Forty years of theory, design and construction of thin cells

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1 Introduction

The accelerated development which the application and the theory of shells and shell structures has undergone during the past forty years does not stem from a number of simple causes. Historical, social and personal influences and circumstances, together with scientific technologic and economic possibilities and new achievements, have largely determined this expansive development.

The following may be included among the intrinsic causes of this rapid expansion of both the building and the investigation of shells:

- the beauty and serviceability of shells and shell structures in nature;
- the need to build shells and shell structures from aesthetic, technical and economic motives;
- the challenge which the effectiveness of the load-carrying capacity of shells presents to human ingenuity.

The shell forms in nature, such as the shell of nuts and eggs, shells and shields of crustaceans and turtles, snails' houses, skulls of men and animals, diatomites, hides, membranes and (soap) films, etc. have always inspired those who devote themselves to the fine arts and to architecture. In the building of vaults and domes, thin roofs or awnings for arenas, halls and theatres, shell roofs on houses and factories, etc. architecture interprets its effort to accomplish the interweaving of beauty and serviceability, as is often attained in the natural shell forms.

The free spatial form, light weight, great strength and stability of form, or just the great bending flexibility, and hence the suitability for bridging large coverage areas, are properties of shells and shell structures of which technology and engineering have already made frequent use, and from which they have derived great advantage. To begin with a few everyday examples: casks and jugs, drums, barrels and boilers, safety helmets, cycle mudguards, motor-car bodies, plastic light domes, etc.

In the sphere of civil engineering mention may be made of: shelters of industrial premises and dwellings, sports and market hall roofs, north-light shell roofs, barrel-vault roofs, water towers, cooling towers, tanks and silos, arch dams, lock gates, barrages, factory chimneys, pressure pipelines, reservoirs, etc. In aircraft and shipbuilding: the hulls of ships and planes, airplane wings, screws and propeller blades, airfoil boats and submarines, pressure vessels, tanks and pipelines. In the field of mechanical and chemical engineering: engine construction, cylindrical and spherical storage tanks, tank lorries, pressure vessels, pipelines, oil and chemical plant columns, boilers, chimneys, etc.

In space technology and nuclear industries: rocket nose, hull and nozzles, fuel tanks, pressure chambers, re-entry capsules, space and satellite vehicles, reactor vessels, reactor domes, etc.

The great usefullness of shells and shell structures in many technical respects need not *per se* coincide with the highest attainable results from an economic standpoint. But for many applications of shells the economic criterion applies without qualification, i.e. the cost of the shell structure must be competitive, as compared with other structural possibilities, if any.

In the case of shell structures, however, economic motives need not always be decisive. Aesthetic value, uniqueness in design and construction, and the simple lack of a feasible alternative may be as many sound reasons why shells are built. The economics of a shell structure are determined, in the main, by the cost of construction or manufacture in the vast majority of cases. Other cost factors include design and calculation, full-scale or model testing, material specification, supporting and foundation construction, thermal, acoustic and radiation insulation, translucency, gas and water sealing, resistance to corrosion, weather, heat and fire, maintenance and/or renovation, etc.

1.1 *Structural design and analysis*

In the middle of the field of forces evoked by shell building and research, the developments which these two sectors undergo, and the practical, aesthetic, technical and economic demands which are made, stands the structural engineer. The activities of the structural engineer in the field of design and analysis of shell structures tend to overlap with the work of the architect and that of the builder or contractor and the manufacturer, on the one hand, and the sphere of operation of the shell theorist, the applied scientist, the experimentalist and the mathematician, on the other.

For purely technical applications of shells, the principal or the executive in charge of the building operations directly assumes the function of the architect.

From the science and applied science angle, the structural engineer has at his disposal a vast and rich arsenal of theoretical knowledge, analytical and experimental experience in the field of shells and shell structures. He can apply this knowledge and experience to fathom and to solve the practical problems issuing from the design, analysis and construction or manufacture of technically and economically sound shell structures. The frame of reference of the structural engineer must include the ability to move about easily in the three spheres of separate disciplines indicated above and to co-operate harmoniously with their typical representatives. With due reference to time and place, the work of the structural engineer must lead to a well-balanced synthesis of all the determining demands and wishes, all contiguous possibilities and impossibilities and all conclusive pros and cons. A creative synthesis too, aiming at shell structures which are suitable and not devoid of beauty, as well as representing the optimum in respect of design, structural and physical behaviour, construction and manufacture, and the cost.

For making a good structural design and an adequate, but not too complicated calculation – one which does not tend to choke the essentials – of stresses and deformations, it is necessary that the structural engineer has a real insight into and a great knowledge of the behaviour of shells and shell structures under the influence of many and various loads and dependent on all kinds of spatial forms, physical and edge conditions. He can acquire this essential insight by means of practical experience of the building of shells and intuitive perception of their load-carrying capacities, experimental research on models of shells, on a reduced scale and full size, and, last but not least, from the study and evaluation of theoretical and computational models of shells and their mechanical behaviour.

1.2 *Modelling and computation of shell behaviour*

The sphere of the theoretical and computational models of shells is a comprehensive one. Generally, we can make a distinction between an engineering and a mathematical approach to shell theories and the modelling of shell behaviours.

The engineering approach includes:

- shells of linen, coated fabrics, synthetic rubber, fibre reinforced plastics, steel, aluminium and other alloys, wood, ferro-cement, reinforced, prefabricated, prestressed and post-tensioned concrete, etc.;
- shells of uniform, variable and abruptly changing thickness, laminated, sandwich and stiffened shells, cable-net covers, etc.;
- thin and thick, closed or complete, open or incomplete, monocoque and multi-shells, wholostic and additional shell shapes, etc.;
- shells with free, hinged, simply and elastically supported and built-in edges, concentrated, discontinuous and continuous edge supports, etc.;
- uniformly and non-uniformly distributed, concentrated, symmetrical and unsymmetrical loads, etc.;
- static, dynamic and transient load conditions, ultimate and collapse loads, etc.;
- temperature gradients, shrinkage, creep and relaxation effects, etc.;
- cable-net, membrane, bending, yield-line and limit-strength shell theories, etc.;
- elastic and inelastic, yield and post-yield, buckling and post-buckling, stable and unstable behaviour of shells, etc.;
- infinitesimal, small, large and finite-deflection and rotation theories of shell behaviour, etc.;
- minimum-weight, pneumatic, pressurized and vacuum shells, shells of constant strength, etc.;

The mathematical approach includes:

- continuous and discontinuous media, classical and generalized continua, Cosserat surfaces, etc.;

- homogeneity and non-homogeneity, isotropy and anisotropy, orthotropy and aeolotropy of materials, elastically ideal and perfectly plastic solids, viscosity, time-, strain- or work-hardening of materials, thermo-elastic, thermo-plastic and thermo-viscous materials, etc.;
- elasticity and plasticity theory, flow and fracture theories, visco-elasticity and visco-plasticity, creep and relaxation theories, yield-line theory and limit analysis, couple-stress elasticity, dislocation theory and field theories of mechanics, etc.;
- kinematic and intrinsic, linearized, geometrically and physically non-linear theories, buckling, general instability and vibration theories of shells, etc.;
- membrane, bending and edge-effect shell theories, cable-net and momentless shells, inextensional deformation of shells and semi-membrane theory, symmetrical and asymmetrical deformations of shells, etc.;
- shell theories based on *a priori* assumptions regarding the behaviour of shells, such as
  - linear filaments of the shell initially normal to the middle surface remain straight and normal to the middle surface after deformation,
  - points of the shell lying initially on a normal-to-the-middle surface remain on the normal-to-the-middle surface of the deformed shell, and normal stresses in the direction transverse to the shells can be disregarded,
  - the state of stress is approximately plane, i.e. the effect of transverse shear stresses and the transverse normal stress may be neglected, etc.;
- shell theories based on two-dimensional shell model concepts, such as
  - two-dimensional oriented media and generalized continua, in particular deformable and Cosserat surfaces, embedded in three-dimensional Euclidean space, etc.
  - supplementation of compatibility and equilibrium equations with suitable constitutive laws relating the middle-surface changes in length and curvature to the integrated actions across the thickness of the shell, i.e. the stress resultants and the stress couples, etc.;
- shell theories derived from three-dimensional continuum theories by means of
  - thickness-coordinate Taylor series and Legendre polynomial expansions of displacement, stress and strain components,
  - *a priori* estimates for the derivatives of stress and strain components,
  - formal asymptotic expansions in terms of some small shell parameter of displacement, stress and strain components, etc.;
- thin-, thick- and double-walled shells, open and closed shells, spherical, conical, toroidal and general shells of revolution, cylindrical and general translational shells, elliptic- and hyperbolic-paraboloidal shells, conoidal and helicoidal shells, shallow and non-shallow shells, negative, zero and positive Gaussian curvature shells, shells of arbitrary shape, etc.

