table for several values of ψ and corresponding α the maximum deflection is given.

ψ	α	$Z=\frac{y_{p\,max}}{Nl^4}$	ψ	α	$Z=\frac{y_{p\,max}}{Nl^4}$
1,00	0,50	0,01302	−1,00	0,76	−0,00041
0,00	0,48	0,00651	−1,25	0,63	−0,00182
−0,50	0,44	0,00331	−1,50	0,59	−0,00339
−0,75	0,39	0,00175	−2,00	0,56	−0,00661
−1,00	0,24	0,00041	−3,00	0,54	−0,01311

For further arithmetical treatment equation (27) is linearized:

$$\begin{aligned}
0{,}00 < \psi &\leqslant 1{,}00 \rightarrow Z=0{,}00651\cdot\psi+0{,}00651\\
-0{,}50 < \psi &\leqslant 0{,}00 \rightarrow Z=0{,}00640\cdot\psi+0{,}00651\\
-0{,}75 < \psi &\leqslant -0{,}50 \rightarrow Z=0{,}00624\cdot\psi+0{,}00643\\
-1{,}00 \leqslant \psi &\leqslant -0{,}75 \rightarrow Z=0{,}00536\cdot\psi+0{,}00577\\
-1{,}25 \leqslant \psi &< -1{,}00 \rightarrow Z=0{,}00564\cdot\psi+0{,}00523\\
-1{,}50 \leqslant \psi &< -1{,}25 \rightarrow Z=0{,}00628\cdot\psi+0{,}00603\\
-2{,}00 \leqslant \psi &< -1{,}50 \rightarrow Z=0{,}00664\cdot\psi+0{,}00667\\
-3{,}00 \leqslant \psi &< -2{,}00 \rightarrow Z=0{,}00650\cdot\psi+0{,}00639
\end{aligned}$$

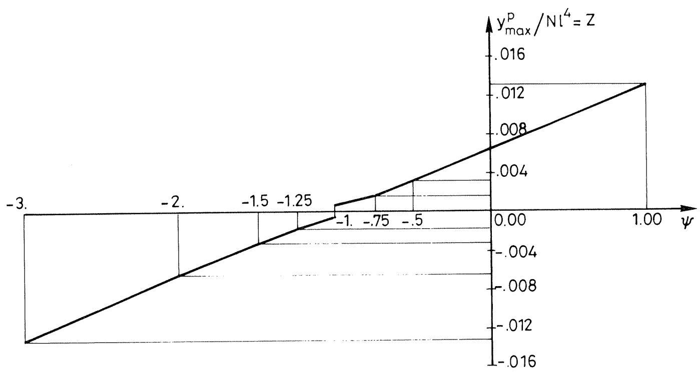


Fig. 4. Magnitude of the maximum deflection.

The initial deflected shape of the transverse stiffener will now be described with the following expression:

$$y_i(\alpha) = \frac{[3(\psi-1)\alpha^5 + 15\alpha^4 - \{10(\psi-1)+30\}\alpha^3 + \{7(\psi-1)+15\}\alpha]Nl^4}{360 \cdot Z \cdot Nl^4} \cdot e \qquad (28)$$

where $e = l/\varepsilon$ a fraction of the stiffener length as the chosen initial imperfection. $y_i(x)$ can be derived from (28) by putting $\alpha = x/l$. Note $y_i(\alpha)$ has the dimension of length.

2.3.3 Determination of the deflected shape of a transverse stiffener with initial deflected shape y_i and subjected to a transverse load q

The differential equation for the behaviour of the imperfect transverse stiffener is a fourth-order linear differential equation with non-constant coefficients.

$$EIy''''(x) = q(x) \qquad (29)$$

A power series can be adopted as the solution for y. In that case it is preferable to make the solution non-dimensional.

Put

$$\alpha = \frac{x}{l} \rightarrow \frac{\mathrm{d}y(\alpha)}{\mathrm{d}x} = \frac{\mathrm{d}y(\alpha)}{\mathrm{d}\alpha} \cdot \frac{\mathrm{d}\alpha}{\mathrm{d}x} = \frac{\mathrm{d}y(\alpha)}{\mathrm{d}x} \cdot \frac{1}{l}$$

$$\frac{d^n y(\alpha)}{\mathrm{d}x^n} = \frac{d^n y(\alpha)}{\mathrm{d}x^n} \cdot \frac{1}{l^n}$$

Substitute (5) into (29):

$$EIy''''(x) = \frac{4\delta'\sigma_1}{a} \cdot \left\{(\psi-1) \cdot \frac{x}{l} + 1\right\} \cdot \{y(x) + y_i(x)\} \qquad (30)$$

The non-dimensional form is:

$$\frac{d^4 y(\alpha)}{\mathrm{d}\alpha^4} = \frac{4\delta'\sigma_1 l^4}{EIa} \cdot \{(\psi-1)\cdot\alpha + 1\} \cdot \left\{y(\alpha) + \frac{y_i^0(\alpha)}{l}\right\} \qquad (31)$$

or:

$$\frac{d^4 y(\alpha)}{\mathrm{d}\alpha^4} = (\gamma \cdot \alpha + \lambda) \cdot y(\alpha) + (\gamma \cdot \alpha + \lambda) \cdot y_i(\alpha) \qquad (32)$$

This is a fourth-order linear differential equation with no-constant coefficients, where

$$\gamma = \frac{4\delta'\sigma_1 l^4}{EIa} \cdot (\psi - 1)$$

$$\lambda = \frac{4\delta'\sigma_1 l^4}{EIa}$$

$$y_i(\alpha) = \frac{3(\psi-1)\alpha^5 + 15\alpha^4 - \{10(\psi-1)+30\}\alpha^3 + \{7(\psi-1)+15\}\alpha}{360\cdot Z\cdot \varepsilon} \tag{33}$$

Note: $y(\alpha) = y_i(x)$ and $y_i^0(\alpha)$ have the dimension of length. $y(\alpha)$ and $y_i(\alpha)$ are non-dimensional.

General solution of the homogeneous differential equation

$$\frac{d^4 y(\alpha)}{d\alpha^4} - \gamma\alpha\cdot y(\alpha) - \lambda\cdot y(\alpha) = 0 \tag{34}$$

Put

$$y(\alpha) = \sum_{n=0}^{\infty} A_n \alpha^n \tag{35}$$

then (33) becomes:

$$\sum_{n=4}^{\infty} \frac{n!}{(n-4)!} A_n \alpha^{n-4} - \gamma \sum_{n=0}^{\infty} A_n \alpha^{n+1} - \lambda \sum_{n=0}^{\infty} A_n \alpha^n = 0 \tag{36}$$

This equation is satisfied for all powers of α separately:

$$\alpha^0:\ \frac{4!}{0!}\cdot A_4 \qquad - \lambda A_0 = 0 \rightarrow A_4 = \qquad \lambda A_0 \cdot \frac{0!}{4!}$$

$$\alpha^1:\ \frac{5!}{1!}\cdot A_5 - \gamma A_0 - \lambda A_1 = 0 \rightarrow A_5 = (\gamma A_0 + \lambda A_1)\cdot \frac{1!}{5!}$$

$$\alpha^2:\ \frac{6!}{2!}\cdot A_6 - \gamma A_1 - \lambda A_2 = 0 \rightarrow A_6 = (\gamma A_1 + \lambda A_2)\cdot \frac{2!}{6!}$$

$$\alpha^3:\ \frac{7!}{3!}\cdot A_7 - \gamma A_2 - \lambda A_3 = 0 \rightarrow A_7 = (\gamma A_2 + \lambda A_3)\cdot \frac{3!}{7!}$$

$$\alpha^4:\ \frac{8!}{4!}\cdot A_8 - \gamma A_3 - \lambda A_4 = 0 \rightarrow A_8 = (\gamma A_3 + \lambda A_4)\cdot \frac{4!}{8!}$$

$$\alpha^5:\ \frac{9!}{5!}\cdot A_9 - \gamma A_4 - \lambda A_5 = 0 \rightarrow A_9 = (\gamma A_4 + \lambda A_5)\cdot \frac{5!}{9!}$$

etc.

$$n \geqslant 5:\ \alpha^{n-4}:\ \frac{n!}{(n-4)!}\cdot A_n - \gamma A_{n-4} - \lambda A_{n-4} = 0 \rightarrow A_n = (\gamma A_{n-5} + \lambda A_{n-4})\cdot \frac{(n-4)!}{n!}$$

A_0, A_1, A_2 and A_3 are the four independent constants of integration. All the other constants $A_n\,(n \geqslant 4)$ can be expressed in these independent ones:

$$A_n = \lambda\,(\varrho A_{n-5} + A_{n-4})\cdot \frac{(n-4)!}{n!} \tag{37}$$

where

$$\varrho = \frac{\gamma}{\lambda}$$

and

$$\lambda = \frac{4\delta'\sigma_1 l^4}{EIa}$$

Particular solution of the non-reduced differential equation
Equation (32) can be written as

$$\frac{d^4y(\alpha)}{d\alpha^4} - \gamma\alpha \cdot y(\alpha) - \lambda \cdot y(\alpha) = (\gamma\alpha + \lambda) \cdot y_i(\alpha) \tag{38}$$

with

$$y_i(\alpha) = \beta_0 + \beta_1\alpha + \beta_2\alpha^2 + \beta_3\alpha^3 + \beta_4\alpha^4 + \beta_5\alpha^5 \tag{39}$$

Comparing (39) with (33) leads to:

$$\beta_0 = 0 \tag{39a}$$

$$\beta_1 = \frac{7(\psi - 1) + 15}{360 \cdot Z \cdot \varepsilon} \tag{39b}$$

$$\beta_2 = 0 \tag{39c}$$

$$\beta_3 = \frac{-10(\psi - 1) - 30}{360 \cdot Z \cdot \varepsilon} \tag{39d}$$

$$\beta_4 = \frac{15}{360 \cdot Z \cdot \varepsilon} \tag{39e}$$

$$\beta_5 = \frac{3(\psi - 1)}{360 \cdot Z \cdot \varepsilon} = \beta_4 \cdot \frac{(\psi - 1)}{5} \tag{39f}$$

Equation (38) now becomes:

$$\frac{d^4y(\alpha)}{d\alpha^4} - \gamma\alpha \cdot y(\alpha) - \lambda \cdot y(\alpha) = \lambda\beta_1\alpha + \gamma\beta_1\alpha^2 + \lambda\beta_3\alpha^3 + (\gamma\beta_3 + \lambda\beta_4)\alpha^4 + \\ + (\gamma\beta_4 + \lambda\beta_5)\alpha^5 + \gamma\beta_5\alpha^6 \tag{40}$$

Put

$$y(\alpha) = \sum_{n=0}^{5} B_n \alpha^n;$$

then (40) becomes

$$\sum_{n=4}^{5} \frac{n!}{(n-4)!} B_n \alpha^{n-4} - \gamma \sum_{n=0}^{5} B_n \alpha^{n+1} - \lambda \sum_{n=0}^{5} B_n \alpha^n = \\ \lambda\beta_1\alpha + \gamma\beta_1\alpha^2 + \lambda\beta_3\alpha^3 + (\gamma\beta_3 + \lambda\beta_4)\alpha^4 + (\gamma\beta_4 + \lambda\beta_5)\alpha^5 + \gamma\beta_5\alpha^6 \tag{41}$$

This equation is satisfied if the coefficients of α^n, with $n = 0, 1, 2, 3, 4, 5$ and 6 on the left are equal to those on the right.

$$\alpha^6: \quad -\gamma B_5 = \gamma\beta_5 \qquad \rightarrow B_5 = -\beta_5 \tag{41a}$$

$$\alpha^5: \quad -\gamma B_4 - \lambda B_5 = \gamma\beta_4 + \lambda\beta_5 \rightarrow B_4 = -\beta_4 \tag{41b}$$

$$\alpha^4: \quad -\gamma B_3 - \lambda B_4 = \gamma\beta_3 + \lambda\beta_4 \rightarrow B_3 = -\beta_3 \tag{41c}$$

$$\alpha^3: \quad -\gamma B_2 - \lambda B_3 = \lambda\beta_3 \qquad \rightarrow B_2 = 0 \tag{41d}$$

$$\alpha^2: \quad -\gamma B_1 - \lambda B_2 = \gamma\beta_1 \qquad \rightarrow B_1 = -\beta_1 \tag{41e}$$

$$\alpha^1: \quad \frac{5!}{1!} B_5 - \gamma B_0 - \lambda B_1 = \lambda\beta_1 \rightarrow \frac{5!}{1!} B_5 - \gamma B_0 = 0 \tag{41f}$$

$$\alpha^0: \quad \frac{4!}{0!} B_4 - \lambda B_0 = 0 \tag{41g}$$

From (41g):

$$B_0 = -\frac{4!}{0!}\beta_4 \cdot \frac{1}{\lambda} \tag{41h}$$

Substitute (41a) into (41f):

$$-\frac{5!}{1!}\beta_5 - \gamma B_0 = 0 \tag{41i}$$

Substitute (39f) and $\gamma = \lambda \cdot (\psi - 1)$ into (41i):

$$B_0 = -\frac{5!}{1!}\beta_5 \cdot \frac{1}{\gamma} = -\frac{5!}{1!} \cdot \beta_4 \cdot \frac{\psi - 1}{5} \cdot \frac{1}{\lambda \cdot (\psi - 1)}$$

$$B_0 = -\frac{4!}{1!} \cdot \beta_4 \cdot \frac{1}{\lambda} \tag{41j}$$

As $0! = 1$ and also $1! = 1$ (41h) is identical to (41j).

General solution of the non-reduced differential equation

Adding together the general and the particular solution gives:

$$y(\alpha) = A_0 + A_1\alpha + A_2\alpha^2 + A_3\alpha^3 + \sum_{n=4}^{\infty} A_n\alpha^2 + \ldots$$

$$\ldots - \frac{4!}{0!}\beta_4 \cdot \frac{1}{\lambda} - \beta_1\alpha - \beta_3\alpha^3 - \beta_4\alpha^4 - \beta_5\alpha^5 \tag{42}$$

The constants of integration can be determined by using the boundary conditions.

Boundary conditions:

$$1°: \; \alpha = 0 \rightarrow y(\alpha = 0) = 0 \rightarrow A_0 = \frac{4!}{0!}\beta_4 \cdot \frac{1}{\lambda}$$

$$2°:\ \alpha = 0 \rightarrow \left(\frac{d^2y}{\mathrm{d}\alpha^2}\right)_{\alpha=0} = 0 \rightarrow A_2 = 0$$

Applying these two boundary conditions lead to:

$$y(\alpha) = A_1\alpha + A_3\alpha^3 + \sum_{n+4}^{\infty} A_n\alpha^n - \beta_1\alpha - \beta_3\alpha^3 - \beta_4\alpha^4 - \beta_5\alpha^5 \tag{43}$$

$$3°:\ \alpha = 1 \rightarrow y(\alpha = 1) = 0$$

$$4°:\ \alpha = 1 \rightarrow \left(\frac{d^2y}{\mathrm{d}\alpha^2}\right)_{\alpha=1} = 0$$

Applying the third and fourth boundary conditions lead to

$$A_1 + A_3 + \sum_{n=4}^{\infty} A_n - \beta_1 - \beta_3 - \beta_4 - \beta_5 = 0 \tag{44}$$

$$6A_3 + \sum_{n=4}^{\infty} A_n \cdot n \cdot (n-1) - 6\beta_3 - 12\beta_4 - 20\beta_5 = 0 \tag{45}$$

with

$$A_n = \lambda \cdot (\varrho A_{n-5} + A_{n-4}) \cdot \frac{(n-4)!}{n!} \tag{46a}$$

where

$$\lambda = \frac{4\delta'\sigma_1 l^4}{EIa} \tag{46b}$$

$$\varrho = \psi - 1 \tag{46c}$$

The terms

$$\sum_{n=4}^{\infty} A_n \quad \text{and} \quad \sum_{n=4}^{\infty} A_n \cdot n \cdot (n-1)$$

from (44) and (45) can be expressed in the constants of integration A_0, A_1 and A_3 as follows

$$\begin{aligned}\sum_{n=4}^{\infty} A_n = A_0 &\sum_{m=0}^{\infty}\sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_0)} + \\ + A_1 &\sum_{m=0}^{\infty}\sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_1)} + \\ + A_3 &\sum_{m=0}^{\infty}\sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_3)}\end{aligned} \tag{47}$$

$$\sum_{n=4}^{\infty} A_n = A_0 \sum_{m=0}^{\infty} \varrho^m \xi_m^{(A_0)} + A_1 \sum_{m=0}^{\infty} \varrho^m \xi_m^{(A_1)} + A_3 \sum_{m=0}^{\infty} \varrho^m \xi_m^{(A_3)} \tag{48}$$

where

$$\xi_m^{(A_i)} = \sum_{n=4}^{\infty} C_{n,m}^{(A_i)} \qquad (48a)$$

$$\sum_{n=4}^{\infty} A_n \cdot n \cdot (n-1) = A_0 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_0)} \cdot n \cdot (n-1) +$$

$$+ A_1 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_1)} \cdot n \cdot (n-1) +$$

$$+ A_3 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_3)} \cdot n \cdot (n-1) \qquad (49)$$

$$\sum_{n=4}^{\infty} A_n \cdot n \cdot (n-1) = A_0 \sum_{m=0}^{\infty} \varrho^m \eta_m^{(A_0)} + A_1 \sum_{m=0}^{\infty} \varrho^m \eta_m^{(A_1)} +$$

$$+ A_3 \sum_{m=0}^{\infty} \varrho^m \eta_m^{(A_3)} \qquad (50)$$

where

$$\eta_m^{(A_i)} = \sum_{n=4}^{\infty} C_{n,m}^{(A_i)} \cdot n \cdot (n-1) \qquad (50a)$$

Put:

$$\overline{\xi^{(A_i)}} = (\xi_0^{(A_i)} \xi_1^{(A_i)} \xi_2^{(A_i)} \xi_3^{(A_i)} \xi_4^{(A_i)} \ldots\ldots) \text{ matrix } 1 \times \infty \qquad (51)$$

$$\overline{\eta^{(A_i)}} = (\eta_0^{(A_i)} \eta_1^{(A_i)} \eta_2^{(A_i)} \eta_3^{(A_i)} \eta_4^{(A_i)} \ldots\ldots) \text{ matrix } 1 \times \infty \qquad (52)$$

$$f = \begin{vmatrix} \varrho^0 \\ \varrho^1 \\ \varrho^2 \\ \varrho^3 \\ - \\ - \\ - \\ - \\ \varrho^\infty \end{vmatrix} \quad \text{vector} \qquad (53)$$

The components of the $\xi^{(A_i)}$ and $\eta^{(A_i)}$ in themselves form series for which it can be shown that they are convergent. This can be done by majorating the series and then comparing them with the convergent series

$$S_n = \sum_{n=1}^{\infty} \frac{\lambda^n}{n!}$$

$$\lim_{n \to \infty} \delta_n = e^\lambda - 1 \quad \text{with} \quad e = 2{,}718281828$$

The equations (44) and (45) can now be written in terms of A_0, A_1, A_3 and $\beta_1, \beta_3, \beta_4, \beta_5$.

From (44):

$$A_1 + A_3 + A_0\overline{\xi^{(0)}\underline{\varrho}} + A_1\overline{\xi^{(1)}\underline{\varrho}} + A_3\overline{\xi^{(3)}\underline{\varrho}} - \beta_1 - \beta_3 - \beta_4 - \beta_5 = 0 \tag{54}$$

From (45):

$$6A_3 + A_0\overline{\eta^{(0)}\underline{\varrho}} + A_1\overline{\eta^{(1)}\underline{\varrho}} + A_3 \cdot \overline{\eta^{(3)}\underline{\varrho}} - 6\beta_3 - 12\beta_4 - 20\beta_5 = 0 \tag{55}$$

From the first boundary condition followed

$$A_0 = \frac{4!}{0!}\beta_4 \cdot \frac{1}{\lambda}$$

So (54) and (55) become:

$$(1 + \overline{\xi^{(1)}\underline{\varrho}}) \cdot A_1 + (1 + \overline{\xi^{(3)}\underline{\varrho}}) \cdot A_3 = \beta_1 + \beta_3 + \left(1 - \frac{4!}{\lambda}\overline{\xi^{(0)}\underline{\varrho}}\right)\beta_4 + \beta_5 \tag{56}$$

$$(\overline{\eta^{(1)}\underline{\varrho}}) \cdot A_1 + (6 + \overline{\eta^{(3)}\underline{\varrho}}) \cdot A_3 = 6\beta_3 + \left(12 - \frac{4!}{\lambda}\overline{\eta^{(0)}\underline{\varrho}}\right)\beta_4 + 20\beta_5 \tag{57}$$

Put:

$$K_1 = \overline{\xi^{(1)}\underline{\varrho}} \qquad K_3 = \overline{\xi^{(3)}\underline{\varrho}} \qquad K_0 = \overline{\xi^{(0)}\underline{\varrho}}$$

$$E_1 = \overline{\eta^{(1)}\underline{\varrho}} \qquad E_3 = \overline{\eta^{(3)}\underline{\varrho}} \qquad E_0 = \overline{\eta^{(0)}\underline{\varrho}}$$

and:

$$C_{11} = (1 + K_1)$$
$$C_{12} = (1 + K_3)$$
$$C_{21} = \lambda E_1$$
$$C_{22} = (6 + E_3)$$
$$B_1 = \beta_1 + \beta_3 + \left(1 - \frac{24}{\lambda}K_0\right) \cdot \beta_4 + \beta_5$$
$$B_2 = 6\beta_3 + \left(12 - \frac{24}{\lambda}E_0\right) \cdot \beta_4 + 20\beta_5$$

The equations (56) and (57) become:

$$C_{11} \cdot A_1 + C_{12} \cdot A_3 = B_1 \tag{58}$$

$$C_{21} \cdot A_1 + C_{22} \cdot A_3 = B_2 \tag{59}$$

Put:

$$D = C_{11} \cdot C_{22} - C_{12} \cdot C_{21}$$
$$D_1 = B_1 \cdot C_{22} - C_{12} \cdot B_2$$
$$D_3 = C_{11} \cdot B_2 - B_1 \cdot C_{21}$$

Now from (58) and (59) the constants of integration A_1 and A_3 can be calculated from

$$A_1 = \frac{D_1}{D} \tag{60}$$

$$A_3 = \frac{D_3}{D} \tag{61}$$

The complete expression for the non-dimensional deflection of the transverse stiffener is known.

$$y(\alpha) = A_1\alpha + A_3\alpha^3 + A_0 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_0)}\alpha^n +$$
$$+ A_1 \cdot \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_1)}\alpha^n + A_3 \cdot \sum_{m=4}^{\infty} \sum_{n=4}^{\infty} \varrho^m \cdot C_{n,m}^{(A_3)}\alpha^n +$$
$$- \beta_1\alpha - \beta_3\alpha^3 - \beta_4\alpha^4 - \beta_5\alpha^5 \tag{62}$$

2.3.4 Location and magnitude of the largest moment in the transverse stiffener

$$- EIy''(x) = M(x) \tag{63}$$

The location of this largest moment can be ascertained by solving from the equation (62) the third derivative

$$\frac{d^3y(\alpha)}{d\alpha^2} = 0$$

the root α for particular values of ϱ and λ.

$$-2 \leqslant \varrho = \psi - 1 \leqslant 0 \quad \text{since} \quad -1 \leqslant \psi \leqslant 1 \tag{64}$$

$$0 < \lambda = \frac{4\delta'\sigma_1 l^4}{EIa} \leqslant \lambda \tag{65}$$

From equation (62) the third derivative is:

$$\frac{d^3y(\alpha)}{d\alpha^3} = 6A_3 + A_0 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m C_{n,m}^{(A_0)} \cdot n(n-1)(n-2) \cdot \alpha^{n-3} +$$
$$+ A_1 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m C_{n,m}^{(A_1)} \cdot n(n-1)(n-2) \cdot \alpha^{n-3} +$$
$$+ A_3 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m C_{n,m}^{(A_3)} \cdot n(n-1)(n-2) \cdot \alpha^{n-3} +$$
$$- 6\beta_3 - 24\beta_4\alpha - 60\beta_5\alpha^2 = 0 \tag{66}$$

The root $\alpha(\varrho, \lambda)$ of this equation is substituted into (63) to calculate the extreme moment.

