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**Contents**

# DURABILITY OF BUILDINGS: A RELIABILITY ANALYSIS

*A. J. M. Siemes, A. C. W. M. Vrouwenvelder, A. van den Beukel*
Institute TNO for Building Materials and Building Structures
P.O. Box 49, 2600 AA Delft, The Netherlands

**Abstract**

The scatter in observed service lifes of building structures is very high. Designing an optimal structure in relation to durability is therefore not an easy task. This paper investigates whether reliability analysis can be used to solve the problem. Essentially the same techniques are used as those which have proven to be successful as a design tool for ultimate and serviceability limit states without deterioration effects. Attention is given to the mathematical modelling of deterioration mechanisms, the probabilistic modelling of uncertainties and scatter and the economic consequences of design, maintenance and damage. As an example a reinforced concrete slab for an outdoor gallery of an apartment building is analysed for the limit state of corrosion of the reinforcement.

# Durability of buildings: a reliability analysis

## 1 Introduction

In principle, a building has a finite service life. After a number of years a building is judged to be obsolete and is then demolished or radically altered (renovation). Aging and deterioration of component parts of a building may also occur, leading to their repair or replacement. Deterioration may relate to various aspects such as aesthetic appearance, comfort, serviceability and safety of structural members, finishes and fittings, installations and services (piping, wiring, etc.).

In new building construction, but also in the case of conversion or repair, it is important to have a proper conception of the various deteriorative aging processes already in the design stage. In this way some idea of the durability of the building is obtained, i.e., its ability to function satisfactorily for a certain length of time. These processes are associated with external influences, the mechanisms through which deterioration is brought about, and the rate at which deterioration proceeds. This rate can never be accurately predicted. It involves stochastic processes, so that it is necessary to take account both of the average service life and of the scatter with respect to this average. In this context the concept of technical service life is applied, as distinct from other types of service life such as the economic or the social service life.

Besides having insight into the deteriorative aging processes, it is also necessary to know how the durability of different parts or components should be interadjusted. This knowledge, plus information on construction cost and on maintenance and operating expenses, will provide elements for the solution of optimization problems arising from the need to build economically.

These matters show a strong similarity [1] with another set of problems associated with building construction, namely, the safety of buildings and other structures. In this context, safety denotes: the capacity of a structure (or part thereof) to resist, with a sufficient degree of certainty, the occurrence of failure in consequence of various potential hazards to which the structure is exposed. In that case, too, the problem involves a number of structural members and various loads, with different ways in which the structure as a whole (or its parts) will respond to the loads. The loads, responses and strength are likewise stochastic quantities. The safety criteria for the various members and for the various loads must be duly interadjusted. The fundamental approach adopted for the purpose comprises, among other principles, the optimization of the cost associated with achieving a particular degree of safety.

In order to apply a systematic and rational approach to assessing the safety of a structure, risk and reliability analyses are employed. It has been found that a similar methodology can advantageously be applied to the assessment of the serviceability of structures. In that case serviceability denotes the extent to which a structure satisfies the requirements associated with the normal utilization of it. The requirements may, for example, relate to deflections of structural members or to aesthetic features.

Experience gained in connection with this has highlighted a number of advantages. One of the features of the method is that it leads to a sensitivity analysis, i.e., the most relevant parameters relating to the scatter in a problem are quickly distinguished. For this purpose it is not even particularly important to operate with very accurate input data. The principal parameters can be determined even when comparatively poor available data are used. Attention can subsequently be focused in these in order to improve the accuracy of the analysis.

Another advantage is obtained in connection with the harmonization of codes of practice. Both at national and international level this is simplified by the consistent and rational approach adopted.

Applications of reliability analysis have hitherto been confined more particularly to problems in which time plays only a subordinate part. The use of this technique is, however, now increasingly advocated for dealing also with durability and service life problems [2].

## 2 Reliability analysis of building structures

### 2.1 *Elements of reliability analysis*

The risk and reliability analysis of structures centres upon three key words: hazard, mechanism and effect.

Hazards may be classified into those which are of a mechanical, a physical, a chemical or a biological nature, as exemplified respectively by forces, temperature, acids, fungi, etc. The mechanisms indicate how the structure responds to the hazards. By effect is understood the loss of function that ensues or the emergence of a fresh hazard.

The first step in carrying out the reliability analysis consists in listing the hazards and mechanisms. This is usually done in the form of a "Failure Mode and Effect Analysis" (FMEA). For this purpose the hazards, mechanisms and effects are systematically assembled in a table, as exemplified in principle in Table 1. Possible counter-measures may also be included.

After the particulars have thus been listed, the first (fairly rough) quantification is applied: which hazards are important enough to warrant further analysis and which are not? So long as a particular hazard is of infrequent occurrence, or a structural member is rather insentitive to it, or the effects associated with that hazard are minor ones, no

Table 1. Example of a Failure Mode and Effect Analysis (FMEA) with respect to the durability of concrete

| hazard | mechanism | effect |
|---|---|---|
| 1 alternating load | fatigue | cracking, failure |
| 2 flowing water | erosion | surface deterioration |
| 3 frost | expansion | cracking |
| 4 carbon dioxide | carbonation | corrosion of reinforcement |
| 5 chloride | depassivation | pitting corrosion |

further analysis need to be carried out. The hazard under consideration may also be accepted if alternative construction methods or protective measures are uneconomical. Mechanisms of greater complexity, in which several hazards play a part and/or the effect of a mechanism in turn constitutes a hazard for another, can be represented with the aid of fault trees and event trees (Fig. 1).

The next step will in general have to consist in applying a selection in the multiplicity of hazards, circumstances and mechanisms. The basis for this quantitative selection is sought in the magnitude of the probability of failure and the extent of the damage. The combination of probability and damage, expressed as the mathematical product of the two, is called the risk.

The final step in the reliability analysis is: quantification. This will involve assigning values to entities, indicating scatters, designating probabilities and performing calculations. In the following treatment of the subject this quantitative part of the analysis will be considered in greater detail.

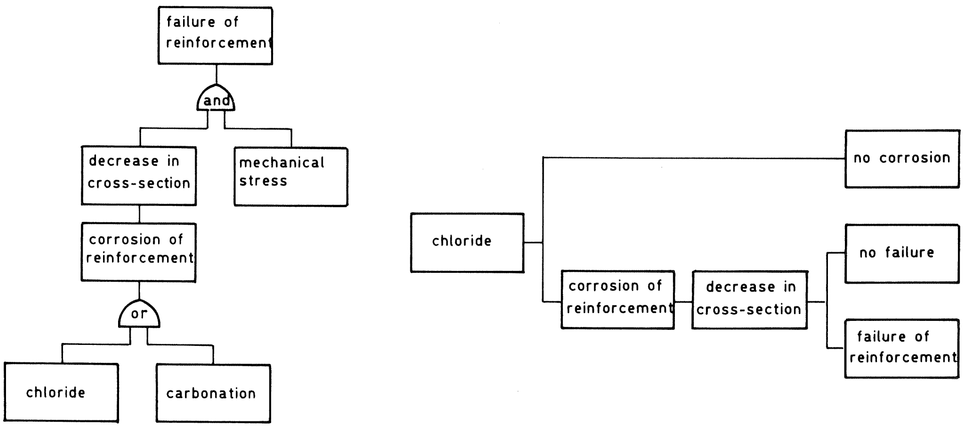


Fig. 1. Fault tree (a) and event tree (b) relating to the corrosion mechanism.

## 2.2 *Calculation of failure probability*

In connection with reliability analysis the failure of loadbearing structures has so far more particularly been in the focus of attention. The aim is to have a low probability of failure. In such cases the planned end of the service life is linked to the point of time when the failure probability becomes unacceptably high. The actual failure of the structure will in general occur a good deal later in time than this, so that the service life aspect is usually of secondary importance.

With reference to functional requirements and aesthetic aspects the planned service life will be much more closely linked with the moment in time when the structure, in its original form, ceases to exist. By means of maintenance, replacement, renovation and other such measures an extension period can be added to the service life. The durability aspect is therefore emphatically associated with this.

In reliability analysis within the context of durability the theory of probability, on the one hand, and materials technology, on the other, play a special part. This is so because reliability analysis starts from the description of the failure of a structure, i.e., the description of a constitutional change. Knowledge of the time-dependent behaviour of the relevant materials under the influence of the external circumstances and the uncertainties therein plays an important part in connection with this. For building structures this knowledge must relate to:

- mechanical processes such a failure, deformation, wear, erosion and weathering;
- physical processes such as temperature and moisture variations (causing deformation), absorption and emission;
- chemical processes such as corrosion and attack;
- biological processes such as fungal growth and rotting.

On the basis of this it is, for the relevant hazards to which structures are exposed, essential to gain insight into the service life distribution and thus into the probability of failure within the lifetime envisaged.

In practice there are three different methods of determining the probability of failure:

- on the basis of experience (statistical data) the failure probability or the service life distribution is known: this situation is encountered, for example, in the case of fire and window-pane breakage;
- on the basis of a probabilistic calculation of the distribution of the service life $t_L$;
- on the basis of a probabilistic calculation of the failure probability with the aid of a reliability function.

If the failure probability is determined on the basis of statistical information, it is not absolutely necessary to have exact data at one's disposal. Quite often it will suffice to make a reasonable estimate of the mean value and the standard deviation of the service life. The desired accuracy will of course depend on the contribution of the risk to the overall cost.

In many cases it is possible to express the service life $t_L$ explicitly in a number of stochastic quantities $X_i$:

$$t_L = f(X_1, X_2, \dots X_n) \tag{1}$$

The mean $\mu(t_L)$ and the standard deviation $\sigma(t_L)$ can be calculated with the aid of the known probabilistic methods such as the "mean value" approximation and the "advanced" approximation [1]. The form of the distribution is not calculated in this way. Quite often, for determining the type of distribution it will suffice to make a reasonable assumption, e.g., a log-normal distribution (this means that $\log t_L$ is normally distributed).

The direct calculation of the failure probability ties up most directly with the probabilistic approach to safety. The simplest mathematical model for describing the event "failure" comprises a load parameter $S$ and a resistance (loadbearing capacity) parameter $R$.

If $R$ and $S$ are independent of time, then failure is expressed by:

$$\{F_i\} = \{\text{failure}\} = \{R < S\} \tag{2}$$

The failure probability $P\{F_i\}$ then follows [1] from the convolution integral:

$$P\{F_i\} = \int_0^\infty F_R(s) \cdot f_S(s)\, \mathrm{d}s \tag{3}$$

where:

$F_R(s)$ = cumulative distribution of $R$
$f_S(s)$ = probability density function of $S$

In many cases the exact mathematical determination of $P\{F_i\}$ is too time-consuming, so that in such cases approximation methods such as the mentioned "mean value" and the "advanced" approximation are also applied.

In durability problems, $R$ and/or $S$ are time-dependent (see Fig. 2). Failure occurs if

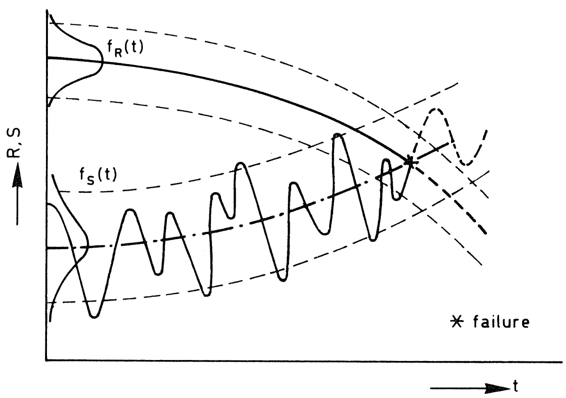


Fig. 2. *R* and *S* both time-dependent.

for at least one value of the time $\tau$ in the period $(0-t)$ under consideration the resistance $R(\tau)$ is less than the load $S(\tau)$ at that instant:

$$P\{\text{failure in } (0-t)\} = P\{R(\tau) < S(\tau) \text{ for at least one } \tau \text{ in } (0,t)\} \tag{4}$$

Since the event {failure in $(0-t)$} is identical with the event $\{t_L < t\}$, equation (4) describes the distribution function $F_{t_L}(t)$ (see Fig. 3) for the service life $t_L$. By differen-

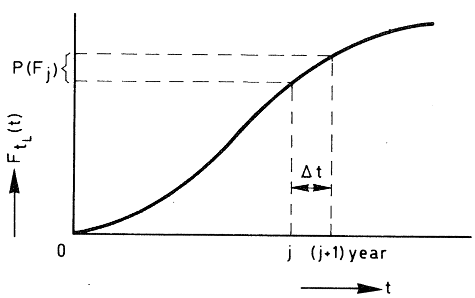


Fig. 3. Distribution of $t_L$.

tiating this function once with respect to $t$ we obtain the probability density function $f_{t_L}(t)$, which is represented in Fig. 4.

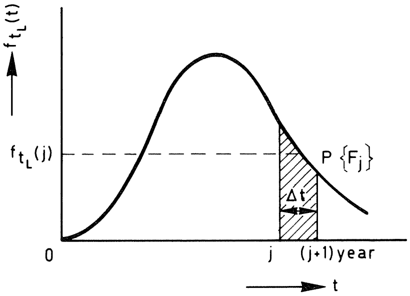


Fig. 4. Probability density of $t_L$.