Shell theories and models of shell behaviour, both mathematical and engineering models, represent a very lively and comprehensive branch of the mechanics of continuous and discretized media in its many forms of theoretical and practical appearance.

The basic skills for developing, apprehending and improving shell theories are the same as those generally applied in general mechanics. The foremost descriptive and modelling aids in the case of shells are: differential geometry, vector and tensor analysis, integral transformation theorems and general principles, calculus of variations, theory of ordinary and partial differential equations, complex variable and function theory, kinematics, dynamics and thermodynamics, numerical analysis, methods of asymptotic integration, finite-difference and finite-element solution techniques, trigonometric, parametric or formal asymptotic and iterative approximation procedures, matrix methods, digital and analog computer utilization, experimental investigation and testing, etc.

These basic tools of analysis in shell theory enable workers in this field to aim at and to realize a great variety of objectives.

1.3 *Implications of shell theory, design and construction*

The structural engineer is the key figure of what we have called the engineering approach to shells and their behaviour. From the scientific-mathematical angle, just as from the "architectural" and the "building" angle, he is supplied with qualitative and quantitative information: data, knowledge, experience gained and insight into the nature of the things. His direct objectives are the design, analysis and building of shells and shell structures which are beautiful and suitable for the end in view, and which are technically and economically sound. As we have already seen, the design and construction of shells call in the first place for a clear understanding of and insight into the behaviour of and the force pattern in shells. The shell designer can do nothing with complicated theories and unwieldy computational methods. They confuse his view of the structural objectives which must constitute the basis of any important design. For that reason in particular, the design calculations must not be complicated or unwieldy.

In the second place, insight into the problems is also required for the proper selection and development of an adequate method of final or check computation, or to think out and work out a well-directed experimental investigation.

We have in mind, for instance, the final calculation of a shell structure by means of the finite element method, in which the choice of the dimensions of the element in a certain domain of the shell must be attuned to the rate of variation of the ultimate solution in that domain.

Or another example: experiments with models on a reduced scale are subject to a number of rules which in the general case make demands which are incompatible with one another. The connection between the various effects in a structure which is to be examined must determine which model rules should be strictly observed and which rules are to be set aside deliberately, i.e. though playing their full part in the interpretation of the results, to ensure the attainment of the preconceived intention of the model tests, or not to endanger it.

In the following, we propose to consider, in succession, the principal, trendsetting fates and facts during the past forty years of:

- shell theories, the modelling and computation;

- the design, and
- the construction and fabrication of thin shells,

from the point of view of the possibilities for direct use and proper application of the theoretical results and practical achievements, by the architect, the designing and the building engineer in the day-to-day practice of his profession. These observations are by no means intended to cover the whole ground, throwing light on every facet of the subject, or to be systematically presented.

In any process of design and analysis, real insight and wide knowledge, side by side with intuition and inventiveness or creativeness, are the pillars which support these typical activities of the human mind and its powers of expression.

This applies to a marked degree to the design of shells and shell structures. Insight into the characteristic behaviour of and the effective force pattern in shells, as well as wide knowledge of the types, loads and edge conditions of shells, together enable the (structural) architect and the designing engineer to make shell designs which:

- answer the purpose in view,
- can be built or fabricated, and that
- without unnecessary expense.

Finally, there remains the inspection, investigation and judgement of shell failures and damage to shells, the exposure of the cause of the failures and damage and/or drawing up of recommendations for their prevention. Within this framework too, great expertise, based on knowledge of the subject and experience is required of the practising engineer.

Not infrequently, shell failures and many minor and major difficulties and complications with shells prove to be attributable to the design and the design stage of the shell or shell structure. We have also indicated above that difficult and unwieldy shell analyses are often the result of a faulty or unpractical design. Thus this indicates that also within the sphere of shell theory and practice the greatest possible stress must be laid upon the making and devising of a good design, and that much, if not all, depends upon it. This makes it clear, also for the sphere which comprises the design, analysis and construction of shells and shell structures, that it is no exception to what is generally perceived: *design is the core of engineering.*

2 Thin shell theory

The theories of thin shells and shell structures: modelling, computation and experimentation, and the verification of hypotheses and theories generally rely on the following fundamentals.

A shell is defined as a body having one dimension – the thickness – small compared with the other two dimensions. The general shape of the shell wall can be represented by a curved surface in space, usually termed the reference or midsurface. Thus the shell geometry may be determined from the shape of the reference surface, the shell wall thickness, and the shape of the boundary or edges. The analysis of shells is based on the fundamental laws of solid continuum mechanics. The assumptions listed below (which are called Kirchhoff assumptions) are generally admissible because of the thinness of the shell wall:

a. Straight lines normal to the shell reference surface before deformation remain straight and normal after deformation, and
b. Stresses normal to the shell reference surface are negligible in comparison with the other stresses in the shell wall.

Integration through the thickness permits a two-dimensional formulation of the theory of shells in terms of the coordinates of the reference surface. This formulation transfers attention from stresses to crosssectional forces and moments which are fundamental quantities in shell analysis.

Solutions of problems in the above sphere were attempted as early as 1744 by Euler. Such problems were among those which motivated the formulation of the general equations of elasticity by Navier in 1821. In 1850 Kirchhoff developed the theory of plates, and this theory was used by Aron in 1874 to develop the first theory of shells. Some inaccuracies in Aron's theory were found and corrected by Love in 1888. The theory of shells based on the hypotheses of Kirchhoff and the development by Love is not unique, and many other formulations have been developed. In the 1960s, criteria were defined for judging the accuracy of linear shell theories, and consequently it was shown that most other theories differ from Love's by insignificant terms only. Finally, at the end of that decade the general equations of linear thin shell theory were rigorously derived from the general three-dimensional equations of linear elasticity theory on the basis of asymptotic expansion with respect to a small thickness parameter.

In applications, progress was made principally in the area of some specialized problems involving surfaces of revolution - cylinders, cones and spheres. In each case, the problem was made somewhat more tractable because of the nature of the surface: constant curvature in the case of spheres, and zero curvature in one direction in the case of cylinders and cones. Nevertheless, specific numerical solutions were still difficult to achieve, except for some geometries and load and edge conditions. At the same time, the linear problem of stability of these shapes was solved, equivalent to the Euler solution of the problem of stability of a column.

The period immediately after World War II was characterized by the development prompted by the needs of the construction industry. Geckeler, Finsterwalder and Dischinger in Germany, Torroja in Spain, Aas-Jacobsen in Norway, Jenkins in Great Britain, Bouma in the Netherlands, all contributed most significantly to the theory of shells, analytically and experimentally, as a result of the need for a practical formulation of shell problems occurring in engineering and construction.