Note: $y(x) = y(\alpha) \cdot l$

$$\frac{d^2y(x)}{dx^2} : \frac{d^2y(\alpha)}{dx^2} \cdot l = \frac{d^2y(\alpha)}{d\alpha^2} \cdot \frac{l}{l^2} \tag{67}$$

Substitute (67) into (63):

$$-\frac{EI}{l} \cdot \frac{d^2y(\alpha)}{d\alpha^2} = M(\alpha) \tag{68}$$

From equation (63) the second derivative is:

$$\begin{aligned}\frac{d^2y(\alpha)}{d\alpha^2} = 6A_3\alpha + A_0 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m C_{n,m}^{(A_0)} \cdot n(n-1) \cdot \alpha^{n-2} + \\ + A_1 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m C_{n,m}^{(A_1)} \cdot n(n-1) \cdot \alpha^{n-2} + \\ + A_3 \sum_{m=0}^{\infty} \sum_{n=4}^{\infty} \varrho^m C_{n,m}^{(A_3)} \cdot n(n-1) \cdot \alpha^{n-2} + \\ - 6\beta_3\alpha - 12\beta_4\alpha^2 - 20\beta_5\alpha^3\end{aligned} \tag{69}$$

The extreme moment is:

$$M_{\max}(\alpha = \propto) = -\left\{\frac{d^2y(\alpha)}{d\alpha^2}\right\}_{\alpha=\propto} \cdot \frac{EI}{l} \leqslant M_{\text{limit transverse stiffener}} \tag{70}$$

The limit moment for the transverse stiffener may correspond to the attainment of a particular limit stress in the extreme fibre, e.g., the yield stress of lateral torsional buckling stress

$$M_{\text{limit}} = \sigma_{\text{limit}} \cdot W \tag{71}$$

$$W = \frac{I}{\mu_1 H} \tag{72}$$

$$-\frac{d^2y(\alpha)}{d\alpha^2(\alpha = \propto)} \cdot \frac{EI}{l} \leqslant \sigma_{\text{limit}} \cdot \frac{I}{\mu_1 H} \tag{73}$$

$$-\frac{d^2y(\alpha)}{d\alpha^2(\alpha = \propto)} \cdot \frac{E}{l} \leqslant \frac{\sigma_{\text{limit}}}{\mu_1 \mu_2 l} \tag{74}$$

For the transverse stiffener the following equation must hold:

Fig. 5. Transverse stiffener.

$$-\frac{d^2y(\alpha)}{d\alpha^2\{\alpha = \propto(\psi,\lambda)\}} \cdot E \cdot \mu_1 \cdot \mu_2 \leqslant \sigma_{\text{limit}} \tag{75}$$

or

$$\Phi E \mu_1 \mu_2 \leqslant \sigma_{\text{limit}} \tag{76}$$

with

$$\psi = \varrho = \psi - 1$$

$$\lambda = \frac{4\delta' \sigma_1 l^4}{EIa}$$

$$\mu_1 = \frac{H}{h}$$

$$\mu_2 = \frac{h}{l}$$

being the parameters which describe the loading condition of the plate panel and the geometry of the transverse stiffener.

If the transverse stiffener is also subjected to a directly acting transverse load and/or to end moments, the following requirement will have to be satisfied:

$$\left(1 + \frac{\varepsilon}{\varepsilon^1}\right) \Phi E \mu_1 \mu_2 + \frac{M_D}{W} \leqslant \sigma_{\text{limit}} \tag{77}$$

where:

M_D = maximum moment in the transverse stiffener due to the transverse load and/or end moments

W = section modulus of the transverse stiffener

ε = length of the transverse stiffener divided by the maximum deflection due to the initial imperfection

ε^1 = length of the transverse stiffener divided by the first-order deflection due to the transverse load and/or end moments

In this case the imperfection must be taken as the sum of the geometric imperfection $1/E$ of the length of the transverse stiffener and the first-order deflection due to the transverse load. In the formula this is taken into account by the factor $(1 + \varepsilon/\varepsilon')$.

In order to make the checking criterion for the transverse stiffeners, described by the expression (76), convenient to handle, a parameter analysis will be performed with respect to the term:

$$\Phi = -\left(\frac{d^2y(\alpha)}{d\alpha^2}\right)_{\{\alpha = \propto(\psi,\lambda)\}} \tag{78}$$

Φ is a non-dimensional quantity depending on ψ and λ and can be determined for various values of ε, i.e., in principle for $e = l/\varepsilon$.

2.4 *Iterative method of calculation with the aid of a fourth-order differential equation with constant coefficients*

It is possible to avoid using a fourth-order differential equation with *non*-constant coefficients by solving a series of successive fourth-order differential equations with constant coefficients.

A finite series will be used for describing the initial deflected shape:

$$y_i(\alpha) = \sum_{m=0}^{n} B_m \alpha^m \tag{79}$$

This initial shape satisfies the boundary conditions:

$$\alpha = 0 : y_i(\alpha = 0) = 0 \rightarrow B_0 = 0 \tag{80a}$$

$$\alpha = 0 : \left(\frac{d^2 y_i(\alpha)}{\mathrm{d}\alpha^2}\right)_{\alpha = 0} = 0 \rightarrow B_2 = 0 \tag{80b}$$

Equation (79) becomes

$$y_i(\alpha) = \sum_{m=1}^{n} B_m \alpha^m \tag{81}$$

The deflected shape is represented by a finite series

$$y(\alpha) = \sum_{p=0}^{q} A_p \alpha^p \tag{82}$$

where $q = n + 5$.

The following fourth-order differential equation with constant coefficients can be established:

$$\frac{d^4 y(\alpha)}{\mathrm{d}\alpha^4} = (\gamma \cdot \alpha + \lambda) \cdot y_p(\alpha) \tag{83}$$

where

$$\gamma = \frac{4\delta' \sigma_1 l^4}{EIa} \cdot (\psi - 1)$$

$$\lambda = \frac{4\delta' \sigma_1 l^4}{EIa}$$

General solution of the homogeneous differential equation

$$\frac{d^4 y(\alpha)}{\mathrm{d}\alpha^4} = 0 \tag{84}$$

Put

$$y(\alpha) = \sum_{p=0}^{q} C_p \alpha^p \tag{85}$$

Substitute (85) into (84):

$$\sum_{p=4}^{q} \frac{p!}{(p-4)!} C_{p\alpha}^{p-4} = 0 \tag{86}$$

The series satisfies this if

$$C_p = 0 \quad \text{with} \quad 4 \leqslant p \leqslant q$$

Particular solution of the non-reduced differential equation

$$\frac{d^4 y(\alpha)}{\mathrm{d}\alpha^4} = (\gamma \cdot \alpha + \lambda) \cdot y_i(\alpha) \tag{87}$$

Put

$$y(\alpha) = \sum_{p=0}^{q} D_p \alpha^p \tag{88}$$

Substitute (88) into (87):

$$\sum_{p=4}^{q} \frac{p!}{(p-4)!} D_p \alpha^{p-4} = (\gamma \cdot \alpha + \lambda) \cdot \sum_{m=1}^{n} B_m \alpha^m \tag{89}$$

This condition can be satisfied by equating the coefficients of the same powers of α.

D_0, D_1, D_2 and D_3 are zero. (89a)

$$\alpha^0 : \frac{4!}{0!} D_4 = 0 \rightarrow D_4 = 0 \tag{89b}$$

$$\alpha^1 : \frac{5!}{1!} D_5 = \lambda B_1 \tag{89c}$$

$$\alpha^2 : \frac{6!}{2!} D_6 = \gamma B_1 + \lambda B_2 \tag{89d}$$

$$\alpha^3 : \frac{7!}{3!} D_7 = \gamma B_2 + \lambda B_3 \tag{89e}$$

$$\alpha^{q-4} = \alpha^{n+1} : \frac{q!}{(q-4)!} D_q = \gamma B_n \tag{89f}$$

Hence

$$y(\alpha) = C_0 + C_1\alpha + C_2\alpha^2 + C_3\alpha^3 + \sum_{p=5}^{q} D_p \alpha^p \tag{90}$$

Boundary conditions:

$$1°:\ \alpha = 0:\ y(\alpha = 0) = 0 \quad \rightarrow \quad C_0 = 0$$

$$2°:\ \alpha = 0:\ \left(\frac{d^2y}{d\alpha^2}\right)_{(\alpha = 0)} = 0 \quad \rightarrow \quad C_2 = 0$$

$$3°:\ \alpha = 1:\ y(\alpha = 1) = 0 \quad \rightarrow \quad C_1 + C_3 = -\sum_{p=5}^{q} D_p$$

$$4°:\ \alpha = 1:\ \left(\frac{d^2y}{d\alpha^2}\right)_{(\alpha = 1)} = 0 \quad \rightarrow \quad 6C_3 = -\sum_{p=5}^{q} D_p \cdot p \cdot (p-1)$$

From the 3° and 4° boundary condition it follows that:

$$C_3 = -\tfrac{1}{6} \cdot \sum_{p=5}^{q} D_p \cdot p \cdot (p-1)$$

$$C_1 = -\sum_{p=5}^{q} D_p + \tfrac{1}{6} \cdot \sum_{p=5}^{q} D_p \cdot p \cdot (p-1)$$

Hence

$$y(\alpha) = C_1\alpha + C_3\alpha^3 + \sum_{p=5}^{q} D_p \alpha^p \tag{91}$$

Now all the coefficients of (82) are known and can be written as

$$y(\alpha) = \sum_{p=1}^{q} A_p \alpha^p \tag{92}$$

Equation (92) is the deflected shape which is obtained in accordance with linear elastic theory. By adjusting ("updating") the initial shape as follows:

$$y_{i(I+1)}(\alpha) = y_{i(I)}(\alpha) + y_{(I)}(\alpha) \tag{93}$$

it is possible again to establish a fourth-order differential equation with constant coefficients and solve it. This process can be repeated as many times as necessary for determining the final deflected shape $y(\alpha)$.

If the initial deflected shape is described with, for example, five terms, then the first-order deflected shape is described with a series

$$y(\alpha) = \sum_{p=1}^{10} A_p \alpha^p \tag{94}$$

Each time the process is repeated, five more terms are added. Therefore

$$y_{(I)}(\alpha) = \sum_{p=1}^{10 + I \cdot 5} A_p \alpha^p$$

where I is the number of iterations.

In this way an iterative process has been developed which can be stopped when equilibrium is satisfied within a predetermined accuracy.

On completion of this process for determining the deflected shape the location and magnitude of the maximum moment can be determined in the same way as has been described in Section 2.3.4.

2.5 *Parameter analysis*

The theory described in Section 2.4 was used for setting up a FORTRAN computer program for carrying out a parameter analysis with respect to the factor described by equation (78).

(78): $$\Phi = -\left(\frac{d^2 y(\alpha)}{d\alpha^2}\right)_{\{\alpha = \propto(\psi,\lambda)\}}$$

The largest value for λ was estimated as follows.
According to formula (76):

$$\Phi E \mu_1 \mu_2 \leqslant \sigma_{\text{limit}}$$

$$\mu_1 = \frac{H}{h} \qquad \mu_1 \text{ is not less than } 0{,}5$$

$$\mu_2 = \frac{h}{l} \qquad \text{put } \mu_2 \simeq 0{,}1$$

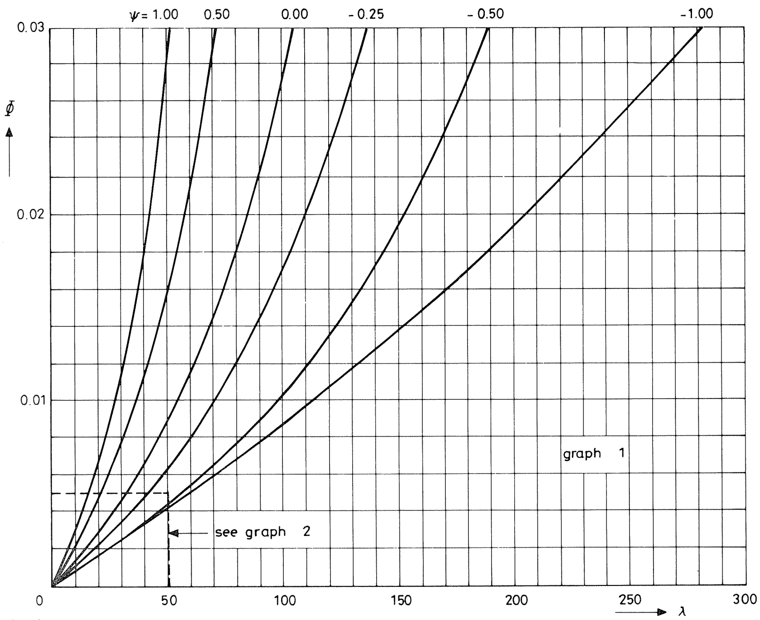
$$E = 2{,}1 \cdot 10^5 \text{ N/mm}^2 \quad \text{and} \quad \sigma_{\text{limit}} = 360 \text{ N/mm}^2$$

The largest value for Φ is found to be:

$$\Phi \leqslant \frac{360}{2.1 \cdot 10^5 \cdot 0{,}5 \cdot 0{,}1} \simeq 0.03 \tag{95}$$

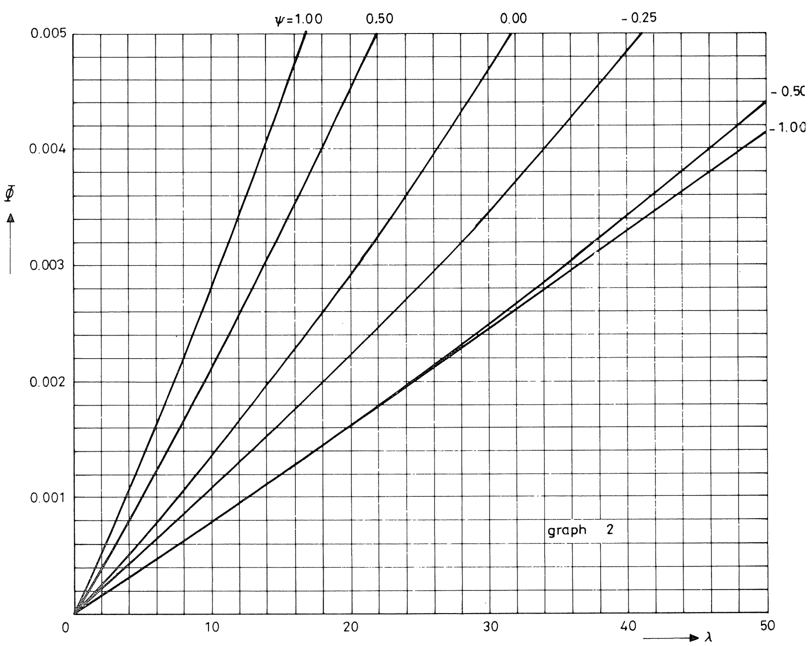
The parameter λ was increased until Φ attained the value 0.03. The results have been represented in graph form.

Note: In graphs 6 and 7 the curves for the values of ψ in the range $-3.00 \leqslant \psi \leqslant -1.25$ have been plotted only to a limited extent. Because of the numerical instability of the computational process only a limited number of values could be computed. Since the tensile stresses in the plate panel predominate for these values of ψ, the transverse stiffeners are, as it were, "pulled straight". The maximum curvature that then occurs has therefore a limit value.



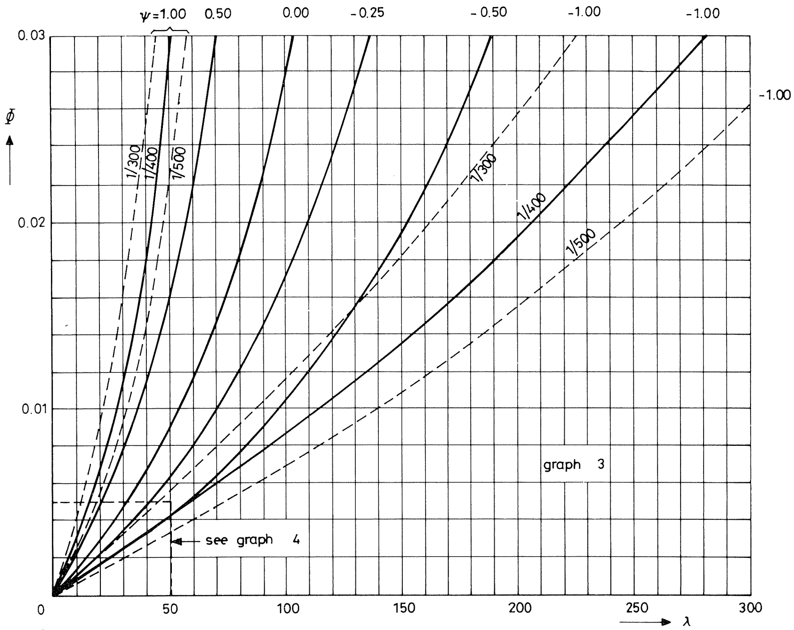
a. Graph 1

Φ as a function of λ for $-1.00 \leqslant \psi \leqslant 1.00$. The initial deflected shape is dependent on ψ. The maximum initial deflection is: $e = \frac{1}{400} l$.

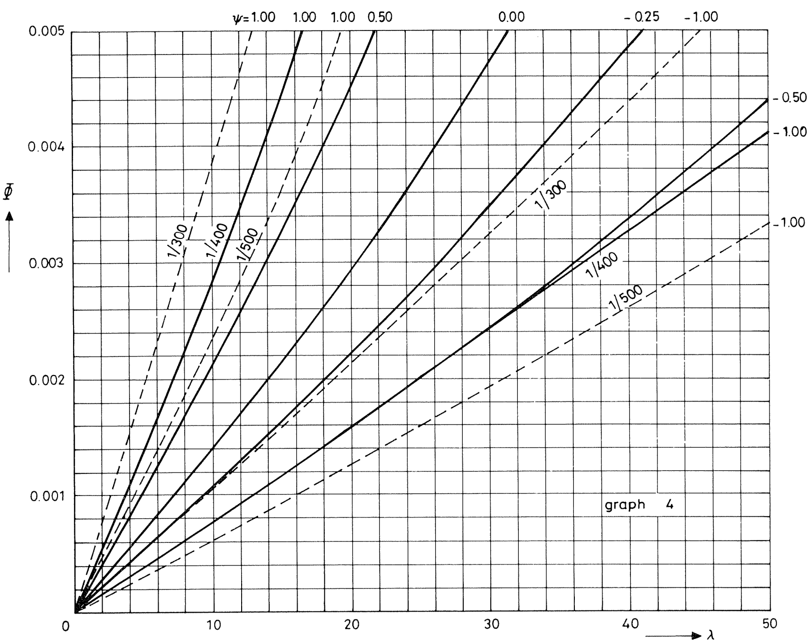


b. Graph 2

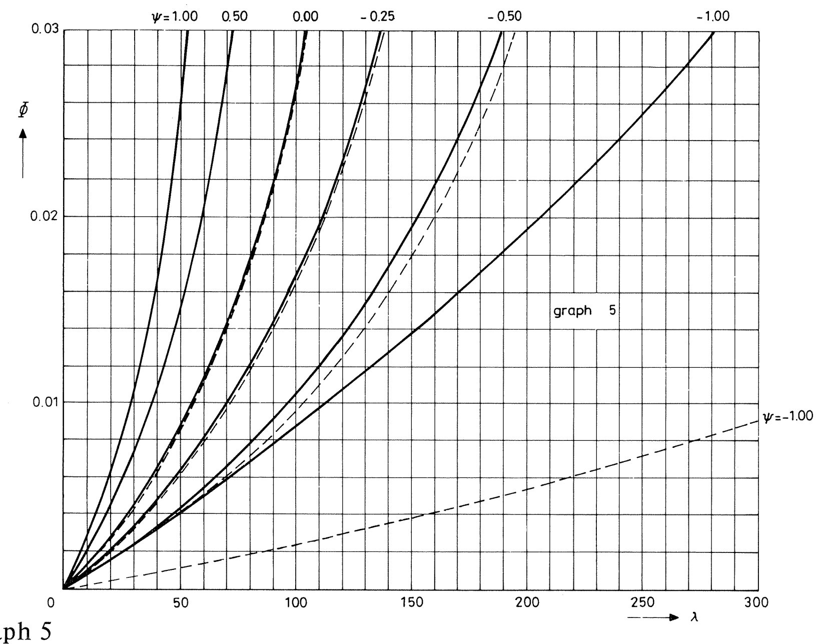
Larger-scale representation of graph 1 for the ranges $0 \leqslant \lambda \leqslant 50$ and $0.000 \leqslant \Phi \leqslant 0.005$.



c. Graph 3
Φ as a function of λ for $-1.00 \leqslant \psi \leqslant 1.00$. The initial deflected shape is dependent on ψ. Maximum initial deflection:
* full line ——— $e = \frac{1}{400} l$
* dash line --------- $e = \frac{1}{300} l$ and $e = \frac{1}{500} l$ respectively.



d. Graph 4
Larger-scale representation of graph 3 for the ranges $0 \leqslant \lambda \leqslant 50$ and $0.000 \leqslant \Phi \leqslant 0.005$.

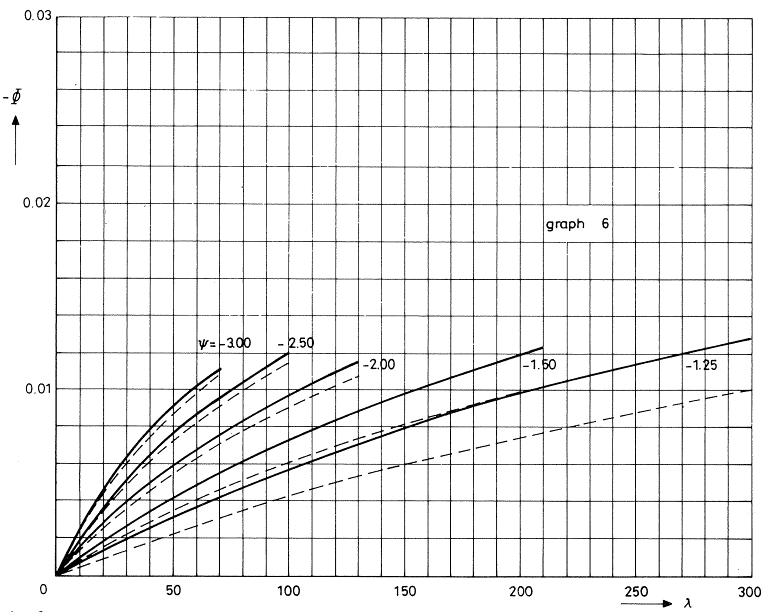


e. Graph 5

Φ as a function of λ for $-1.00 \leqslant \psi \leqslant 1.00$. The initial deflected shape is:

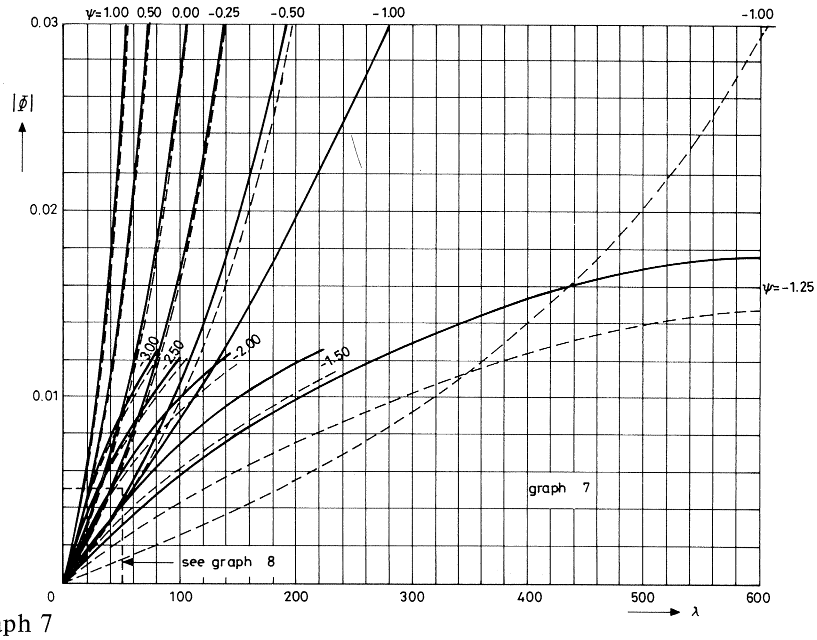
* full line ——— dependent on ψ
* dash lines ---------- constant deflected shape.

The maximum initial deflection is $e = \frac{1}{400} l$.

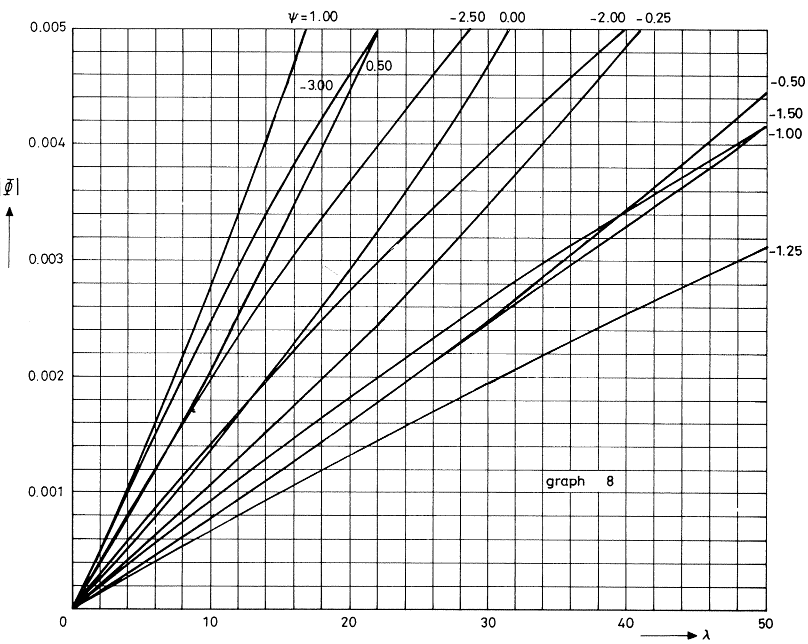


f. Graph 6

$-\Phi$ as a function of λ for $-3.00 \leqslant \psi \leqslant 1.00$. Otherwise similar to graph 5.



g. Graph 7
$|\Phi|$ as a function of λ for $-3.00 \leqslant \psi \leqslant 1.00$. Otherwise similar to graph 5.



h. Graph 8
Larger-scale representation of graph 7 for the ranges $0 \leqslant \lambda \leqslant 50$ and $0.000 \leqslant |\Phi| \leqslant 0.005$.