## 3 Economic considerations

### 3.1 *General*

Building a structure general involves considerable capital expenditure. The interest and writing-off for depreciation associated with this, as well as insurance, are assignable to the fixed operating costs. These moreover comprise:
- maintenance expenses, e.g., in respect of cleaning and technical maintenance (including repair and replacement);
- energy expenses;
- administration expenses;
- specific running expenses, e.g., caretaker staff and security.

These costs are largely determined by the design and manner of construction of the structure. Therefore they should be duly taken into account already in the decision-making stage relating to design and construction. If correctly applied, higher investment expenditure can result in longer service life and a reduction of maintenance, so that the overall operating costs can be reduced.

As the depreciation period for a building structure can in general be very long (average order of magnitude: fifty years), the average annual amount to be written off is relatively low.

In estimating the service life a number of rather complex problems are encountered – in addition to the fact that the service life is a stochastic quantity – because:
- the various parts of a building may have different service lives; if a particular part or member performs several functions, each function may be associated with a service life peculiar to that function;
- the various members exhibit interactions; replacement of members sometimes necessarily involves replacement of other members if they are not sufficiently demountable;

- building codes and regulations cater only in part for the functional requirements of building structures; besides, to indicate in the design stage for how long it will be possible to satisfy these requirements is a complex problem;
- the maintenance, the workmanship (quality of execution) and the care bestowed on detailing (in elaborating the design) also play a part in determining the service life; these factors are, however, difficult to assess in the design stage.

In present-day building practice little, if any, account is taken of the fact that service life is linked to probability. Furthermore, there are hardly any quantitative (numerical) records relating to service life. For example, even for a notably durable material like concrete the technical literature dealing with durability [3] gives no figures at all for its service life. And if information concerning service life is indeed available, it is often not clear what probability is associated with the life stated.

### 3.2 *Principles of cost analysis*

The cost analysis for a building may be based on annual cost or on capitalized cost. In the former case the investment cost of the building is translated into an annual amount comprising interest and redemption. In the latter case, i.e., involving capitalization of cost, every future expenditure is converted back to an amount which can be added to the direct investment. In principle, both approaches lead to the same conclusions.

In particular cases the termination of the service life of a part, member or component of a structure is determined solely by its ability to perform its function: as soon as it fails to satisfy the functional requirements (and assuming that repair is too expensive), it is replaced (window-pane, electric bulb, motor). External factors may also determine the service life, however. For example, an offshore drilling platform need not have a service life of more than 30 years because the oil well in question will be exhausted by the end of that period. All that is required of the structure and its members is that they will not give rise to problems within this service life. A similar approach may be adopted with regard to housing or industrial building construction, basing oneself on an intended service life of, say, 50 years. If a member of the structure fails in one way or another within that period, it means that damage will have occurred, necessitating replacement or repair.

In the present section concerning capitalized cost only the case of the intended "target" service life will be considered. For cases where a free service life or a service life of indefinite duration is envisaged the reader is referred to Appendix A. For many purposes it will suffice to conceive the anticipated capitalized cost as being composed of three terms: the direct investment (initial expenditure), the cost of maintenance and the risk (anticipated loss, being the cost to be incurred in remedying deterioration or damage). Both the maintenance cost and the loss due to deterioration or damage should be calculated for each year of the planned service life and then be capitalized and added:

$$E\{C_{cap}\} = S + \sum_{j=1}^{t_G} \frac{V_j}{(1+r')^j} + \sum_{j=1}^{t_G} \frac{P\{F_j\} \cdot D_j}{(1+r')^j} \tag{5}$$

where:

$E\{C_{cap}\}$ = expectation of the capitalized cost $C_{cap}$
$S$ = direct investment
$V_j$ = maintenance cost and administration expenses for year $j$
$P\{F_j\}$ = probability of failure in year $j$
$D_j$ = loss due to failure in year $j$
$r'$ = real rate of interest (nominal rate minus inflation)
$t_G$ = target service life (in years)

The probability of failure $P\{F_j\}$ in year $j$ follows from the probability distribution (see Figs. 3 or 4) for the service life $t_L$ (in years)

$$P\{F_j\} = P\{(j-1) < t_L < j\} = F_{t_L}(j) - F_{t_L}(j-1) \tag{6}$$

Alternatively we can start from the probability density function $f_{t_L}(t)$ as follows:

$$P\{F_j\} = [f_{t_L}\{j\} - f_{t_L}\{(j-1)\}] \cdot 1 \text{ year} \tag{7}$$

$$P\{F_j\} = f_{t_L}(t) \cdot \Delta t \tag{8}$$

where

$t = j$ years
$\Delta t = 1$ year

The real interest rate, i.e., the nominal interest rate corrected for inflation, is adopted in equation (5). In determining $D_j$ and $V_j$ it is therefore not necessary to take account of a rise in costs due to inflation. The term $(1 + r')^j$ indicates that the value $V_j$ and $D_j$ in year $j$ correspond to $V_j/(1+r')^j$ and $D_j/(1+r')^j$ in the initial year.

*Exemple*

Suppose that the annual cost of maintenance $V_j$ is 1.2% of the direct investment $S$, that the probability of loss (due to damage or deterioration) is $10^{-4}$ for each year and that the loss $D_j$ is equal to $2S$. Such a constant probability of loss occurs in the case of fire, for example. For working out the calculation the sum of the following geometric progression is used:

$$\sum_{i=1}^{t_G} \frac{1}{(1+r)^i} = \frac{1}{r}\left\{1 - \left(\frac{1}{1+r}\right)^{t_G}\right\} \tag{9}$$

For the present example:

$$E\{C_{cap}\} = S + \frac{0.012S}{r'}\left\{1 - \left[\frac{1}{1+r'}\right]^{t_G}\right\} + \frac{0.001 \cdot 2S}{r'}\left\{1 - \left[\frac{1}{1+r'}\right]^{t_G}\right\}$$

With $r' = 0.02$ and $t_G = 50$ this expression yields:

$$E\{C_{\text{cap}}\} = S + 0.38S + 0.06S = 1.44S$$

It thus appears that, in this example, maintenance and risk amount for 44% of the direct investment.

### 3.3 *Cost calculation on annual basis*

First the case will be considered where there is only direct investment and where every year a certain constant value $X$ (annuity) is paid throughout the entire service life of $t_G$ years. The amount $X$ paid in year $j$ corresponds to a direct redemption equal to $X/(1 + r')^j$. Since the total redemption must be equal to $S$, it is thus possible to calculate $X$ from:

$$S = \sum_{j=1}^{t_G} \frac{1}{(1 + r')^j} = \frac{X}{r'} \left\{ 1 - \frac{1}{(1 + r')^{t_G}} \right\} \tag{10}$$

or:

$$X = S \cdot r' \left\{ \frac{(1 + r')^{t_G}}{(1 + r')^{t_G} - 1} \right\} \tag{11}$$

Example: If $r' = 0.02$ and the intended service life $t_G = 50$, then it follows that $X = 0.0318S$ (if no interest were payable, then $X = S/50 = 0.02S$).

If $r'$ in equation (10) denotes the nominal interest, this represents a normal annuity (i.e., nominally the same amount every year). However, for the purpose of the present report it is preferable to work with the real interest because then the annual increase of an amount keeps pace with the price level.

The cost of maintenance and risk can, if they are the same for each year, be directly added to $X$. But if these costs are not constant, they must be capitalized and added to $S$, after which the value of $X$ can be determined with the aid of equation (11). For the optimization problems considered in this report the capitalized cost is therefore often a more suitable criterion.

In the foregoing, the real interest has always been introduced and cost increases due to inflation have been left out of account. The drawback of this procedure is that the calculated amounts of money do not directly correspond to the amounts actually payable.

### 3.4 *Examples*

By way of illustration some optimization problems will be dealt with in this section. The following will successively be considered:

a. a simple carbonation problem in which, for a given service life, the optimum depth of concrete cover is determined;
b. a problem in which the service life of a wearing course is optimized;
c. analytical treatment of b;

d. similar to problem b, but taking account of the stochastic character of the service life;
e. a system with two elements liable to fail.

### 3.4.1 Carbonation of concrete

Carbonation refers to the binding of the free lime in concrete by carbon dioxide from the atmosphere, as a result of which the pH of the concrete is lowered. If the concrete thus ceases to have sufficient alkalinity, its protective effect is lost and the reinforcing steel may corrode, which can be regarded as constituting the end of the service life.

In Chapter 4 the following approximation for the progress of carbonation is given:

$$d^2 = a \cdot t \tag{12}$$

where

$d$ = the depth of carbonation
$a$ = a constant
$t$ = the exposure time

The service life $t_L$ is attained when the carbonation depth $d$ becomes equal to the depth of concrete cover $c$:

$$t_L = c^2/a \tag{13}$$

For $a = 10$ mm$^2$/year the service life $t_L$ of the concrete is given as a function of $c$ in Fig. 5. It appears from this diagram that 24 mm cover corresponds to a service life of 60 years. Conversely, to obtain a service life of 60 years it will be necessary to provide 24 mm cover. If the cost of the cover is f 0.40 per m$^2$/mm (which is a reasonable figure for, say, a gallery slab), the required cover will cost:

$$C_{cap} = 24 \times 0.4 = \text{f}\,9.60 \text{ per m}^2$$

It is, however, not very realistic to conceive the service life as a deterministic quantity, for the carbonation does not proceed everywhere at the same rate, and the depth of cover itself will never exactly conform to the nominal value indicated on the drawing.

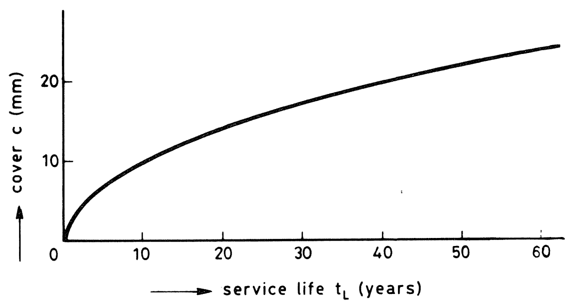


Fig. 5. Service life $t_L$ as a function of the depth of cover $c$.

Assuming that Fig. 5 represents the mean service life and that the actual values display a scatter in relation to this with a coefficient of variation of 33%, we obtain Fig. 6.

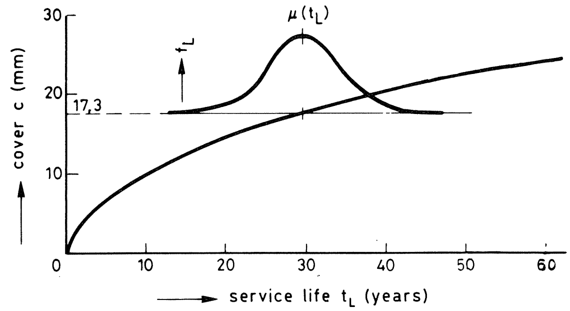


Fig. 6. Service life of the cover as a stochastic quantity.

Now, for a given depth of concrete cover, it is possible to calculate with what probability the entire cover will become carbonated within a certain period. The total capitalized cost is then, according to equation (5):

$$E\{C_{\mathrm{cap}}\} = S + \sum_{j=1}^{t_G} \frac{V_j}{(1+r')^j} + \sum_{j=1}^{t_G} \frac{P\{F_j\} \cdot D_j}{(1+r')^j} \qquad (5)$$

With $S = 0.4c$, $V_j = 0$, $D_j = D$ and an intended service life $t_G = 60$ years this reduces to:

$$E\{C_{\mathrm{cap}}\} = 0.4c + D \cdot \sum_{j=1}^{60} \frac{P\{F_j\}}{(1+r')^j} \qquad (14)$$

Because $P\{F_j\}$ is not constant, the summation is difficult to carry out. Therefore the following approximation can suitably be adopted for equation (14):

$$E\{C_{\mathrm{cap}}\} = 0.4c + \frac{D}{(1+r')^{60}} \sum_{j=1}^{60} P\{F_j\} \qquad (15)$$

What this approximation comes down to is that, technically from the financing point of view, in each of the 60 years failure is considered to occur in the 60th year. The financial loss is thereby somewhat underestimated.

The sum of the probabilities $P\{F_i\}$ for the years 1 to 60 is equal to the probability that the service life is less than 60 years, so that:

$$E\{C_{\mathrm{cap}}\} = 0.4c + \frac{D}{(1+r')^{60}} \cdot P\{t_L < 60\} \qquad (16)$$

If a normal distribution is adopted for $t_L$, the probability that $t_L$ is less than 60 years can be determined from:

$$P\{t_L < 60\} = \Phi_N(-\beta) \qquad (17)$$

where:

$$\beta \;=\frac{\mu(t_L)-60}{\sigma(t_L)}$$

$\Phi_N$ = standard normal distribution ($\mu = 0$ and $\sigma = 1$)

For example, putting $c = 24$ mm (as found earlier on), we obtain $\mu(t_L) = 60$ years, so that $\beta = 0$. The probability of failure is then 50%. For loss amounting to f 50.-/m$^2$ and a real interest rate $r' = 0.02$ it follows that:

$$E\{C_{cap}\} = 9.60 + \frac{50}{1.02^{60}} \cdot 0.50 = 9.60 + 7.60 = \text{f } 17.20 \text{ per m}^2$$

If a 30 mm depth of concrete cover is chosen, the following results are obtained:

$$\mu(t_L) = \frac{c^2}{a} = \frac{900}{10} = 90 \text{ years}$$

$$\sigma(t_L) = 0.33 \cdot 90 = 30 \text{ years}$$

$$\beta \;= \frac{\mu(t_L)-60}{\sigma(t_L)} = \frac{90-60}{30} = 1$$

$$P\{t_L < 60 \text{ years}\} = \Phi_N(-1) = 0.16 \qquad \text{(see [20])}$$

$$E\{C_{cap}\} = 0.4 \cdot 30 + \frac{50}{1.02^{60}} \cdot 0.16 = 12 + 2.4 = \text{f } 14.40 \text{ per m}^2$$

Increasing the cover results in an increase in direct cost and a reduction of the risk, while the total cost expectation decreases. The optimum cover can be found by repeating this calculation for a number of values of $c$, as indicated in Fig. 7. The optimum is obtained for $c = 32$ mm, the failure probability then being 10%, in the present example.