Because of the complexity of shell theory, only the simplest cases could be solved before the advent of the digital computer. However, the analysis of shell structures has expanded in quantity and scope as the capabilities of the computer have grown. A study of the relevant journals indicates that initially the application of the computer to shell analysis was gradual. In the early 1960s computers were used for some problems, but it was not until the mid-1960s that the words "computer solution" appeared in titles and that the operations involved were tailored for calculations carried out by computers rather than by hand.

An indication of the relative difficulty of solving shell problems is represented by the order of the differential equations involved in shell theory. Many difficult problems in mathematical physics deal with equations of second order in the coordinate variables. The equations of plate theory are of fourth order. The equations of shell theory, however, are of eighth order. Another consideration which may complicate the solution of shell problems is the fact that nonlinearities are often important. Usually, the elastic deformation of solid bodies leads to small displacements and linear differential equations. In shell problems, however, the shell wall may displace several times its thickness under load, and in this circumstance, even though the strains may remain small, as is usual in solid bodies, they depend nonlinearly on the displacements.

Various types of problems must be faced by the structural designer of shells. In general, the strength and stiffness of the shell structure are of foremost importance. Assessment of strength and stiffness requires analysis of the deformations of and the forces and moments in the shell wall under all pertinent loading conditions and comparison of these values with appropriate allowable values. Shells are often subject to bending and compression and are, therefore, prone to structural instability (buckling). Where oscillating load inputs are present, knowledge of the vibration behaviour of a shell structure is of vital importance to prevent resonances which might damage the structure. If the structure is subjected to very suddenly applied (or dynamic) loadings, the transient response could be of importance.

2.1 *Buckling of shells and its growing importance*

Over the last twenty-five years the problem of buckling of shells has grown in importance because of several interrelated developments. The spans of shell structures - or, rather, the areas covered without intermediate supports - have become very large. It has become fairly common to see large halls or sports arenas with a plan diameter of 100 or even 200 metres.

At the same time, the requirements of economy and the desire to save materials led to the use of ever thinner shells with a very small ratio of the weight of structural material to the unit of area covered. In terms of shell design these developments meant that the ratio of shell thickness to its radius of curvature became significantly smaller, changing from approximately 1 : 150 to as little as 1 : 500, or even less. In consequence, where former shells had a thickness largely dictated by the requirements of construction technology, and hence had a great reserve of strength and stiffness, the new shells may have no such reserve and their buckling capacity may well be the determining factor of shell design: thickness and, indeed, its very shape.

In general, three different manifestations of shell buckling are possible: bifurcation, maximum load and snap-through, as shown in Figure 1. In each case, it is possible that the shell may experience an increase in its load-carrying capacity in the postbuckling

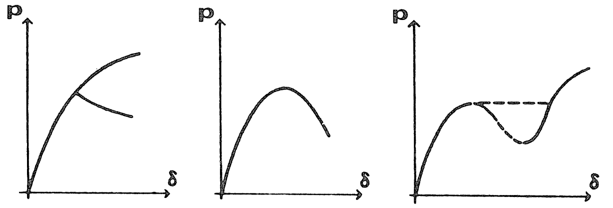

Fig. 1. a. Bifurcation. b. Maximum load. c. Snap-through.

range. However, the associated deflections are large, and usually inadmissible in structures of importance in building applications.

The mode of buckling may very not only from shell to shell; it may vary for a shell of given geometry and load, subject only to a change in some parameter.

The same dependencies govern the ultimate load-carrying capacity of shells after the occurrence of buckling: the post-buckling behaviour of shells.

The importance of the post-buckling behaviour lies in the fact that on it depends the sensitivity of the structure to initial imperfections. Thus shells are divided into two classes: those which are and those which are not sensitive to initial imperfections. A shell sensitive to initial imperfections suffers a significant reduction in buckling capacity. The importance of this to a designer is obvious, in that the factor of safety of a structure sensitive to initial imperfections, based on the buckling load of an ideal structure, must be significantly greater.

Because many shells are built of concrete it may be appropriate to mention some of the additional factors which influence – and reduce – the buckling load. These include the creep effect, the effect of cracking of concrete as well as the amount and the location of reinforcement, the effect of plasticity of materials, aggregate interlock, and the effect of bond between the reinforcement and concrete.

The investigation of buckling over the years has developed along the following paths:
- analytical solutions;
- experimental solutions;
- computer-based solutions.

Analytical solutions

As in the case of all problems of mechanics, the formulation entails composing a mathematical model, setting up the governing equations – usually a system of differential equations – and then solving these equations exactly, if possible, or approximately. The term “exactly” should be understood in the sense that the solution represents exactly the behaviour of the assumed mathematical model.

In the field of buckling of shells, classical solutions in the case of spherical and cylindrical shells represent essentially an extension of the linear Euler approach. As has been known for some time, this approach is inadequate, on several grounds. Firstly, the bifurcation mode is not the only possible one in shells. Secondly, the classical Euler model provides no information about post-buckling behaviour, and hence about a factor of safety. To gain this knowledge, it is necessary to examine the problem, including the effect of initial imperfections, using a geometrically nonlinear formulation, with the attendant difficulties in obtaining a solution.

Experimental solutions

Experimental analysis, of unquestioned importance in all structural problems, has proved of particular significance in studying the problems of buckling of shells.

Firstly, it was thanks to the experimental investigations that the importance of initial imperfections on the buckling capacity of shells was finally understood. Secondly, the great difficulties in obtaining solutions of nonlinear problems, and in composing the mathematical models incorporating all of the important material parameters, meant that, for several decades, experimental analysis based on model testing of shells on a reduced scale was possibly the only practical way of obtaining information about the buckling capacity of specific shells.

In fact this is still the case, to a certain extent, in investigating the problems of the stability of reinforced concrete shells, since the important material properties such as concrete cracking are difficult to model mathematically.

Computer-based solutions

Significant progress in obtaining solutions of problems of buckling of shells occurred after the advent of computers permitted a systematic attack on problems involving nonlinear formulations.

Finally, it is important to note that safety from buckling represents only one aspect of stability, while stability represents only one aspect of structural adequacy. Structural adequacy, in turn, is only one of the objectives of design as stated in the foregoing. The art of designing is as important as is its science.

2.2 *Vibrations and general dynamic behaviour*

Shell structures may be exposed to dynamic loads: oscillations, wind gusts, earthquakes, etc. To avoid resonant conditions which might cause structural damage, it is important to know the natural vibration behaviour of such shells. The equations required to determine vibration behaviour of shell structures are linear and homogeneous and, in fact, are quite similar to the equations required for bifurcation buckling calculations. The eigenvalues of the system are now the natural frequencies of vibration; one key difference between the vibration and the bifurcation buckling problem lies in the fact that several natural frequencies of vibration are of importance to the designer whereas, generally, only *the* lowest buckling load is of interest.

Some illustrative results for vibration of a shell structure are shown in Fig. 2, where natural frequencies for a simply supported cylindrical shell are plotted as a function of the number of axial half waves m. Each value of m, the number of axial half waves, and n, the number of circumferential waves, determine a natural frequency for the cylinder.

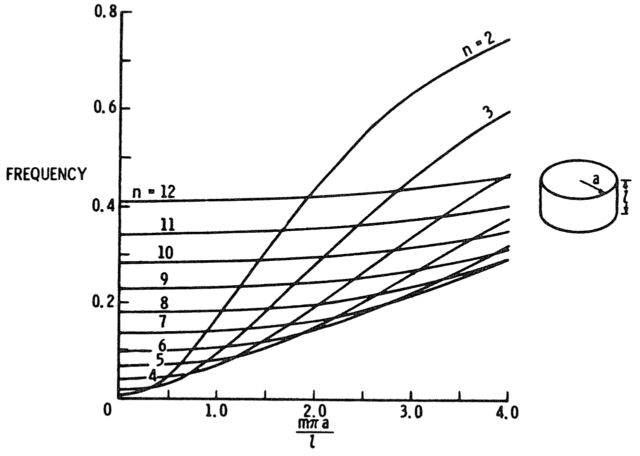

Fig. 2. Nondimensional natural frequencies for a simply supported cylindrical shell; thickness-radius ratio is 0.01; *m* is the number of axial half waves; *n* is the number of circumferential waves.