An approximation for this limit may be calculated as follows:
Suppose that the deflected shape is:

$$y = \tfrac{1}{400} \cdot \sin(\alpha\pi)$$

$$|\Phi| = \frac{d^2y}{d\alpha^2} = \tfrac{1}{400} \cdot \pi^2 \cdot \sin(\alpha\pi)$$

$$|\Phi| = \tfrac{1}{400} \cdot \pi^2 = 0.0247$$

The maximum magnitude of the pre-deflection has a relatively great effect on the associated values of Φ if it is varied between $e = \frac{1}{300}l$ and $e = \frac{1}{500}l$.

As a basis for establishing design rules a value $e = \frac{1}{300}l$ may be adopted, which corresponds to the tolerance in the straightness of the transverse stiffeners as envisaged in the German code DASt Richtlinie 12 "Plattenbeulen".

The shape of the initial deflection is of considerable effect on the values of Φ if $\psi = -0.50$ and $\psi = -1.00$ respectively. For the other values of Φ the effect is greatly reduced. The adoption of an initial deflected shape dependent on ψ will in all cases yield a higher value of $|\Phi|$ than when a constant initial deflected shape is adopted. Therefore an initial deflected shape which is dependent on ψ can most suitably be chosen as a basis for design rules.

2.6 *Transverse stiffeners under general loading state*

2.6.1 General

If a plate panel provided with transverse stiffeners is loaded in two directions within its plane and moreover perpendicularly to its plane, these stiffeners will thus be loaded as follows:

1. A directly-acting transverse load of constant magnitude.
2. A deflection-dependent transverse load arising from the loading within the plane of the plate, attributable to geometric imperfections of the transverse stiffeners.
3. The normal force and moment acting at the ends of the transverse stiffeners and caused by various possible circumstances, e.g., loading in the plane of the plate in a direction parallel to these stiffeners, cantilevering of the stiffeners, etc.

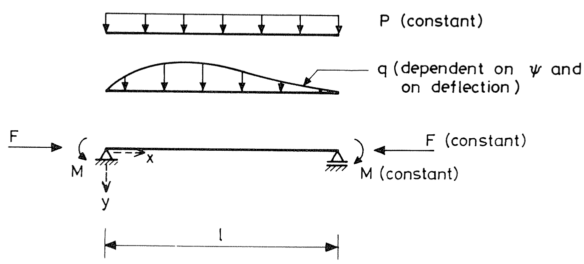


Fig. 6. Load acting on the transverse stiffener.

In the final state of the loaded transverse stiffener (i.e., for equilibrium in the deflected state) the magnitude of the maximum curvature then occurring has to be determined.

Suppose that this curvature is Φ, in analogy with the approach adopted in Section 2.3.4.

The requisite stiffness of the transverse stiffener is determined as follows.

Section 2.3.4 deals with the loading case $p = 0$, $F = 0$, $M = 0$, $q \neq 0$. The checking rule in this latter case is

(76): $\Phi E \mu_1 \mu_2 \leqslant \sigma_{\text{limit}}$

In the general case $P \neq 0$, $F \neq 0$, $M \neq 0$ and $q \neq 0$ the following condition must apply:

$$\Phi E \mu_1 \mu_2 + \frac{F}{A} \leqslant \sigma_{\text{limit}} \tag{96}$$

The factor Φ is in this case dependent on the type of loading.

2.6.2 Constitution of the deflected shape

a. Initial deflection in the unloaded state. This deflected shape is dependent on ψ and has a maximum value of $e = \frac{1}{400} l$.
b. A first-order deflection due to the direct transverse load P.
c. A first-order deflection due to the end moments M; for the purpose of this analysis the two end moments are assumed to produce a constant bending moment in the transverse stiffener.
d. A geometrically non-linear deflection due to the – in itself constant – normal force F.
c. A geometrically non-linear deflection due to the – in itself deflection-dependent – transverse load q.

2.6.3 Further treatment in non-dimensional form

a. The initial deflection is to be determined according to Section 2.3.2.

On the basis of equation (33) and the linearization of equation (27), the initial deflection can be written as:

$$y_i(\alpha) = \sum_{n=0}^{\infty} A_n \alpha^n \tag{97}$$

with

$$A_0 = 0$$

$$A_1 = \frac{7(\psi - 1) + 15}{360 \cdot Z \cdot \varepsilon}$$

$$A_2 = 0$$

$$A_3 = \frac{-\{10(\psi - 1) + 30)}{360 \cdot Z \cdot \varepsilon}$$

$$A_4 = \frac{15}{360 \cdot Z \cdot \varepsilon}$$

$$A_5 = \frac{3(\psi - 1)}{360 \cdot Z \cdot \varepsilon}$$

b. A first-order deflection due to the direct transverse load *P.*

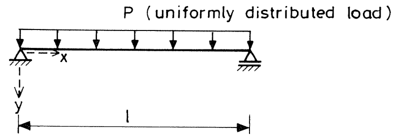


Fig. 7. Transverse stiffener with transverse load.

The first order deflection can be written as:

$$y(x) = \sum_{b=0}^{\infty} b_n x^n = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \ldots \tag{98}$$

The differential equation is:

$$EIy''''(x) = P(x) = P(\text{constant}) \tag{99}$$

Boundary conditions:

1°: $x = 0 \rightarrow y(x = 0) = 0 \rightarrow b_0 = 0$
2°: $x = 0 \rightarrow y''(x = 0) = 0 \rightarrow b_2 = 0$

This means that all the coefficients in the series $y(x) = \ldots$ associated with x^n for $n \geqslant 5$ must be zero, while:

$$b_4 = \frac{P}{24EI}$$

Therefore P (98) becomes:

$$y(x) = b_1 x + b_3 x^3 + \frac{P}{24EI} x^4 \tag{100}$$

Boundary conditions:

3°: $x = l \rightarrow y(x = l) = 0 \rightarrow b_1 + b_3 l^3 + \dfrac{Pl^4}{24EI} = 0$

4°: $x = l \rightarrow y''(x = l) = 0 \rightarrow 6b_3 + \dfrac{Pl^2}{2EI} = 0$

$$b_3 = -\frac{Pl}{12EI}$$

$$b_1 = \frac{1}{l}\left(\frac{Pl^4}{12EI} - \frac{Pl^4}{24EI}\right) = \frac{Pl^3}{24EI}$$

$$y(x) = \frac{Pl^3}{24EI}x - \frac{Pl}{12EI}x^3 + \frac{P}{24EI}x^4$$

$$y(x) = \frac{Pl^4}{24EI}\left(\frac{x}{l}\right) - \frac{Pl^4}{12EI}\left(\frac{x}{l}\right)^3 + \frac{Pl^4}{24EI}\left(\frac{x}{l}\right)^4$$

Put

$$\frac{x}{l} = \alpha$$

$$y^\circ(\alpha) = \frac{Pl^4}{24EI}\alpha - \frac{Pl^4}{12EI}\alpha^3 + \frac{Pl^4}{24EI}\alpha^4$$

$$y^\circ(\alpha) = y(\alpha)\cdot l$$

$$y(\alpha) = \frac{Pl^3}{24EI}\alpha - \frac{Pl^3}{12EI}\alpha^3 + \frac{Pl^3}{24EI}\alpha^4$$

$$y_p(\alpha) = \sum_{n=0}^{4} B_n \alpha^n \tag{101}$$

with

$$B_0 = 0$$

$$B_1 = \frac{Pl^3}{24EI}$$

$$B_2 = 0$$

$$B_3 = -\frac{Pl^3}{12EI}$$

$$B_4 = \frac{Pl^3}{24EI}$$

c. A first-order deflection due to a constant bending moment along the stiffener

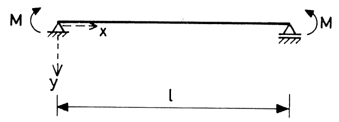


Fig. 8. Transverse stiffener with end moments.

$$y(x) = \sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 \ldots \tag{102}$$

The differential equation is:

$$EIy''''(x) = 0 \tag{103}$$

all coefficients C_n with $n \geqslant 4$ are zero

$$y(x) = \sum_{n=0}^{3} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3$$

Boundary conditions:

1°: $x = 0 \rightarrow y(x = 0) = 0 \rightarrow C_0 = 0$

2°: $x = l \rightarrow y(x = l) = 0 \rightarrow C_1 l + C_2 l^2 + C_3 l^3 = 0$

$y''(x) = 2C_2 + 6C_3 x$

3°: $x = 0 \qquad y''(X = 0) = -\dfrac{M}{EI} \rightarrow 2C_2 = -\dfrac{M}{EI} \rightarrow C_2 = \dfrac{M}{2EI}$

4°: $x = y''(x = l) = -\dfrac{M}{EI} \rightarrow 2C_2 + 6C_3 l = -\dfrac{M}{EI} \rightarrow C_3 = 0$

$$C_1 = \frac{1}{l}\left(\frac{Ml^2}{2EI}\right) = \frac{Ml}{2EI}$$

$$y(x) = \frac{Ml}{2EI} x - \frac{M}{2EI} x^2$$

$$y(x) = \frac{Ml^2}{2EI}\left(\frac{x}{l}\right) - \frac{Ml^2}{2EI}\left(\frac{x}{l}\right)^2$$

Put

$$\frac{x}{l} = \alpha$$

$$y^0(\alpha) = \frac{Ml^4}{2EI}\alpha - \frac{Ml^2}{2EI}\alpha^2$$

$$y^0(\alpha) = y(\alpha) \cdot l$$

$$y(\alpha) = \frac{Ml}{2EI}\alpha - \frac{Ml}{2EI}\alpha^2$$

$$y_M(\alpha) = \sum_{n=0}^{2} C_n \alpha^n \tag{104}$$

with

$$C_0 = 0$$

$$C_1 = \frac{Ml}{2EI}$$

$$C_2 = -\frac{Ml}{2EI}$$

The deflected shape produced by the initial deflection plus the first-order deflections due to P and M is described by:

$$y^1(\alpha) = \sum_{n=1}^{5} E_n \alpha^n \tag{105}$$

with

$$E_1 = \frac{7(\psi - 1) + 15}{350 \cdot Z \cdot \varepsilon} + \frac{Pl^3}{24EI} + \frac{Ml}{2EI}$$

$$E_2 = \qquad\qquad\qquad\qquad - \frac{Ml}{2EI}$$

$$E_3 = -\frac{10(\psi - 1) + 30}{360 \cdot Z \cdot \varepsilon} - \frac{Pl^3}{12EI}$$

$$E_4 = \frac{15}{360 \cdot Z \cdot \varepsilon} + \frac{Pl^3}{24EI}$$

$$E_5 = \frac{3(\psi - 1)}{360 \cdot Z \cdot \varepsilon}$$

d. A geometrically non-linear deflection due to the – in itself constant – normal force F

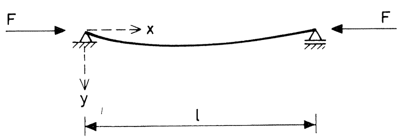


Fig. 9. Transverse stiffener with normal force.

The first-order moment distribution is obtained from:

$$M(x) = F \cdot y^1(x)$$

The first-order deflection due to this is obtained from:

$$\frac{d^2y}{dx^2} = -\frac{M(x)}{EI} = -\frac{F}{EI} \cdot y^1(x) \tag{106}$$

$$\alpha = \frac{x}{l}$$

$$y(x) = y^{0}(\alpha) = y(\alpha) \cdot l$$

$$y^{1}(x) = y^{01}(\alpha) = y^{1}(\alpha) \cdot l$$

Equation (106) becomes:

$$\frac{d^2 y(\alpha)}{\mathrm{d}\alpha^2} = -\frac{Fl^2}{EI} \cdot y^{1}(\alpha)$$

Homogeneous differential equation:

$$\frac{d^2 y(\alpha)}{\mathrm{d}\alpha^2} = 0$$

This is satisfied by

$$y_F(\alpha) = \sum_{n=0}^{1} f_n \alpha^n = f_0 + f_1 \alpha$$

Non-reduced differential equation:

$$\frac{d^2 y(\alpha)}{\mathrm{d}\alpha^2} = -\frac{Fl^2}{EI} \cdot y^{1}(\alpha)$$

$$\frac{d^2 y(\alpha)}{\mathrm{d}\alpha^2} = -\frac{Fl^2}{EI} \cdot \sum_{n=1}^{5} E_n \alpha^n$$

$$\frac{d^2 y(\alpha)}{\mathrm{d}\alpha^2} = -\frac{Fl^2}{EI} \cdot \{E_1 \alpha + E_2 \alpha^2 + E_3 \alpha^3 + E_4 \alpha^4 + E_5 \alpha^5\}$$

particular solution:

$$y(\alpha) = \sum_{n=3}^{7} g_n \alpha^n = g_3 \alpha^3 + g_4 \alpha^4 + g_5 \alpha^5 + g_6 \alpha^6 + g_7 \alpha^7$$

$$\frac{d^2 y(\alpha)}{\mathrm{d}\alpha^2} = 6g_3 \alpha + 12 g_4 \alpha^2 + 20 g_5 \alpha^3 + 30 g_6 \alpha^4 + 42 g_7 \alpha^5$$

$$\cdot \frac{Fl^2}{EI} \cdot \{E_1 \alpha + E_2 \alpha^2 + E_3 \alpha^3 + E_4 \alpha^4 + E_5 \alpha^5\}$$

This relation is satisfied if the coefficients of the same powers of α are equal; hence:

$$6g_3 = -\frac{Fl^2}{EI} E_1 \quad \rightarrow \quad g_3 = -\frac{Fl^2}{6EI} E_1$$

$$12g_4 = -\frac{Fl^2}{EI} E_2 \quad \rightarrow \quad g_4 = -\frac{Fl^2}{12EI} E_2$$

$$20g_3 = -\frac{Fl^2}{EI} E_3 \quad \rightarrow \quad g_5 = -\frac{Fl^2}{20EI} E_3$$

$$30g_6 = -\frac{Fl^2}{EI}E_4 \quad \rightarrow \quad g_6 = -\frac{Fl^2}{30EI}E_4$$

$$42g_4 = -\frac{Fl^2}{EI}E_5 \quad \rightarrow \quad g_7 = -\frac{Fl^2}{42EI}E_5$$

General solution of the non-reduced differential equation:

$$y_F(\alpha) = f_0 + f_1\alpha + g_3\alpha^3 + g_4\alpha^4 + g_5\alpha^5 + g_6\alpha^6 + g_7\alpha^7$$

Boundary conditions:

1°: $\alpha = 0 \rightarrow y(\alpha = 0) = 0 \rightarrow f_0 = 0$

2°: $\alpha = 1 \rightarrow y(\alpha = 1) = 0 \rightarrow f_1 = -(g_3 + g_4 + g_5 + g_6 + g_7)$

$$y_F(\alpha) = f_1\alpha + g_3\alpha^3 + g_4\alpha^4 + g_5\alpha^5 + g_6\alpha^6 + g_7\alpha^7$$

$$y_F(\alpha) = \sum_{n=1}^{7} G_n\alpha^n \qquad (107)$$

General formulation of the iterative process

$$y^1(\alpha) = \sum_{m=1}^{n} E_m\alpha^m = y_i(\alpha) + y_p(\alpha) + y_M(\alpha) \qquad (108)$$

A finite series

$$y_F(\alpha) = \sum_{p=0}^{q} G_p\alpha^p$$

where $q = n + 2$, is adopted as the deflected shape.

The following second-order differential equation with constant coefficients can be established:

$$\frac{d^2y(\alpha)}{d\alpha^2} = \xi_F \cdot y^1(\alpha) \qquad (109)$$

with

$$\xi_F = -\frac{Fl^2}{EI}$$

General solution of the homogeneous differential equation:

$$\frac{d^2y(\alpha)}{d\alpha^2} = 0$$

Put

$$y(\alpha) = \sum_{p=0}^{q} G_p\alpha^p$$

$$\frac{d^2y(\alpha)}{d\alpha^2} = \sum_{p=2}^{q} \frac{p!}{(p-2)!} G_p\alpha^{p-2} = 0$$

This is satisfied by the series if

$$G_p = 0 \quad \text{with} \quad 2 \leqslant p \leqslant q$$

Particular solution of the non-reduced differential equation:

$$(109): \quad \frac{d^2 y(\alpha)}{d\alpha^2} = \xi_F y^1(\alpha) = \xi_F \sum_{m=1}^{n} E_m \alpha^m \tag{110}$$

Put

$$y(\alpha) = \sum_{p=0}^{q} G_p^0 \alpha^p$$

$$\sum_{p=2}^{q} \frac{p!}{(p-2)!} G_p^0 \alpha^{p-2} = \xi_F \sum_{m=1}^{n} E_m \alpha^m$$

This condition can be satisfied by equating the coefficients of the same powers of α.

G_0^0 and G_1^0 are zero

$$\alpha^0: \frac{2!}{0!} G_2^0 = 0 \quad \rightarrow \quad G_2^0 = 0$$

$$\alpha^1: \frac{3!}{1!} G_3^0 = \xi_F E_1 \quad \rightarrow \quad G_3^0 = \frac{1!}{3!} \xi_F E_1$$

$$\alpha^2: \frac{4!}{2!} G_4^0 = \xi_F E_2 \quad \rightarrow \quad G_4^0 = \frac{2!}{4!} \xi_F E_2$$

$$\alpha^3: \frac{5!}{3!} G_5^0 = \xi_F E_3 \quad \rightarrow \quad G_5^0 = \frac{3!}{5!} \xi_F E_3$$

$$\alpha^{q-2} = \alpha^n \cdot \frac{q!}{(q-2)!} G_q^0 = \xi_F E_n \quad \rightarrow \quad G_q^0 = \frac{(q-2)!}{q!} \xi_F E_n$$

General solution of the non-reduced differential equation:

$$y_F(\alpha) = G_0 + G_1 + \sum_{p=3}^{q} G_p^0 \alpha^p \tag{111}$$

Boundary conditions:

1°: $\alpha = 0 \quad y(\alpha = 0) = 0 \rightarrow G_0 = 0$

2°: $\alpha = 1 \quad y(\alpha = 1) = 0 \rightarrow G_1 = -\sum_{p=3}^{q} G_p^0$

Equation (111) becomes:

$$y_F(\alpha) = G_1 + \sum_{p=3}^{q} G_p^0 \alpha^p = \sum_{p=1}^{q} G_p \alpha^p \tag{112}$$

This is the deflected shape which is obtained in accordance with linear elastic theory.

Now, by adjusting ("updating") the initial deflected shape, in the same way as given in equation (93) it is possible once more to establish a second-order differential equation with constant coefficients and solve it. This process can be repeated as many times as necessary for determining the final deflected shape $y(\alpha)$. Each time the process is repeated, two more terms are added.

e. A geometrically non-linear deflection due to the – in itself deflection-dependent – transverse load q.

The first-order deflected shape is described by the differential equation

$$\frac{d^4y(\alpha)}{d\alpha^4} = (\gamma \cdot \alpha + \lambda) \cdot y^1(\alpha) \tag{113}$$

where

$$\gamma = \frac{4\delta'\sigma_1 l^4}{EIa} \cdot (\psi - 1)$$

$$\lambda = \frac{4\delta'\sigma_1 l^4}{EIa}$$

$$y^1(\alpha) = \sum_{n=1}^{5} E_n \alpha^n$$

Homogeneous differential equation

$$\frac{d^4y(\alpha)}{d\alpha^4} = 0$$

This is satisfied by:

$$y_q(\alpha) = \sum_{n=0}^{3} h_n \alpha^n = h_0 + h_1\alpha + h_2\alpha^2 + h_3\alpha^3$$

Non-reduced differential equation

$$\frac{d^4y(\alpha)}{d\alpha^4} = (\gamma \cdot \alpha + \lambda) \cdot y^1(\alpha)$$

$$\frac{d^4y(\alpha)}{d\alpha^4} = (\gamma \cdot \alpha + \lambda) \cdot \{E_1\alpha + E_2\alpha^2 + E_3\alpha^3 + E_4\alpha^4 + E_5\alpha^5\}$$

$$\frac{d^2y(\alpha)}{d\alpha^4} = \lambda E_1\alpha + (\lambda E_1 + \lambda E_2)\alpha^2 + (\gamma E_2 + \lambda E_3)\alpha^3 + (\gamma E_3 + \lambda E_4)\alpha^4 +$$

$$+ (\gamma E_4 - \lambda E_5)\alpha^5 + \gamma E_5\alpha^5$$

Particular solution:

$$y_q(\alpha) = \sum_{n=5}^{10} j_n \alpha^n$$

$$\frac{d^4 y}{d\alpha^4} = 120 j_5 \alpha + 360 j_6 \alpha^2 + 840 j_7 \alpha^3 + 1680 j_8 \alpha^4 + 3024 j_9 \alpha^5 + 5040 j_{10} \alpha^6$$

$$= \lambda E_1 \alpha + (\gamma E_1 + \lambda E_2)\alpha^2 + (\gamma E_2 + \lambda E_3)\alpha^3 + (\gamma E_3 + \lambda E_4)\alpha^4 + (\gamma E_4 + \lambda E_5)\alpha^5 + \gamma E_5 \alpha^6$$

This relation is satisfied if the coefficients of the same powers of α are equal; hence:

$$120 j_5 = \lambda E_1 \quad \rightarrow \quad j_5 = \frac{\lambda E_1}{120}$$

$$360 j_6 = (\gamma E_1 + \lambda E_2) \quad \rightarrow \quad j_6 = \frac{(\gamma E_1 + \lambda E_2)}{360}$$

$$840 j_7 = (E_2 + E_3) \quad \rightarrow \quad j_7 = \frac{(\gamma E_2 + \lambda E_3)}{840}$$

$$1680 j_8 = (\gamma E_3 + \lambda E_4) \quad \rightarrow \quad j_8 = \frac{(\gamma E_3 + \lambda E_4)}{1680}$$

$$3024 j_9 = (\gamma E_4 + \lambda E_5) \quad \rightarrow \quad j_9 = \frac{(\gamma E_4 + \lambda E_5)}{3024}$$

$$5040 j_{10} = \gamma E_5 \quad \rightarrow \quad j_{10} = \frac{\gamma E_5}{5040}$$

General solution of the non-reduced differential equation

$$y_q(\alpha) = h_0 + h_1 \alpha + h_2 \alpha^2 + h_3 \alpha^3 + j_5 \alpha^5 + j_6 \alpha^6 + j_7 \alpha^7 + j_8 \alpha^8 + j_9 \alpha^9 + j_{10} \alpha^{10}$$

Boundary conditions:

$1°:\ \alpha = 0 \rightarrow y(\alpha = 0) = 0 \rightarrow h_0 = 0$

$2°:\ \alpha = 0 \left(\frac{d^2 y}{d\alpha^2}\right)_{(\alpha = 0)} = 0 \rightarrow h_2 = 0$

$3°:\ \alpha = 1 \rightarrow y(\alpha = 1) = 0 \rightarrow h_1 + h_3 = -(j_5 + j_6 + j_7 + j_8 + j_9 + j_{10})$

$4°:\ \alpha = 1 \rightarrow \left(\frac{d^2 y}{d\alpha^2}\right)_{(\alpha = 1)} = 0 \rightarrow 6h_3 = -(20 j_5 + 30 j_6 + 42 j_7 + 56 j_8 + 72 j_9 + 90 j_{10})$

$$h_3 = \frac{-1}{6} \cdot \sum_{p=5}^{10} j_p \cdot p \cdot (p-1)$$

$$h_1 = -\sum_{p=5}^{10} j_p \cdot p \cdot (p-1)$$

Equation (114) becomes

$$y(\alpha)=h_1+h_3\alpha^3+j_5\alpha^5+j_6\alpha^6+j_7\alpha^7+j_8\alpha^8+j_9\alpha^9+j_{10}\alpha^{10}$$

$$y(\alpha)=\sum_{n=1}^{10} J_n\alpha^n$$

General formulation of the iterative process

$$y^1(\alpha)=\sum_{m=1}^{n} E_m\alpha^m=y_i(\alpha)+y_p(\alpha)+y_M(\alpha)$$

A finite series

$$y_q(\alpha)=\sum_{p=0}^{q} J_p\alpha^p$$

where $q=n+5$, is adopted as the deflected shape.

The following fourth-order differential equation can be established:

$$\frac{d^4y(\alpha)}{d\alpha^4}=(\gamma\cdot\alpha+\lambda)\cdot y_i(\alpha) \qquad (115)$$

with

$$\gamma=\frac{4\delta'\sigma_1 l^4}{EIa}\cdot(\psi-1)$$

A finite series

$$y_q(\alpha)=\sum_{p=0}^{q} J_p\alpha^p$$

where $q=n+5$, is adopted as the deflected shape.

General solution of the homogeneous differential equation:

$$\frac{d^4y(\alpha)}{d\alpha^4}=0$$

$$\sum_{p=4}^{q}\frac{p!}{(p-4)!}J_p\alpha^{p-4}=0$$

This is satisfied if:

$$J_p=0 \quad \text{with} \quad 4\leqslant p\leqslant q$$

Particular solution of the non-reduced differential equation:

$$\frac{d^4y(\alpha)}{d\alpha^4}=(\gamma\cdot\alpha+\lambda)\cdot y_i(\alpha)$$

Put

$$y_q(\alpha) = \sum_{p=0}^{q} J_p^0 \alpha^p$$

$$\sum_{p=4}^{q} \frac{p!}{(p-4)!} \cdot J_p^0 \alpha^{p-4} = (\gamma \cdot \alpha + \lambda) \cdot \sum_{m=1}^{n} E_m \alpha^m$$

These conditions can be satisfied by equating the coefficients of the same powers of α.