This example will be further considered in Chapter 5, where a more detailed reliability analysis will be given. It will also be investigated whether protective painting may offer a more economical alternative to increasing the concrete cover.

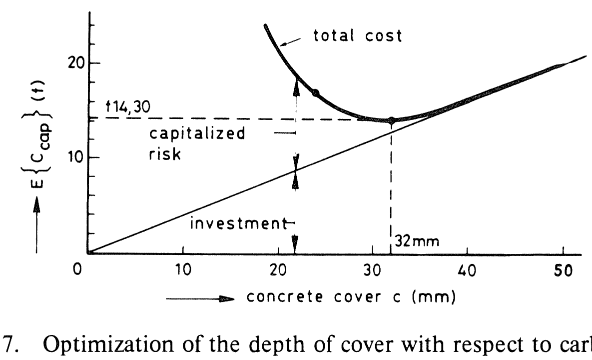


Fig. 7. Optimization of the depth of cover with respect to carbonation.

### 3.4.2 A wear problem

The problem for analysis relates to a flooring tile provided with a wearing course of thickness $d_0$ (Fig. 8). The tile will have to be replaced by a new one as soon as the wearing course has worn away.

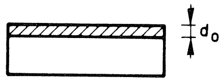


Fig. 8. Tile with wearing course of thickness $d_0$.

The problem consists in so determining this thickness $d_0$ that the total cost is a minimum. For a small value of $d_0$ the cost of manufacture is low, but early replacement of the tile will be necessary. For a greater thickness $d_0$ the tile will cost more to manufacture, but it will have a longer service life.

The reliability function for this case is:

$$Z = d_0 - v \cdot t \tag{18}$$

where

$d_0$ = the design thickness
$v$ = the rate of wear
$t$ = the time

The service life $t_L$ of the tile is reached when $Z = 0$:

$$t_L = d_0 / v \tag{19}$$

Suppose that the cost of manufacturing a tile can be split up into a fixed portion and portion that increases linearly with $d_0$ (see als Fig. 9):

$$S = A + B \cdot d_0 \tag{20}$$

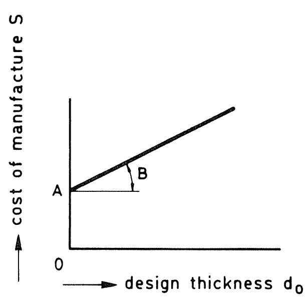


Fig. 9. Cost of manufacture as a function of $d_0$.

To begin with, the the rate of wear $v$ is assumed to be deterministic. With $t_L = d_0/v$ and $S = A + B \cdot d_0$ we obtain the following expression for the cost $X$ on an annual basis:

$$X = Sr'\left\{\frac{(1+r')^{t_G}}{(1+r')^{t_G}-1}\right\} = (A+B\cdot d_0)\cdot r'\cdot\left\{1+\frac{1}{(1+r')^{d_0/v}-1}\right\} \tag{21}$$

Note: This problem, in which the service life is determined by the structure itself, can alternatively be solved by capitalizing the cost. The formula for $C_{cap}$ differs only by a factor $r'$ from $X$ (see Appendix A).

In Fig. 10 the mean annual cost $X$ has been plotted as a function of the design parameter $d_0$. The solid curve is based on:

$A = 1.0$ cost unit (cu)
$B = 0.1$ cu/mm
$v = 0.5$ mm/year
$r' = 0.02$/year (real interest rate)

The minimum for $X$ is obtained when $d_0 = 19.5$ mm and is equal to 0.11 cost unit per year. The optimum service life is then $t_L = d_0/v = 19.5/0.5 = 39$ years. For this case the curve in Fig. 10 slopes only so slightly that there is little objection to choosing a somewhat larger or smaller design thickness $d_0$ (or a longer or shorter service life). For example, if $d_0 = 10$ mm is adopted, the service life will be $t_L = 10/0.5 = 20$ years, and the mean annual cost will be $X = 0.12$ cu/year, which is only a little above the optimum.

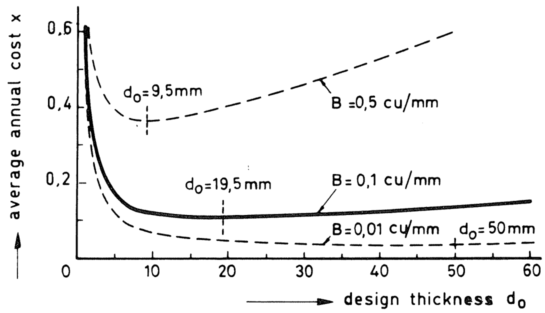


Fig. 10. Average annual cost $x$ as a function of the design thickness $d_0$ for the case: $A = 1.0$ cu, $B =$ variable, $v = 0.5$ mm/year and $r' = 0.02$.

If the variable cost $B$ is very low in relation to the fixed cost $A$, e.g., $B = 0.01$ cu/mm, the optimum design thickness can be expected to increase. In that case (as appears from Fig. 10): $d_0 = 50$ mm and $t_L = 100$ years.

If the variable cost $B$ is high in relation to the fixed cost $A$, the importance of a "sharply selective" choice of the design thickness becomes greater. In Fig. 10 this is illustrated for $B = 0.5$ cu/mm. The optimum design thickness is then $d_0 = 9.5$ mm, while the service life is $t_L = 19$ years and the mean annual cost $X = 0.37$ cu/year.

### 3.4.3 Analytical treatment

The equation (21) for the mean annual cost contains the term $(1+r')^{d_0/v}$. For small values of $r'$ we may write as an approximation:

$$(1+r')^{t_L} = e^{r' t_L}$$

Then equation (21) becomes:

$$X = (A + B \cdot t \cdot v) \cdot r' \cdot \left\{1 + \frac{1}{e^{r' \cdot t} - 1}\right\} \qquad (22)$$

The optimum service life is attained when $dX/dt = 0$:

$$\frac{dX}{dt} = B \cdot v \cdot r' \left\{1 + \frac{1}{e^{r' \cdot t} - 1}\right\} - (A + B \cdot t \cdot v) \cdot \frac{r'^2 \cdot e^{r' \cdot t}}{(e^{r' \cdot t} - 1)^2} = 0$$

or:

$$B \cdot v(e^{r' \cdot t} - 1)e^{r' \cdot t} = (A + B \cdot t \cdot v) \cdot r' e^{r' \cdot t}$$
$$B \cdot v(e^{r' \cdot t} - 1) = (A + B \cdot t \cdot v) \cdot r'$$

Expansion in a series gives:

$$e^{r' \cdot t_L} = 1 + r' \cdot t + \tfrac{1}{2}(r' \cdot t)^2 + \ldots$$

whence we obtain:

$$B \cdot v[r' \cdot t + \tfrac{1}{2}(r' \cdot t)^2] = (A + B \cdot t_L \cdot v) \cdot r'$$
$$\tfrac{1}{2} B \cdot v(r' \cdot t)^2 = A \cdot r'$$

so that:

$$t_L = \frac{2A}{B \cdot v \cdot r'} \qquad (23)$$

For $A = 1.0$ cu, $B = 0.1$ cu/mm, $v = 0.5$ mm/year and $r' = 0.02$ (as in the previous example) we obtain from equation (23):

$$t_L = \sqrt{\frac{2 \cdot 1.0}{0.1 \cdot 0.5 \cdot 0.02}} = \sqrt{200} = 45 \text{ years}$$

In example b the optimum was found to be 39 years. For practical purposes the difference between the two results is negligible. Closer agreement will be obtained for smaller values of the product $r' \cdot t_L$. Thus, in the case where the variable cost is high, e.g., $B = 0.5$ cu/mm, equation (33) gives:

$$t_L = \sqrt{\frac{2 \cdot 1.0}{0.5 \cdot 0.5 \cdot 0.02}} = \sqrt{400} = 20 \text{ years}$$

This is in good agreement with the 19 years obtained earlier.

If the cost structure and the rate of wear are known, the optimum service life of the optimum design thickness can be calculated from equation (23).

### 3.4.4 Rate of wear as a stochastic quantity

In the foregoing treatment of the problem the rate of wear was assumed to be deterministic. Now what are the consequences if the rate is not accurately known because both the wear resistance and the load are stochastic? Suppose that the rate of wear is the same for every tile within one and the same consignment of tiles. This means that uncertainties with regard to average wear resistance and average load level will predominate in relation to change differences between individual tiles.

In those cases where the end of the service life is governed by a mechanism of cumulative damage the Weibull distribution is often applied. The distribution function and density function of the Weibull distribution for the stochastic service life $t_L$ are then:

$$F_{t_L}(t) = 1 - e^{-(t/u)^k} \quad (24)$$

$$f_{t_L}(t) = \frac{k}{u}\,(t/u)^{k-1} \cdot e^{-(t/u)^k} \quad (25)$$

Fig. 11 represents the density function for several values of $k$. The mean and the standard deviation of the Weibull distribution are obtained from:

$$\mu(t_L) = \mu\Gamma(1 + 1/k) \quad (26)$$

$$\mu^2(t_L) + \sigma^2(t_L) = u^2\Gamma(1 + 2/k) \quad (27)$$

where $\Gamma(\,)$ is the gamma function for which $\Gamma(z+1) = z!$ if $z$ is an integer. As a rough approximation we may adopt:

$$\mu(t_L) \approx \mu \quad \text{and} \quad V = \frac{\sigma(t_L)}{\mu(t_L)} \approx \frac{1}{k} \quad (28)$$

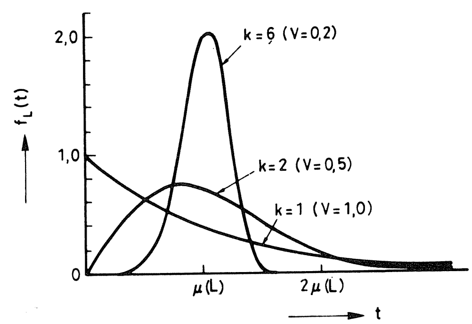


Fig. 11. Weibull distribution for various values of $k$.

Since the service life is stochastic, the average annual cost is also stochastic. Only when the service life has ended it is possible to calculate what the actual cost per year has been. The annual cost itself therefore cannot be optimized, but the expectation value $E\{X\}$ of that cost can:

$$E\{X\} = \sum_{j=1}^{\infty} X(j) \cdot P\{F_j\} \quad (29)$$

If the probability of failure (end of service life) is equal to $\frac{1}{3}$ for 9, 10 and 11 years, then $E\{X\}$ should be determined from:

$$E\{X\} = \tfrac{1}{3}\{X(9) + X(10) + X(11)\}$$

With the aid of equation (7) we can write equation (29) as:

$$E\{X\} = \sum_{j=1}^{\infty} X(j) \cdot f_{t_L} \cdot \Delta t \tag{30}$$

where $t = j$ years and $\Delta t = 1$ year.

Obviously, if there is a continuous distribution for the service life, equation (30) must be replaced by:

$$E\{X\} = \int_0^{\infty} X(t) \cdot f_{t_L}(t) \cdot \mathrm{d}t \tag{31}$$

For numerical evaluation the integral will of course have to discretized, so that we revert to equation (30).

The above presentation has been kept very succinct in order not to weary the reader with detailed mathematical derivations. Suffice it merely to give the result corresponding to:

$A \quad = 1$ cost unit (cu)
$B \quad = 0.1$ cu/mm
$r' \quad = 0.02$
$d \quad = 0.5$ mm/year
$V(t_L) = 0.33$

It now follows that $E\{X\}$ attains a minimum for $d_0 = 25$ mm instead of the previously determined value of 39 mm. The cost will then amount to 0.14 cu/year (instead of 0.11 cu/year). The behaviour of $E\{X\}$ as a function of $d_0$ is entirely comparable to Fig. 10 for the deterministic case.

### 3.4.5 A system with two members liable to fail

Consider a system comprising two members and suppose that the whole system must be replaced as soon as one of the members fails. Hence we here have a serial system. The direct investment for the system is expressed by:

$$S = (A_1 + B_1 \cdot d_1) + (A_2 + B_2 \cdot d_2) \tag{32}$$

where $d_1$ and $d_2$ are design parameters of the members 1 and 2. (In this context it is not necessary to conceive these solely in terms of the thickness of wearing courses). The service lives of the members are expressed by:

$$t_1 = d_1/v_1 \quad \text{and} \quad t_2 = d_2/v_2 \tag{33}$$

The mean annual cost can thus be written as:

$$X=(A_1+A_2+B_1\cdot d_1+B_2\cdot d_2)\cdot r'\cdot\left\{1+\frac{1}{(1+r')^{t_L}-1}\right\} \tag{34}$$

where $t_L=\min(t_1,t_2)$.