For this simple problem elementary functions satisfy the differential equations and the edge conditions, and exact results are easy to obtain.

For particular values of the axial wave number parameter, the natural frequencies tend to cluster together; in this case they cluster near the lowest frequency. For more complex shells, where numerical methods are required, the closeness of the eigenvalues can lead to numerical difficulties such as slow convergence or failure to determine all frequencies in the range of interest.

Another consideration which may increase computational difficulties in shell vibration problems is also illustrated in Fig. 2. Note that the lowest frequencies do not necessarily correspond to the lowest wave numbers in contrast to the behaviour of simpler structures such as beams and plates where the lowest frequencies almost always are associated with the simplest wave forms.

Shell structures are sometimes subjected to very suddenly applied or dynamic loads. In such cases the inertia of the shell may be important, and calculation of the transient response of the structure may be necessary to determine whether or not forces or deflections remain within acceptable limits. In transient response problems an additional independent variable, time, is introduced.

From the computational standpoint, a significant feature of transient response problems in shells is that, effectively, a complete static force and deflection analysis must be performed at each time increment, and often many time increments must be taken to establish meaningful results. Computation times for transient response problems are, therefore, substantially longer than for corresponding problems in static analysis, buckling, or in vibrations.

2.3 *Computerized shell analysis methods*

In problems as complicated as those dealing with shells, almost all methods of analysis will involve numerical calculations.

There are three important approaches in numerical methods of shell analysis: the finite element method, the finite difference method, and the forward integration method.

In the finite element approach, the shell structure is broken up into a finite number of relatively simple geometrical and physical elements and the set of equations for each element is solved approximately, except for a group of constants. These constants are determined to satisfy conditions of continuity and/or equilibrium among the elements. The use of a variational procedure automatically provides a best choice of the finite element equations governing shell behaviour within the limits assumed for the elements.

In the finite difference approach, derivatives in the equations are simply replaced by difference expressions and integrals by sums. In the forward integration method, the problem is converted into an initial value problem and the solution is projected forward in space by a technique such as the standard Runge-Kutta method.

All three approaches give solutions approaching the exact solution if enough properly defined elements or enough differences or integration stations are used.

At present, the finite element method by far dominates the scene. For practical problems, no single element type has been found that can be used to advantage in all cases. The better performance of some higher-order elements is often outweighted by their greater complexity in application. Conversely, the use of simpler flat plate elements with a coarse mesh layout may lead to unsatisfactory results. In the absence of an ideal element, many general-purpose computer programs have been developed with families of elements from which the analyst can choose those which best model his problem based on his experience, intuition, and engineering judgement.

Some of the more widely used and available programs at computer centres in the United States and Europe are: ADINA, ANSYS, ASAS, ASKA, EASE, MARC, NASTRAN, SAP, SESAM, STRUDL, GENESYS, GEMINIX/DIANA, IDEAS, etc.

The finite element analysis of thin shells was introduced in 1961 using flat rectangular plate elements to analyse cylindrical shells. This was soon followed by the use of flat triangular plate elements for thin shells of general form. Curved thin shell elements were introduced in 1966, and these were followed in 1968 by the use of three-dimensional isoparametric solid elements.

During the past 15 years, many new or modified elements of these three types have been proposed in an attempt to improve their performance and accuracy. All of them have certain advantages and disadvantages, and the analyst must trade these off, one against the other, when making a selection for a particular problem.

Element modelling has been based on:

1. assumed displacement fields;
2. assumed force fields, or
3. a mixture of 1 and 2.

However, the preponderance of elements being used in today's general purpose programs are based on assumed displacement fields.

In the early works on displacement field elements, it was stated that to ensure convergence with a refinement of element mesh, the assumed element displacement fields should satisfy:

1. continuity at the inter-element surfaces;
2. include constant strain states; and
3. include rigid body displacements.

Very few shell elements satisfy all three of these requirements completely, yet they give satisfactory results for most cases. Much research has been directed toward investigating these problems and, also, to improving the performance of elements by the use of reduced or selected integration.

There is a need for correlation and verification for the various finite element models being proposed, especially for nonlinear problems. Standard numerical test cases with known and reliable experimental data need to be established for numerical comparisons in nonlinear applications. The whole subject of effect of the formulation and size of elements and of load steps and convergence criteria on accuracy or results obtained for nonlinear applications still needs much more study.

Numerical methods have the advantage of very general application; that is, a formulation may be applied to wide varieties of problems with minor modifications. Practical shell problems invariably have complications such as variable thickness, wall stiffening, a variety of loading conditions or combinations of loads, a variety of edge conditions or complicated shapes not easily specified by equations. Such complications are almost impossible to handle by analytical methods, but can be handled in almost routine fashion by numerical methods.

There are some disadvantages to numerical methods. Obviously, there must be a computer of adequate capacity available. The output of a computer using numerical methods is often a vast array of numbers, and this situation sometimes obscures trends that might be obvious from an algebraic formula. Finally, numerical methods are sometimes difficult to check and limiting cases may not be as easily obtained as with the use of analytical methods.

Numerical methods could not be used extensively until computer capability had been increased to present-day levels. Only now is the shell analyst able to use general purpose computer programs that handle wide classes of shell configurations. Of course, the computer also has expanded the analytical capability.

However, computer supported analysis of shells will be no more accurate than the theory on which the analysis is based. Therefore, a few major weaknesses of theory should be mentioned.

A criterion for establishing the relative merits of various versions of linear shell theory exists, but a corresponding criterion for establishing the relative merits of the various versions of nonlinear shell theory has not been derived as yet. With the number of such theories growing and their extreme complexity undiminished, the analyst needs some convenient basis for a rational choice.

Experimental research has not kept up with the ability to solve theoretically shell problems of great complexity with the advanced numerical methods and the help of the computer. Therefore, the application of the latter by the practical designer will be inhibited by his natural reluctance to use unproven methods. There is an obvious deficiency here, and time urges that experimental programs be accelerated to study the limitations of computerized methods.

3 Thin shell design

In the human activity which is called "design" of structures in general, and thin shells in particular, the attainment of a number of different goals is aimed at. Among these the most important are:

- serviceability and functionality;
- reliability, safety and durability;
- constructability and maintainability;
- economy;
- aesthetic value.

All structures, to be successful, must meet these objectives. The tenet of the necessity to design and build well is best illustrated by the many successful and delightful shell structures built in the past in many parts of the world.

There are a number of reasons why shells play such an important role in modern building. From the utilitarian point of view, they permit large areas to be covered without intermediate supports, and with reasonable economy. Such large spaces are almost a condition *sine qua non* of our times, with their insistence on very large gatherings of a commercial, political, sport and religious nature. Thus there is an ever growing number of ever larger projects of stadia, etc. showing their silhouettes on the skyline of cities. The structure of such large-scale projects defines to a very large extent the space within them, as well as the outside space which surrounds them.

The basic ideas underlying design as seen by a great shell designer, Felix Candela, were expressed by him on the occasion of the twentieth anniversary of the IASS at Madrid in September 1979: .... quote, "I would like to address myself to three ideas that I consider essential for the design and construction of any kind of structure. Both processes cannot be separated. The ideal would be to return to that "Golden Age" of the construction history, when the whole work was carried out by one single person, by the ancient Roman's "Magister Operis". Unfortunately, the complexity of modern life, makes it impossible. We are now forced to separate the old unified architectural labour in many different trades and the architect became the coordinator of specialists.