J_0^0, J_1^0, J_2^0 and J_3^0 are zero

$$\alpha^0 : \frac{4!}{0!} J_4^0 = 0$$

$$\alpha^1 : \frac{5!}{1!} J_5^0 = \qquad \lambda E_1$$

$$\alpha^2 : \frac{6!}{2!} J_6^0 = \gamma E_1 + \lambda E_2$$

$$\alpha^3 : \frac{7!}{3!} J_7^0 = \gamma E_2 + \lambda E_3$$

$$\alpha^{q-4} = \alpha^{n+1} : \frac{q!}{(q-4)!} \cdot J_q^0 = \gamma E_4$$

General solution of the non-reduced differential equation:

$$y(\alpha) = J_0 + J_1\alpha + J_2\alpha^2 + J_3\alpha^3 + \sum_{p=5}^{q} J_p^0 \alpha^p \tag{116}$$

Boundary conditions:

1°: $\alpha = 0$ $\quad y(\alpha = 0) = 0 \rightarrow J_0 = 0$

2°: $\alpha = 0$ $\quad \left(\frac{d^2y}{d\alpha^2}\right)_{(\alpha=0)} = 0 \rightarrow J_2 = 0$

3°: $\alpha = 1$ $\quad y(\alpha = 1) = 0 \rightarrow J_1 + J_3 = -\sum_{p=5}^{q} J_p^0$

4°: $\alpha = 1$ $\quad \left(\frac{d^2y}{d\alpha^2}\right)_{(\alpha=1)} = 0 \rightarrow 6J_3 = -\sum_{p=5}^{q} J_p^0 \cdot p \cdot (p-1)$

$$J_3 = -\tfrac{1}{6} \cdot \sum_{p=5}^{q} J_p^0 + \sum_{p=5}^{q} J_p^0 \cdot p \cdot (p-1)$$

Equation (116) becomes:

$$y_1(\alpha) = J_1\alpha + J_3\alpha^3 + \sum_{p=5}^{q} J_p^0 \alpha^p = \sum_{p=1}^{q} J_p \alpha^p \tag{117}$$

This is the deflected shape which is obtained in accordance with linear elastic theory.

Now, by adjusting ("updating") the initial deflected shape in the same way as given in equation (93) it is possible once more to establish a second-order differential equation with constant coefficients and solve it. This process can be repeated as many times as necessary for determining the final deflected shape $y(\alpha)$. Each time the process is repeated, five more terms are added.

2.6.4 Iterative process

The iterative process for determining the deflected shape of the transverse stiffener under the general loading state proceeds as follows:

a. Determining the stressless initial deflected shape:

(97): $$y_i(\alpha) = \sum_{n=1}^{5} A_n \alpha^n$$

with

$$A_1 = \frac{7(\psi - 1) + 15}{360 \cdot Z \cdot \varepsilon}$$

$$A_2 = 0$$

$$A_3 = -\frac{10(\psi - 1) + 30}{360 \cdot Z \cdot \varepsilon}$$

$$A_4 = \frac{15}{360 \cdot Z \cdot \varepsilon}$$

$$A_5 = \frac{3(\psi - 1)}{360 \cdot Z \cdot \varepsilon}$$

b. Determining the first-order deflected shape due to direct transverse load P

(101): $$y_P(\alpha) = \sum_{n=1}^{4} B_n \alpha^n$$

with

$$B_1 = \frac{\xi_P}{24}$$

$$B_2 = 0$$

$$B_3 = -\frac{\xi_P}{12}$$

$$B_4 = \frac{\xi_P}{24}$$

where

$$\xi_P = \frac{Pl^3}{EI}$$

c. Determining the first-order deflected shape due to the end moments M

(104): $$y_M(\alpha) = \sum_{n=1}^{2} C_n \alpha^n$$

with

$$C_1 = \frac{\xi_M}{2}$$

$$C_2 = -\frac{\xi_M}{2}$$

where

$$\xi_M = \frac{Ml}{EI}$$

d. Establishing the initial deflected shape and the two first-order deflected shapes

(105): $$y^1(\alpha) = \sum_{n=1}^{5} E_n \alpha^n$$

with

$$E_1 = A_1 + B_1 + C_1$$
$$E_2 = A_2 + B_2 + C_2$$
$$E_3 = A_3 + B_3$$
$$E_4 = A_4 + B_4$$
$$E_5 = A_5$$

e. Determining the first-order deflected shape due to the normal force F

(112): $$y_F(\alpha) = \sum_{p=1}^{q} G_p \alpha^p$$

f. Determining the first-order deflected shape due to the deflection-dependent transverse load q

(117): $$y_q(\alpha) = \sum_{p=1}^{q} J_p \alpha^p$$

g. Adjusting the initial deflected shape ("updating")

$$y^1_{(I+1)}(\alpha) = y^1(\alpha) + y_{F(I)}(\alpha) + y_{q(I)}(\alpha)$$

h. In repeating the steps e, f and g as many times as necessary for attaining equilibrium within a predetermined accuracy, the final deflected shape of the transverse stiffener under general loading is calculated.

2.6.5 Determining the magnitude of the greatest curvature in the transverse stiffener

The magnitude of the greatest curvature in the transverse stiffener due to the loading must be determined only on the basis of the deflected shape that causes stresses.

$$y(\alpha) = y_p(\alpha) + y_M(\alpha) + y_F(\alpha) + y_q(\alpha) \tag{118}$$

$$y(\alpha) = \sum_{p=1}^{Q} Y_p \alpha^p \tag{119}$$

$$\frac{dy(\alpha)}{d\alpha} = \sum_{p=1}^{Q} p \cdot Y_p \cdot \alpha^{p-1} \tag{120}$$

$$\frac{d^2y(\alpha)}{d\alpha^2} = \sum_{p=1}^{Q} p \cdot (p-1) \cdot Y_p \cdot \alpha^{p-2} \tag{121}$$

$$\frac{d^3y(\alpha)}{d\alpha^3} = \sum_{p=1}^{Q} p \cdot (p-1) \cdot (p-2) \cdot Y_p \cdot \alpha^{p-3} \tag{122}$$

The location of the greatest curvature can be determined by solving the equation

$$\frac{d^3y(\alpha)}{d\alpha^3} = 0$$

for the root $\alpha = \propto$.

Then this root $\alpha = \propto$ is substituted into (121), which leads to:

$$\Phi = \left(\frac{d^2y(\alpha)}{d\alpha^2}\right)_{\{\alpha = \propto(\psi, \lambda, \xi_P, \xi_M, \xi_F)\}} \tag{123}$$

The process for determining the greatest curvature in the transverse stiffener, as set forth in Sections 2.6.4 and 2.6.5, has been embodied in a computer program by means of which the calculations can be carried out. First, the program determines the maximum curvature in the stiffener under the influence of the predetermined loading state and geometry. Next, it is verified whether the relation (96) is satisfied.

2.6.6 Parameter analysis with the aid of the mathematical model

With the aid of the mathematical model described in the foregoing the value of Φ for the relation (96) is calculated for a number of variations of the parameters. For this purpose the relation (96) can be rewritten in a somewhat different form:

$$\frac{F}{A} = \frac{Fi^2}{I} = \frac{Fl^2}{I} \cdot \frac{1}{\lambda_x^2} \quad \text{and} \quad \xi_F = \frac{Fl^2}{EI} \tag{124}$$

Substitution of (124) into (96) leads to:

$$\frac{E}{\sigma_{cr}} \cdot \left(\Phi \mu_1 \mu_2 + \frac{\xi_F}{\lambda_x^2}\right) \leqslant 1 \tag{125}$$

where Φ is dependent on the parameters

$$\psi = 1{,}00;\ 0{,}50;\ 0{,}00;\ -0{,}25;\ -0{,}50;\ -1{,}00$$

$$0 \leqslant \lambda = \frac{4\delta'\sigma_1 l^4}{EIa} \leqslant \lambda$$

$$0 \leqslant \xi_P = \frac{Pl^3}{EI} \leqslant 1.10 \qquad (\xi_P = 0{,}1 \text{ to } 1{,}1 \text{ with step magnitude } 0{,}2)$$

$$0 \leqslant \xi_M = \frac{Ml}{EI} \leqslant .14 \qquad (\xi_M = 0{,}0 \text{ to } 0{,}15 \text{ with step magnitude } 0{,}02)$$

$$0 \leqslant \xi_F = \frac{Fl^2}{EI} \leqslant \pi^2 \simeq 10 \qquad (\xi_F = 0{,}0 \text{ to } 10{,}0 \text{ with step magnitude } 2{,}0)$$

in such a way that, for certain values of the other input variables, the value of $\Phi \leqslant 0{,}14$.

The final values of the various parameters have been so chosen that, if the corresponding load occurs separately, the ultimate stress in the transverse stiffener is equal to the yield stress for the values $\mu_1 = 0.5$ and $\mu_2 = 0.025$.

This parameter analysis results in a comprehensive set of tables for the determination of Φ (see [4]).

2.6.7 Solution via a fourth-order differential equation with non-constant coefficients

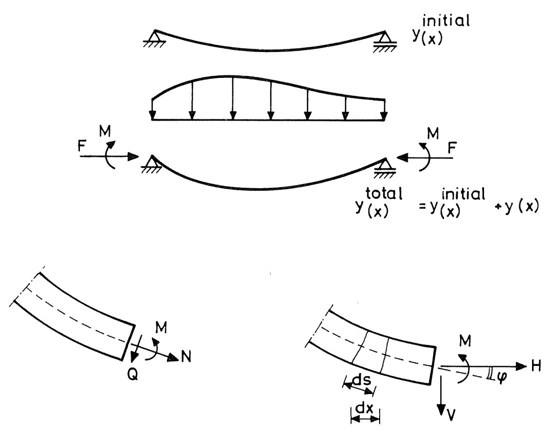


Fig. 10. Transverse stiffener with general loading.

$$H = N\cos\varphi - Q\sin\varphi$$
$$V = N\sin\varphi + Q\cos\varphi$$
$$M = M$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$1 - \sin \varphi \simeq \Phi; \quad \cos \varphi \simeq 1; \quad \tan \varphi \simeq \Phi \quad \text{and} \quad dx \simeq ds \tag{126}$$

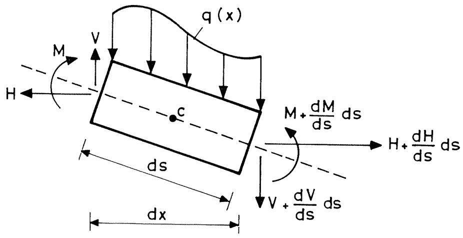


Fig. 11. Forces on an element of the transverse stiffener.

Horizontal equilibrium:

$$H + \frac{dH}{ds}\, ds - H = 0 \tag{127}$$

Vertical equilibrium:

$$V + \frac{dV}{ds}\, ds - V + q(x)\, ds = 0 \tag{128}$$

Moment equilibrium about point C:

$$M + \frac{dM}{ds}\, ds - M - \frac{ds \cos \varphi}{2}\left(V + \frac{dV}{ds}\, ds + V\right) + \frac{ds \sin \varphi}{2}\left(H + \frac{dH}{ds}\, ds + H\right) = 0 \tag{129}$$

Neglecting $(dV/ds)\, ds$ in relation to V and $(dH/ds)\, ds$ in relation to H.

$$(127): \quad \Sigma H = 0 \rightarrow \frac{dH}{ds} = 0 \tag{130}$$

$$(128): \quad \Sigma V = 0 \rightarrow \frac{dV}{ds} + q(x) = 0 \tag{131}$$

$$(129): \quad \Sigma M = 0 \rightarrow \frac{dM}{ds} - V \cos \Phi + H \sin \Phi = 0 \tag{132}$$

On the basis of (126) the equations (130), (131) and (132) can be rewritten as:

$$\frac{dH}{dx} = 0 \tag{133}$$

$$\frac{dV}{dx} + q(x) = 0 \tag{134}$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} - V + H\frac{\mathrm{d}Y^{\text{total}}}{\mathrm{d}x} = 0 \tag{135}$$

Moment-curvature relation:

$$M = -EI\frac{d^2y}{\mathrm{d}x^2} \tag{136}$$

Equations (133) to (136) are four differential equations of the first and second order for determining the unknowns H, V, M and $y(x)$.

From (133) follows:

$$H = \text{constant} = -F \tag{137}$$

where F is the externally acting axial force. From (135) follows:

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} + H\frac{\mathrm{d}y^{\text{total}}}{\mathrm{d}x} \tag{138}$$

Substitution of (136) and (137) into (138) gives

$$V = \frac{d\left(-\dfrac{EId^2y}{\mathrm{d}x^2}\right)}{\mathrm{d}x} - F\frac{\mathrm{d}y^{\text{total}}}{\mathrm{d}x} \tag{139}$$

$$V = -EI\frac{d^3y}{\mathrm{d}x^3} - F\frac{\mathrm{d}y^{\text{total}}}{\mathrm{d}x} \tag{140}$$

From (134) follows:

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -q(x) \tag{141}$$

Substitution of (140) into (141) gives

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -EI\frac{d^4y}{\mathrm{d}x^4} - F\frac{d^2y^{\text{total}}}{\mathrm{d}x^2} = -q(x)$$

$$EI\frac{d^4y(x)}{\mathrm{d}x^4} + F\cdot\frac{d^2(y_i(x) + y(x))}{\mathrm{d}x^2} = q(x)$$

$$EI\frac{d^4y(x)}{\mathrm{d}x^4} + F\frac{d^2y(x)}{\mathrm{d}x^2} = q(x) - F\cdot\frac{d^2y_i(x)}{\mathrm{d}x^2} \tag{142}$$

where

$$q(x) = \frac{4\delta'\sigma_1}{a}\cdot\left\{(\psi - 1)\cdot\frac{x}{l} + 1\right\}\cdot\{y(x) + y_i(x)\} + p \tag{143}$$

The differential equation (142) must be written in non-dimensional form

$$y(x) = y^0(\alpha) \cdot l \quad \text{with} \quad \alpha = \frac{x}{l} \tag{144}$$

$$\frac{d^n y(x)}{dx^n} = l \cdot \frac{d^n y(\alpha)}{d\alpha^n} \cdot \frac{d^n \alpha}{dx^n} = l \cdot \frac{d^n y(\alpha)}{d\alpha^n} \cdot \frac{1}{l^n} = \frac{1}{l^{n-1}} \cdot \frac{d^n y(\alpha)}{d\alpha^n} \tag{145}$$

$$\frac{d^4 y(x)}{dx^4} = \frac{1}{l^3} \cdot \frac{d^4 y(\alpha)}{d\alpha^4}; \qquad \frac{d^2 y(x)}{dx^2} = \frac{1}{l} \cdot \frac{d^2 y(\alpha)}{d\alpha^2} \tag{146}$$

Substitution of (144) and (146) into (142) and (143) gives:

$$\frac{EI}{l^3} \cdot \frac{d^4 y(\alpha)}{d\alpha^4} + \frac{F}{l} \cdot \frac{d^2 y(\alpha)}{d\alpha^2} = q(\alpha) - \frac{F}{l} \cdot \frac{d^2 y_i(\alpha)}{d\alpha^2} \tag{147}$$

where

$$q(\alpha) = \frac{4\delta' \sigma_1}{\alpha} \cdot \{(\psi - 1) \cdot \alpha + 1\} \cdot l \cdot \{y(\alpha) + y_i(\alpha)\} + P \tag{148}$$

Equation (147) can be rewritten as:

$$\frac{d^4 y(\alpha)}{d\alpha^4} + \frac{Fl^2}{EI} \cdot \frac{d^2 y(\alpha)}{d\alpha^2} = (\gamma \cdot \alpha + \lambda) \cdot \{y(\alpha) + y_i(\alpha)\} + \frac{Pl^3}{EI} - \frac{Fl^2}{EI} \cdot \frac{d^2 y(\alpha)}{d\alpha^2} \tag{149}$$

$$\frac{d^4 y(\alpha)}{d\alpha^4} + \xi_F \frac{d^2 y(\alpha)}{d\alpha^2} = (\gamma \cdot \alpha + \lambda) \cdot \{y(\alpha) + y_i(\alpha)\} + \xi_P - \xi_F \frac{d^2 y(\alpha)}{d\alpha^2} \tag{150}$$

with

$$\xi_F = \frac{Fl^2}{EI}$$

$$\xi_P = \frac{Pl^3}{EI}$$

$$\gamma = \frac{4\delta' \sigma_1 l^4}{EI\alpha} \cdot (\psi - 1)$$

$$\lambda = \frac{4\delta' \sigma_1 l^4}{EI\alpha}$$

Equation (150) is a linear fourth-order differential equation with non-constant coefficients. The independent parameters are:

ψ, λ, ξ_F, ξ_P and ξ_M

The independent parameter ξ_M is introduced through the boundary conditions.

The solution of (150) can be found by using power series of α as done in previous sections.

3 Longitudinal stiffeners

3.1 *General*

Longitudinally stiffened plate panels loaded in compression may undergo so-called stiffener-induced failure in consequence of a critical state of stress being attained in the stiffener before the panel itself becomes critical.

A possible critical state of stress for a stiffener may consist in the attainment of the level of stress at which torsional buckling occurs. This phenomenon may more particularly occur with "open"-section stiffeners consisting of flat bars, *T*-sections or *L*-sections (Fig. 12).

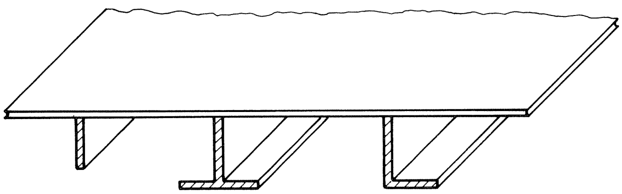


Fig. 12. Various types of open-section stiffeners.

For checking the stability of the stiffeners it is therefore necessary to calculate the torsional buckling stress $\sigma_{\text{t.b.}}$. On the basis of linear elastic theory the so-called Euler torsional buckling stress $\sigma_{\text{e.t.b.}}$ can be determined, and then the initial imperfections and the elasto-plastic behaviour of the material can be taken into account in accordance with a chosen convention. In this way the Euler torsional buckling stress $\sigma_{\text{e.t.b.}}$ is reduced to the torsional buckling stress $\sigma_{\text{t.b.}}$.

The method presented here asumes a complete separated behaviour of the plate and the longitudinal stiffeners leading to safe results. In the analysis of a stiffened plate the stiffeners are designed as beam columns and have to remain straight so that the unstiffened panel can reach initial buckling.

3.2 *Determining the Euler torsional buckling stress* $\sigma_{\text{e.t.b.}}$

If a stiffened plate panel is loaded in compression and the compressive stress is uniformly distributed over the cross-sectional area of the plate and of the stiffener, the line of action of the resultant of the compressive stresses will be located between the centroidal axis of the plate and the centroidal axis of the stiffener (Fig. 13).

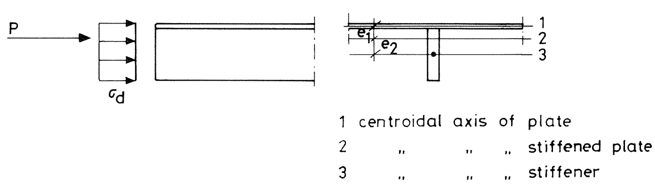


Fig. 13. Stiffened plate under uniform compressive stress.

In practical cases the resultant of the stress will always be located at a distance

$$e \leqslant e_1 = \frac{\Sigma A_{\text{stiff}} \cdot e_2}{A_{\text{pl}} + \Sigma A_{\text{stiff}}}$$

from the plane of reference of the plate.

For determining the Euler torsional buckling stress it is therefore a safe assumption to base oneself on an axially loaded stiffener which is connected at one longitudinal edge to the plate.

The derivation of the Euler torsional buckling stress starts from an axially loaded bar which is, along one axis parallel to the longitudinal axis of the bar, elastically supported with respect to displacements and rotations in planes perpendicular to the axis of the bar (Fig. 14).

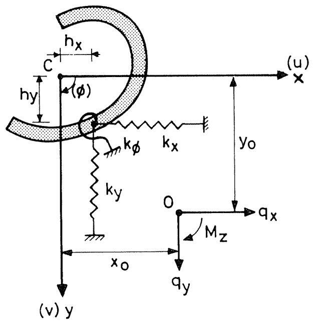


Fig. 14. Elastically supported bar.
C = centroid of the cross-section
N = point of application of the elastic support
O = shear centre

The displacements and the rotation of the centroid are designated by u, v and Φ respectively. The displacements of the N-axis are then, assuming a non-deformable section:

$$u_N = u + (y_0 - h_y)\Phi \tag{151}$$

$$v_N = v - (x_0 - h_x)\Phi \tag{152}$$

$$\Phi_N = \Phi \tag{153}$$

These displacements are associated with the following reactions per unit length:

$$n_x \doteq - k_x \{u + (y_0 - h_y)\Phi\} \tag{154}$$

$$n_y = - k_y \{v + (x_0 - h_x)\Phi\} \tag{155}$$

$$m_z = - k_\Phi \Phi \tag{156}$$

The compressive stress acting in the longitudinal direction will, for a small deformation of the axis of the bar, give rise to forces acting transversely to that axis:

$$n_x' = -\int_A \sigma \left\{\frac{d^2u}{dz^2} + (y_0 - y)\frac{d^2\Phi}{dz^2}\right\} dA \tag{157}$$

$$n_y' = -\int_A \sigma \left\{\frac{d^2v}{dz^2} - (x_0 - x)\frac{d^2\Phi}{dz^2}\right\} dA \tag{158}$$

By definition the following holds:

$$\int_A \sigma \, dA = P; \qquad \int_A x \, dA = \int_A y \, dA = 0 \tag{159}$$

Substitution of (159) into (157) and (158) gives:

$$n_x' = -P\left(\frac{d^2u}{dz^2} + y_0\frac{d^2\Phi}{dz^2}\right) \tag{160}$$

$$n_y' = -P\left(\frac{d^2v}{dz^2} - x_0\frac{d^2\Phi}{dz^2}\right) \tag{161}$$

With respect to the shear centre the reactions n_x and n_y, as expressed by equations (154) and (155), produce the following torsional moment load per unit length:

$$m_z'' = -k_x\{u + (y_0 - h_y)\Phi\}(y_0 - h_y) + k_y\{v - (x_0 - h_y)\Phi\}(x_0 - h_x) \tag{162}$$

The "internal" forces n_x' and n_y', as expressed by equations (157) and (158), likewise produce a torsional moment:

$$m_z' = -\int_A \sigma t \, ds\,(y_0 - y)\left\{\frac{d^2u}{dz^2} + (y_0 - y)\frac{d^2\Phi}{dz^2}\right\} + \int_A \sigma t \, ds\,(x_0 - x)\left\{\frac{d^2v}{dz^2} - (x_0 - x)\frac{d^2\Phi}{dz^2}\right\} \tag{163}$$

By definition the following holds:

$$\int_A y^2 \, dA = \int_A y^2 t \, ds = I_x \quad \text{and} \quad \int_A x^2 \, dA = \int_A x^2 t \, ds = I_y$$

and put

$$I_0 = I_x + I_y + A(x_0^2 + y_0^2) \tag{164}$$

Substitution of (159) and (164) into (163) gives:

$$m_z' = P\left\{x_0\frac{d^2v}{dz^2} - y_0\frac{d^2u}{dz^2}\right\} - \frac{I_0}{A}P\frac{d^2\Phi}{dz^2} \tag{165}$$

The forces acting in the directions x, y and Φ are now:

$$n_x^* = n_x + n_x' = -k_x\{u + (y_0 - h_y)\Phi\} - P\left(\frac{d^2u}{dz^2} + y_0\frac{d^2\Phi}{dz^2}\right) \tag{166}$$

$$h_y^* = n_y + n_y' = -k_y\{v - (x_0 - h_x)\Phi\} - P\left(\frac{d^2v}{dz^2} - x_0\frac{d^2\Phi}{dz^2}\right) \tag{167}$$

$$m_z^* = m_z + m_z' + m_z'' = -k_\Phi\,\Phi + P\left\{x_0\frac{d^2v}{dz^2} - y_0\frac{d^2u}{dz^2}\right\} - \frac{I_0}{A}P\frac{d^2\Phi}{dz^2} +$$

$$-k_x\{u + (y_0 - h_y)\Phi\}(y_0 - h_y) + k_y\{V - (x_0 - h_y)\Phi\}(x_0 - h_x) \tag{168}$$

For a beam the following deformation equation can be established:

bending about the y-axis: $EI_y\dfrac{d^4u}{dz^4} = n_x^*$ (169)

bending about the x-axis: $EI_x\dfrac{d^4v}{dz^4} = n_y^*$ (170)

torsion about the z-axis: $C_1\dfrac{d^4\Phi}{dz^4} - C\dfrac{d^2\Phi}{dz^2} = m_z^*$ (171)

Substitution of (166) to (168) into (169) to (171) yields a set of three simultaneous fourth-order differential equations for describing the combined torsional and flexural buckling of an elastically supported bar.