So long as the service lives of the members can be accurately calculated, i.e., when a deterministic approach is adopted, the problem is simple to solve. In that case there is in fact no advantage in trying to obtain a longer service life for one member as compared with the other. Therefore it is appropriate to adopt $t_1=t_2=t_L$, so that equation (34) can be written as:

$$X=(A_1+A_2+B_1\cdot v_1\cdot t_1+B_2\cdot v_2\cdot d_2)\cdot r'\cdot\left\{1+\frac{1}{(1+r')^{t_L}-1}\right\} \tag{35}$$

Then, in a manner similar to that in Section 3.4.3, the optimum service life is obtained:

$$t=\sqrt{\frac{2(A_1+B_2)}{(B_1\cdot v_1+B_2\cdot v_2)\cdot r'}} \tag{36}$$

If scatter has to be taken into account in the determination of the service life, the problem becomes more interesting. It is a well known design principle that, if one member is relatively expensive and the other relatively cheap, it must be ensured that replacement will not be governed by the cheaper member. This principle can be suitable demonstrated with the aid of the model developed here and be further quantified.

Starting from equation (31), the following would have to be minimized:

$$E\{X\}=\int_0^\infty X(t)\cdot f_{t_L}(t)\cdot dt \tag{37}$$

In determining the density function for $t_L$ it must be taken into account that $t_L$ is the minimum of two service lives $L_1$ and $L_2$. Suppose that $L_1$ and $L_2$ conform to a Weibull distribution. Then the density function of $t_L$ has still to be determined. For this purpose the complement of the distribution function is first calculated, i.e., the probability that $t_L$ is greater than a value $t$. The service life $t_L$ can exceed a value $t$ only if both $L_1>t$ and $L_2>t$. Assuming $L_1$ and $L_2$ to be independent, we have:

$$P\{t_L<t\}\cdot P\{L_1>t \quad \text{and} \quad L_2>t\}=P\{L_1>t\}\cdot P\{L_2>t\} \tag{38}$$

On the basis of the definition of a distribution function it then follows that:

$$1-F_{t_L}(t)=\{1-F_{L_1}(t)\}\{1-F_{L_2}(t)\} \tag{39}$$

or:

$$F_{t_L}(t)=F_{L_1}(t)+F_{L_2}(t)-F_{L_1}(t)\cdot F_{L_2}(t) \tag{40}$$

The density function is obtained by once differentiating equation (40):

$$f_{t_L}(t)=f_{L_1}(t)+f_{L_2}(t)-\{F_{L_1}(t)\cdot f_{L_2}(t)+F_{L_2}(t)\cdot f_{L_1}(t) \tag{41}$$

Substitution of this expression into (31) gives:

$$E\{X\} = \int_0^\infty X(t) \cdot \{[1 - F_{L_1}(t)] f_{L_2}(t) + [1 - F_{L_2}(t)] \cdot f_{L_1}(t)\} \, dt \tag{42}$$

Evaluation of equation (42) is almost impracticable except with the aid of a computer. The results for a particular example are presented in Fig. 12. The data for this example were:

$A_1 + A_2 \quad = 1$ cu
$B_1 \quad = 0.1$ cu/mm
$B_2 \quad = 0.01$ cu/mm
$r' \quad = 0.02$ per year
$v_1 = v_2 \quad = 1$ mm/year
$V(t_1) = V(t_2) = 0.33$

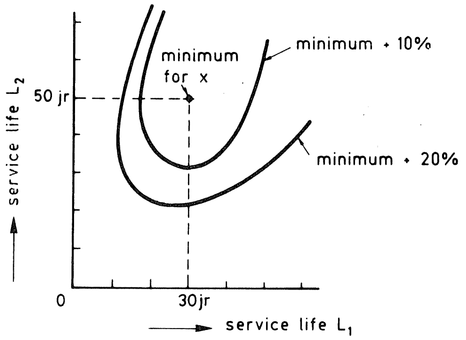


Fig. 12. Optimum service life of two members of a serial system.

The difference between the members therefore that for member 1 the variable cost per unit of additional service life is 10 times as high as for member 2.

With the deterministic approach the optimum service life would be expressed by:

$$t_L = \sqrt{\frac{2(A_1 + A_2)}{(B_1 \cdot d_1 + B_2 \cdot d_2) \cdot r'}} = \sqrt{\frac{2}{(0.1 + 0.02) \cdot 0.02}} = 30 \text{ years}$$

As appears from Fig. 12, the optimum design service life for member 1 is indeed 30 years, but it is advisable to adopt a design service life of 50 years for member 2.

The cost associated with $L_1 = L_2 = 30$ years is approximately 10% higher. Of course, greater differences than this may occur when different ratios of $B_1$ and $B_2$ are considered.

## 4 Failure mechanisms for reinforced concrete

### 4.1 *General*

In order to make quantified statements as to the service life of a material, a member or a structure it is necessary to know the factors that affect the service life. Accordingly, this chapter takes stock of hazards, mechanisms and effects relating to service life. Its purpose is illustrative and, because of the necessity to limit the scope of the investigation, it relates only to reinforced concrete.

If the hazards, mechanisms and effects are identified and known, it becomes possible to estimate the service life by calculation. First, the safety and serviceability aspects will be briefly considered, without explicitly paying attention to service life.

Next, the durability aspect will be examined, with reference to a summary of the failure mechanisms described in the literature. Finally, at the end of this chapter, a table of hazards, mechanisms and effects for reinforced concrete is presented in the form of a Failure Mode and Effect Analysis (FMEA). It has been endeavoured to present a summarizing, but nevertheless as complete as possible, review of the subject.

## 4.2 *Limit states relating to safety and serviceability*

Mechanical influences in the form of forces and, to a lesser extent, imposed deformations play an important part in connection with the design and dimensioning of reinforced concrete structures. The principal types of load to be taken into account are:
- the dead weight of the structure;
- the live load on floors, stairs, landings, galleries, etc.;
- the snow load;
- the wind load.

The imposed deformations are in general associated with shrinkage of the concrete and with changes in length due to temperature variations.

The concrete structure is considered to fail when the load to be taken into account exceeds the ultimate load capacity of the structure. This relates to the limit state with respect to failure. The structure also fails when the deflection or the crack width becomes too great, this being taken to constitute the limit state with respect to serviceability.

In these considerations the service life of the structure plays only a subordinate part. The maximum value of the load which is taken into account is - as a rule: implicitly - related to a particular reference period. A period of 50 years is often adopted for this. This does not mean to say that the service life will then also be 50 years. It is only shown that the probability of failure, excessive deflection or cracking is sufficiently low during this period.

Check calculations for the crack width in reinforced concrete structures are performed from aesthetic and durability considerations. At a crack the (aggressive) environment has direct access to the reinforcement. This may give rise to corrosion, but this possibility can be limited by imposing a limit upon the maximum crack width.

## 4.3 *Durability aspects*

*Fatigue*

When concrete, reinforcing steel or the bond zone is subjected to a large number of stress variation cycles, fatigue failure may occur in course of time [4], i.e., failure then occurs at a level of stress which is considerably lower than the failure stress for static loading (when the load is increased in a single operation up to failure).

Fatigue in concrete as well as in reinforcing steel is a process of gradually progressing internal cracking. Via the number of stress cycles a direct relation with the service life is established. In this respect fatigue is indeed a durability problem.

In general, Miner's rule is adopted as the mathematical model for fatigue. This rule is based on the service life $N_i$, being the number of cycles up to fatigue failure, for a stress with a constant amplitude and a constant average value. At the instant of failure due to $n$ non-constant stress cycles, it applies:

$$M_S = \sum_{i=1}^{\infty} \frac{1}{N_i} = M_M \tag{43}$$

In this formula $M_S$ is the (hypothetical) damage caused by the stress cycles, while $M_M$ is the so-called Miner number, which constitutes a measure of the resistance that the material offers to cyclic loading. The structure fails when $M_S$ becomes equal to $M_M$. Both $M_S$ and $M_M$ are stochastic quantities, $M_S$ being primarily related to the (stochastic) loading and $M_M$ to the (stochastic) strength of the material. There is a reasonable measure of insight into both of them in relevant cases.

*Erosion*

Erosion is a process in which the surface of the concrete is worn away by mechanical influences such as those due, among others, to sliding, scraping or knocking [3]. These may occur in air or under water [5].

The following erosion mechanisms can be distinguished:

a. wear due to mechanical influences upon concrete surfaces exposed to the air;
b. wear due to mechanical influences caused by solid particles carried along by water;
c. cavitation.

There is no generally accepted model for the mechanisms a and c, and a clear conception of their stochastic character is lacking. For mechanism a it is, however, possible to indicate service lives for certain properly identifiable conditions of service.

In [5] the following expression for the wear $s$ is given as the model for mechanism b:

$$s = a \cdot t + b \cdot v^k \cdot t \tag{44}$$

where $a$ is a measure of the proportion of solid particles performing a scouring or rolling motion, $b$ represents the effect of the particles impinging on the concrete surface, $v$ is the velocity of the water and $t$ is the time, while $k$ is a constant which remains to be determined. It is not clear whether this model is applicable also to non-laboratory conditions (or to large values of $t$).

*Damage due to frost and de-icing salts*

The mechanism explaining the damage caused to concrete by frost and di-icing salts does not appear to have been fully elucidated [3]. A number of explanations have been put forward, the most familiar of which is based on the 9% volume increase that water undergoes on freezing.

Another possible explanation is the ice lens mechanism. From considerations of thermodynamics it can be shown that equilibrium can exist between large and small ice crystals only if the pressure on the larger crystals is higher than on the smaller ones. This means that water in narrow pores will not freeze until sufficient build-up of pressure is possible.

A third explanation for the damage due to frost is provided by the phenomenon of contraction associated with a lowering of temperature, resulting in detachment of the pore walls. In consequence of diffusion and sublimation (direct conversion from ice to water vapour), water can freeze in the voids thus formed until they are completely filled. When the temperature subsequently rises, the ice expands and causes fracturing of the concrete.

If de-icing salts are present, they are found often to create conditions in which damage occurs more rapidly [6]. Osmosis and the absorption of water of hydration are also mentioned as possible causes of damage to concrete exposed to frost action.

The degree of saturation, the rate of freezing, the type of cement used, the use of de-icing salts – all these are factors which govern the degree and type of attack.

*Chemical attack*

In connection with the chemical attack of reinforced concrete [3] the following broad distinctions can be drawn:

a. attack of concrete due to the dissolving of certain material constituents (leaching) or to the expansion of certain constituents;
b. corrosive attack of the reinforcement.

Corrosion of the reinforcement is generally associated with the loss of the protective action that the surrounding concrete should provide for the steel. This will not necessarily be accompanied by the loss of other functions of the concrete (e.g., its strength, rigidity and fire protecting capacity).

a. *Attack of concrete*

Attack occurring in the form of dissolving or expansion usually affects the hardened cement paste. With some kinds of aggregate these actions may, in principle, also involve the aggregate particles. It would be outside the scope of this report to attempt a complete survey of all possible types of chemical attack. A brief outline must suffice:

- inorganic acids: the aggressiveness depends on the concentration of the acid, the temperature, and the nature of the reaction products (soluble or insoluble and expansive: gypsum, ettringite); acid-forming gases must also be mentioned in this context: carbon dioxide, sulphur dioxide, sulphur trioxide and nitric oxide;
- weak organic acids such as lactic acid, acetic acid, formic acid and tannic acid; some acids, such as oxalic acid, tartaric acid and humic acids sometimes react with calcium

hydroxide to give a poorly soluble product, so that further attack is arrested;
- other substances which form salts with calcium hydroxide, such as sugar, phenol and glycerine;
- organic and inorganic salts, including more particularly sulphates and chlorides, which often cause problems (those due to chlorides mostly consist in attack of the reinforcement and not of the concrete itself, however);
- soft water which dissolves calcium hydroxide; the aggressiveness may be increased when carbon dioxide or sulphur dioxide is dissolved in the water, or the hardness is lower or the flow velocity of the water is higher;
- alkaline substances may react with the aggregate, resulting in the formation of expansive products; in this respect alkali-silicate and alkali-carbonate reactions (jointly referred to as alkali-aggregate reaction) may play a part; with high alkalinity, hydrates of calcium aluminate ($C_3A$) may dissolve, and even the reinforcing steel may be attacked by dissolution.

b. *Corrosion of reinforcement*
Under normal conditions, a so-called passivation layer is formed on the surface of reinforcing bars. It is a thin but very dense layer of hydroxide which prevents further corrosion. This film may, however, be attacked by the surrounding concrete environment, so that electrical potential differences may develop along the bars. Electrochemical corrosion will then take place. The corrosion products take up a considerably larger volume than the original iron, which may result in fracturing of the concrete by expansive pressure (spalling).
In practice, attack of the passivation layer occurs chiefly in two different ways:
- carbonation of the concrete surrounding the reinforcement;
- presence of chloride.

Carbonation is said to occur when the calcium hydroxide in the concrete reacts with carbon dioxide from the atmosphere, as follows:

$$Ca(OH)_2 + CO_2 \overset{\text{water}}{\rightarrow} CaCO_3 + H_2O \qquad (45)$$

As a result, the alkalinity of the concrete is lowered and the passivation can no longer be preserved, so that corrosion of the reinforcement is then possible. Carbonation is a diffusion process. The depth of carbonation $d$ can therefore be expressed as a function of time:

$$d^2 = a \cdot t \qquad (46)$$

In this formula the constant $a$ is dependent on, among other factors:
- the permeability of the concrete, which is bound up with:
  - the water-cement ratio
  - the cement content
  - the type of cement
  - the particle size of the aggregate

- the curing of the concrete
- the humidity

- the content of carbon dioxide in the air.