The first of the three concepts I want to speak about is the one of SCALE. Each kind of structure has a different limit for the span it can cover. This limit depends, of course, on the shape of the structure, and on many other factors. We all know, more or less, the limit of span that can be covered with a flat slab, or with usual beams. A folded plate, for instance, has a much lower limit than a long cylindrical vault, or than a short one, or than a dome, etc.

But usually, and from a purely subjective standpoint (I have not the slightest intention of being objective on this point) 30 metres seem to be the sensible limit for reinforced concrete shell structures. Of course, larger shells have been and will be built, but precisely on this boundary between what is possible and what is reasonable rests one of the greatest problems of our civilization: everything possible has to be done, and what happens later does not matter; we may use a nuclear weapon, and see what happens later. I mean, nobody worries about the possible consequences or results of their investigations or acts.

This matter of the scale is quite important. As Galileo said, size is always limited by physical laws. I may say, for my own part, that as a structure becomes bigger, the problems related to it became bigger too, but growing in geometrical proportion.

The second of the concepts is that of ECONOMY and by it I mean not only building costs but the previous analysis as well. Obviously, these last ones should represent a very low percentage of the whole construction costs. It does not make much sense to carry out calculations which in themselves are as expensive as the subsequent building process.

The last idea or concept, is, perhaps, that of BEAUTY, a concept which, as it cannot be measured, is never mentioned in any of the technical texts or articles. Beauty is for me a very important matter which does not necessarily cost any money. Its achievement requires only some talent or perhaps good luck. In many cases it is just a question of proportioning and sensitivity in the treatment of details. It cannot be measured nor taught, but, anyway, it is of the greatest importance; many of the structures shown and explained by Mr. Isler, with their extraordinary beauty; what I mean: that it is something that we must attempt to incorporate in any kind of structure ...," unquote.

Thin shells have been built in a great variety of shapes, governed by many influences. A shell is a structure which, in the large, has the form of a surfaces or a combination of surfaces in space. The thickness of a shell is small compared to its other dimensions, including its radii of curvature. It is of interest to note that in the shell structures that have been built the ratio of the shell thickness to typical radii of curvature varies from approximately 1 : 24, in the case of the Roman Pantheon, to approximately 1 : 500 and less in some modern shells. The same ratio in the shell of a hen's egg is approximately 1 : 60. Since a plane is a particular case of a surface, a combination of planar elements in space meets the definition of a shell given above. This means that the so-called folded plates or, more generally, faceted plate structures, are a special case of shells.

The definition above refers to a shell only as a surface in the large. No mention is made of the properties of the surface in the small, in the sense of the manner in which the material is distributed over the surface. In a typical shell, the material distribution is smooth, and the thickness of the shell is constant or smoothly varying from point to point. In ribbed shells additional material is placed along certain preferred lines - the ribs. Finally, in the case of reticulated shells, all the material is placed along the rib lines, leaving the spaces between the ribs open, possibly to be filled with glazing or some other non-structural material.

However, in the large, such reticulated or ribbed shells still have the form of a surface or a combination of surfaces in space. Shells of this type have been used to a great advantage by such masters as Torroja and Nervi.

The geometrical shape of shells endows them with their most important characteristic: the strength and stiffness which permit them to cover large areas without intermediate supports. Further, since a shell is thin, only a small amount of material is required for its construction; utilized to transfer loads as well as to enclose space.

Thus thin shells can be expected to be quite economical as compared to other structural systems of similar spans. However, from the point of view of structural shape the most important characteristic of shells is the major influence that they exert on the definition of space. The large spans, the generally curved surfaces perceivable by the observer, all tend to define space to a greater extent than, perhaps, can do other structural systems - both the interior space within and the exterior space without the building. This tends to make thin shell structures the dominant element of the architecture of a building or structural project, as has been recognized by many designers.

3.1 *Factors which influence the shapes of shells*

A great many geometrical shapes have been used in the design and construction of shell structures. Although this may be sometimes difficult to believe, each structure was designed - in the broad sense of the word - by someone. In each case someone - an individual or a committee - made the final choice of the shape. In each case the selection was the final result of a chain of events, of a decision-making process, conscious or subconscious. The final choice is inevitably influenced by a number of factors - tradition, nature, function of the project, design theories, considerations of economics and technology of construction, structural theory - all these play a role.

Tradition - The structural shapes previously used in theory, design and construction affect the shapes that follow. Possibly the best example is the dome of the Pantheon, which influenced innumerable buildings of later eras.

Design theory - By this term is understood an established body of precepts generally accepted as valid and binding at a given time, in a given locality or area, or at least accepted as such by the designer of a given project.

Practicability and economics - Here, the emphasis is on constructability. Is the shape of the shell such that it can be built, and built within a stated budget? There are all too many projects which, beautiful and functional though they be, are never realized because the cost of construction exceeds the available resources, or because the required technology does not exist. This tends to work against innovation - a form proven practical in the past, a traditional form, is more likely to lead to a succesful completion of a project; sad though it may be, practice and reality tend to remain on the conservative side.

Building codes - In the modern era of construction the form of buildings is dictated to a surprisingly large extent by the building codes and standards. This is more pronounced, perhaps, at the level of details, but it is felt quite strongly at the level of the overall form also.

Theories of structural mechanics - Clearly, the requirements of structural analysis based on theories of structural mechanics do exert an influence on the shapes of structures, including thin shells. This influence will be discussed in more detail later on (see Section 3.4).

3.2 *Modern shell shapes*

A great many of the shapes which occur in modern shell structures can be traced to the traditional shapes - domical, barrel vaults, cloister vaults, and groin vaults. Nevertheless, all these were subjected to modifications resulting from the unique needs of the society, and from the technology of the contemporary construction industry. Some of these modifications were but minor, so that the resulting shape is a clear derivative from tradition.

Others were fundamental in character leading to shapes which should be classified as new, the influence of tradition barely perceptible.

The modifications referred to above affected the shape of shells in the large. In addition, there also occurred modifications of the shape in the small. In many cases, the modifications in the small occurred in consequence of the methods of construction utilized in any given project. For example, the reticulated shells of Nervi, with their delicate tracery of intersecting ribs, are a vivid reminder of the importance the method of construction can have.

Similarly, the folded plate and faceted plate structures can be thought of as being shell structures in the large, as noted before. In the small, however, such structures present to the observer a series of plane, faceted surfaces, and the overall effect is markedly different from that presented by the smooth surface of a "classical" shell. As in the case of ribbed structures, the initial stimulus for their development emanated from the needs of the construction industry - the search for economical methods of construction.

Another modification of the traditional dome is possible by changing the shape of the generating curve. The traditional shape was generated by a segment of a circle. This can be changed to be a parabola, an ellipse, etc., thus changing the shape of the dome.

The same type of shape modification occurs if the generating curve is concave, the resulting change in the shape, its visual characteristics and its structural behaviour being much more drastic. Since the product of the principal curvatures of such surfaces is negative, the surface is said to be "anticlastic". Except for some minor examples, such as the "onion" domes of the Eastern rite, or some domical forms of the Near East, and except for fragments of surfaces encountered in Roman architecture, the anticlastic surfaces did not appear in traditional architecture.

In modern construction these shapes have become familiar through their use as cooling towers (Fig. 3), the characteristic hyperboloid of revolution shape being dictated by the functional demands of the draft. Towers up to 200 m in height have been built.