$$EI_y\frac{d^4u}{dz^4} + P\left(\frac{d^4u}{dz^2} + y_0\frac{d^2\Phi}{dz^2}\right) + k_x\{u + (y_0 - h_y)\Phi\} = 0 \tag{172}$$

$$EI_x\frac{d^4v}{dz^4} + P\left(\frac{d^2v}{dz^2} - x_0\frac{d^2\Phi}{dz^2}\right) + k_y\{(v - (x_0 - h_x)\Phi\} = 0 \tag{173}$$

$$C_1\frac{d^4\Phi}{dz^4} - \left(C - \frac{I_0}{A}\right)\frac{d^2\Phi}{dz^2} - P\left(x_0\frac{d^2v}{dz^2} - y_0\frac{d^2u}{dz^2}\right) +$$

$$k_x[u + (y_0 - h_y)\Phi](y_0 - h_y) - k_y[v - (x_0 - h_x)\Phi](x_0 - h_x) + k\Phi = 0 \tag{174}$$

where:

E = modulus of elasticity
I_x = moment of inertia about the x-axis
I_y = moment of inertia about the y-axis
A = cross-sectional area
$I_0 = I_x + I_y + A(x_0^2 + y_0^2)$ = polar moment of inertia about the shear centre
$C_1 = EC_w$, with C_w = warping constant
$C = GI_t$, with I_t = torsional moment of inertia
G = shear modulus $= \dfrac{E}{2(1+\nu)}$
ν = Poisson's ratio = 0,3

For stiffened plate panels it can be assumed that the support given to the stiffener perpendicularly to the plane of the plate is zero and the displacements of the stiffener at the connection with the panel within the plane of the latter are zero. Hence it follows that:

$$k_y = 0 \quad \text{and} \quad k_x = \infty \tag{175}$$

The general solution of the homogeneous differential equation is obtained by introducing:

$$u = A_1 \sin \frac{n\pi z}{l}; \qquad v = A_2 \sin \frac{n\pi z}{l}; \qquad \Phi = A_3 \sin \frac{n\pi z}{l} \tag{176}$$

Substitution of (176) into (172) to (174) gives:

$$\left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right) A_1 + 0A_2 + \left\{- Py_0 \frac{\pi^2 n^2}{l^2} + k_x(y_0 - h_y)\right\} A_3 = 0 \tag{177}$$

$$0A_1 + \left(EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_y\right) A_2 + \left\{Px_0 \frac{\pi^2 n^2}{l^2} - k_y(x_0 - h_x)\right\} A_3 = 0 \tag{178}$$

$$\left\{Py_0 \frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)\right\} A_1 + \left\{Px_0 \frac{\pi^2 n^2}{l^2} - k_y(x_0 - h_x)\right\} A_2 +$$

$$\left\{+ C_1 \frac{n^4\pi^4}{l^4} + \left(C - \frac{I_0}{A} P\right)\frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)^2 + k_y(x_0 - h_x)^2 + k_\Phi\right\} A_3 = 0 \tag{179}$$

A non-trivial solution can be obtained by equation the determinant of the matrix of coefficients to zero; also, the condition $k_y = 0$ can be utilized:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \tag{180}$$

with

$$a_{11} = \left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right)$$

$$a_{12} = a_{21} = 0$$

$$a_{13} = a_{31} = \left\{- Py_0 \frac{\pi^2 n^2}{l^2} + k_x(y_0 - h_y)\right\}$$

$$a_{22} = \left(EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2}\right)$$

$$a_{23} = a_{32} = Px_0 \frac{\pi^2 n^2}{l^2}$$

$$a_{33} = \left\{C_1 \frac{n^4\Phi^4}{l^4} + \left(C - \frac{I_0}{A} P\right)\frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)^2 + k_\Phi\right\}$$

Worked out, (180) yields a third-degree equation in P, the smallest root of which gives the critical force P_{cr}. The Euler torsional buckling stress is

$$\sigma_{\text{e.t.b.}} = P_{cr}/A \tag{181}$$

The condition $k_x = \infty$ can be utilized by first working out (180), then dividing the third-degree equation in $Pk_x \neq 0$ and finally putting $k_x = \sim$.

3.3 *Convention for reducing the Euler torsional buckling stress $\sigma_{\text{e.t.b.}}$ to the torsional buckling stress $\sigma_{\text{t.b.}}$*

The reduction of $\sigma_{\text{e.t.b.}}$ to $\sigma_{\text{t.b.}}$ proceeds analogously to the approach adopted for flexural buckling 171.

$$\text{For flexural buckling holds } \lambda = \sqrt{\frac{\pi^2 E}{\sigma_E}} \tag{182}$$

$$\text{Put for torsional buckling } \lambda = \sqrt{\frac{\pi^2 E}{\sigma_{\text{i.t.k.}}}} \tag{183}$$

The limiting slenderness ratio is defined as:

$$\lambda_g = \pi \sqrt{\frac{E}{\sigma_y}} \tag{184}$$

This is the slenderness ratio where the squash load is equal to the linear Euler buckling load.

The relative slenderness ratio is defined as:

$$\bar{\lambda} = \frac{\lambda}{\lambda_g} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} \tag{185}$$

The reduction proceeds as a function of the relative slenderness ratio.

$$\frac{\sigma_{\text{t.k.}}}{\sigma_y} = \frac{1 + 0{,}339(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2}{2\bar{\lambda}^2} - \frac{1}{2\bar{\lambda}^2} \cdot \sqrt{\{1 + 0{,}339(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2\}^2 - 4\bar{\lambda}^2} \tag{186}$$

This is the reduction curve b for flexural buckling according to the Eurocode 3 for steel structures 171, now used as reduction curve for torsional buckling.

3.4 *Flat bar stiffener*

The flat stiffener has a section which is symmetrical about two axes. Consequently, the shear centre coincides with the centroid of the section.

$$x_0 = y_0 = 0 \tag{187}$$

The stiffener is connected along one edge to the plate, so that:

$$h_x = 0 \quad \text{and} \quad h_y \neq 0 \tag{188}$$

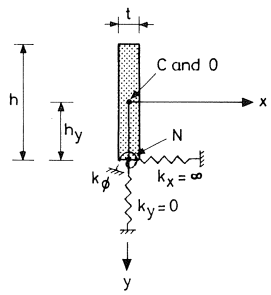


Fig. 15. Flat bar stiffener.

Substitution of (187) and (188) into (180) gives:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \tag{189}$$

with

$$a_{11} = \left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x \right)$$

$$a_{12} = a_{21} = a_{23} = a_{32} = 0$$
$$a_{13} = a_{31} = -\,k_x h_y$$

$$a_{22} = \left(EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} \right)$$

$$a_{33} = \left\{ C_1 \frac{n^4\pi^4}{l^4} + \left(C - \frac{I_0}{A} P \right) \frac{n^2\pi^2}{l^2} + k_x h_y^2 + k_\Phi \right\}$$

The decoupling between flexural buckling about the x-axis, on the one hand, and coupled flexural buckling about the y-axis and torsional buckling about the z-axis, on the other, is directly evident from (189).

Flexural buckling:

$$EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} = 0 \tag{190}$$

$$P_E = \frac{n^2\pi^2 EI_x}{l^2} \tag{191}$$

The lowest value will be obtained if the bar buckles in one half-wave,

$$n = 1:\ P_E = \frac{\pi^2 EI_x}{l^2} \tag{192}$$

In determining I_x the effective width of the plate may be taken into account.

Coupled flexural-torsional buckling:

$$\left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right) \cdot \left\{C_1 \frac{n^4\pi^4}{l^4} + \left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_x h_y^2 + k_\Phi\right\} - k_x^2 h_y^2 = 0 \qquad (193)$$

$$EI_y C_1 \frac{n^8\pi^8}{l^8} + EI_y\left(C - \frac{I_0}{A}P\right)\frac{n^6\pi^6}{l^6} + EI_y k_x h_y^2 \frac{n^4\pi^4}{l^4} + EI_y k_\Phi \frac{n^4\pi^4}{l^4} +$$

$$- PC_1 \frac{n^6\pi^6}{l^6} - P\left(C - \frac{I_0}{A}P\right)\frac{n^4\pi^4}{l^4} - Pk_x h_y^2 \frac{n^2\pi^2}{l^2} - Pk_\Phi \frac{n^2\pi^2}{l^2} +$$

$$+ k_x C_1 \frac{n^4\pi^6}{l^4} + k_x\left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_x^2 h_y^2 + k_x k_\Phi - k_x^2 h_y^2 = 0 \qquad (194)$$

Dividing (194) by k_x and putting $k_x = \infty$ gives:

$$EI_y h_y^2 \frac{n^4\pi^4}{l^4} - Ph_y^2 \frac{n^2\pi^2}{l^2} + C_1 \frac{n^4\pi^4}{l^4} + \left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_\Phi = 0 \qquad (195)$$

$$P_{cr} = \frac{(EI_y h_y^2 + C_1)\dfrac{n^2\pi^2}{l^2} + C + k_\Phi \dfrac{l^2}{n^2\pi^2}}{\left(h_y^2 + \dfrac{I_0}{A}\right)}$$

$$\sigma_{\text{e.t.b.}} = \frac{(EI_y h_y^2 + C_1)\dfrac{n^2\pi^2}{l^2} + C + k_\Phi \dfrac{l^2}{n^2\pi^2}}{(Ah_y^2 + I_0)} \qquad (196)$$

It is possible to derive a condition subject to which the stiffener remains effective up to the attainment of the yield point.

According to (186):

$$\sigma_{\text{t.b.}} = \sigma_y \quad \text{if} \quad \bar{\lambda} \leqslant 0{,}2$$

(40): $$\bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} \leqslant 0{,}2 \qquad (197)$$

$$\sigma_{\text{e.t.b.}} \geqslant \frac{\sigma_y}{0{,}2^2} = 25\sigma_y \qquad (198)$$

Furthermore:

$$\left.\begin{aligned} C_w &= 0 \rightarrow C_1 = 0 \\ C &= GI_t = \frac{1}{2(1+\nu)} EI_t \\ h_y &= \tfrac{1}{2}h \end{aligned}\right. \qquad (199)$$

$$I_0 = I_x + I_y + A(x_0^2 + y_0^2) = I_x + I_y$$

A safe assumption is:

$$k_\Phi = 0, \text{ so-called piano hinge} \tag{200}$$

Substitution of (196), (199) and (200) into (198) gives:

$$\frac{\frac{1}{4} E I_y h^2 \frac{n^2 \pi^2}{l^2} + \frac{1}{2(1+\nu)} E I_t}{A(\frac{1}{2}h)^2 + I_x + I_y} \geqslant 20.9 \sigma_y \tag{201}$$

For flat bar stiffeners $I_y << I_x$, so that, neglecting I_y, this expression becomes:

$$\frac{\frac{1}{2(1+\nu)} E I_t}{A(\frac{1}{2}h)^2 + I_x} \geqslant 25 \sigma_y \tag{202}$$

Suppose that $I_N = I_x + A(\frac{1}{2}h)^2$ is the moment of inertia of the section about the point N where the stiffener is connected to the plate; then (202) becomes:

$$\frac{I_t}{I_N} \geqslant 65 \frac{\sigma_y}{E} \tag{203}$$

$$\begin{array}{|l|} \hline \text{Fe 360 } \frac{I_t}{I_N} \geqslant 0{,}074 \\ \text{Fe 430 } \frac{I_t}{I_N} \geqslant 0{,}087 \\ \text{Fe 510 } \frac{I_t}{I_N} \geqslant 0{,}111 \\ \hline \end{array} \tag{204}$$

I_t nd I_N are referred to the stiffener (i.e., excluding the effective width of the plate)..

It is not in all cases necessary for the stiffener to remain effective up to the attainment of the yield point. Hence it is useful to be able to determine $\sigma_{\text{e.t.b.}}$, and thus $\sigma_{\text{t.b.}}$, in a convenient manner. There are two methods of achieving this, namely:

a. neglecting the same quantities as in the derivation of (203), or:

b. not neglecting any quantities.

Re a:

$$\sigma_{\text{e.t.b.}} = \frac{\frac{1}{2(1+\nu)} E I_t}{I_N} \tag{205}$$

Substituting (205) into (185):

$$\bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} = \sqrt{\frac{\sigma_y}{0.81 \cdot 10^5} \cdot \frac{I_N}{I_t}} \tag{206}$$

$$\bar{\lambda} = 0{,}0035 \cdot \sqrt{\sigma_y \cdot \frac{I_N}{I_t}} \qquad |\sigma_y| = |\mathrm{N/mm^2}| \tag{207}$$

$0{,}25\,(t/h)^2$ is small in relation to 1 for flat bar stiffeners with commonly employed dimensions.

$$\bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1}{0{,}881\left(\frac{t}{l}\right)^2 \cdot n^2 + 0{,}551\left(\frac{t}{h}\right)^2 + 0{,}434\,\frac{k_\Phi}{Eth}\left(\frac{l}{h}\right)^2 \cdot \frac{1}{n^2}}} \tag{219}$$

With the aid of (186) the reduction can be determined numerically, with the parameters:

$$\frac{\sigma_y}{E};\quad \frac{t}{l};\quad \frac{t}{h} \quad \text{and} \quad \frac{k_\Phi}{Eth}$$

Neglecting the restraint stiffness k_Φ, we obtain:

$$\bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1}{0{,}881\left(\frac{t}{l}\right)^2 + 0{,}551\left(\frac{t}{h}\right)^2}} \tag{220}$$

The reduction can be determined with the aid of (186). $\sigma_{\text{t.b.}}$ can then be determined with the aid of (186).

Also, it is possible to derive a condition subject to which the stiffener will not fail earlier in torsional buckling than the adjacent panels will fail in plate buckling.

The requirement is then:

$$\sigma_{\text{t.b.}} \geqslant \sigma_{\text{plate buckling}} \tag{208}$$

From (208) it can be deduced that the following condition must then apply:

$$\sigma_{\text{e.t.b.}} \geqslant \beta\,\sigma_{\text{e. plate buckling}} \tag{209}$$

The factor β takes account of the difference in reduction from Euler torsional buckling and Euler plate buckling to torsional buckling and plate buckling (see Fig. 16).

It appears from Fig. 5 that the greatest difference in reduction exists for

$$\frac{\sigma_{\text{eb}}}{\sigma_y} = \frac{\sigma_{\text{e. plate buckling}}}{\sigma_y} = 0.6$$

In that case:

$$\beta = \frac{\text{reduction for plate buckling}}{\text{reduction for flexural buckling}} = \frac{1}{0{,}72} = 1{,}39 \tag{210}$$

Therefore it is safe to assume:

$$\sigma_{\text{e.t.b.}} \geqslant 1{,}39\,\sigma_{\text{e. plate buckling}} \tag{211}$$

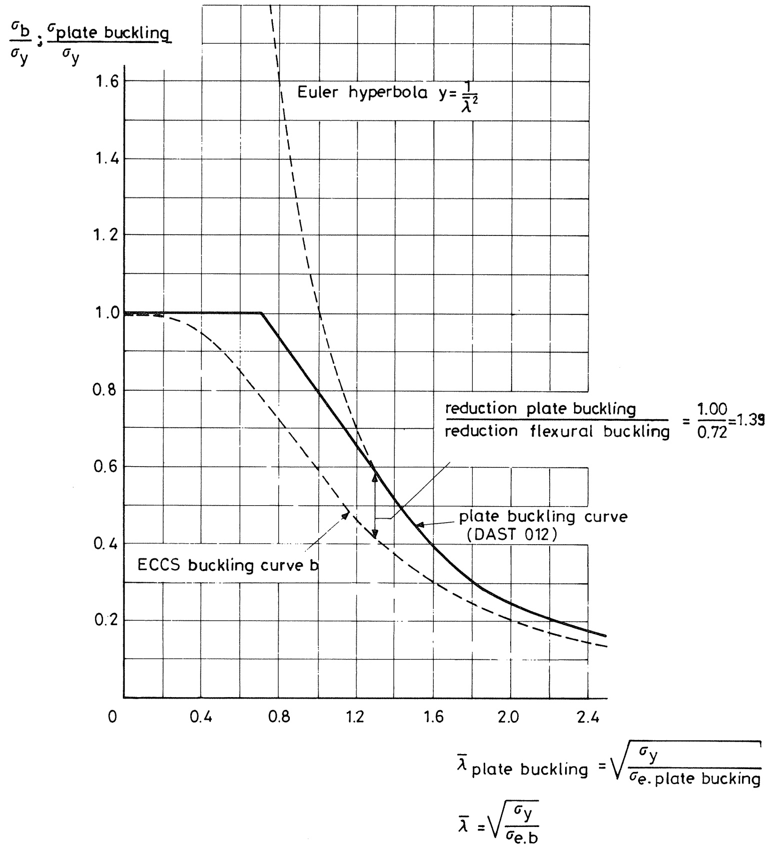


Fig. 16. Reduction for plate buckling and for flexural buckling.

$$\sigma_{\text{e. plate buckling}} = k_\sigma \sigma_E = k_\sigma \cdot \frac{\pi^2 E}{12(1-\nu)} \cdot \left(\frac{t_{pl}}{b}\right)^2 \tag{212}$$

The factor for plate buckling for a plate panel subjected to constant compression in one direction:

$$k_\sigma = 4$$

Substitution of (212) and (205) into (211) leads to

$$\frac{1}{2(1+\nu)} \cdot \frac{EI_t}{I_N} \geqslant 1{,}39 \cdot 4 \cdot \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t_{pl}}{b}\right)^2 \tag{213}$$

$$\frac{I_t}{I_N} \geqslant 13 \cdot \left(\frac{t_{pl}}{b}\right)^2 \tag{214}$$

Flat bar stiffener:

$$\frac{I_t}{I_N} = \frac{\frac{1}{3} \cdot h \cdot t^3}{\frac{1}{12} \cdot t \cdot h^3 + t \cdot h \cdot (\frac{1}{2}h)^2} = \left(\frac{t}{h}\right)^2 \tag{215}$$

Substitution of (69) into (68) gives:

$$\left(\frac{t}{h}\right)_{\text{flat bar stiffener}} \geqslant 3{,}61 \left(\frac{t_{pl}}{b}\right)_{\text{adjacent plate panel of least width}}$$

Re b:
According to (196):

$$\sigma_{\text{e.t.b.}} = \frac{(EI_y h_y^2 + C_1)\,\dfrac{n^2\pi^2}{l^2} + C + k_\Phi\,\dfrac{l^2}{n^2\pi^2}}{Ah_y^2 + I_0}$$

Substitution of (199) into (196) gives:

$$\sigma_{\text{e.t.b.}} = \frac{EI_y (\frac{1}{2}h)^2\,\dfrac{n^2\pi^2}{l^2} + GI_t + k_\Phi\,\dfrac{l^2}{n^2\pi^2}}{A(\frac{1}{2}h)^2 + I_x + I_y} \tag{216}$$

$$\sigma_{\text{e.t.b.}} = \frac{\frac{1}{4}E\frac{1}{12}ht^3h^2\,\dfrac{n^2\pi^2}{l^2} + G\frac{1}{3}ht^3 + k_\Phi\,\dfrac{l^2}{n^2\pi^2}}{\frac{1}{4}thh^2 + \frac{1}{12}th^3 + \frac{1}{12}ht^3}$$

$$\sigma_{\text{e.t.b.}} = \frac{0.20562\,\dfrac{t^3h^3}{l^2}\,n^2E + \frac{1}{3}ht^3G + 0.10132\,\dfrac{k_\Phi l^2}{n^2}}{\frac{1}{3}th^3 + \frac{1}{12}ht^3} \tag{217}$$

$$\bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} = 1{,}195\sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1 + 0{,}25\left(\dfrac{t}{h}\right)^2}{0{,}881\left(\dfrac{t}{l}\right)^2 \cdot n^2 + 0{,}551\left(\dfrac{t}{h}\right)^2 + 0{,}434\,\dfrac{k_\Phi}{Eth}\left(\dfrac{l}{h}\right)^2 \cdot \dfrac{1}{n^2}}} \tag{218}$$

3.5 *T-section stiffener*

The *T*-section stiffener has a section which is symmetrical about one axis. Consequently, the shear centre does not coincide with the centroid of the section.

Let the *y*-axis be the axis of symmetry; then:

$$x_0 = 0 \qquad y_0 \neq 0 \tag{221}$$

The stiffener is connected along the edge of its web to the plate.

$$h_x = 0 \qquad h_y \neq 0 \tag{222}$$

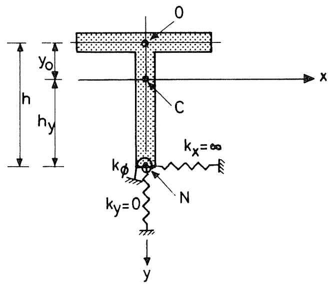


Fig. 17. *T*-section stiffener.

Substitution of (221) and (222) into (180) gives:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \tag{223}$$

with

$$a_{11} = \left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right)$$

$$a_{12} = a_{21} = a_{23} = a_{32} = 0$$

$$a_{13} = a_{31} = \left\{- Py_0 \frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)\right\}$$

$$a_{23} = \left(EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2}\right)$$

$$a_{33} = \left\{C_1 \frac{n^4\pi^4}{l^4} + \left(C - \frac{I_0}{A} P\right)\frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)^2 + k_\Phi\right\}$$

From (223) it is apparent that, just as in the case of the flat bar stiffener, there is decoupling between flexural buckling about the x-axis, on the one hand, and coupled flexural buckling about the y-axis and torsional buckling about the z-axis, on the other.

Flexural buckling:

$$EI_x \frac{n^2\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} = 0 \tag{224}$$

$$P_E = \frac{n^2\pi^2 EI_x}{l^2} \tag{225}$$

The lowest value will be obtained if the bar buckles in one half-wave, i.e., $n=1$:

$$P_E = \frac{\pi^2 EI_x}{l^2} \tag{226}$$

In determining I_x the co-operating width of the plate may be taken into account.

Coupled flexural-torsional buckling:

$$\left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right)\cdot\left\{C_1 \frac{n^4\pi^4}{l^4} + \left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)^2 + k_\Phi\right\} +$$
$$-\left\{-Py_0 \frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)\right\}^2 = 0 \tag{227}$$

$$EI_y C_1 \frac{n^8\pi^8}{l^8} + EI_y\left(C - \frac{I_0}{A}P\right)\frac{n^6\pi^6}{l^6} + EI_y k_x (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + EI_y k_\Phi \frac{n^4\pi^4}{l^4} +$$
$$- PC_1 \frac{n^6\pi^6}{l^6} - P\left(C - \frac{I_0}{A}P\right)\frac{n^4\pi^4}{l^4} - Pk_x(y_0 - h_y)^2 \frac{n^2\pi^2}{l^2} - Pk_\Phi \frac{n^2\pi^2}{l^2} +$$
$$+ k_x C_1 \frac{n^4\pi^4}{l^4} + k_x\left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_x^2(y_0 - h_y)^2 + k_x k_\Phi +$$
$$- P^2 y_0^2 \frac{n^4\pi^4}{l^4} + 2Py_0 k_x (y_0 - h_y)\frac{n^2\pi^2}{l^2} - k_x^2(y_0 - h_y)^2 = 0 \tag{228}$$

Put $k_x = \infty$

$$EI_y(y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} - P(y_0 - h_y)^2 \frac{n^2\pi^2}{l^2} + C_1 \frac{n^4\pi^4}{l^4} +$$
$$+\left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_\Phi + 2Py_0(y_0 - h_y)\frac{n^2\pi^2}{l^2} = 0 \tag{229}$$

$$P_{cr} = \frac{\{EI_y(y_0 - h_y)^2 + C_1\}\dfrac{n^2\pi^2}{l^2} + C + k_\Phi \dfrac{l^2}{n^2\pi^2}}{(y_0 - h_y)^2 + \dfrac{I_0}{A} - 2y_0(y_0 - h_y)} \tag{230}$$

$$\begin{aligned} C_1 &= 0 \\ I_0 &= I_x + I_y + Ay_0^2 \end{aligned} \tag{231}$$

Substitution of (231) into (230) gives:

$$\sigma_{cr} = \frac{EI_y(y_0 - h_y)^2 \dfrac{n^2\pi^2}{l^2} + C + k_\Phi \dfrac{l^2}{n^2\pi^2}}{I_x + I_y + A(y_0^2 + y_0^2 - 2y_0 h_y + h_y^2 - 2y_0^2 + 2y_0 h_y)} \tag{232}$$

$C = GI_t$
$y_0 - h_y = -h_y$
$I_x + I_y + Ah_y^2 = I_{PN}$ (233)
polar moment of inertia of the stiffener
about its connection to the plate (point N)

Substitution of (233) into (232) gives:

$$\sigma_{\text{e.t.b.}} = \frac{EI_y h^2 \frac{n^2\pi^2}{l^2} + E\frac{1}{2(1+\nu)} I_t + k_\Phi \frac{l^2}{n^2\pi^2}}{I_{PN}} \tag{234}$$

As in the case of the flat bar stiffener it is possible to derive a condition subject to which the stiffener remains effective up to attainment of the yield point.