In a literature study by De Sitter [7] the constant $a$ in equation (46) is expressed by the formula:

$$a=\frac{46w-17.6}{2.7}R\cdot K \tag{47}$$

where $w$ is the water-cement ratio ($<0.6$), $R$ is the influence of the cement and $K$ is the influence of the climatic conditions.

The following approximate values are stated:

$R=1.0$ for portland cement class A
$R=0.6$ for portland cement class B
$R=1.4$ for portland blastfurnace cement with 30–40% slag
$R=2.2$ for portland blastfurnace cement with 60% slag
(the usual proportion for Dutch cements)
$K=0.3$ for wet concrete
$K=0.5$ under average outdoor conditions
$K=0.7$ under protected outdoor conditions
$K=1.0$ under indoor conditions

The formula (47) yields an average value. The maximum depth of carbonation is in general 5 to 10 mm greater (Fig. 13). It must moreover be borne in mind that the permeability of concrete decreases as a result of, among other factors, the advanced hardening that occurs in course of time. The carbonation process will then proceed at a slower rate than that corresponding to formula (47). Because of inhomogeneities in the concrete (large pores, cracks) the permeability may locally be higher, however.

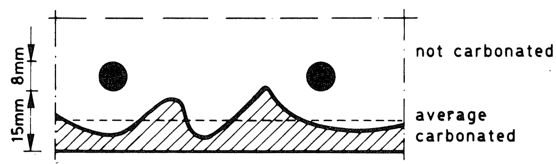

Fig. 13. Irregular carbonation front in concrete.

For existing structures it is therefore advisable to calculate the constant $a$ by working back from the measured actual depth(s) of cabonation and the known age of the concrete. With this procedure the uncertainties associated with the calculation of $a$ can substantially be eliminated.

After disruption of the passivation layer – whether by carbonation or by chloride attack – corrosion of the reinforcing steel may occur, subject to the presence of water and oxygen. The rate of corrosion will depend on the availability of these agents.

In an indoor environment there is generally not enough moisture in the concrete for the corrosion process to occur at a significantly high rate. Concrete in an outdoor environment will, however, generally contain enough moisture for this to occur. But under such conditions the actual period of corrosion leading to deterioration or damage is relatively so short that it can be neglected in comparison with the duration of the carbonation process or of the chloride penetration process.

The detrimental effects of corrosion are:
- reduction of steel cross-sectional areas, so that strength and rigidity are diminished;
- spalling of the concrete cover, because the corrosion products occupy a larger volume than the original steel.

## 4.4 *Overview*

In general, concrete structures may justifiably be regarded as durable structures. Their primary property is that they are able to support stochastic loads safely over a very long period of time. With concrete of suitable composition and workmanship this property is accompanied by high resistance to other influences such as cyclic load variations, erosion, frost and chemical attack.

In the Netherlands, deteriorative damage to concrete structures is generally associated with corrosion of the reinforcement. The cause must often be sought in the use of a hardening accelerator, perfunctory workmanship resulting in porous concrete, or inadequate depth of concrete cover. Particular circumstances such as the presence of de-icing salts or marine salts may give rise to corrosion.

To provide a good insight into the phenomena relating to the durability of concrete structures, Table 2 has been compiled from the information presented in the foregoing.

Table 2. FMEA for the durability of reinforced concrete

| hazard | mechanism | effect |
|---|---|---|
| 1 alternating load | fatigue | cracking, failure |
| 2 flowing water | erosion | surface deterioration |
| 3 turbulent water | cavitation | formation of cavities |
| 4 walking, vehicles | wear | unserviceability |
| 5 frost | expansion | cracking |
| 6 de-icing salts | heat extraction | scaling |
| 7 frost/de-icing salts | heat extraction | scaling |
| 8 acids | neutralization | corrosion |
| 9 acid-forming gases | neutralization | corrosion |
| 9a carbon dioxide | carbonation | corrosion |
| 10 sugar, glycerine | formation of acids | corrosion |
| 11 micro-organisms | production of acids | corrosion |
| 12 soft water | neutralization | corrosion |
| 13 chloride, etc. | depassivation | pitting corrosion |
| 14 sulphate | crystallization | disintegration |
| 15 corrosion | rusting | cracking of concrete |
| 16 corrsion | reduction of bar dia. | deformation/failure |
| 17 contamination of aggregate | crystallization | explosive spalling (pop-outs) |
| 18 alkaline aggregate | alkali-silica reaction | expansion |

It lists the various hazards, mechanisms and effects thereof in the form of a Failure Mode and Effect Analysis (FMEA).

## 5 Case study: durability of a gallery slab

### 5.1 *Introduction*

As a demonstration of an optimization analysis relating to durability a reinforced concrete outdoor gallery slab will be investigated (see Fig. 14). In order to keep the

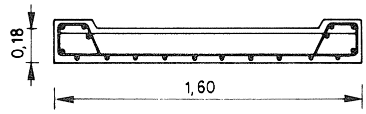


Fig. 14. Data for gallery slab.

problem within conveniently manageable limits, only the hazard due to carbon dioxide will be considered. The mechanism concerned is the carbonation of the free lime and the effect is corrosion of the main reinforcement at the underside, manifesting itself in spalling of the concrete cover (Fig. 15).

Four design alternatives are to be distinguished:

1. 15 mm cover, no coating;
2. 30 mm cover, no coating;
3. 15 mm cover, coating, maintenance every 20 years;
4. 15 mm cover, coating, maintenance every 10 years.

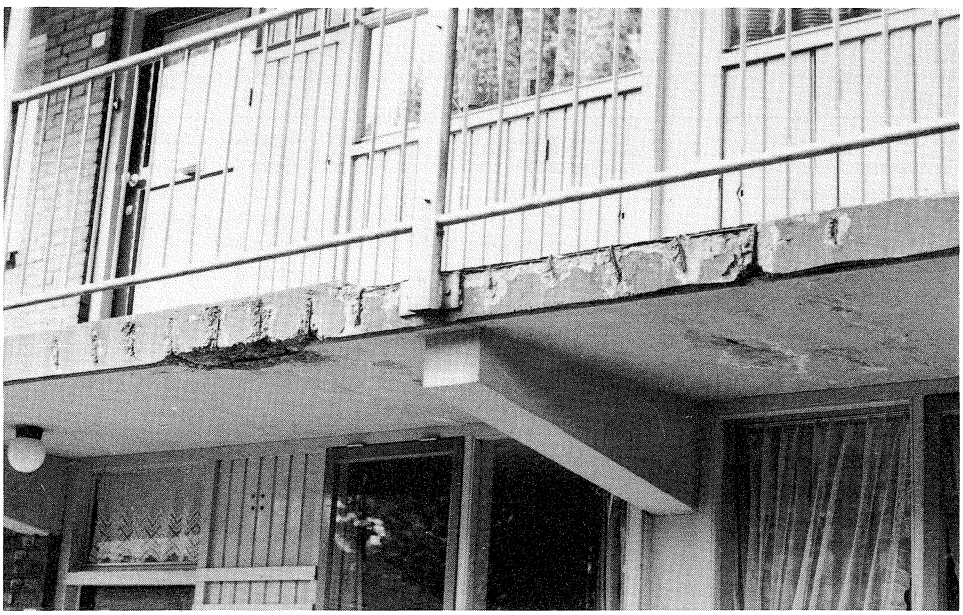

Fig. 15. Damage due to carbonation of a gallery slab.

In principle both the cover and the maintenance period are of course continuous variables. Actually a much greater number of alternatives would therefore have to be investigated, in the same way as in Section 3.4, example *a*, where a similar but more highly schematized problem was dealt with. However, for a preliminary examination of the problem an analysis of the four above-mentioned alternatives will suffice.

As soon as the deteriorative damage visibly manifests itself, remedial action will have to be taken. The following possibilities may be considered:

1. complete replacement;
2. removal of the entire carbonated zone and application of sprayed concrete;
3. local repair of the visible damage with polymer mortar.

Which of these methods will offer the optimum in any particular case will depend also on the point of time when the repairs are effected: radical repair work is meaningful only if the structure still has a sufficiently long unexpired service life to complete. The planned service life for the building as a whole is 60 years.

## 5.2 *Mathematical models for carbonation*

If the concrete is not provided with a protective coating, the depth of carbonation ($d$ in mm) can be calculated with the aid of equations (46) and (47):

$$d = \left(\frac{46w - 17.6}{2.7}\right) R \cdot K \cdot \sqrt{t} \tag{48}$$

For the case under present consideration (and with reference to [8]) we have:

$$w = 0.50 \qquad R = 2.0 \qquad K = 0.7$$

The depth of carbonation is not the same at every point of the slab, however. Formula (48) gives the average depth over the slab (see Fig. 13).

The start of corrosion is governed, not by the average depth of carbonation, but by the advanced peaks of the carbonation front. According to [7], this may make a difference of 5–10 mm. On the other hand, not every peak encounters a reinforcing bar, and it moreover takes some time for the corrosion to manifest itself in an externally visible manner (spalling of the concrete cover). For this the following formula is given in [12]:

$$t_1 = \frac{0.08 \cdot d}{\varnothing \cdot v_c} \tag{49}$$

where

$c$ = the cover
$\varnothing$ = the bar diameter
$v_c$ = the rate of rusting

All in all, the time that elapses before corrosion damage becomes visibly manifest can be calculated from the following formula (with replacement of $x$ in equation (48) by $(d - \Delta)$):

$$t_L = \frac{(d-\Delta)^2}{R^2 \cdot K^2} \left\{ \frac{2.7}{46w - 17.6} \right\}^2 + \frac{0.08d}{\varnothing \cdot v_c} E \tag{50}$$

The influence of fluctuation of the carbonation depth is taken into account by means of $\Delta$, for which an average value of about 5 mm may be adopted. The rusting rate $v_c$ is affected by very considerable scatter. In the literature [9, 10] values ranging from 0.015 to 0.09 mm/year are found. $E$ is a constant with a value of 0.08 mm. Its purpose is to make equation (50) non-dimensional.

The question as to what is the appropriate model to be adopted for describing the carbonation process in concrete is still a subject of much discussion in the literature. In giving formula (50) it is not the intention of the present report to declare a position in this discussion. The only purpose is to show that reliability analysis can be used for solving durability problems. The formula in question has, in this context, been chosen more or less arbitrarily.

In the slab considered in this example the nominal cover is 15 mm. The actual cover can be assumed to be somewhat greater: 20 mm, say. The reinforcement consists of 8 mm diameter bars ($\varnothing = 8$ mm) spaced at 150 mm centres. If the previously stated values are adopted for $w$, $C$, $K$, $E$ and $t$ and if $\Delta$ and $v_c$ are taken as 5 mm and 0.04 mm/year respectively, the following result is obtained for the service life:

$$t_L = \frac{(20-5)^2}{2.0^2 \cdot 0.7^2} \left\{ \frac{2.7}{46 \cdot 0.5 - 17.6} \right\}^2 + \frac{0.08 \cdot 20}{8 \cdot 0.04} = 29 + 5 = 34 \text{ years}$$

The main contribution comes from the first term, the so-called initiation time.

If, in accordance with the second design alternative, the cover is taken as nominally 30 mm (practically 35 mm), then:

$$t_L = 115 + 7 = 122 \text{ years}$$

In that case there is a marked increase in durability.

If the underside of the concrete slab is provided with a coating highly impermeable to carbon dioxide (chloro-vinyl or epoxy paint), the length of time up to the start of attack of the reinforcement can be considerably increased. In Appendix C it is shown, on the basis of formulae in [7], that this increase is expressed by:

$$\Delta t_L = \frac{(d-\Delta)s}{180f} \tag{51}$$

where $s$ is the thickness of the coating and $f$ is the fraction of the surface not covered by the coating. By way of example an epoxy paint with $s = 0.18$ mm and $f = 10^{-5}$ is considered. Then:

$$\Delta t_L = \frac{(20-5) \cdot 0.18}{180 \cdot 10^{-5}} = 1500 \text{ years}$$

The protective action of the coating is thus clearly demonstrated. This does presuppose, however, that the values of $s$ and $f$ do not vary in course of time and also that the coating remains intact. This (ideal) situation can be approximately achieved if frequent inspection and maintenance are carried out.

As regards the thickness $s$ of the coating, a rate of surface disintegration of 3 microns/year is mentioned in [7] based on [11]. For a normal maintenance cycle of 8–12 years this rate is virtually negligible: it would mean that the thickness of the coating decreases from, say, 0.18 mm to 0.15 mm in 10 years. Only when longer intervals between maintenance operations are considered does this aspect begin to be important.

Of greater significance is the time-dependent behaviour of $f$, at least if phenomena such as damage and scaling-off are considered to be taken into account by $f$. There are, however, no data at present available for this. Meanwhile, pending further information on the subject, the behaviour of $f$ as a function of time will be assumed to be as represented in Fig. 16.