These are special cases of the general class of shell forms of negative double cur-

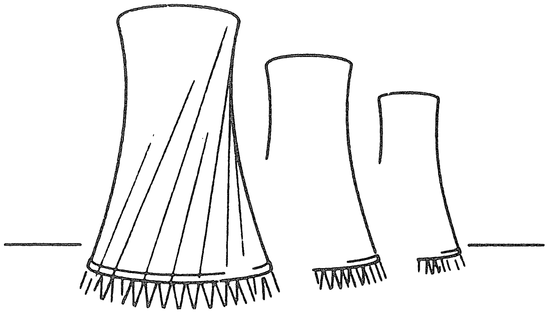

Fig. 3. One-sheet hyperboloidal cooling towers.

vature, i.e., the anticlastic surfaces. The most common among these shells is the hyperbolic paraboloid. These, and in general all anticlastic shapes may be considered the only new structural shapes of modern time (Fig. 4).

The decisive progress in design and construction occurred when Candela, then practising in Mexico, designed and built a great many projects with the hyperbolic paraboloid as the dominant shape. His example spread very quickly to other countries, including the United States – the use of this shell shape became almost a cult with some designers, and it might be useful to examine the reasons for its rapid acceptance.

Firstly, the cost of construction of hyperbolic paraboloidal shells of the general proportions used by Candela proved to be remarkably competitive with other structural

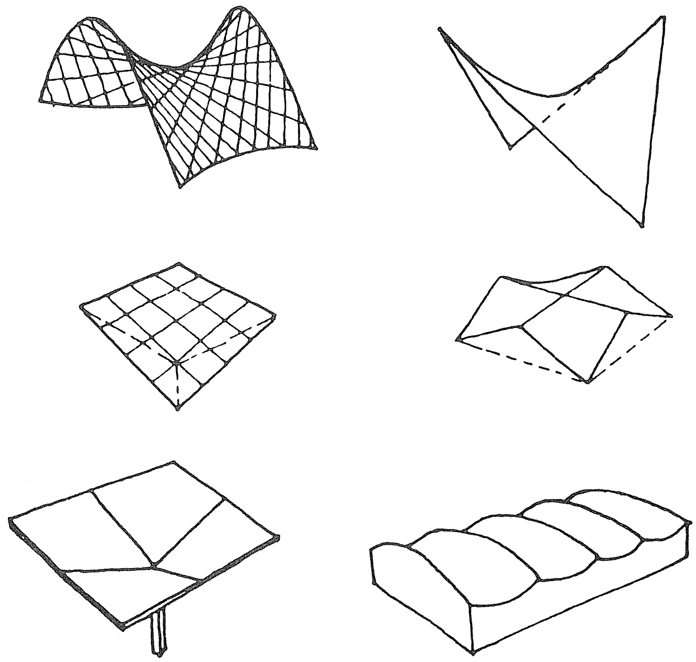

Fig. 4. Hyperbolic paraboloid shapes.

systems. A large part of the reason for this was the fact that the formwork for such shells can be built with the aid of straight planks, since the hyperbolic paraboloid is a doubly ruled surface – the cost of the formwork being a large part of the cost of construction of a concrete shell. In his ability to construct thin shells economically, Candela was aided by the general state of technology of the area where his projects were being realized. Nevertheless, the hyperbolic paraboloidal shell proved to be economic and competitive also in other areas, under differing technological conditions.

Secondly, the analysis of hyperbolic paraboloidal shells was placed on the level of a statically determinate system, so that it could be tackled by most competent engineers. This seemingly removed the need for the highly sophisticated analytical and computational techniques necessary in the design of other, more traditional shell forms. This was true for hyperbolic paraboloidal shell dimensions utilized by Candela, not necessarily for the larger shell projects of the same type.

Finally, and what was perhaps most important, the geometry of the hyperbolic paraboloid lends itself, singly or in combinations, to the creation of a great wealth of endlessly new shapes and endlessly new spaces. This had and still has an enormous appeal to many shell designers and modern structural architects.

3.3 *Methods of determining the shapes of shells (formfinding)*

The following are among the methods used to find new (and rediscover the traditional) shapes of shells:

Geometry	Analytical methods
Mathematics	
Membranes under tension	Experimental methods
Pneumatic membranes	
Flowing shapes	
Hanging reversed membranes	
Sculptured shapes	Other methods
Simulation of shells in nature	
Etc.	

According to the architectural demands and the idea in mind, the designer can use one of these methods to approach, elaborate and finally realize the formfinding objective. Modern man is very fond of having something new and originally special, if it is no more expensive or more risky than what he already has within his reach.

Light, thin shell structures and the above formfinding methods are, in the hands of creative as well as receptive structural architects or engineers, the ideal means to this end. When working steadfastly on these lines – deepening the understanding, following attentively the methods in their various diversifications – a virtually unlimited area of possibilities is unfolded, leading to a vast variety of new, economical, durable and beautiful shell structures of an extreme simplicity.

In this respect, the following statements are relevant:

- formfinding is one of the most important factors in shell design. It is among the utmost important;
- each of the methods above leads to an unlimited number of shapes;
- the method of the hanging reversed membrane seems to be the most efficient one;
- the deformation pattern - calculated or measured on scale-models or full-scale - might be a criterion of qualification of the shape;
- a high precision in formfinding investigation is worthwhile for shells of medium span (30 m), but is indispensable for shells of large span.

Indispensable conditions for successful design and construction in the sphere of new shell shapes are:

- exact and openminded work in the experiments;
- experienced and skilled contractors who can build what is designed. They also must have specific knowledge; the materials required and the willingness to approach new problems with new ideas.

3.4 *The influence of theory on shell shapes*

Clearly, the development of structural mechanics during the second half of the nineteenth and during the twentieth century permitted the use of structures of a much greater scope and size then was possible before. However, in this section primary interest will be placed on the shape of the structure, more specifically the shape of shells, not on their size. The discussion of the preceding sections indicates that the theories of structural mechanics have an influence on the development of shell shapes - in fact, by extension, of all structural systems. Several different modes of this influence can be distinguished.

1. Theory indicates that a given shell shape is structurally advantageous and should therefore be used. This is essentially design based on tradition, with the theory confirming existing tendencies. Examples of this type of influence are as follows:
   - shape in the large: use of doubly curved shells because they are more likely to transfer forces through in-plane membrane action;
   - shape in the small: use of reduced curvature in some areas; the thickening of a shell and the use of edge beams; the use of ribs (or reticulated shells) to improve stability performance of a shell;
   - the use of shapes derived from experimentation, as was currently done by H. Isler (see preceding Section).
2. Theory shows that solutions are difficult to obtain, hence the shapes are not being used, or are avoided. This is essentially the case of "play safe" design.
3. Theory provides design aids in the form of closed-form solutions, tables, graphs or computer programmes. This might be described as the "least work" approach to design. Thus a circular cylindrical shell is more likely to be used than a barrel vault of some other shape, and a spherical dome is more common than a dome formed with

the aid of some other generating curve. It should be remembered that this approach is not always safe. Design tables appropriate to a particular range of shell dimensions may become invalid when that range is exceeded (inherent scale limitations).

All these ways in which the influence of the theory of structural mechanics on the development of the shapes of shells is felt have this in common: in each case the shape is devised first; the theoretical studies of the structural implications of this shape come later.

Thus, structural mechanics is not a leader in the development of shell shapes, it is a follower.

Conversely, the development of theories of shells was spurred by the practical problems of the shapes important in practical applications. The only possible exception is the experimental finding of shell shapes, but even this technique initially was used before a theory of shells could point out the path. The shape itself comes largely from the totality of human, social, technological and economical needs and conditions of a given time and place. Via the *act* of designing of an individual designer, these needs evolve into a geometry of the (structural) shape and into a specific spatial shaping of the inner and outer appearance of the building, the structure, etc.

4 Thin shell construction

Concrete shell structures, in-situ or precast, reinforced concrete, prefabricated, pre- or post-tensioned, etc., which have been developed as thin-walled roofings to cover large spaces, are one of the finest technological outcomes of the decades since World War II.