(197): $\bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} \leqslant 0{,}2$

(198): $\sigma_{\text{e.t.b.}} \geqslant \frac{\sigma_y}{0{,}2^2} = 25\sigma_y$

Assuming $k_\Phi = 0$, so-called piano hinge, and neglecting I_y in the numerator of the expression for $\sigma_{\text{e.t.b.}}$ (234), we obtain in analogy with (203):

$$\frac{I_t}{I_{PN}} \geqslant 65\,\frac{\sigma_y}{E} \tag{235}$$

		(236)
Fe 360	$\frac{I_t}{I_{PN}} \geqslant 0{,}074$	
Fe 430	$\frac{I_t}{I_{PN}} \geqslant 0{,}087$	
Fe 510	$\frac{I_t}{I_{PN}} \geqslant 0{,}111$	

I_t and I_{PN} are referred to the stiffener (i.e., excluding the effective width of the plate).

In analogy with (207) the following expression can be derived for the T-section stiffener:

$$\bar{\lambda} = 0{,}0035 \cdot \sqrt{\sigma_y \cdot \frac{I_{PN}}{I_t}} \qquad |\sigma_y| = |\text{N/mm}^2| \tag{237}$$

$\sigma_{\text{t.b.}}$ can then be determined with the aid of (186).

In analogy with (214) it can be shown that if

$$\frac{I_t}{I_{PN}} \geqslant 13 \cdot \left(\frac{t_{pl}}{b}\right)^2 \tag{238}$$

the stiffener will not fail earlier in torsional buckling than the adjacent panels will fail in plate buckling.

Without neglecting any quantities, we obtain:

(234): $$\sigma_{\text{e.t.b.}} = \frac{EI_y h^2 \dfrac{n^2\pi^2}{l^2} + E\dfrac{1}{2(1+\nu)} I_t + k_\Phi \dfrac{l^2}{n^2\pi^2}}{I_{PN}}$$

$$\bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1}{14{,}099\,\dfrac{I_y}{I_{PN}} \cdot \left(\dfrac{h}{l}\right)^2 n^2 + 0{,}551\,\dfrac{I_t}{I_{PN}} + 0{,}145\,\dfrac{k_\Phi l^2}{EI_{PN}} \cdot \dfrac{1}{n^2}}} \tag{239}$$

With the aid of (186) the reduction can be determined numerically, with the parameters:

$$\frac{\sigma_y}{E};\quad \frac{I_y}{I_{PN}} \cdot \left(\frac{h}{l}\right)^2;\quad \frac{I_t}{I_{PN}} \quad \text{and} \quad \frac{k_\Phi l^2}{EI_{PN}}$$

Neglecting the restraint stiffness k_Φ, we obtain:

$$\bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1}{14{,}099\,\dfrac{I_y}{I_{PN}} \cdot \left(\dfrac{h}{l}\right)^2 + 0{,}551\,\dfrac{I_t}{I_{PN}}}} \tag{240}$$

The reduction can be determined with the aid of (186).

3.6 *L-section stiffener*

The *L*-section stiffener has an asymmetrical cross-section. Consequently, the shear centre does not coincide with the centroid of the section.

$$x_0 \neq 0 \qquad y_0 \neq 0 \tag{241}$$

The stiffener is connected along the edge of one (usually the longer) leg of its *L*-section to the plate.

$$k_x \neq 0 \qquad h_y \neq 0 \tag{242}$$

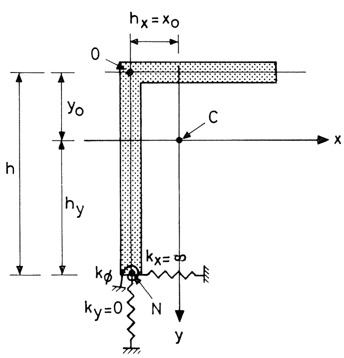


Fig. 18. *L*-section stiffener.

For this case equation (180) is applicable in full.
Substituting $C_1 = 0$ gives:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \tag{243}$$

with

$$a_{11} = \left(EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right)$$

$$a_{12} = a_{21} = 0$$

$$a_{13} = a_{31} = \left\{- Py_0 \frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)\right\}$$

$$a_{22} = \left(EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2}\right)$$

$$a_{23} = a_{32} = Px_0 \frac{n^2\pi^2}{l^2}$$

$$a_{33} = \left\{\left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)^2 + k_\Phi\right\}$$

$$\left\{EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right\} \cdot \left\{EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2}\right\} \cdot \left\{\left(C - \frac{I_0}{A}P\right)\frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)^2 + k_\Phi\right\} - \left\{- Py_0 \frac{n^2\pi^2}{l^2} + k_x(y_0 - h_y)\right\}^2 \cdot \left\{EI_x \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2}\right\} + - \left\{EI_y \frac{n^4\pi^4}{l^4} - P\frac{n^2\pi^2}{l^2} + k_x\right\} \cdot \left\{Px_0 \frac{n^2\pi^2}{l^2}\right\}^2 = 0 \tag{244}$$

$$E^2 I_x I_y \left(C - \frac{I_0}{A}P\right)\frac{n^{10}\pi^{10}}{l^{10}} - PE(I_x + I_y)\left(C - \frac{I_0}{A}P\right)\frac{n^8\pi^8}{l^8} + (P^2 + k_x EI_x) \cdot \left(C - \frac{I_0}{A}P\right)\frac{n^6\pi^6}{l^6} - Pk_x\left(C - \frac{I_0}{A}P\right)\frac{n^4\pi^4}{l^4} + E^2 I_x I_y k_x (y_0 - h_y)^2 \frac{n^8\pi^8}{l^8} - PE \cdot (I_x + I_y)k_x(y_0 - h_y)^2 \frac{n^6\pi^6}{l^6} + P^2 k_x (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + k_x^2 EI_x (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + - Pk_x^2(y_0 - h_y)^2 \frac{n^2\pi^2}{l^2} + E^2 I_x I_y k_\Phi \frac{n^8\pi^8}{l^8} - PE(I_x + I_y)k_\Phi \frac{n^6\pi^6}{l^6} + (P^2 + k_x EI_x) \cdot$$

$$k_\Phi \frac{n^4\pi^4}{l^4} - Pk_x k_\Phi \frac{n^2\pi^2}{l^2} - P^2 y_0^2 EI_x \frac{n^8\pi^8}{l^8} + 2Py_0 k_x EI_x (y_0 - h_y) \frac{n^6\pi^6}{l^6} +$$

$$- EI_x k_x^2 (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + P^3 y_0^2 \frac{n^6\pi^6}{l^6} - 2P^2 y_0 k_x (y_0 - h_y) \frac{n^4\pi^4}{l^4} + Pk_x^2 \cdot$$

$$(y_0 - h_y)^2 \frac{n^2\pi^2}{l^2} - P^2 x_0^2 EI_y \frac{n^8\pi^8}{l^8} + P^3 x_0^2 \frac{n^6\pi^6}{l^6} - P^2 x_0^2 k_x \frac{n^4\pi^4}{l^4} = 0 \qquad (245)$$

Dividing (245) by k_x and putting $k_x = \infty$

$$EI_x \left(C - \frac{I_0}{A} P\right) \frac{n^6\pi^6}{l^6} - P\left(C - \frac{I_0}{A} P\right) \frac{n^4\pi^4}{l^4} + E^2 I_x I_y (y_0 - h_y)^2 \frac{n^8\pi^8}{l^8} +$$

$$- PE(I_x + I_y)(y_0 - h_y)^2 \frac{n^6\pi^6}{l^6} + P^2 (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + k_\Phi EI_x \frac{n^4\pi^4}{l^4} - Pk_\Phi \frac{n^2\pi^2}{l^2} +$$

$$+ 2Py_0 EI_x (y_0 - h_y) \frac{n^6\pi^6}{l^6} - 2P^2 y_0 (y_0 - h_y) \frac{n^4\pi^4}{l^4} - P^2 x_0^2 \frac{n^4\pi^4}{l^4} = 0 \qquad (246)$$

$$EI_x C \frac{n^6\pi^6}{l^6} - EI_x \frac{I_0}{A} P \frac{n^6\pi^6}{l^6} - PC \frac{n^4\pi^4}{l^4} + P^2 \frac{I_0}{A} \frac{n^4\pi^4}{l^4} + E^2 I_x I_y (y_0 - h_y)^2 \frac{n^8\pi^8}{l^8} +$$

$$- PE(I_x + I_y)(y_0 - h_y)^2 \frac{n^6\pi^6}{l^6} + P^2 (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + k_\Phi EI_x \frac{n^4\pi^4}{l^4} - Pk_\Phi \frac{n^2\pi^2}{l^2} +$$

$$+ 2Py_0 EI_x (y_0 - h_y) \frac{n^6\pi^6}{l^6} - 2P^2 y_0 (y_0 - h_y) \frac{n^4\pi^4}{l^4} - P^2 x_0^2 \frac{n^4\pi^4}{l^4} = 0 \qquad (247)$$

$$P^2 \cdot \left\{\frac{I_0}{A} \cdot \frac{n^4\pi^4}{l^4} + (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} - 2y_0 (y_0 - h_y) \frac{n^4\pi^4}{l^4} - x_0^2 \frac{n^4\pi^4}{l^4}\right\} +$$

$$+ P \cdot \left\{- EI_x \frac{I_0}{A} \frac{n^6\pi^6}{l^6} - C \frac{n^4\pi^4}{l^4} - E(I_x + I_y)\ (y_0 - h_y)^2 \frac{n^6\pi^6}{l^6} - k_\Phi \frac{n^2\pi^2}{l^2} +\right.$$

$$\left.+ 2y_0 EI_x (y_0 - h_y) \frac{n^6\pi^6}{l^6}\right\} + \left\{EI_x C \frac{n^6\pi^6}{l^6} + E^2 I_x I_y (y_0 - h_y)^2 \frac{n^8\pi^8}{l^8} + k_\Phi EI_x \frac{n^4\pi^4}{l^4}\right\} = 0 \qquad (248)$$

Dividing (248) by $\frac{n^4\pi^4}{l^4}$ gives:

$$P^2 \cdot \left\{\frac{I_0}{A} + (y_0 - h_y)^2 - 2y_0 (y_0 - h_y) - x_0^2\right\} +$$

$$+ P \cdot \left\{- EI_x \frac{I_0}{A} \frac{n^2\pi^2}{l^2} - GI_t - E(I_x + I_y)(y_0 - h_y)^2 \frac{n^2\pi^2}{l^2} - k_\Phi \frac{l^2}{n^2\pi^2} + 2y_0 EI_x \cdot\right.$$

$$\left.(y_0 - h_y) \frac{n^2\pi^2}{l^2}\right\} + \left\{EI_x GI_t \frac{n^2\pi^2}{l^2} + E^2 I_x I_y (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + k_\Phi EI_x\right\} = 0 \qquad (249)$$

$$\left(\frac{P}{A}\right)^2 \cdot \{AI_0 + A^2(y_0 - h_y)^2 - 2A^2 y_0(y_0 - h_y) - A^2 x_0^2\} +$$

$$+\left(\frac{P}{A}\right) \cdot E \cdot \left\{- I_x I_0 \frac{n^2\pi^2}{l^2} - 0.386 AI_t - A(I_x + I_y)(y_0 - h_y)^2 \frac{n^2\pi^2}{l^2} + \right.$$

$$\left. - \frac{Ak_\Phi}{E} \frac{l^2}{n^2\pi^2} + 2AI_x y_0(y_0 - h_y) \frac{n^2\pi^2}{l^2}\right\} + E^2 \left\{0{,}386 I_x I_t \frac{n^2\pi^2}{l^2} + \right.$$

$$\left. + I_x I_y (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + \frac{k_\Phi I_x}{E}\right\} = 0 \qquad (250)$$

Substituting $\frac{P}{A} = \sigma$ into (250) and further working out:

$$\sigma^2 \cdot A\{I_x + I_y + A(x_0^2 + y_0^2 + y_0^2 - 2y_0 h_y + h_y^2 - 2y_0^2 + 2y_0 h_y - x_0^2)\} +$$

$$+ \sigma \cdot E \cdot \left\{\frac{n^2\pi^2}{l^2} (- I_x^2 - I_x I_y - AI_x x_0^2 - AI_x y_0^2 - AI_x y_0^2 + 2AI_x y_0 h_y - AI_x h_y^2 + \right.$$

$$- AI_y y_0^2 + 2AI_y y_0 h_y - AI_y h_y^2 + 2AI_x y_0^2 - 2AI_x y_0 h_y) +$$

$$\left. - 0{,}386 AI_t - \frac{Ak_\Phi}{E} \frac{l^2}{n^2\pi^2}\right\} +$$

$$+ E^2 \cdot \left\{0{,}386 I_x I_t \frac{n^2\pi^2}{l^2} + I_x I_y (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + \frac{k_\Phi I_x}{E}\right\} = 0 \qquad (251)$$

$$\sigma^2 \cdot A \cdot (I_x + I_y + Ah_y^2) +$$

$$+ \sigma \cdot E \cdot \left[\frac{n^2\pi^2}{l^2} \{- I_x^2 - I_x I_y - AI_x (x_0^2 + h_y^2) - AI_y (y_0^2 - 2y_0 h_y + h_y^2)\} + \right.$$

$$\left. - 0{,}386 AI_t - \frac{Ak_\Phi}{E} \frac{l^2}{n^2\pi^2}\right] +$$

$$+ E^2 \cdot \left\{0{,}386 I_x I_t \frac{n^2\pi^2}{l^2} + I_x I_y (y_0 - h_y)^2 \frac{n^4\pi^4}{l^4} + \frac{k_\Phi I_x}{E}\right\} = 0 \qquad (252)$$

$h_x = x_0$ and $y_0 - h_y = - h$

$I_x + I_y + A(h_x^2 + h_y^2) = I_{PN}$ polar moment of inertia about point N (253)

$I_x + I_y + Ah_y^2 = I_{Py}$ polar moment of inertia about the point of intersection of the y-axis with the plate

Substitution of (253) into (252) gives:

$$\sigma^2 \cdot AI_{Py} + \sigma \cdot E \left\{(- I_x I_{PN} - I_y Ah^2) \frac{n^2\pi^2}{l^2} - 0{,}386 AI_t - \frac{Ak_\Phi}{E} \frac{l^2}{n^2\pi^2}\right\} +$$

$$+E^2\left\{0{,}386 I_x I_t \frac{n^2\pi^2}{l^2}+I_x I_y h^2 \frac{n^4\pi^4}{l^4}+\frac{I_x k_\Phi}{E}\right\}=0 \qquad (254)$$

Neglecting the terms with I_x, I_y and k_Φ, we obtain from (254):

$$\sigma^2 A I_{Py} - \sigma G A I_t = 0 \qquad (255)$$

$\sigma \neq 0$ and dividing (255) by $\sigma \cdot A$ (256)

$$\sigma I_{PY} - G I_t = 0$$

$$\sigma_{\text{e.t.b.}} = \frac{G I_t}{I_{Py}} \qquad (257)$$

As in the case of the flat bar stiffener it is possible to derive a condition subject to which the stiffener remains effective up to attainment of the yield point.

(197): $\bar{\lambda} = \frac{\sigma_y}{\sigma_{\text{e.t.b.}}} \geqslant 0{,}2$

(198): $\sigma_{\text{e.t.b.}} \geqslant 25\sigma_y$

Then, in analogy with (203):

$$\frac{I_t}{I_{Py}} \geqslant 65\,\frac{\sigma_y}{E} \qquad (235)$$

$$\begin{array}{|l|}\hline \text{Fe } 360 \ \dfrac{I_t}{I_{Py}} \geqslant 0{,}074 \\ \text{Fe } 430 \ \dfrac{I_t}{I_{Py}} \geqslant 0{,}087 \\ \text{Fe } 510 \ \dfrac{I_t}{I_{Py}} \geqslant 0{,}111 \\ \hline \end{array} \qquad (259)$$

I_t and I_{PY} are referred to the stiffener (i.e., excluding the effective width of the plate).

In analogy with (207) the following expression can be derived for the *L*-section stiffener:

$$\bar{\lambda} = 0{,}0035 \cdot \sqrt{\sigma_y \cdot \frac{I_{Py}}{I_t}} \qquad |\sigma_y| = |\text{N/mm}^2| \qquad (260)$$

$\sigma_{\text{t.b.}}$ can then be determined with the aid of (186).

In analogy with (213) it can be shown that if

$$\frac{I_t}{I_{Py}} \geqslant 13 \cdot \left(\frac{t_{pl}}{b}\right)^2 \qquad (261)$$

the stiffener will not fail earlier in torsional buckling than the adjacent panels will fail in plate buckling.

Without neglecting any quantities, we obtain:

$$(254):\quad \sigma^2 - \sigma \cdot E \left\{ \left(\frac{I_x I_{PN}}{A I_{Py}} + \frac{I_y h^2}{I_{Py}} \right) \frac{n^2 \pi^2}{l^2} + 0{,}386 \frac{I_t}{I_{Py}} + \frac{k_\Phi}{E I_{PY}} \frac{l^2}{n^2 \pi^2} \right\} +$$

$$+ E^2 \left\{ 0{,}386 \frac{I_x I_t}{A I_{Py}} \frac{n^2 \pi^2}{l^2} + \frac{I_x I_y h^2}{A I_{Py}} \frac{n^4 \pi^4}{l^4} + \frac{I_x k_\Phi}{E A I_{Py}} \right\} = 0$$

$$\sigma^2 - \sigma E B + E^2 C = 0 \tag{262}$$

with

$$B = \left(\frac{I_x I_{PN}}{A I_{Py}} + \frac{I_y h^2}{I_{Py}} \right) \frac{n^2 \pi^2}{l^2} + 0{,}386 \frac{I_t}{I_{Py}} + \frac{k_\Phi}{E I_{Py}} \frac{l^2}{n^2 \pi^2} \tag{263}$$

$$C = 0{,}386 \frac{I_x I_t}{A I_{Py}} \frac{n^2 \pi^2}{l^2} + \frac{I_x I_y h^2}{A I_{Py}} \frac{n^4 \pi^4}{l^4} + \frac{I_x k_\Phi}{E A I_{Py}} \tag{264}$$

$$\sigma_{\text{i.t.k.}} = \frac{EB \pm \sqrt{E^2 B^2 - 4E^2 C}}{2} \tag{265}$$

$$\sigma_{\text{i.t.k.}} = E \cdot \left\{ \frac{B}{2} \pm \tfrac{1}{2} \sqrt{B^2 - 4C} \right\} \tag{266}$$

Equation (266) yields two values for the Euler torsional buckling stress, the lower of which is relevant:

$$\sigma_{\text{e.t.b.}} = E \cdot \left\{ \frac{B}{2} - \frac{\sqrt{B^2 - 4C}}{2} \right\} \tag{267}$$

$$\bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1{,}4}{B - \sqrt{B^2 - 4C}}} \tag{268}$$

With the aid of (186) the reduction can be determined numerically, with the parameters:

$$\frac{\sigma}{E};\quad \frac{I_x I_{PN}}{A I_{Py} l^2};\quad \frac{I_y}{I_{Py}} \cdot \left(\frac{h}{l} \right)^2;\quad \frac{I_t}{I_{Py}};\quad \frac{k_\Phi l^2}{E I_{Py}}$$

$$\frac{I_x I_t}{A I_{Py} l^2};\quad \frac{I_x I_y}{A I_{Py} l^2} \cdot \left(\frac{h}{l} \right)^2;\quad \frac{I_x k_\Phi}{E A I_{Py}}$$

Neglecting the restraint stiffness k_Φ, we obtain:

$$B = \left(\frac{I_x I_{PN}}{A I_{Py}} + \frac{I_y h^2}{I_{Py}} \right) \frac{\pi^2}{l^2} + 0{,}386 \frac{I_t}{I_{Py}} \tag{269}$$

$$C = 0{,}386 \frac{I_x I_t}{A I_{Py}} \cdot \frac{\pi^2}{l^2} + \frac{I_x I_y h^2}{A I_{Py}} \cdot \frac{\pi^4}{l^2} \tag{270}$$

The reduction can be determined with the aid of (268) and (186).

3.7 *Summary of the formulae*

3.7.1 Stiffener effective up to attainment of yield point

The formula which has been derived is valid on the assumption of the so-called piano hinge between the stiffener and the plate to be stiffened.

$\sigma_{\text{t.b.}} = \sigma_y$ if the following conditions are satisfied:

$$\frac{I_t}{I_{Py}} \geqslant 0{,}074 \text{ for Fe 360}$$

$$\frac{I_t}{I_{Py}} \geqslant 0{,}087 \text{ for Fe 430} \qquad (271)$$

$$\frac{I_t}{I_{Py}} \geqslant 0.111 \text{ for Fe 510}$$

I_{Py} is the polar moment of inertia of the stiffener about the intersection of y-axis of the stiffener with the plate.

For a flat bar stiffener:

$$I_{Py} = I_N = I_x + A\left(\tfrac{1}{2}h\right)^2$$

where I_N is the moment of inertia of the flat bar stiffener about a line through the point N, parallel to the plate.

For a T-section stiffener:

$$I_{Py} = I_{PN} = I_x + I_y + Ah_y^2$$

where I_{PN} is the polar moment of inertia of the T-section stiffener about the point N.

For a L-section stiffener:

$$I_{Py} = I_x + I_y + Ah_y^2$$

where I_{Py} is the polar moment of inertia of the L-section stiffener about the point of intersection between the y-axis and the plate.

3.7.2 Stiffener does not fail earlier in torsional buckling than the adjacent panels in plate buckling

This derivation has also been based on the assumption of the so-called piano hinge between the stiffener and the plate to be stiffened.

The requirement $\sigma_{\text{t.k.}} \geqslant \sigma_{\text{plate buckling}}$ is fulfilled if the following condition is satisfied:

$$\frac{I_t}{I_{Py}} \geqslant 13 \cdot \left(\frac{t_{pl}}{b}\right)^2 \text{ adjacent panel with least width} \qquad (272)$$

The same further particulars as those relating to (271) in Section 3.7.1 apply here.

3.7.3 Explicit determination of the relative slenderness ratio $\bar{\lambda}$ of a stiffener, based on the piano hinge between the stiffener and the plate

Flat bar stiffener

$$(220): \quad \bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1}{0{,}881 \left(\frac{t}{l}\right)^2 + 0{,}551 \left(\frac{t}{h}\right)^2}}$$

where:

t = thickness of flat bar stiffener
h = depth of stiffener cross-section
l = length of stiffener

T-section stiffener

$$(240): \quad \bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1}{14{,}099 \frac{I_y}{I_{PN}} \cdot \left(\frac{h}{l}\right)^2 + 0{,}551 \frac{I_t}{I_{PN}}}}$$

where:

h = depth of stiffener cross-section
l = length of stiffener
I_y = moment of inertia of the *T*-section about of the $y-y$ axis
I_t = torsional moment of inertia of the *T*-section
I_{PN} = polar moment of inertia of the *T*-section about the point *N*, i.e., the point at which the stiffener cross-section is connected to the plate

L-section stiffener:

$$(268): \quad \bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_{\text{i.t.k.}}}} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \sqrt{\frac{1{,}4}{B - \sqrt{B^2 - 4C}}}$$

with:

$$(269): \quad B = \left(\frac{I_x I_{PN}}{A I_{Py}} + \frac{I_y h^2}{I_{Py}}\right) \frac{\pi^2}{l^2} + 0{,}386 \frac{I_t}{I_{Py}}$$

$$(270): \quad C = 0{,}386 \frac{I_x I_t}{A I_{Py}} \cdot \frac{\pi^2}{l^2} + \frac{I_x I_y h^2}{A I_{Py}} \cdot \frac{\pi^4}{l^2}$$

where:

A = cross-sectional area of *L*-section stiffener
h = depth of stiffener cross-section
l = length of stiffener

I_x = moment of inertia of the *L*-section about the $x-x$ axis
I_y = moment of inertia of the *L*-section about the $y-y$ axis
I_t = torsional moment of inertia of the *L*-section
I_{PN} = polar moment of inertia of the *L*-section about the point *N*, i.e., the point at which the stiffener cross-section is connected to the plate
$I_{PN} = I_x + I_y + A(h_x^2 + h_y^2)$
I_{Py} = polar moment of inertia of the *L*-section about the intersection of the *y*-axis of the stiffener cross-section with the plate
$I_{Py} = I_x + I_y + Ah_y^2$

3.7.4 Explicit determination of the relative slenderness ratio $\bar{\lambda}$ without neglecting the restraint stiffness k_Φ

For all three types of stiffener considered here the relative slenderness ratio $\bar{\lambda}$ can be determined with the formula:

$$\bar{\lambda} = 1{,}195 \sqrt{\frac{\sigma_y}{E}} \cdot \eta \tag{273}$$

where η is a numerical value depending on the geometry of the stiffener and on the restraint stiffness k_Φ.