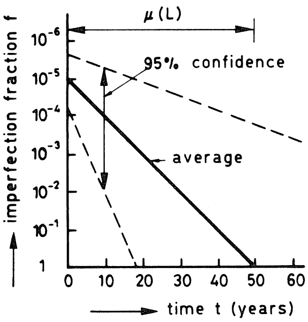


Fig. 16. Assumed behaviour of the fraction $f$ as a function of time $t$ if no maintenance is carried out.

On this assumption the logarithm of $f$ decreases linearly with time and the coating will have entirely disappeared after about 50 years. This is expressed by the formula:

$$f(t) = f_0 \cdot e^{-\alpha t} \tag{52}$$

where $f_0$ is the value of $f$ for $t = 0$, while $\alpha = \ln f_0 / T_0$ in which $T_0 = 50$ years.

If maintenance is periodically carried out, $f(t)$ will show the familiar saw-tooth behaviour as represented in Fig. 17.

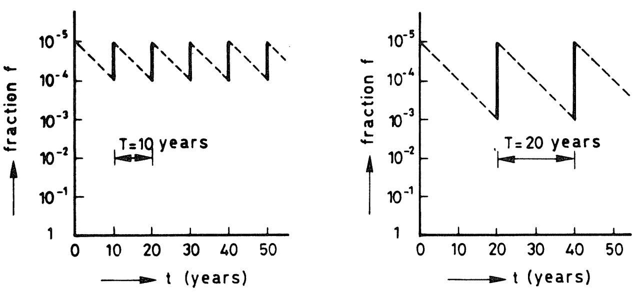


Fig. 17. Behaviour of $f$ as a function of time $t$ for maintenance every 10 and 20 years respectively (saw-tooth model).

For calculating the corresponding service life it will be presupposed that the contribution made by the concrete itself is negligible in comparison with the extra time gained by the coating. Since $f(t)$ is time-dependent, it is not possible to make direct use of equation (52). Some differential calculus will have to be applied. During a very short period $\mathrm{d}t$ the carbonation front will advance a very short distance $\mathrm{d}y$. Then, in analogy with equation (51), we have:

$$\mathrm{d}t = \frac{s \cdot \mathrm{d}y}{180 \cdot f(t)}$$

or:

$$f(t) \cdot \mathrm{d}t = \frac{s \cdot \mathrm{d}y}{180}$$

Integration of both sides of the equation, from the commencement of carbonation ($t = 0$, $y = 0$) to the time when it reaches the reinforcement ($t = t_L$, $y = d - \Delta$), yields:

$$\int_0^{t_L} f(t) \cdot \mathrm{d}t = \int_0^{d-\Delta} \frac{s}{180} \cdot \mathrm{d}y \tag{53}$$

The right-hand integral is equal to $s(d - \Delta)/180$. For the integral on the left it is necessary to take account of the logarithmic saw-tooth pattern displayed by $f(t)$ according to Fig. 17. If the maintenance period is $T$, the number of maintenance cycles in the service life is $t_L/T$, so that the following approximation can be made for this integral:

$$\int_0^{t_L} f(t) \cdot \mathrm{d}t = \frac{t_L}{T} \int_0^{T} f_0 \cdot e^{-\alpha t} \, \mathrm{d}t = \frac{t_L}{T} \cdot \frac{f_0}{\alpha} \{1 - e^{-\alpha T}\} \tag{54}$$

By equation the two expressions (53) and (54) we obtain the following formula for the service life:

$$t_L = \frac{(d - \Delta) \cdot s}{180 f_0} \left\{ \frac{\alpha T}{1 - e^{-\alpha T}} \right\}$$

For $T = 10$ years: $\alpha T = (T/T_0) \ln f_0 = (10/50) \cdot (-11.15) = -2.3$, whence we obtain:

$$t_L = \frac{(d - \Delta) \cdot s}{180 f_0} \cdot 0.26$$

The earlier estimate of 1500 years for the service life thus reduces to:

$$t_L = \frac{(20 - 5) \cdot 0.18}{180 \cdot 10^{-5}} \cdot 0.26 = 1500 \cdot 0.26 = 390 \text{ years}$$

For $T = 20$ years: $\alpha T = (T/T_0) \ln f_0 = (20/50) \cdot (-11.5) = -4.6$

$$t_L = \frac{(20 - 5) \cdot 0.08}{180 \cdot 10^{-5}} = 1500 \cdot 0.047 = 70 \text{ years}$$

For this maintenance period of 20 years the resistance that the concrete itself offers to the advance of carbonation, previously neglected, would certainly appear to be worth taking into account. According to Appendix C, the carbonation times are allowed to be added together, so that:

for $T = 10$ years: $t_L = 390 + 40 = 430$ years
for $T = 20$ years: $t_L = \ \ 70 + 40 = 110$ years

### 5.3 *Quantification of the uncertainties*

In probabilistic calculations a variable is described with the aid of a probability density function. Such a function is in many cases characterized by a particular type of distribution (e.g., a normal distribution or a Weibull distribution), a mean value $\mu$ and a standard deviation $\sigma$. Instead of the standard deviation the coefficient of variation ($V = \sigma/\mu$) is often used.

In the ideal case there are statistical data available for all the variables, enabling the type of distribution, the mean value and the standard deviation to be unambiguously determined. Mostly, however, as also in this example, the reverse of this situation exists, namely, that there are no or, at best, scanty data available. The statistical properties will then have to be entirely estimated.

For many variables it is possible, basing oneself on information published in the literature, on experience or on intuition, to specify values above or below which the variable in question will in all probability not be situated. It is well known that, for the normal distribution, there is 95% probability that a variable will have a value situated between $\mu - 2\sigma$ and $\mu + 2\sigma$. It is thus possible to estimate the mean value and the standard deviation:

$$\mu(x) = \tfrac{1}{2}\{x_{\text{high}} + x_{\text{low}}\} \tag{56}$$

$$\sigma(x) = \tfrac{1}{4}\{x_{\text{high}} - x_{\text{low}}\} \tag{57}$$

For a log-normal distribution the corresponding formulae, so long as $x_{\text{high}}/x_{\text{low}} < 10$ (see Appendix B), are:

$$\mu(x) = \sqrt{x_{\text{high}} \cdot x_{\text{low}}} \tag{58}$$

$$V(x) = \tfrac{1}{4} \ln \{x_{\text{high}}/x_{\text{low}}\} \tag{59}$$

The choice between using the normal or the log-normal distribution will depend on the physical nature of the stochastic variable. Many variables are by nature unable to take on negative values, and in such cases the log-normal distribution is to be preferred. For small values of the coefficient of variation ($V < 0.10$) the difference between normal and log-normal is negligible for practical purposes. As for deciding between normal or log-normal, on the one hand, and other potentially available distributions (e.g., extreme value distributions), on the other, the present example hardly seems to provide relevant arguments. The consequences of choosing a particular type of distribution for the stochastic variables will be considered later on.

In Table 3 a choice has been made for the type of distribution, the mean value and the standard deviation for the various problem variables. Some of the variables have been taken as deterministic because the scatter affecting them is considered to be very small.

Table 3. Review of the carbonation variables and the stochastic properties

| description | type designation | mean | c.o.v. |
|---|---|---|---|
| $d$ concrete cover nominal 15 mm | log-normal | 20 | 0.25 |
| $d$ concrete cover nominal 30 mm | log-normal | 35 | 0.14 |
| $\Delta$ distance maximum-mean carb. depth mm | log-normal | 5 | 0.20 |
| $R$ influence factor for the type of cement | log-normal | 2.0 | 0.15 |
| $K$ climate factor | log-normal | 0.7 | 0.20 |
| $w$ water-cement ratio | log-normal | 0.5 | 0.05 |
| $E$ constant mm | deterministic | 0.08 | – |
| $\varnothing$ diameter of reinforcement bar mm | deterministic | 8 | – |
| $v_c$ rate of corrosion mm/year | log-normal | 0.04 | 0.50 |
| $s$ thickness of coating mm | deterministic | 0.18 | – |
| $f_0$ damage coefficient for coating mm/year | log-normal | 0.00001 | 1.00 |
| $T_0$ durability parameter for coating year | log-normal | 50 | 0.50 |
| $T$ maintenance period year | deterministic | 10/20 | – |

The mean values adopted are those values which, in the preceding section, were rated as providing the best estimates. The coefficients of variation are largely based on estimates of the type indicated above. Only in the case of the concrete cover is it possible to base oneself on statistical information obtained from measurements of the depths of cover on existing structures [12].

The least scatter is presumed to occur in the water-cement ratio. The coefficient of variation $V(w) = 0.05$ indicates that with 95% probability the value of $w$ is between 0.45 and 0.55. The amounts of scatter in, for example, $R$ and $K$ are considered to be much greater. For the rate of corrosion $v_c$ the values 0.015 and 0.09 have already been mentioned as the upper and the lower limit. On applying equations (58) and (59) we obtain:

$$\mu(v_c) = \sqrt{0.015 \cdot 0.090} = 0.036 \text{ mm/year}$$
$$V(v_s) = \tfrac{1}{4} \ln 6 = 0.45$$

Note: A more exact calculation yields $\mu(v_c) = 0.040$ mm/year and $V(v_c) = 0.47$. These values have been adopted in Table 3, where $V(v_c)$ has been rounded off to 0.50.

The large coefficient of variation $V(f_0) = 1$ for the imperfection parameter $f_0$ indicates a ratio $f_{0\,(\text{high})}/f_{0\,(\text{low})}) = 50$. The estimate for $V(f_0)$ is based on [18], where the information given suggests that for any particular paint system it is difficult to estimate $f_0$ to an accuracy within a factor of 10. Finally, $V(T_0) = 0.50$ indicates that this durability value is in all probability between 20 and 100 years.

## 5.4 *Probabilistic analysis*

On the basis of the mathematical model according to equation (55) and the stochastic properties listed in Table 3 it is possible to determine a probability density function of the service life. This analysis will be confined to an approximate calculation for the mean service life and the coefficient of variation. The procedure adopted for the purpose is called the "level II/mean value approximation" or the "first order, second moment approximation" (FOSM) in the literature [1].

According to this procedure the mean service life is obtained by calculating the life on the basis of mean values for all the variables. This calculation has in fact already been performed in Section 5.2. The standard deviation is determined by linearizing the function $t$ with the aid of a Taylor series. We then obtain:

$$\sigma^2(t_L) = \sum_{j=1}^{t_G} \left\{ \frac{\partial t_L}{\partial X_j} \cdot \sigma(X_j) \right\}^2 \tag{60}$$

The partial derivatives $\partial t_L/\partial X_j$ are calculated for the mean values of the stochastic variables. For all four alternatives this calculation has been performed and the results given in Table 4.

Table 4. Calculation of $\sigma^2(t_L) = \sum \left( \frac{\partial t}{\partial X_j} \cdot \sigma(X_j) \right)^2$

| | | alternative | | | |
|---|---|---|---|---|---|
| $X_j$ | description | 1 | 2 | 3 | 4 |
| $d$ | cover | 415 | 1561 | 1905 | 21994 |
| $\Delta$ | distance between max. and mean carbonation depth | 15 | 59 | 72 | 864 |
| $R$ | influence factor for type of cement | 74 | 1187 | 74 | 74 |
| $K$ | climate factor | 132 | 2111 | 132 | 132 |
| $w$ | water-cement ratio | 150 | 2395 | 150 | 150 |
| $\varnothing$ | bar diameter | – | – | – | – |
| $v_c$ | rate of corrosion | 6 | 19 | 6 | 6 |
| $s$ | coating thickness | – | – | – | – |
| $f_0$ | imperfection coefficient for coating | – | – | 2304 | 112084 |
| $T_0$ | durability value for coating | – | – | 16178 | 89366 |
| $T$ | maintenance period | – | – | – | – |
| | $\sigma^2(t_L)$ | 792 | 7333 | 20820 | 224672 |

The scatter in the service life is found to be very great. Table 5 shows which variables are most responsible for this scatter. Thus it appears that with alternative (1) particularly the depth of cover is of major influence. If the cover is increased, criteria such as the water-cement ratio and the climate factor become more important. If a coating is applied, the durability value $T_0$ predominates if maintenance is carried out every 20 years, whereas the initial imperfection coefficient $f_0$ predominates if maintenance is carried out every 10 years. It is particularly these variables affected by a large amount of scatter that should receive most attention in further research. A table such as Table 5 reveals that a probability analysis is essentially a sensitivity analysis, in which the sensitivity $\partial t/\partial X_j$ of the variable $X_j$ is weighed with a measure for the variation, the standard deviation $\sigma(X_j)$, in relation to the scatter in the service life.

By means of the calculations in Sections 5.2 and 5.4 the mean value and the standard

Table 5. Relative contributions of the stochastic variables to the variance of the service life for the four design alternatives

| | | alternative | | | |
|---|---|---|---|---|---|
| $X_j$ | description | 1 | 2 | 3 | 4 |
| $d$ | cover | 52% | 21% | 9% | 10% |
| $\Delta$ | distance between max. and mean carbonation depth | 2 | 1 | 0 | 0 |
| $R$ | influence factor for type of cement | 9 | 16 | 0 | 0 |
| $K$ | climate factor | 17 | 29 | 1 | 0 |
| $w$ | water-cement ratio | 19 | 33 | 1 | 0 |
| ⌀ | bar diameter | – | – | – | – |
| $v_0$ | rate of corrosion | 1 | 0 | 0 | 0 |
| $s$ | coating thickness | – | – | – | – |
| $f_0$ | imperfection coefficient for coating | – | – | 11 | 50 |
| $T_0$ | durability value for coating | – | – | 78 | 40 |
| $T$ | maintenance period | – | – | – | – |
| | total | 100% | 100% | 100% | 100% |

deviation of the service life $t_L$ have now been determined. The distribution of $t_L$ is as yet not known.