In the construction of large reinforced concrete shell structures, many innovative techniques have been utilized to rationalize and economize the construction. One of the main difficulties in concrete shell construction is undoubtedly the formwork, they are complex and represent a relatively high percentage of the total costs. That is one of the reasons why concrete shell structures are reputed to be very expensive.

That this is not necessarily true is evidenced by a number of outstanding shell designers and builders who provided the exceptions to this seeming rule. Their success, however, is invariably preconditioned by the following prerequisites:

- sound design;
- proper shape, making use of modern scientific methods for shape finding;
- knowledge of limits, allowing minimalization without taking too high risks;
- execution by highly trained working groups;
- continuity in the work of these groups (first training period is most expensive);
- rational formwork (good design, high quality, mobility for repetitions or alterations);
- high quality of concrete work (to avoid maintenance or coating, etc.).

Nevertheless one of the main factors for economy remains the formwork. Thin shell structures need a very small amount of material; 100 mm of concrete is not expensive; 100 N of reinforcing steel per projected square metre is not expensive either. What is not cheap traditionally is the formwork. Shells are curved, must be curved.

Several systems of formgiving elements have proved to be expedient for shells. These include the following.

Flexible beams - Trusses with adjustable diagonals can be bent to any curvature. Once the screws and bolts are tightened, they form stiff curved girders, which can be placed on the scaffolding.

Vertical boards - Simple wooden boards, having been sawn curved on one side, when placed vertical give inexpensive formwork. This method is used for single applications, where there is no re-use of the form.

Laminated binders - When several re-uses of formwork are intended glued laminated wooden binders can be used. They are light, resistant to weather and need the least amount of manhours to be placed and removed. If protected by a good varnish and stacked in a dry place, they can endure many applications over many years.

Their comparatively high production costs are largely recouped by the number of re-uses, which can range up to 15–20 times.

Inflated fabrics - For smaller shell buildings of the pneumatic-shape type inflated fabrics give a quick procedure.

The inflated fabric is the most expensive. It costs many times more per covered square metre, but when handled carefully it has many times more applications than rigid systems and is very quick in handling. It therefore becomes economic when re-uses over 50 times are to be expected.

The shell construction technique called “lattice-shell-technique” was developed and used for the in-situ construction of ferrocement hyperbolic paraboloidal shells. With this special technique no scaffolding nor forms are required, which means a significant advantage since the erection of formwork for double curvature shells is difficult and represents an important percentage of the structure total cost.

The main steps of construction of a ferrocement lattice-shell roof can be summarized as follows:

- erection of a permanent or temporary external frame following the contour of the roof;
- construction of the lattice made of steel wires tensioned in several directions between the supporting contour frame beams;
- placing of welded wire meshes above and below the lattice. These layers of mesh and tensioned steel wires constitute the shell reinforcement;
- installation of a polythene sheet under the reinforcement to retain fresh mortar during casting. To support this sheet another welded wire mesh is temporarily fixed underneath;
- casting of cement mortar;
- after hardening of the mortar, the external frame can be removed if not required to support the roof. The polythene sheet and its supporting welded wire mesh are also removed.

Ferrocement has been known for quite some time as a construction material for thin-walled components, usually 10 to 20 mm thick; it consists of cement mortar with embedded layers of wire mesh. Modern development started some 40 years ago when this material was used in boat-building and as a material for self-supporting permanent formwork for widespan roof systems in structural engineering. Further developments of ferrocement after 1960 occurred outside western Europe, especially in eastern Europe and developing countries.

Ferrocement is a material suited for thin-walled spatial structures of any shape because of its inherent mechanical properties due to the advantageous combination of fine-mesh reinforcement and fine-aggregate concrete, the latter showing sufficient workability even at a comparatively low water-cement ratio.

A number of characteristics of ferrocement attract the interest of designers: the efficient utilization of material, the availability of high quality sands in many parts of the world, the high density and water resistance, and the possibility for industrialized production, to name just a few. Various methods of production of ferrocement elements have been developed and tested under factory conditions, leading to the promise of mass production in the future.

On the other hand, however, current knowledge and methods do not as yet attain the standards of design, analysis and construction as available for reinforced concrete. An essential reason for this is the fact that the mechanism of interactions and effects of mesh reinforcement and mortar matrix in the direct vicinity of the steel surface which determine the overall structural material properties (load-bearing capacity, deformation and cracking properties) have not yet sufficiently been explored. Additional information from basic research is also required with respect to corrosion behaviour with 2 mm mortar cover, behaviour in fire, and impact resistance.

The development and fabrication of fabrics for use as formwork in constructing concrete shells represent only an insignificant market segment of structural fabric technology and industry nowadays. A decade ago, no one could have predicted the extremely vigorous development of the new fabric membrane technology in its use for permanent buildings. There is no longer any doubt about the important role structural fabrics will play in the built environment. Not only have existing membrane materials and predicted forms been proved successful on a major scale, but a second generation of materials has been developed, and new forms and buildings have been created, making use of the integrated properties of strength, seamless construction, translucency and reflectivity to produce building spaces never achieved before.

Fabric membrane structures, as we now know them, have a number of roots in the past. Spatial structures, such as the domes of the Arabs and Romans, first explored the use of shape to help span large spaces more economically and more gracefully. The first concrete shell structures of the 1930s continued that tradition, expanding the capacity tremendously by introducing scientific design methods and vastly superior materials.

Tensile structures as an architectural form were first explored and presented by Frei Otto whose many publications have familiarized the architectural and engineering profession with the potentials of this technology and its inherent-spatial and structural shapes.

The new technology, beyond achieving conventional goals better, opens the way to totally new architectural and structural concepts.

4.1 *Ferrocement shell structures*

Pier Luigi Nervi, the well-known Italian civil engineer and architect, who died in 1979, started experimenting with wire-reinforced mortar in 1942/43 and he called this material "Ferrocemento" after its main components steel and cement.

Nervi recognized the advantages of this composite material for plane load-bearing structures and specialized in experimenting with thin shells which he used as folded-plate systems for wide-span roofs and self-supporting permanent formwork. Between 1945 and 1960 numerous festival halls, exhibition halls, sports pavillons, factory buildings and warehouses were built which enjoy world-wide reputation because of the harmony of their structural and architectural shaping.

The last great examples of this construction method are the domes of two sport pavillons in Rome built for the 1960 Olympics. Around this time, the first buildings for industrial and agricultural use were erected with ferrocement in the USSR. Investigations and developments of thin-walled ferrocement folded-plate roof systems for apartments and public buildings were carried out at the Concrete and Reinforced Concrete Research Institute of Gosstroy/USSR, and standards were eventually established.

By the end of the 1950s and during the 1960s there was a boom in building yachts of ferrocement, individual as well as series models, originating from New Zealand and spreading over Australia, Canada and the US. Even today boatbuilding is still the primary domain of ferrocement. Thousands of boats have been built in China and the USSR and are used on inland waters as workships, pontoons and barges.

Many attempts have been made to define ferrocement and distinguish its essential properties. If one assumes that the definition is intended to be rather a description than a specification of base materials and ways of reinforcement, the most understandable definition will probably be the most useful one. It reads: Ferrocement is a composite construction material of cement mortar and mesh reinforcement and is used for thin-walled load-bearing structures.

The special characteristic is the narrow-mesh and fine diameter continuous mesh reinforcement which is uniformly distributed over the cross section in a specified minimum quantity.

The reinforcement usually consists of endless mats which are either welded, woven, twisted or produced in a similar way. Besides wire, natural or plastic fibres may be used. This clearly distinguishes it from standard reinforced concrete and fibre-reinforced concrete.