Flat bar stiffener

$$\eta_1 = \sqrt{\frac{1}{0{,}881 \left(\frac{t}{l}\right)^2 \cdot n^2 + 0{,}551 \left(\frac{t}{l}\right)^2 + 0{,}434 \frac{k_\Phi}{Eth}\left(\frac{l}{h}\right)^2 \cdot \frac{1}{n^2}}} \tag{274}$$

T-section stiffener

$$\eta_T = \sqrt{\frac{1}{14{,}099 \frac{I_y}{I_{PN}} \cdot \left(\frac{h}{l}\right)^2 \cdot n^2 + 0{,}551 \frac{I_t}{I_{PN}} + 0{,}145 \frac{k_\Phi l^2}{EI_{PN}} \cdot \frac{1}{n^2}}} \tag{275}$$

L-section stiffener

$$\eta_L = \sqrt{\frac{1{,}4}{B - \sqrt{B^2 - 4C}}} \tag{276}$$

with:

(263): $B = \left(\frac{I_x I_{PN}}{AI_{py}} + \frac{I_y h^2}{I_{py}}\right)\frac{n^2\pi^2}{l^2} + 0{,}386 \frac{I_t}{I_{py}} + \frac{k_\Phi}{EI_{py}} \cdot \frac{l^2}{n^2\pi^2}$

(264): $C = 0{,}386 \frac{I_x I_t}{AI_{py}} \cdot \frac{n^2\pi^2}{l^2} + \frac{I_x I_y h^2}{AI_{py}} \cdot \frac{n^4\pi^4}{l^4} + \frac{I_x k_\Phi}{EAI_{py}}$

η is a function of the parameters

$$\eta_1 = \eta_1 \left\{ \frac{t}{l};\ \frac{t}{h};\ \frac{k_\Phi}{Eth};\ n \right\} \tag{277}$$

$$\eta_T = \eta_T \left\{ \frac{I_y}{I_{PN}} \cdot \left(\frac{h}{l}\right)^2;\ \frac{I_t}{I_{PN}};\ \frac{k_\Phi l^2}{EI_{PN}};\ n \right\} \tag{278}$$

$$\eta_L = \eta_L \left\{ \frac{I_x I_{PN}}{AI_{Py} l^2};\ \frac{I_y}{I_{Py}} \cdot \left(\frac{h}{l}\right)^2;\ \frac{I_t}{I_{Py}};\ \frac{k_\Phi l^2}{EI_{Py}};\right.$$

$$\left.\frac{I_x I_t}{AI_{Py} l^2};\ \frac{I_x I_y h^2}{AI_{Py} l^4};\ \frac{I_x k_\Phi}{EAI_{Py}};\ n \right\} \tag{279}$$

The factor n is the number of half sine wave into which the stiffener buckles over the length l; $n = 1, 2, 3$, etc. The value of n must be so chosen that the stiffener buckles at the lowest possible load. From the formulae (274), (275), (263) and (264) it emerges that $n = 1$ if $k_\Phi = 0$. If $k_\Phi \neq 0$, then n must be determined numerically so that the values η_1, η_T and η_L can likewise most simply be determined numerically. In the case were $k_\Phi \neq 0$ it is necessary to take care that the number of waves into which the stiffener buckles (torsional buckling) and the number of buckling waves of the unstiffened plate panel are not equal. If they are equal, it will not be possible to take advantage of the spring stiffness that the panel must provide. There should be a substantial difference between these two numbers of waves.

3.7.5 Numerical determination of the factors η_1, η_T and η_L

A FORTRAN computer program has been produced for the calculation of η_1, η_T and η_L. The results obtained with the program are presented in graph form.

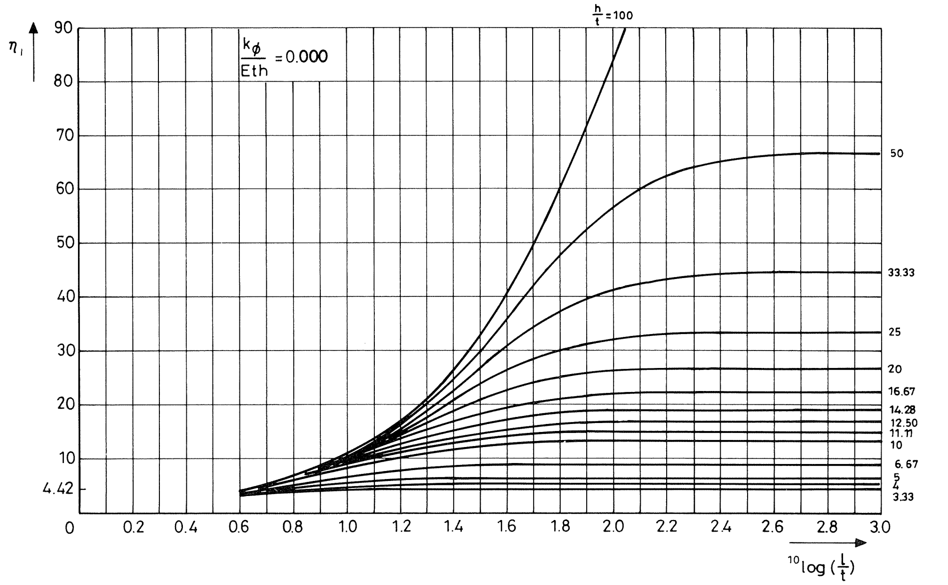
Flat bar stiffeners
Some results for flat bar stiffeners are presented in Graphs 9 to 14. It appears from these graphs that for values of $^{10}\log\ (l/t) > 3$ the value of η_1 is constant. This has been utilized in Graph 15 by eliminating the parameter l/t.

T-section stiffeners
Graphs 16 to 19 contain some results for the T-section stiffener. It appears from these graphs that with increasing magnitude of the parameter

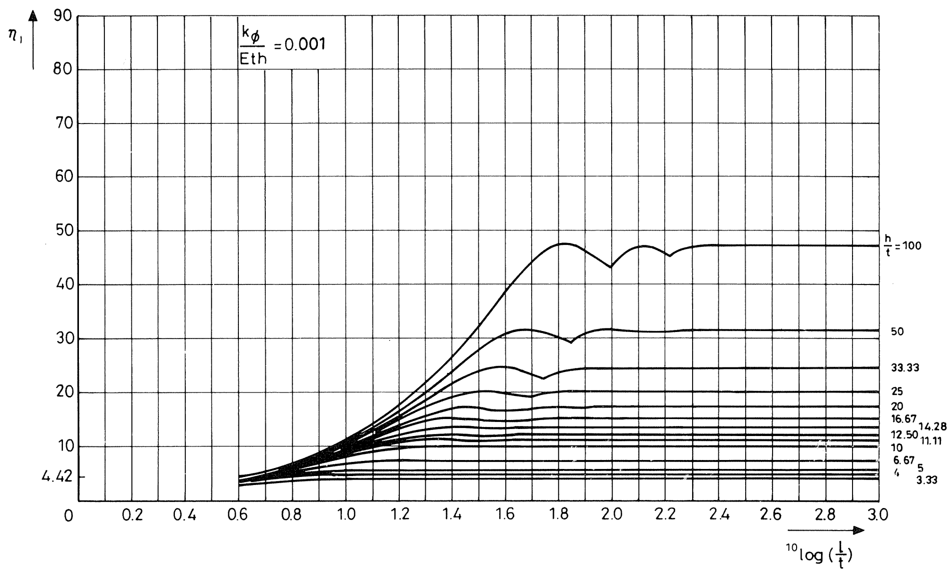
$$^{10}\log \frac{I_{PN}}{I_y} \cdot \left(\frac{l}{h}\right)^2$$

the value of η_T does not become constant, but approaches the corresponding curve for

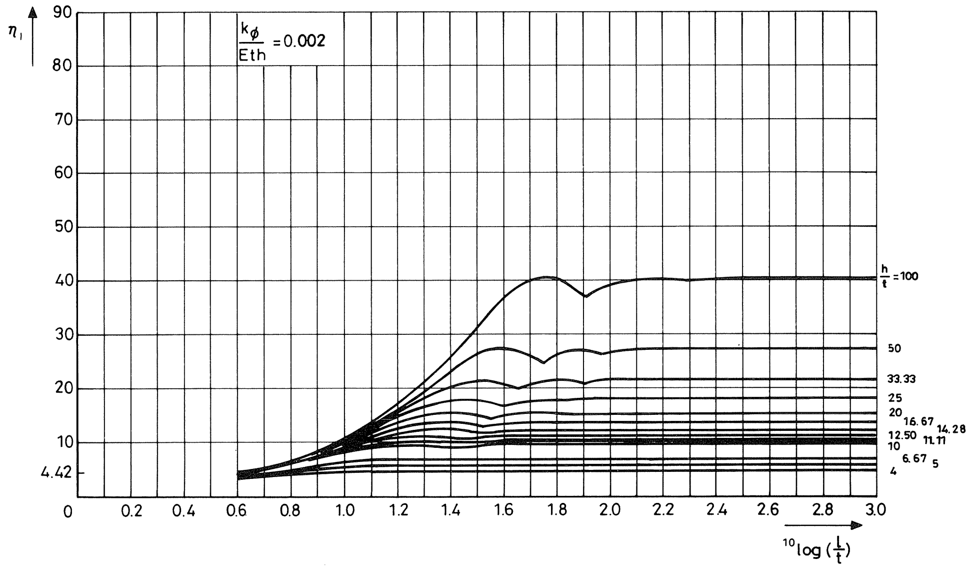
$$\frac{k_\Phi l^2}{EI_{PN}} = 0.$$



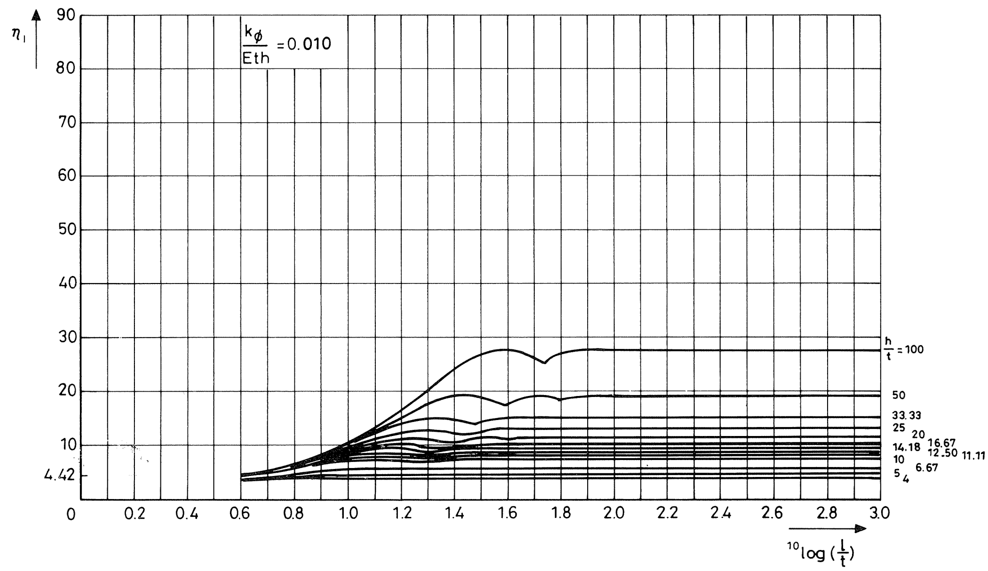
Graph 9: Flat bar stiffeners



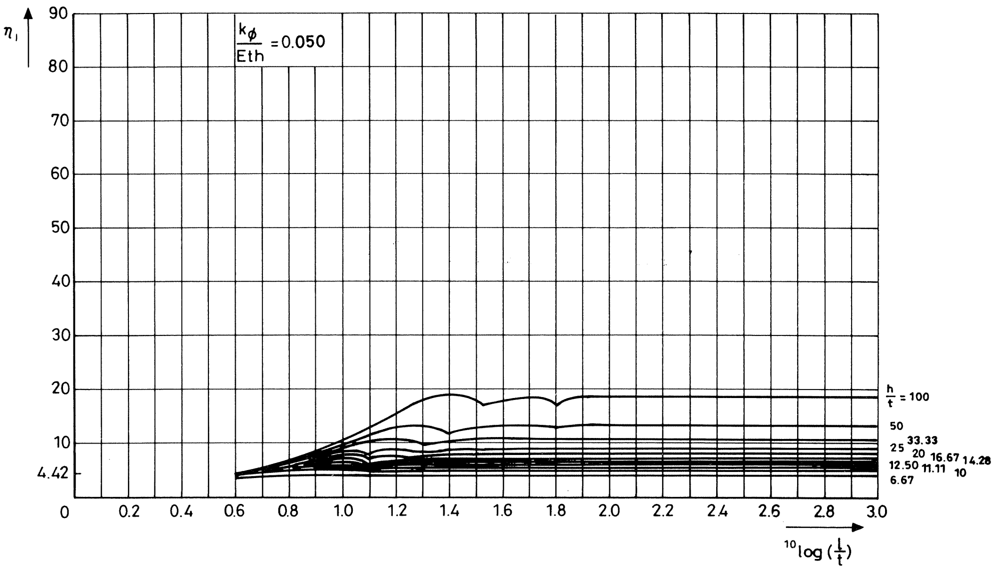
Graph 10: Flat bar stiffeners



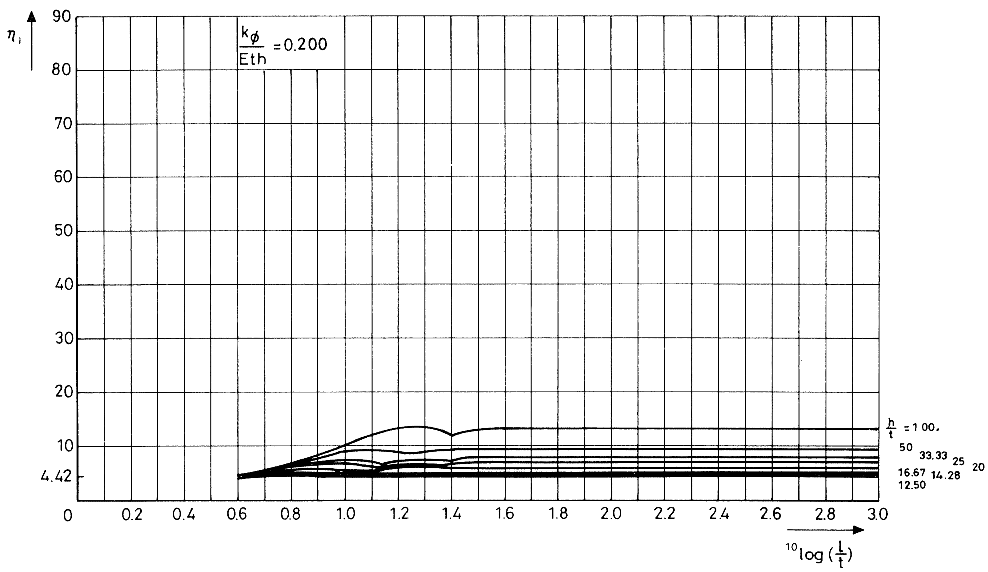
Graph 11: Flat bar stiffeners



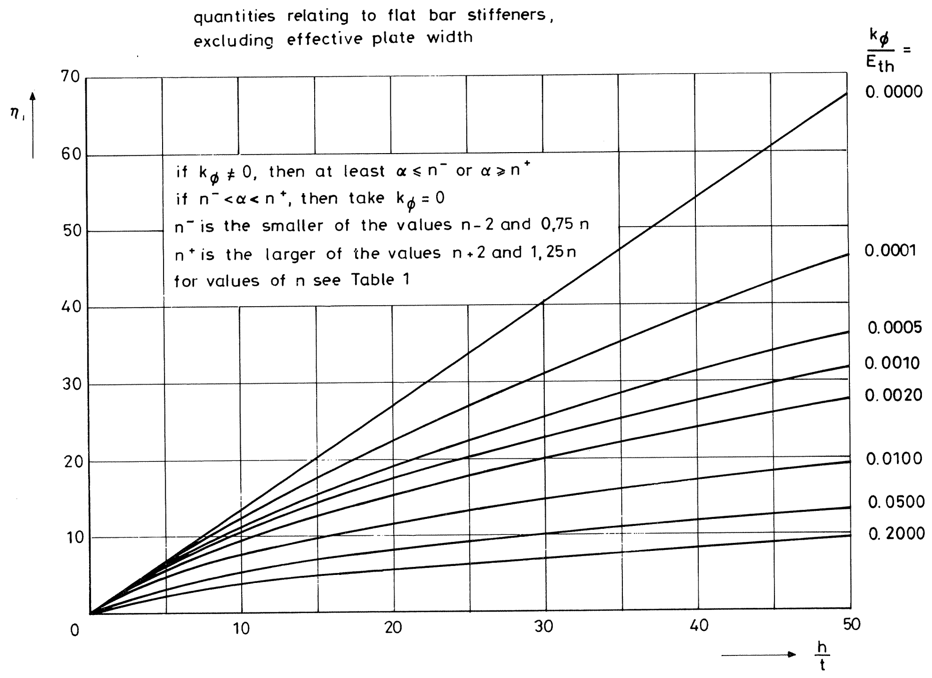
Graph 12: Flat bar stiffeners



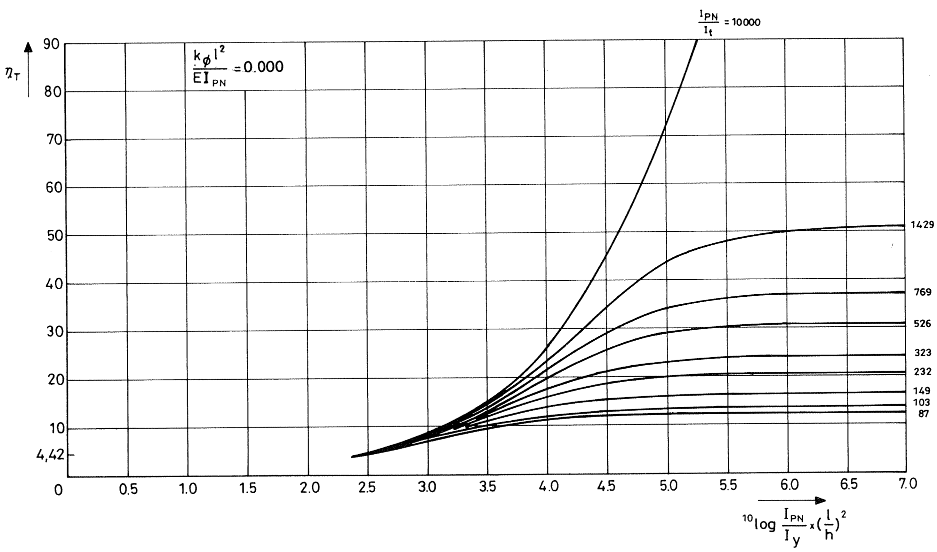
Graph 13: Flat bar stiffeners



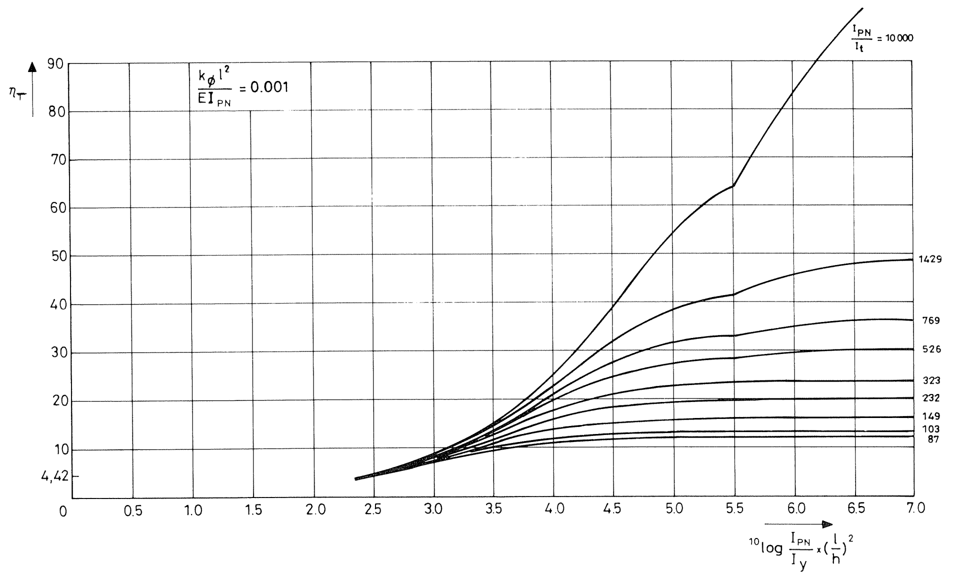
Graph 14: Flat bar stiffeners



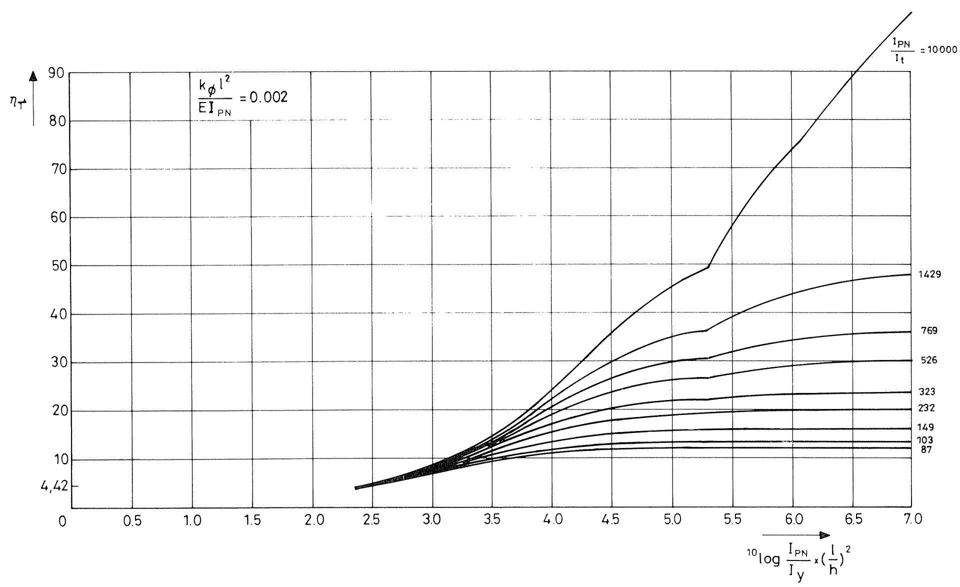
Graph 15: Flat bar stiffeners



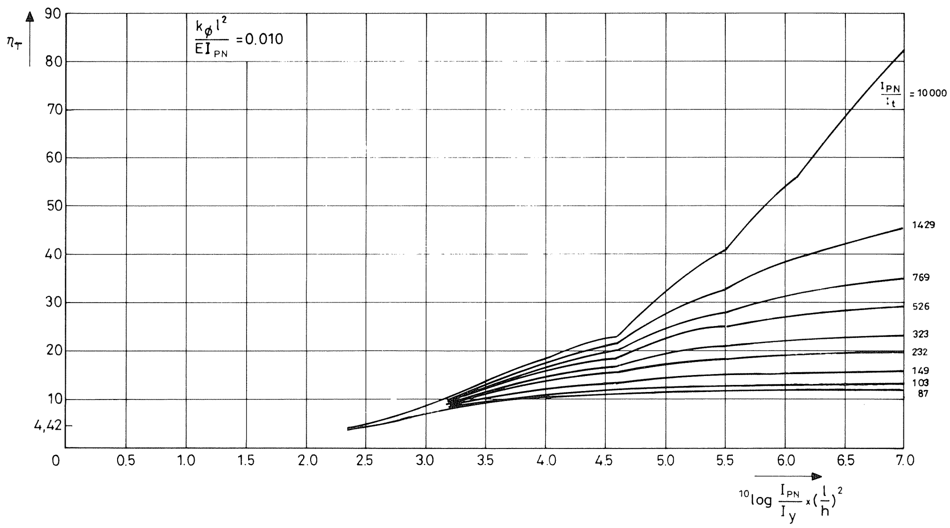
Graph 16: *T*-section stiffeners



Graph 17: *T*-section stiffeners



Graph 18: *T*-section stiffeners



Graph 19: *T*-section stiffeners

In order nevertheless to present a graph in which the parameter

$$\frac{I_{PN}}{I_y} \cdot \left(\frac{l}{h}\right)^2$$

no longer occurs on the horizontal axis, the condition has been introduced that the graph is valid if the parameter

$$\frac{I_{PN}}{I_y} \cdot \left(\frac{l}{h}\right)^2$$

does not exceed a certain value. See Graph 20.