To begin with, the log-normal distribution once again deserves consideration as a possible choice: the service life is definitely positive and the scatter is large. On the basis of theoretical considerations it can be shown that the distribution cannot be exactly log-normal. According to equation (55) the service life is composed of three terms. For each of these terms in itself there is indeed something to be said in favour of the log-normal distribution (multiplication or division of variables conforming to this distribution will again yield a log-normally distributed variable), but the sum of log-normally distributed variables will certainly not be log-normally distributed. However, for the present purpose it does not matter so much what is exactly correct, but what will provide a serviceable model. To this end, the probability that the four alternative solutions will fail within the intended service life of 60 years has been calculated – on the basis of the log-normal model and also with the aid of the "first order, second moment" method (FOSM). In problems of this type the last-mentioned method provides a virtually exact calculation of the failure probability. The results are presented in Table 6. From these it can be inferred that the log-normal distribution yields good results. For the determination of the probability of failure for a log-normal distribution and given mean value and

Table 6. Comparison of results on the basis of the log-normal distribution with those obtained with the FOSM method

| design alternative | $\mu(t_L)$ (year) | $\sigma(t_L)$ (year) | $P\{t_L < 60\}$ (log-normal distribution) | $P\{t_L < 60\}$ (FOSM) |
|---|---|---|---|---|
| 1 | 34 | 28 | 0.86 | 0.76 |
| 2 | 123 | 86 | 0.20 | 0.13 |
| 3 | 103 | 144 | 0.50 | 0.39 |
| 4 | 417 | 474 | 0.05 | 0.05 |

standard deviation, see Appendix B. Finally, the service life distributions obtained for the first two alternatives are represented in Fig. 18 by way of illustration.

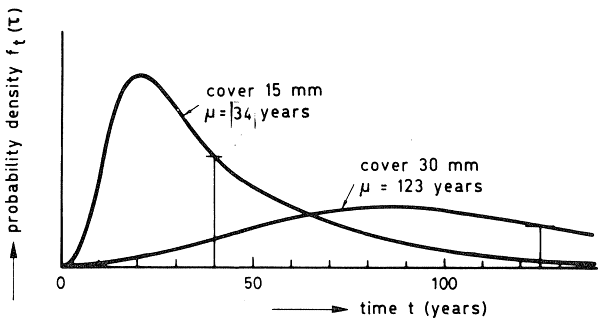


Fig. 18. Service life distribution of two alternatives.

## 5.5 *Cost analysis*

For the purpose of economic optimization it is necessary to obtain insight into the cost of the increased concrete cover, the application and maintenance of protective paint coatings, and repairs. The cost figures adopted in this example are assembled in Table 7. Uncertainties in these costs, and also the uncertainty in the interest rate to be applied, have been ignored for the sake of simplicity.

Table 7. Review of cost data (in guilders/m$^2$)

| | |
|---|---|
| making and installing concrete slabs in new building ($h_t$ = depth of slab in mm) | $40 + 0.4h_t$ |
| replacement at $h_t \sim 150$ mm | 275 |
| removing concrete cover, applying sprayed concrete | 225 |
| removing concrete cover, repairing with polymer mortar | 250 |
| painting new concrete (epoxy) | 50 |
| painting existing concrete | 80 |
| maintenance of coating | 40 |
| repair of coating | 80 |

The expectation of the capitalized cost, according to equation (5), will be adopted as the basis of comparison for optimization and be written as:

$$E\{C_{cap}\} = S + \sum_{j=1}^{t_G} \frac{V_j}{(1+r')^j} + \sum_{j=1}^{t_G} \frac{P\{F_j\} \cdot D_j}{(1+r')^j}$$

where $r'$ (= real rate of interest) is taken as 0.02, while $V_j$ and $D_j$ are respectively the cost of maintenance and the loss (cost associated with damage or deterioration) in the year $j$. The planned service life of 60 years is so interpreted that up to and including the 60th year of service all necessary repairs and maintenance are carried out. Therefore $t_G = 60$ is adopted in the equation. Any unexpired service life of the structure beyond that period, though probably available, will not be considered.

In Table 8 the values for all three terms of equation (61) have been calculated on the basis of the cost data given in Table 7, the service life data given in Section 5.4 and a few additional assumptions concerning the extent of the deterioration or damage. The following explanatory comment is offered:

The slab is 1.60 m wide and 6.30 m long, so that its area is 10 $m^2$. The point of reference for the cost comparison is represented by a slab with 15 mm concrete cover, without coating. For this slab no extra cost is incurred for increasing the durability. For the slab considered as alternative (2), with 30 mm cover, a sum of $15 \times 0.4 = 6$ guilders/$m^2$, i.e., a total of 60 guilders per slab, is spent on improving the durability. Applying a coating to new concrete costs 50 guilders/$m^2$, i.e., 500 guilders for the whole slab. This extra investment occurs also in the alternatives (3) and (4).

Table 8. Expected capitalized costs (in guilders)

| cost item | | design alternative | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| extra concrete cover | guilder | – | 60 | – | – |
| coating | guilder | – | – | 500 | 500 |
| maintenance of coating | guilder | – | – | 675 | 1130 |
| repair expectation | guilder | 860 | 170 | 260 | 40 |
| $E\{C_{cap}\}$ | guilder | 860 | 230 | 1435 | 1670 |

Maintenance is carried out only in the case of these two last-mentioned alternatives. First, consider alternative (4) with maintenance at 10-year intervals. Maintenance costs 40 guilders/$m^2$, i.e., 400 guilders for the slab. The calculation of the capitalized cost of maintenance proceeds as follows:

$$\sum_{j=1}^{t_G} \frac{V_j}{(1+r')^j} = \frac{400}{1.02^{10}} + \frac{400}{1.02^{20}} + \dots + \frac{400}{1.02^{50}} = \frac{400}{1.28} + \frac{400}{1.49} + \frac{400}{1.81} + \frac{400}{2.21} +$$

$$+ \frac{400}{3.69} = 2.83 \cdot 400 = 1130 \text{ guilders}$$

For alternative (3) it is assumed that maintenance at intervals of 20 years will, in half the number of cases that it is carried out, involve repair work costing 800 guilders instead of 400 guilders, i.e., 600 guilders on average. Hence the capitalized cost of maintenance is:

$$\sum_{j=1}^{t_G} \frac{V_j}{(1+r')^j} = \frac{600}{1.02^{20}} + \frac{600}{1.02^{40}} = 675 \text{ guilders}$$

The final item that has to be determined is the loss (the cost associated with remedying damage or deterioration). In a more or less arbitrary manner the following possible cost amounts may be distinguished (see also Section 5.1):

a. complete replacement:
   $10 \times 275 = 2750$ guilders

b. major repairs with sprayed concrete over an area of 4 $m^2$:
   $4 \times 225 = 900$ guilders
c. minor repairs with polymer mortar over an area of 2 $m^2$:
   $2 \times 250 = 500$ guilders

As a rule, the loss arising from refurbishment to remedy the effects of damage or deterioration will be between 500 and 900 guilders; major repairs or complete replacement is of less frequent occurrence. In the further calculation an average figure of 1000 guilders will be adopted for this loss. An exception is the occurrence of premature damage in the case of alternative (1). If a slab with 15 mm cover and no coating is found to be giving trouble (due to deterioration) in a number of places within about 30 years, it is evident that complete replacement is often likely to offer the most rational remedy. Otherwise it is fairly certain that more trouble will occur in other places on the slab after a time. For alternative (1) and $t_L < 30$ years the loss is accordingly taken as 2000 guilders.

To calculate the risk, we must calculate the probability of failure in each year. For convenience, six periods of 10 years will be considered:

$$\sum_{j=1}^{60} \frac{P\{F_j\} \cdot D_j}{(1+r')^j} = \frac{D_{10}}{(1+r')^{10}} \cdot P\{0 < t_L < 10\} + \frac{D_{20}}{(1+r')^{20}} \cdot P\{10 < t_L < 20\} + \dots +$$

$$+ \frac{D_{60}}{(1+r')^{60}} \cdot P\{50 < t_L < 60\}$$

The probability $P\{50 < t_L < 60\}$ is equal to $P\{t_L < 60\} - P\{t_L < 50\}$. All the data needed for evaluating the risk are thus available. For alternative (1), i.e., 15 mm cover and no coating, we obtain:

$$\sum_{j=1}^{60} \frac{P\{F_j\} \cdot D_j}{(1+r')^j} = \frac{2000 \cdot 0.09}{1.02^{10}} + \frac{2000 \cdot 0.25}{1.02^{20}} + \frac{2000 \cdot 0.29}{1.02^{30}} + \frac{1000 \cdot 0.15}{1.02^{40}} +$$

$$+ \frac{1000 \cdot 0.09}{1.02^{50}} + \frac{1000 \cdot 0.04}{1.02^{60}} = 150 + 336 + 265 + 68 + 33 + 12 = 860 \text{ guilders}$$

For the period $0 < t_L < 30\,j$ the loss has been taken as 2000 guilders and for $t_L > 30\,j$ it has been taken as 1000 guilders, as explained above. The factors 0.09, 0.25, etc. are the probabilities of failure in the period 0–10 years, 10–20 years, etc. For the determination of these, see Appendix D. The calculation for the other design alternatives is entirely similar (except that the loss is then 1000 guilders for the entire service life). The results are given in Table 8.

## 5.6 *Conclusion*

By far the most advantageous solution is found to consist in increasing the depth of concrete cover. The direct cost that this entails is low (60 guilders), while the deterioration or damage occurs only after a long time. To remedy these defects it is often sufficient to carry our relatively simple repairs, and the interest ensures a substantial decrease in the capitalized cost.

It is notable that applying and maintaining the coating is more expensive than alternative (1), despite the marked reduction of the probability of deterioration or damage.

In view of this result the coating should be applied primarily for aesthetic reasons. its protective effect being an additional bonus. (Alternatively, cheaper paints for application to concrete may be used if the object is simply to improve the appearance of the structure; such paints are in general not impermeable to carbon dioxide, however, so that in this respect they offer little or no protection). An advantage associated with the use of protective coatings is the reduced probability of inconvenience to the residents or users of the building caused by repair work. This applies only to the alternatives requiring frequent maintenance.

Finally, it should be pointed out that the results of the calculations are based on initial concepts and data which (though representing the most realistic possible assumptions) are still debatable. The example presented here is intended only to open the discussion.

## 6 Summary and conclusions

During its construction and its subsequent lifetime a building has to perform a great many functions with regard to, inter alia, strength and serviceability. The conditions under which it does this will change in course of time, and the properties of the building itself may also be time-dependent. If, after the passage of time, the building ceases to be able to perform a function, maintenance or replacement will become necessary. Otherwise the building will then have reached the end of its technical service life.

Assessing the durability aspect of a building is no simple matter. The conditions under which the building has to function are comparatively uncertain. This also applies to its properties. Because of these uncertainties, the service lives of essentially similar buildings under apparently similar conditions may vary considerably. The concept of service life is therefore difficult to apply in a meaningful way. For this reason a first step to developing a durability philosophy has been attempted in the present report. In order to make it serviceable for practical purposes, a limited list of durability data has been compiled, based on a somewhat arbitrary choice of entries in seeking to restrict it more particularly to reinforced concrete.

In view of the considerable scatter that may occur in the service life, it is by no means easy to take appropriate measures for achieving optimum durability. This report has therefore had recourse to risk and reliability analysis, which has proved its suitability in dealing with problems of structural safety. Despite the scatter in loads and structural properties, a conceptual pattern for dealing with the scatter is established in a rational and orderly manner. As this approach to durability corresponds to the approach adopted in matters of structural safety, the methodology applied can claim to be consistent.

In carrying out a reliability analysis, a list is first compiled of all the hazards, the way in which the structure responds to them (mechanisms) and the effects. By way of illustration an overview relating to concrete structures is given. Next, a selection is made. The hazards which occur frequently or have serious effects are considered in more detail. At the same time the criterion applied is that it must be economically advantageous to take protective measures. In considering the economic aspects it is necessary to take account not only of the investment cost, but also of the operating expenses, interest and depreciation. All these costs are largely determined by the design features and the manner of construction. The report enters into an economic study of capitalized cost, cost on a yearly basis and dynamic cost price rent. For this purpose the service life is assumed to be stochastic. These matters are explained and illustrated with reference to some simple examples. The examples relate to the problem of determining the optimum depth of concrete cover with respect to the carbonation of concrete if the required service life is specified and to the problem of determining the optimum service life in a case where the concrete surface is affected by wear.

A more comprehensive optimization analysis of an outdoor gallery slab built of reinforced concrete is then given. To simplify the problem, only the action of carbon dioxide on the concrete (carbonation), which may result in corrosion of the reinforcement, is considered. Several design alternatives are analysed. Despite the relatively great scatter in the service life associated with each of these alternatives, it is found to be suitably possible to quantify the total cost entailed by a particular alternative. As a subsidiary feature of the result of the analysis, the probability of repair work and the attendant nuisance to residents or users of the building is indicated.