Ferrocement is impermeable to water, impact-resistant, durable and very dense because of its favourable cracking properties. The substantial advantages include low deadweight due to small thickness, low construction material consumption, easy fabrication and base materials which are cheap and available almost everywhere.

Even though ferrocement might be regarded as some kind of reinforced concrete because of its composition of reinforcement and mortar, the type of reinforcement and its distribution over the complete cross-section differ significantly from standard reinforced concrete: this is accordingly revealed by its properties.

For a better classification, a comparison of cross-sectional percentage of reinforcement and specific bond surface of standard reinforcement in reinforced concrete, ferrocement and glass-fibre reinforced plastics may be considered.

	bond surface between mesh reinforcement and mortar matrix per volume of plate or shell element (cm^2/cm^3)	cross-sectional percentage of mesh reinforcement (%)
reinforced concrete	0.04	0.60 to 1.60
ferrocement	1.60 to 6.00	2.00 to 6.00
glass-fibre reinforced plastics	>400	>15

The specific bond surface and the cross-sectional percentage of reinforcement are, in addition to the properties of the base material and the geometrical distribution of reinforcement over the cross-section, the most important factors having an effect on the mechanical properties of ferrocement.

As a result of the characteristics of the base materials, the strength in compression of ferrocement is primarily dependent on the strength in compression of the mortar, whereas the tensile strength in the failure stage is dependent only on the type and cross-sectional area of the mesh reinforcement. In the cracked state, cement mortar under tension helps to stiffen the reinforcement pack, to protect it against corrosion and to transfer the tensile stresses via the bond surface.

As is well known, crack width and crack spacing interrelate directly and both are directly dependent on the specific bond surface of reinforcement. The greater the specific bond surface, the closer is the crack spacing or the faster a crack spacing corresponding to wire spacing is reached.

In the fabrication of ferrocement it is extremely difficult to provide for mechanized or automatic placing of mortar into the reinforcement pack. This is still a handicap for industrial mass fabrication and thus for application in industrialized countries. More or less mechanized fabrication is a precondition for broader application in industrialized countries. Processes have become known from Poland and the USSR for economic manufacture of large quantities of plane, but also curved, folded and three-dimensionally curved load-bearing structures.

A simple technique for producing singly or doubly curved surfaces is “Shaping by lifting”. Here the reinforcement is locked in a horizontal frame one leg of which is movable. During lifting, this leg is moved toward the inside thus permitting a cylindrical sagging of the sheet. The curvature can be controlled amongst others by the stiffness of the reinforcement pack. Doubly curved components can be shaped in a similar way.

Due to its versatility, ferrocement provides advantages and possibilities for sectors where concrete has not yet been used. This has mainly been evidenced by applications in developing countries.

In industrialized countries with their high levels of concrete technologies and the availability of most different construction materials, ferrocement is primarily used with success for wide-span plane load-bearing structures, as permanent formworks and as membrane (skin) reinforcement for load-bearing structures where crack distribution and limited crackwidth are advisable for reasons of utilization and durability.

Of special importance in eastern European countries are mainly material savings and, in comparison to concrete, the low weight in transportation.

4.2 *Teflon- or silicon-coated glass-fibre membranes*

In order to use fabric membranes in permanent buildings, the fabric has to be durable and non-combustible, in addition of having reliable strength properties within the temperature range of outdoor exposure. Among potentially suitable materials teflon-coated glass-fibre satisfied all these requirements. Its additional advantage was high translucency (up to 16% light transmission) and low heat absorption. The inert nature of teflon, furthermore, makes the surface self-cleaning in most circumstances. The combination of heat-sealed and mechanically clamped seams results in watertight structures superior to most roofing systems.

Recently, silicon-coated glass-fibre has emerged as a new material, having similar basic properties, except that it exhibits much higher translucencies (up to 50% for structural fabrics), but with a somewhat reduced dirt rejection. Its tear strength proves to be higher and also its flexurability.

Teflon-coated glass-fibre is expected to last substantially longer than 25 years, based on simulated weathering tests and more than ten years' experience with actual installations which show no significant deterioration in important properties. Similar life spans are expected for silicon-coated glass-fibre with the added potential of on-site re-surfacing of the finish coat by a spray process.

Fabric membrane structures derive their stability and capacity to carry loads directly from their shape. Structure and architecture are therefore one and the same. The membrane defines the space, creates the structural span, forms the enclosure, reflects the heat, lets in light and keeps rain and snow out; and the sculptural external form dominates the architecture. The design process therefore requires an extremely high degree of integration. The most critical element in the design process, however, is the structural shape. It is arrived at by a selection of supporting elements and the curved membrane shapes spanning between them. The membrane generally has synclastic curvatures for air-supported structures and always requires anticlastic curvatures for tension structures. Large air-supported structures require cable reinforcement.

Fabric membrane structures have practically no weight and are made of totally flexible materials. Gravity and rigidity, usually the basis of structural stability, are, therefore, not available to create a load-bearing system. Without internal air pressure as a stabilizing force, tension structures depend entirely upon shape and prestress for their stability and their capacity of carrying superimposed loads.

The membrane surface of any tension structure has to have the general shapes shown

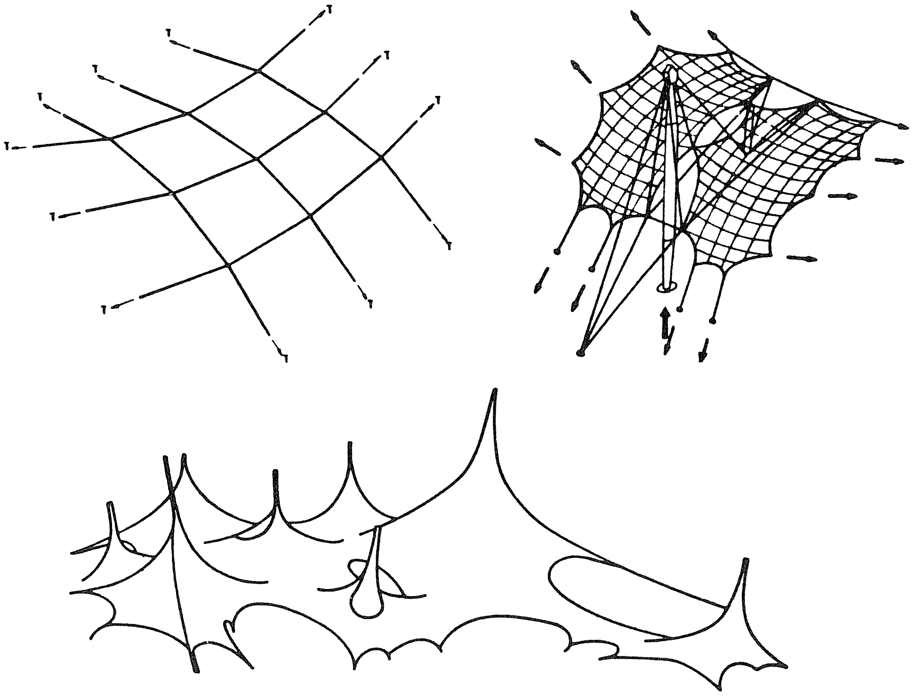

Fig. 5. Shapes of anticlastic curvature surfaces.

in Fig. 5. The shape of the overall structure follows from the selection of supporting system and the configuration of edges.

The shape of the membrane is determined starting from the geometry of supports and edges, and assuming a proper stress pattern over the surface area of the membrane. As a result one particular equilibrium shape of the membrane must be found. A number of mathematical procedures have been developed to handle this shaping process based on computer simulation. Once the equilibrium shape is determined, a non-linear analysis must be carried out to find the stresses under the superimposed loads. Since the deflections are significant and assist the tension structure in its capacity to carry loads, the accuracy of the initial shaping and of the non-linear stress analysis are of decisive importance.