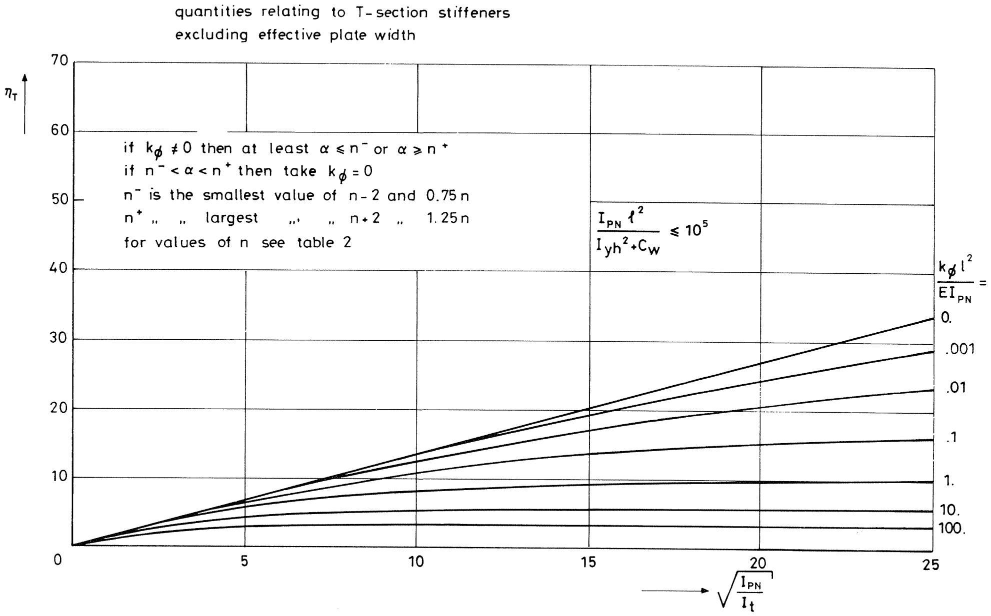
L-section stiffeners

In order to establish a few simple graphs for *L*-section stiffeners, the following basic approach has been adopted:

from (263):

$$B = \frac{I_x}{Al^2} \cdot \frac{I_{PN}}{I_{Py}} \cdot 9{,}8696 \cdot n^2 + \left\{ 9{,}8696 \cdot \frac{I_y}{I_{Py}} \left(\frac{h}{l}\right)^2 \cdot n^2 + 0{,}386 \, \frac{I_t}{I_{Py}} + \right.$$

$$\left. + \frac{k_\Phi l^2}{EI_{Py}} \cdot \frac{1}{9{,}8696} \cdot \frac{1}{n^2} \right\}$$



Graph 20: *T*-section stiffeners

from (264):

$$C = \frac{I_x}{Al^2} \cdot 9{,}8696 \cdot n^2 \cdot \left\{ 9{,}8696 \cdot \frac{I_y}{I_{Py}} \left(\frac{h}{l}\right)^2 \cdot n^2 + 0{,}386 \, \frac{I_t}{I_{Py}} + \right.$$

$$\left. + \frac{k_\Phi l^2}{EI_{Py}} \cdot \frac{1}{9{,}8696} \cdot \frac{1}{n^2} \right\}$$

$$B = \beta P + Q \qquad (280)$$

$$C = P \cdot Q \qquad (281)$$

with

$$\beta = \frac{I_{PN}}{I_{Py}}$$

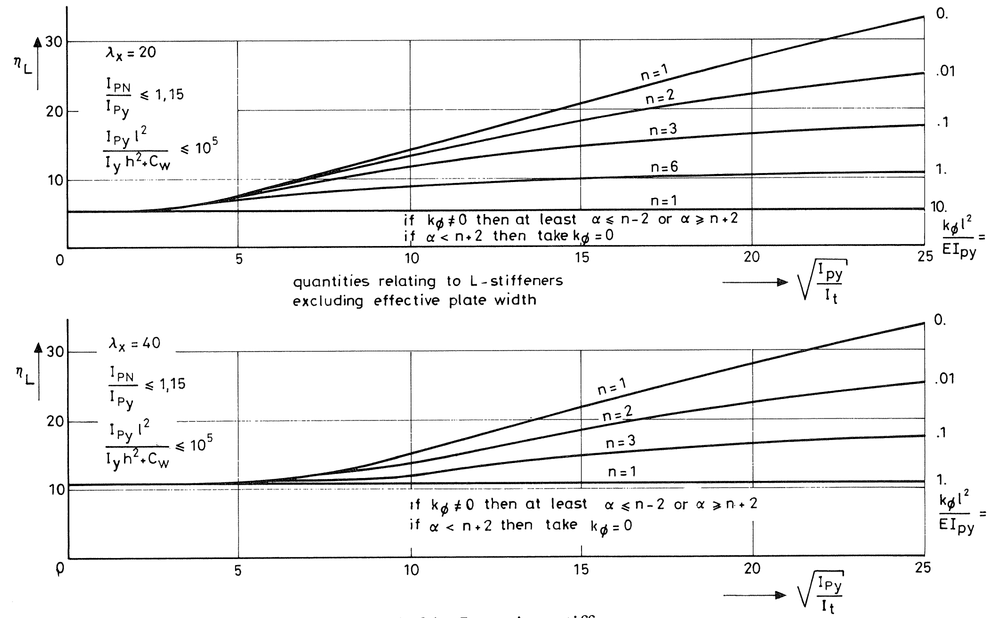
$$P = \frac{I_x}{Al^2} \cdot 9{,}8696 \cdot n^2$$

$$Q = 9{,}8696 \cdot \frac{I_y}{I_{Py}} \left(\frac{h}{l}\right)^2 \cdot n^2 + 0{,}386 \, \frac{I_t}{I_{Py}} + \frac{k_\Phi l^2}{EI_{Py}} \cdot \frac{1}{9{,}8696} \cdot \frac{1}{n^2}$$

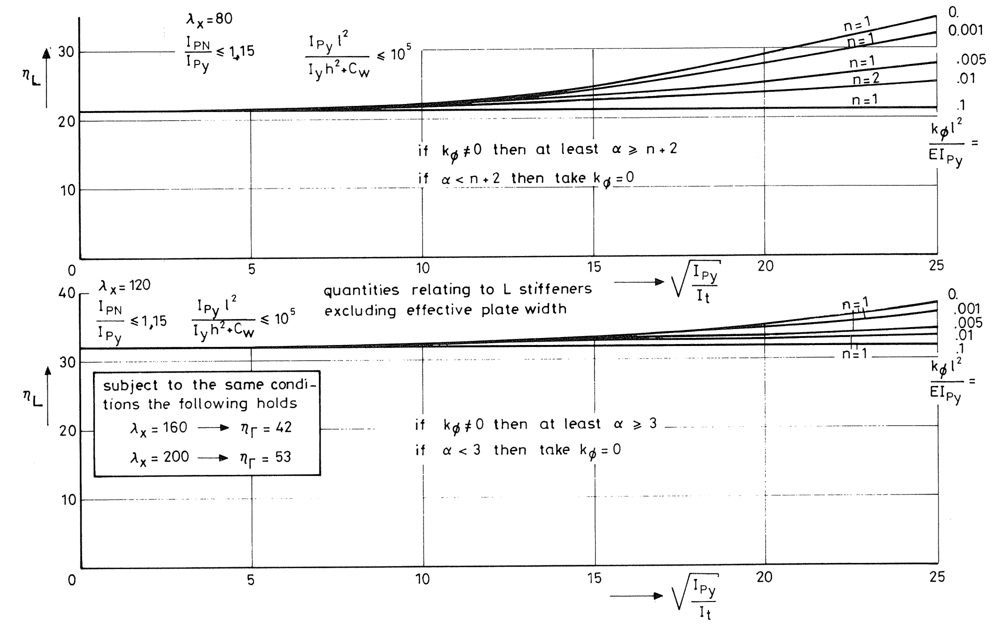
(276): $$\eta_L = \sqrt{\frac{0{,}7}{\frac{B - \sqrt{B^2 - 4C}}{2}}}$$

If it is assumed that steel angle sections (*L*-sections) are to be used as stiffeners only with the longer leg connected to the plate to be stiffened, it emerges that $\beta = I_{PN}/I_{Py}$ is within the range $1{,}10 \leqslant \beta \leqslant 1{,}15$. A safe assumption is $\beta = 1{,}15$. This means that a condition $\beta = I_{PN}/I_{Py} \leqslant 1{,}15$ is introduced, so that β has thus been eliminated as a parameter.

The results are represented in Graphs 21 and 22.



Graph 21: *L*-section stiffeners



Graph 22: *L*-section stiffeners

Table 1. Flat bar stiffeners

values of n				h/t 10	20	30	40	50
$^{10}\log\left(\frac{l}{t}\right)$	1.	$\frac{k_\Phi}{Eth}$	0.0000	1	1	1	1	1
	$(l/t=10)$		0.0001	1	1	1	1	1
			0.0005	1	1	1	1	1
			0.001	1	1	1	1	1
			0.002	1	1	1	1	1
			0,01	1	1	1	1	1
			0.05	1	1	1	1	1
			0,2	2	1	1	1	1
	1.3	$\frac{k_\Phi}{Eth}$	0.0000	1	1	1	1	1
	$(l/t=20)$		0.0001	1	1	1	1	1
			0.0005	1	1	1	1	1
			0.001	1	1	1	1	1
			0.002	1	1	1	1	1
			0.01	2	1	1	1	1
			0.05	3	2	2	1	1
			0.2	4	3	2	2	2
	2.	$\frac{k_\Phi}{Eth}$	0.0000	1	1	1	1	1
	$(l/t=100)$		0.0001	3	2	2	1	1
			0.0005	4	3	2	2	2
			0.001	5	3	3	2	2
			0.002	6	4	3	3	3
			0.01	8	6	5	4	4
			0.05	13	9	7	6	6
			0.2	18	13	10	9	8
	2.3	$\frac{k_\Phi}{Eth}$	0.0000	1	1	1	1	1
	$(l/t=200)$		0.0001	5	4	3	3	2
			0.0005	8	6	5	4	4
			0.001	9	7	5	5	4
			0.002	11	8	6	6	5
			0.01	17	12	10	8	8
			0.05	25	18	14	13	11
			0.2	35	25	20	18	16
	3.	$\frac{k_\Phi}{Eth}$	0.0000	1	1	1	1	1
	$(l/t=1000)$		0.0001	26	19	15	13	12
			0.0005	40	28	23	20	18
			0.001	47	33	27	24	21
			0.002	56	40	32	28	25
			0.01	84	59	48	42	37
			0.05	125	89	72	63	56
			0.2	177	125	102	89	79

Table 2. *T*-section stiffeners

				values of n
$\frac{I_{PN} l^2}{I_y h^2 + C_w}$	10.	$\frac{k_\Phi l^2}{EI_{PN}}$	0.000	1
			0.001	1
			0.01	1
			0.1	1
			1.	1
			10.	1
			100.	2
	100.	$\frac{k_\Phi l^2}{EI_{PN}}$	0.000	1
			0.001	1
			0.01	1
			0.1	1
			1.	1
			10.	2
			100.	3
	1000.	$\frac{k_\Phi l^2}{EI_{PN}}$	0.000	1
			0.000	1
			0.001	1
			0.01	1
			0.1	1
			1.	2
			10.	3
			100.	6
	10000.	$\frac{k_\Phi l^2}{EI_{PN}}$	0.000	1
			0.001	1
			0.01	1
			0.1	2
			1.	3
			10.	6
			100.	10
	100000.	$\frac{k_\Phi l^2}{EI_{PN}}$	0.000	1
			0.001	1
			0.01	2
			0.1	3
			1.	6
			10.	10
			100.	18

4 Summary

Rules for analysing the instability of transverse stiffeners and the torsional buckling instability of longitudinal stiffeners are derived in this report. As the chapters concerned deal in some detail with the theoretical background to the analysis, the rules will here be repeated in the form in which they have been presented in the draft guidelines for the design of stiffened and unstiffened plate panels loaded within the plane of the panel [8]. That publication also explains how the loading state on which the analysis is to be based must be determined.

4.1 *Transverse stiffeners*

Requirements applicable to transverse stiffeners in plates loaded in plane in one direction. It must be shown that transverse stiffeners satisfy the condition:

$$\Phi \cdot E \cdot \mu_1 \mu_2 \leqslant \sigma_{\text{limit}}$$

where (see Fig. 1):

E = modulus of elasticity of the material

μ_1 = H/h, in which H = greatest distance from the centroid of the transverse stiffener to the extreme fibre
h = cross-sectional depth of the transverse stiffener

μ_2 = h/b, in which b = width of the plate panel between the edges, or length of the transverse stiffener

σ_{limit} = limit stress in the extreme fibre of the transverse stiffener, e.g., corresponding to the attainment of the yield stress, the lateral (twist-bend) buckling stress or the local (plate) buckling stress for parts of the transverse stiffener

Φ = factor taking account of the maximum curvature in the transverse stiffener due to the load acting within the plane of the plate; the magnitude of Φ is dependent on ψ and λ; ψ is dependent on the loading state

$$\lambda = \frac{4\delta' \sigma_1 l^4}{EIa}$$

where:

I = moment of inertia of the transverse stiffener

E = modulus of elasticity of the material

a = spacing of the transverse stiffeners

l = width of the plate panel, or length of the transverse stiffener

$\delta' = \delta$ = plate thickness if there are no longitudinal stiffeners

$\delta' = A_{\text{gross}}/l$ = total cross-sectional area of the plate including longitudinal stiffeners, divided by the length of the transverse stiffener

σ_1 = greatest compressive design stress acting in the longitudinal direction of the plate

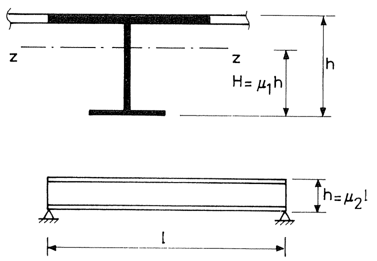


Fig. 19. Transverse stiffener.

The value of Φ can be read with the aid of ψ and λ from Graphs 1 and 2 presented in Chapter 3.

If the analysis for the longitudinal stiffeners shows them not to be loaded to their limiting load, the analysis for the transverse stiffeners may be based on the assumption that their spacing a is equal to the length that the longitudinal stiffeners would need to have so that they were loaded in the limiting load.

Explanatory note: These values of Φ are based on the assumed presence of a geometric imperfection equal to 1/400 of the length of the transverse stiffener. As a more general principle, the value of Φ is proportional to the magnitude of the imperfection. For a detailed treatment of this method of analysis see [1].

Requirements applicable to transverse under general loading condition

It must be shown that the transverse stiffener satisfies the condition:

$$\Phi \cdot E \cdot \mu_1 \mu_2 + \frac{F}{A} \leqslant \sigma_{\text{limit}}$$

where:

Φ = factor taking account of the maximum curvature in transverse stiffener due to a general loading state; the magnitude of Φ is dependent on:

ψ

λ

$$\xi_P = \frac{Pl^3}{EI}$$

$$\xi_M = \frac{Ml}{EI}$$

$$\xi_F = \frac{Fl^2}{EI}$$

l = width of the plate panel between the edges, or length of the transverse stiffener

I = moment of inertia of the transverse stiffener

p = uniformly distributed load acting on a transverse stiffener

M = end moments acting on a transverse stiffener (constant bending moment)

F = normal force (compression) in the transverse stiffener (F as compressive force has positive sign; if F is tensile, substitute $F = 0$)

A = cross-sectional area of the transverse stiffener

The value of Φ can be determined from the tables present in [4].

4.2 *Longitudinal stiffeners*

Requirements applicable to longitudinal stiffeners

Buckling out of the plane of the plate

It must be shown that longitudinal stiffeners satisfy the condition:

$$\sigma_{ls} \leqslant \sigma_{\text{flexural buckling}}$$

where:

σ_{ls} = stress at the longitudinal stiffener

$\sigma_{\text{flexural buckling}}$ = buckling stress of the longitudinal stiffener conceived as a bar with pin-jointed ends; it depends on the slenderness ratio λ_x

$$\lambda_x = \frac{a}{\sqrt{\dfrac{I_x}{A_{ls}}}}$$

with:

I_x = moment of inertia of the longitudinal stiffener, including the co-operating parts of the plate

A_{ls} = cross-sectional area of the longitudinal stiffener, including the co-operating parts of the plate

a = length of the longitudinal stiffener

Explanatory note: If the longitudinal stiffener is subjected to a transverse load, the stiffener should be analysed in accordance with rules for bars loaded in compression and bending. The separate analysis of the unstiffened panel for plate buckling and of the longitudinal stiffeners for bar buckling may prove unfavourable, more particularly for long panels provided with longitudinal stiffeners. In such case the buckled shape of these stiffeners is increasingly compelled to conform to the buckled shape of the plate panel. The buckle waves of the plate then pass through the longitudinal stiffeners, and it is advisable to investigate the buckling of the longitudinally stiffened panel in greater detail.

Torsional buckling of longitudinal stiffeners

Longitudinal stiffeners of open cross-sectional shape must be shown to satisfy the condition:

$$\sigma_{ls} \leqslant \sigma_{\text{t.b.}}$$

where:

σ_{ls} = stress at the longitudinal stiffener

$\sigma_{\text{t.b.}}$ = torsional buckling stress of the longitudinal stiffener, *excluding* the effective width of the plate

Determining the torsional buckling stress for stiffeners of open cross-sectional shape

a. The torsional buckling stress $\sigma_{t.b.}$ of an open-section longitudinal stiffener is equal to the yield stress σ_y if the following condition is satisfied:

$$\frac{I^*}{I_{Py}} \geqslant 0{,}074 \text{ for Fe 360}$$

$$\frac{I^*}{I_{Py}} \geqslant 0{,}087 \text{ for Fe 430}$$

$$\frac{I^*}{I_{Py}} \geqslant 0{,}111 \text{ for Fe 510}$$

For a flat bar stiffener

$$I^* = I_t$$
$$I_{Py} = I_N = I_x + Ah_y^2$$

Because of the assumption of a "piano hinge", for a flat bar stiffener this assumption leads to rather unrealistic values for h/t, namely: $h/t \leqslant 4$ for Fe 360 and $h/t \leqslant 3.3$ for Fe 510. Also, it must be pointed out that flat bar stiffeners are highly sensitive to local imperfections.

For a *T*-section stiffener:

$$I^* = 25{,}66 \cdot \left\{ I_y \left(\frac{h}{l}\right)^2 + \frac{C_w}{l^2} \right\} + I_t$$

$$I_{Py} = I_{PN} = \{I_x + I_y + Ah_h^2\}$$

For an *L*-section stiffener:

$$I^* = 25{,}66 \cdot \left\{ \frac{I_x I_{PN}}{Al^2} + I_y \left(\frac{h}{l}\right)^2 + \frac{C_w}{l^2} \right\} + I_t$$

$$I_{Py} = I_x + I_y + Ah_y^2$$

$$I_{PN} = I_x + I_y + A(h_x^2 + h_y^2)$$

All the section properties are to be calculated for the stiffener cross-section, excluding the effective width of the plate to be stiffened.

b. The torsional buckling stress $\sigma_{t.b.}$ of an open-section longitudinal stiffener is not less than the plate buckling stress for the adjacent panel with the least width, provided that the following condition is satisfied:

$$\frac{I^*}{I_{Py}} \geqslant 13 \cdot \left(\frac{t_{pl}}{b}\right)^2$$

where:

I^* = as defined under a
I_{Py} = as defined under a

t = thickness of the adjacent plate panel
b = width of the adjacent panel with least width

c. Explicit determination of the torsional buckling stress $\sigma_{t.b.}$ of an open-section stiffener in relation to the relative slenderness ratio $\bar{\lambda}$

$$\frac{\sigma_{t.k.}}{\sigma_y} = \frac{1 + 0{,}339(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2}{2\bar{\lambda}^2} - \frac{1}{2\bar{\lambda}^2} \cdot \sqrt{\{1 + 0{,}339(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2\}^2 - 4\bar{\lambda}^2}$$

with

$$\bar{\lambda} = 1{,}61 \sqrt{\frac{I_{Py}}{I^*}} \cdot \sqrt{\frac{\sigma_y}{E}}$$

where:

I^* = as defined under b
I_{Py} = as defined under a

Explanatory note: For determining the torsional buckling stress for open-section stiffeners the following basic assumptions were made:

1. The stiffener is pivotably connected to the plate that it stiffens (so-called piano hinge).
2. The stiffener is of infinite length.
3. The reduction of the Euler torsional buckling stress $\sigma_{e.t.b.}$ to the torsional buckling stress $\sigma_{t.b.}$ proceeds analogously to the reduction of the Euler flexural buckling stress to the buckling stress in accordance with Eurocode 3 Steelstructures [7].
   The basic assumption of the so-called piano hinge between the stiffener and the plate can be dispensed with if, instead, a rotational stiffness k_Φ representing the restraint (fixity) of the connection of the stiffener to the plate is taken into account:

$$k_\Phi = \frac{2EI_{\text{plate}}}{b_{\text{plate}}}$$

where:

EI_{plate} = reckoned per unit width, while b_{plate} denotes the width of the adjacent panel with the greatest width

In this case the relative slenderness ratio $\bar{\lambda}$ will have to be determined numerically:

$$\bar{\lambda} = 1{,}195 \cdot \sqrt{\frac{\sigma_y}{E}} \cdot \eta$$

where η is a numerical value dependent on the geometry of the stiffener and on its restraint k_Φ.

The values of η are obtainable from Graphs 15, 20, 21 and 22 with the aid of Tables 1 and 2, presented in Chapter 4.

Explanatory note: The factor 1,195 in the formulae of $\bar{\lambda}$ is a correction factor between the relative slenderness ratio $\bar{\lambda}$ and the specific slenderness ratio λ_s used in the Netherlands Standard NEN 3851 [2].

The formulae were firstly derived with respect to that standard.

For this paper the Eurocode formulation is used and a correction factor introduced.

$$\lambda_s = \sqrt{\frac{0{,}7\sigma_y}{\sigma_E}}; \qquad \bar{\lambda} = \sqrt{\frac{\sigma_y}{\sigma_E}}$$

$$\bar{\lambda} = 1{,}195\lambda_s$$

5 Acknowledgements

The design method for transverse and for longitudinal stiffeners presented here was developed in the context of the revision of the Netherlands Code of Practice for the Design of Steel Bridges (VOSB '63).

On behalf of the Staalbouwkundig Genootschap (Structural Steelwork Association) the present study was carried out under the guidance of Committee S6-TC-21^{III} "Stability of stiffened plate panels (VOSB)", of which the present author is the Technical Secretary. He wishes to express his indebtedness to the members of that Committee – Messrs. Ir. G. Hardenberg (Chairman), Ing. H. M. C. M. van Maarschalkerwaart, Ir. J. van der Wal and Ir. C. Q. Klap – for their close and pleasant collaboration.

6 List of symbols

Notation employed in Chapter 2 "Transverse stiffeners"

a	spacing of the transverse stiffeners
A	cross-sectional area of the transverse stiffener
A_s	cross-sectional area of the longitudinal stiffener
e	initial imperfection
E	modulus of elasticity
F	compressive force acting on the transverse stiffener
h	cross-sectional depth of the transverse stiffener
H	maximum distance to extreme fibre of the transverse stiffener, measured from the neutral axis
i	radius of gyration
I	moment of inertia of the transverse stiffener with regard to bending out of the plane of the stiffened plate
l	length of the transverse stiffeners
m	scalar
M	moment acting at the ends of the transverse stiffener
$M(x)$	bending moment acting in the transverse stiffener
n	scalar

p	scalar
$p(x)$	load acting within the plane of the plate
P	uniformly distributed transverse load on the transverse stiffener
q	scalar
$q(x)$	transverse load on the transverse stiffener
$y(x)$	deflected shape of the transverse stiffener
$y^0(\alpha)$	deflected shape of the transverse stiffener
$y(\alpha)$	non-dimensional deflected shape of the transverse stiffener
$y_i(x)$	initial deflected shape of the transverse stiffener
$y_i^0(\alpha)$	initial deflected shape of the transverse stiffener
$y_i(\alpha)$	non-dimensional deflected shape of the transverse stiffener
ξ_F	$-Fl^2/EI$
ξ_P	Pl^3/EI
ξ_M	Ml/EI
α	non-dimensional parameter of location on the transverse stiffener
γ	non-dimensional parameter
δ	plate thickness
δ'	average plate thickness
ε	non-dimensional parameter: $\varepsilon = l/e$
λ	non-dimensional parameter
λ_x	slenderness ratio of the transverse stiffener with respect to flexural buckling out of the plane of the plate
φ	angle
Φ	factor taking account of the maximum curvature in the transverse stiffener
μ_1	H/h
μ_2	h/l
ϱ	non-dimensional parameter: $\varrho = \psi - 1$
ψ	proportionality factor: $\sigma_2 = \psi\sigma_1$
σ_1	greatest compressive stress in the plate panel
σ_2	smallest compressive or greatest tensile stress in the plate panel
σ_{limit}	limit stress for the transverse stiffener

Notation employed in Chapter 3 "Longitudinal stiffeners"

A	cross-sectional area of the longitudinal stiffeners
C	centroid of the cross-section
C	GI_t, where I_t = torsional moment of inertia
C_1	EI_w, where I_w = warping resistance constant
E	modulus of elasticity
G	shear modulus: $G = E \cdot \dfrac{1}{2(1+\nu)}$
h	depth of stiffener cross-section
h_x and h_y	location of point N in the co-ordinate system

I_t	torsional moment of inertia
I_x	moment of inertia of the stiffener about the $x-x$ axis
I_y	moment of inertia of the stiffener about the $y-y$ axis
I_0	polar moment of inertia of the stiffener about the shear centre
I_N	moment of inertia of the section about point N
I_{PN}	polar moment of inertia of the section about point N
I_{Py}	polar moment of inertia of the section about the intersection of the y-axis with the face of the plate to which the stiffener is connected
k_x, k_y, k_Φ	spring stiffnesses
n	number of half waves
N	point of application of the elastic support of the stiffener (= where stiffener is connected to plate)
O	shear centre
P	normal force in the stiffener
u, v and Φ	displacements in the x, y, z co-ordinate system (Φ is rotation about the z-axis)
x and y	co-ordinate system with its origin at the centroid of the cross-section of the stiffener
z	co-ordinate for the third dimension; extends along the centroidal axis of the cross-section
x_0 and y_0	location of point 0 in the co-ordinate system
$\bar{\lambda}$	relative slenderness ratio
λ_s	specific slenderness ratio
ν	Poisson ratio
σ_y	yield stress
$\sigma_{e.t.b.}$	Euler torsional buckling stress for the stiffener
$\sigma_{t.b.}$	torsional buckling stress for the stiffener (i.e., the reduced Euler torsional buckling stress)

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