The optimization calculations carried out in the report show that for reinforced concrete it is feasible in principle to graft the durability philosophy on the methodology of reliability analysis. As a result, for reinforced concrete structures the aspects of safety and serviceability, but also durability, can be approached in a consistent and properly coherent manner. For a design which in principle will be based on cost optimization there are then not likely to be contradictions or discrepancies between safety, serviceability and durability. In principle, too, the consistent approach will result in greater simplicity. Hazards, mechanisms and effects are all defined and considered in one and the same way.

The worked examples presented in the report show that the probabilistic approach also leads to a sensitivity analysis. The most relevant parameters in the stated problem are directly recognized. For further investigation it is not necessary to collect more information about all the variables involved. This need only be done for the relevant variables. Thus an improvement of the result is obtained without too much effort.

The preliminary study has, for pragmatic reasons, been confined to the structural material concrete, which is called as a durable material. The examples give here show, however, that this is true only in a qualified sense. All the same, this does not alter the fact that other materials are used whose service life has an entirely different dimension. The consideration of the durability of paint systems applied to concrete surfaces, as envisaged in the examples, suggests that dealing with these and other materials need not entail any major problems. Hence it would appear justifiable to extend the investigation towards other building materials such as masonry, metals, timber, plastics, asphalt, glass, etc. However, it will first have to be verified that the expectations based on this approach are fulfilled.

## 7 Acknowledgements

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## APPENDIX A

### Capitalized cost in the case of free service life

If the service life of an object – a capital asset – is determined only by the condition of the object itself, the following reasoning must be followed when considering the problem on the basis of capitalized cost.

At the beginning of the service life there should, besides the direct investment $S$, be a reserve $R$ available from which, after $n$ years have passed, the replacement structure can be financed. The capitalized cost is then:

$$C_{\mathrm{cap}} = S + R \tag{A1}$$

After $n$ years, $R$ has increased to $R(1 + r')^{\mathrm{n}}$, and the following condition must be satisfied:

$$R(1 + r')^{\mathrm{n}} = S + R \tag{A2}$$

For the sake of a clear understanding, there are three points to note:

1. The real interest is introduced, so that there is no need to take account of cost increases in $S$ due to inflation.
2. In addition to the direct investment $S$ (to pay for the structure initially), there should – on the assumption adopted here – be reserve resources $R$ available at the end of $n$ years in order to finance the next (replacement) structure, etc.
3. It is not necessary actually to possess the amount $R + S$: the purpose is only to establish a basis for comparison, assuming that there will exist a permanent need for the capital asset in question.

From equation (A2) it follows that:

$$R = \frac{S}{(1 + r')^{\mathrm{n}} - 1} \tag{A3}$$

On substitution into equation (A1) this gives:

$$C_{\mathrm{cap}} = S + \frac{S}{(1 + r')^{\mathrm{n}} - 1} = S \left\{ \frac{(1 + r')^{\mathrm{n}}}{(1 + r')^{\mathrm{n}} - 1} \right\} \tag{A4}$$

Example: If $r' = 0.02$ and $n = 50$, then $C_{\mathrm{cap}} = 1.54S$.

Finally, it is to be noted that, except for a factor $r'$, equation (A4) corresponds to equation (21) for the cost analysis on an annual basis. The interesting point about this is that in equation (21) the existence of a permanent need is not presumed. However, in this equation, too, such need is implicitly present: for example, if the service life is optimized, there is no restriction as to avoiding (unnecessarily) long service lives.

## APPENDIX B

### Estimation of $\mu(X)$ and $V(X)$ for a log-normal distribution of $X$

If $X$ has a log-normal distribution with $\mu(X)$ and $\sigma(X)$, then $Y = \ln X$ has a normal distribution with:

$$\mu(Y) = \ln \mu(X) - \tfrac{1}{2}\sigma^2(Y) \tag{B1}$$

$$\sigma^2(Y) = \ln\{1 + V^2(X)\} \tag{B2}$$

where $V(X) = \sigma(X)/\mu(X)$

The estimates $X_{\text{low}}$ and $X_{\text{high}}$ correspond to:

$$Y_{\text{low}} = \ln X_{\text{low}} \tag{B3}$$

$$Y_{\text{high}} = \ln X_{\text{high}} \tag{B4}$$

The mean value of $Y$ is thus obtained from:

$$\mu(Y) = \tfrac{1}{2}(\ln X_{\text{low}} + \ln X_{\text{high}}) = \ln \sqrt{X_{\text{low}} \cdot X_{\text{high}}} \tag{B5}$$

The standard deviation of $Y$ is estimated from:

$$\sigma(Y) = \tfrac{1}{4}(\ln X_{\text{high}} - \ln X_{\text{low}}) \tag{B6}$$

If $V(X)$ is small, then equation (B1) can be approximated by $\mu(Y) = \ln \mu(X) = \mu(\ln X)$ and equation (B2) by $\sigma(Y) = V(X)$. Conversely, it follows directly that:

$$\mu(X) = \sqrt{X_{\text{low}} \cdot X_{\text{high}}} \tag{B7}$$

$$V(X) = \tfrac{1}{2} \ln \left( \frac{X_{\text{high}}}{X_{\text{low}}} \right) \tag{B8}$$

If $V(X)$ is not small, then equations (B1) and (B2) will have to be used.

Putting $a = \ln\{X_{\text{high}}/X_{\text{low}}\}$, we obtain for the estimated value of $\sigma(Y)$ from equation (B6): $\sigma(Y) = \tfrac{1}{4}a$

Therefore it follows from equation (B2) that:

$$V^2(X) = e^{1/16a^2} - 1 \tag{B9}$$

On solving equation (B1) for $\mu(X)$ we obtain:

$$\mu(X) = \mu(Y) + \tfrac{1}{2}\sigma^2(Y)$$

With $\sigma(Y) = \tfrac{1}{4}a$ and $\mu(Y)$ from equation (B5) this gives the following result:

$$\mu(X) = \sqrt{X_{\text{low}} \cdot X_{\text{high}} \cdot e^{1/32a^2}} \tag{B10}$$

## APPENDIX C

### Calculation of the carbonation time for a coating

In [8] the coating is replaced by an equivalent concrete cover:

$$s_0 = \frac{\mu_i \cdot s_i}{\mu_b}$$

With $s_0$ as the equivalent concrete cover, $s_i$ as the coating thickness and $\mu_i$ and $\mu_b$ as the $CO_2$-diffusion coefficients of respectively coating and concrete with respect to air. The total penetration time is in that case:

$$t_L = \frac{(d - \Delta + s_0)^2}{A^2} - \frac{s_0^2}{A^2}$$

with:

$$A = \frac{R \cdot K(46w - 17.6)}{2.7}$$

Working out this relation gives:

$$t_L = \frac{(d - \Delta)^2}{A^2} + \frac{2(d - \Delta) \cdot s_0}{A^2}$$

The total penetration time is also the sum of the two parts, namely the normal concrete carbonation time and an additional time because of the coating. For the present only the second term will be considered:

$$\Delta t_L = \frac{2(d - \Delta) \cdot s_0}{A^2} = \frac{2(d - \Delta)\mu_i s_i}{A^2 \cdot \mu_b}$$

In [8] it is derived that $\mu_b = 360/A^2$, so that follows:

$$\Delta t_L = \frac{2(d - \Delta)\mu_i s_i}{360} = \frac{(d - \Delta)\mu_i s_i}{180}$$

The diffusion coefficient $\mu_i$ is the value for a perfect coating. However, due to deterioration the coating becomes imperfect, and we may take that into account by working with the effective diffusion coefficient $\mu_e$:

$$\mu_e = \frac{\mu_i}{1 + f(\mu_i - 1)}$$

In which $f$ is the degree of imperfection due to pores in the coating. As $\mu_i >> 1$ this can also be written as:

$$\frac{1}{\mu_e} = \frac{1}{\mu_i} + f$$

As $f$ has the order of magnitude of $10^{-5}$ and $\mu_i$ the order $10^6$ it suffices that $\mu_e = 1/f$. In that case $\Delta t_L$ is equal to:

$$\Delta t_L = \frac{(d - \Delta)s_i}{180f}$$

This formulae has been used in Chapter 5.

## APPENDIX D

### Calculation of the expected damage costs of the gallery slab

If $X$ has a lognormal distribution (see Appendix B) with a mean value $\mu(X)$ and a standard deviation $\sigma(X)$, than $Y = \ln(X)$ has a normal distribution with:

$$\sigma^2(Y) = \ln(1 + V_X^2)$$
$$\mu(Y) = \ln \mu(X) - \tfrac{1}{2}\sigma^2(Y)$$
$$V_X = \sigma(X)/\mu(X)$$

The next table gives the mean values and the standard deviations of the service life $t_L$ according to section 5.4 and the values $\mu(Y)$ and $\sigma(Y)$ with $Y = \ln t_L$.

| design alternative | $\mu(t_L)$ (year) | $\sigma(t_L)$ (year) | $\mu(Y)$ | $\sigma(Y)$ |
|---|---|---|---|---|
| 1 | 34 | 28 | 3.27 | 0.72 |
| 2 | 123 | 86 | 4.61 | 0.63 |
| 3 | 103 | 144 | 4.09 | 1.04 |
| 4 | 417 | 474 | 5.62 | 0.91 |

The probability $P\{t_L < t\}$ is calculated according to:

$$P\{t_L < t\} = P\{\ln t_L < \ln t\} = P\{Y < \ln t\} = \Phi(-\beta)$$

with:

$$\beta = \frac{\mu(Y) - \ln t}{\sigma(Y)}$$

$$Y = \ln t_L$$

The next table gives successively the reliability index $\beta$, the probability $P\{t_L < t\}$ and the probabilities $P\{(t-10) < t_L < t\}$ for $t = 10, 20, \ldots 60$ year. The indexes 1, 2, 3 and 4 refer to the design alternatives.

| $t$<br>$\ln t$ | 10j<br>2.30 | 20j<br>3.00 | 30j<br>3.40 | 40j<br>3.69 | 50j<br>3.91 | 60j<br>4.09 |
|---|---|---|---|---|---|---|
| $\beta_1$ | 1.35 | 0.38 | −0.21 | −0.58 | −0.89 | −1.14 |
| $\beta_2$ | 3.67 | 2.56 | 1.92 | 1.46 | 1.11 | 0.83 |
| $\beta_3$ | 1.72 | 1.05 | 0.66 | 0.38 | 0.17 | 0.00 |
| $\beta_4$ | 3.65 | 1.88 | 2.44 | 2.12 | 1.88 | 1.68 |
| $P_1\{t_L < t\}$ | 0.09 | 0.34 | 0.58 | 0.73 | 0.82 | 0.86 |
| $P_2\{t_L < t\}$ | - | 0.006 | 0.03 | 0.07 | 0.14 | 0.20 |
| $P_3\{t_L < t\}$ | 0.05 | 0.15 | 0.25 | 0.34 | 0.43 | 0.50 |
| $P_4\{t_L < t\}$ | - | 0.002 | 0.007 | 0.02 | 0.03 | 0.05 |
| $P_1\{(t-10) < t_L < t\}$ | 0.09 | 0.25 | 0.24 | 0.15 | 0.09 | 0.04 |
| $P_2\{(t-10) < t_L < t\}$ | - | 0.006 | 0.02 | 0.04 | 0.07 | 0.06 |
| $P_3\{(t-10) < t_L < t\}$ | 0.05 | 0.10 | 0.10 | 0.09 | 0.09 | 0.07 |
| $P_4\{(t-10) < t_L < t\}$ | - | 0.002 | 0.005 | 0.01 | 0.01 | 0.02 |

The calculation of the damage costs will be done with $r = 0.02$ and $D_i = 1000$ guilders except for design alternative 1 where $D_i = 2000$ guilders for $t_L \leqslant 30$ years.

General formula:

$$\sum P\{F_i\} \cdot D_i/(1+r)^i = P\{0 < t_L < 10\} \cdot D_{10}/(1+r)^{10} + P\{10 < t_L < 20\} \cdot D_{20}/(1+r)^{20} +$$

$$\ldots + P\{50 < t_L < 60\} \cdot D_{60}/(1+r)^{60}$$

Evaluation for the four alternatives:

$$\text{Alt 1: } \frac{2000 \cdot 0.09}{1.02^{10}} + \frac{2000 \cdot 0.25}{1.02^{20}} + \frac{2000 \cdot 0.24}{1.02^{30}} + \frac{1000 \cdot 0.15}{1.02^{40}} + \frac{1000 \cdot 0.09}{1.02^{50}} + \frac{1000 \cdot 0.04}{1.02^{60}}$$

$$= \frac{180}{1.22} + \frac{500}{1.49} + \frac{480}{1.81} + \frac{150}{2.21} + \frac{90}{2.69} + \frac{40}{3.28} = 860 \text{ guilders}$$

$$\text{Alt 2: } \frac{0}{1.22} + \frac{6}{1.49} + \frac{30}{1.81} + \frac{70}{2.21} + \frac{140}{2.69} + \frac{200}{3.28} = 170 \text{ guilders}$$

$$\text{Alt 3: } \frac{0}{1.22} + \frac{2}{1.49} + \frac{7}{1.81} + \frac{20}{2.21} + \frac{30}{2.69} + \frac{50}{3.28} = 40 \text{ guilders}$$

$$\text{Alt 4: } \frac{50}{1.22} + \frac{100}{1.49} + \frac{100}{1.81} + \frac{90}{2.21} + \frac{90}{2.69} + \frac{70}{3.28} = 260 \text{ guilders}$$