Cross-wind movements of chimneys

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1 Introduction

The cross-wind motion of tall cylindrical structures such as chimneys is a well known phenomenon whose causes have been studied for many years. Nevertheless, so far no reliable means of predicting the amplitude of motion has been available to designers of those structures (see [1], [2] and [6]).

The movements of chimney models have been studied in order to obtain numerical information on their response. These tests demonstrated three important effects [7]:

- The relationship is non-linear between observed movements and those predicted by a method based upon consideration of lift and damping alone.
- The statistical nature of the response amplitudes varied, depending upon their magnitude.
- As response approached resonance at critical wind-speed, the intensity of the excitation increased, whereas no increase was observed in the correlation length.

Using this information, an expression was developed for expected movement amplitude in terms of damping aspect ratio, Scruton number and coefficients, varying with Reynolds number, which were obtained from model and full scale data.

2 Wind-tunnel tests

The models shown in Fig. 1 were tested in the Delft boundary layer wind-tunnel. Having variable stiffness and damping, the models could be tested at various critical wind-speeds. The R.M.S. value (α_x) of the observed top movements were plotted against the predicted movements in each case (see Fig. 1). The predicted movements were calculated using basic dynamic and aerodynamic theory, which yields:

$$w = \frac{C_1 \frac{1}{2} \varrho V_{cr}^2 D^2}{k\zeta} \qquad (1)$$

where:

w = expected top movement
C_1 = constant related to lift coefficient
ϱ = density of air (1.25 kg/m^3)
D = diameter
k = stiffness (load at top giving unit top displacement)

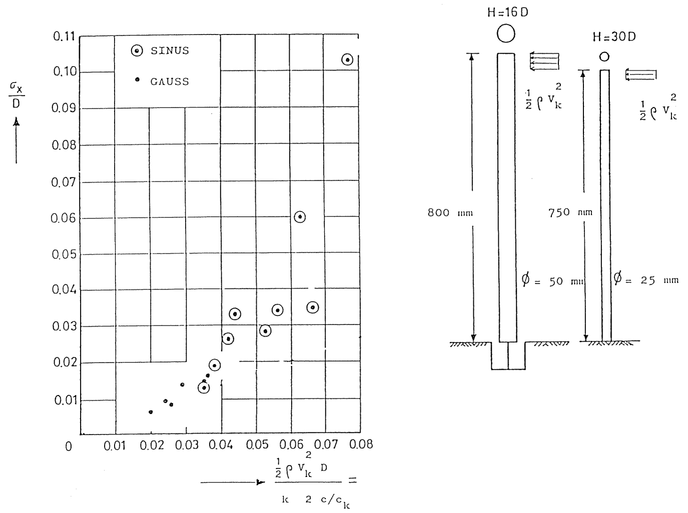


Fig. 1. Standard deviation of the cross-wind movement of some models.

ζ = damping ratio as fraction of critical damping
V_{cr} = critical wind-speed

Fig. 1 shows a non-linearity in the relationship between measured response and predicted response, based on relationship (1). This is due to the so-called "self-excitation" effect and is probably the reason why the search for a clear theory of cross-wind loading has been so difficult.

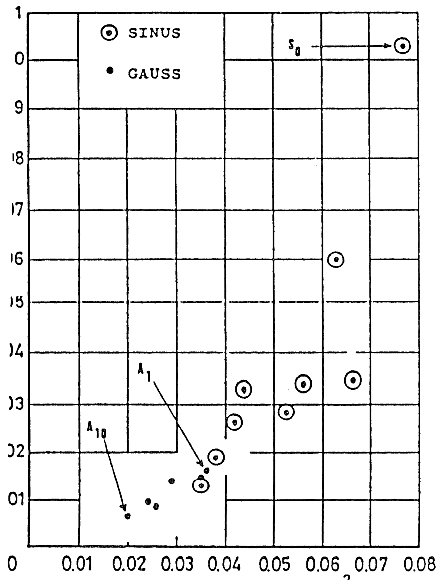
A second phenomenon observed during these tests was the change of statistical distribution of the observed movements as the amplitude increased. Fig. 2 shows that the distribution of amplitude was Gaussian at small maximum amplitudes ($<0.1D$), but tended to a distribution of harmonic movements as the amplitudes increased. In other words, for maximum amplitudes less than $0.1D$, the response was random, whereas it becomes sinusoidal at larger amplitudes.

Fig. 3 illustrates a third phenomenon noted in the tests. This was that, as the wind-speed approached its critical value, the intensity of the excitation increased. No increase of correlation length was noted, even when the so-called self-excitation effect was increasing the response beyond that expected. As a result, the following work considers the disturbing force to be concentrated at the top of the chimney over a length proportional to D.

3 An expression for top movement

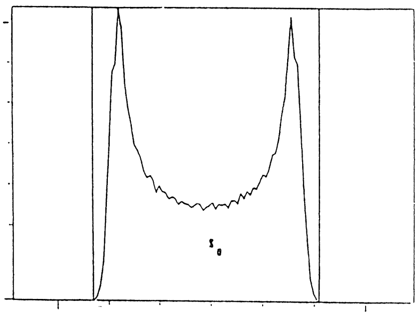
By substituting the following expressions:

$$V_{cr} = \frac{f_1 D}{St}$$



$$\longrightarrow \frac{\frac{1}{2}\rho V_k^2 D}{k \; 2\, c/c_k}$$

Standard deviation of cross-wind movement

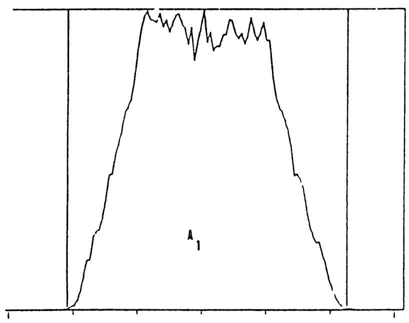


Statistical distribution of large movements (σ > 0.1 D) sin

$$\hat{x} = \frac{8.80}{2} = 4.40 \text{ mm}$$

$$\frac{\hat{x}}{D} = \frac{4.40}{25} = 0.176$$

$$\frac{\sigma_x}{D} = 0.104$$

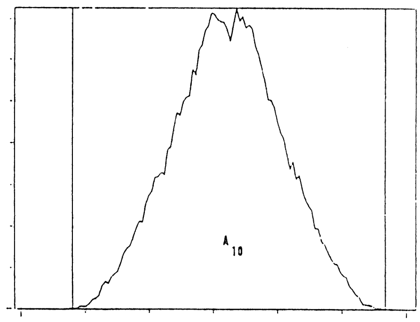


Statistical distribution of movements with $\sigma \approx 0.1$ D

$$\hat{x} = \frac{4.35}{2} = 2.17 \text{ mm}$$

$$\frac{\hat{x}}{D} = \frac{2.17}{50} = 0.043$$

$$\frac{\sigma_x}{D} = 0.016$$



Statistical distribution of small movements (σ < 0.1 D) (random)

$$\hat{x} = \frac{2.43}{2} = 1.22 \text{ mm}$$

$$\frac{\hat{x}}{D} = \frac{1.22}{50} = 0.024$$

$$\frac{\sigma_x}{D} = 0.006$$

Fig. 2.

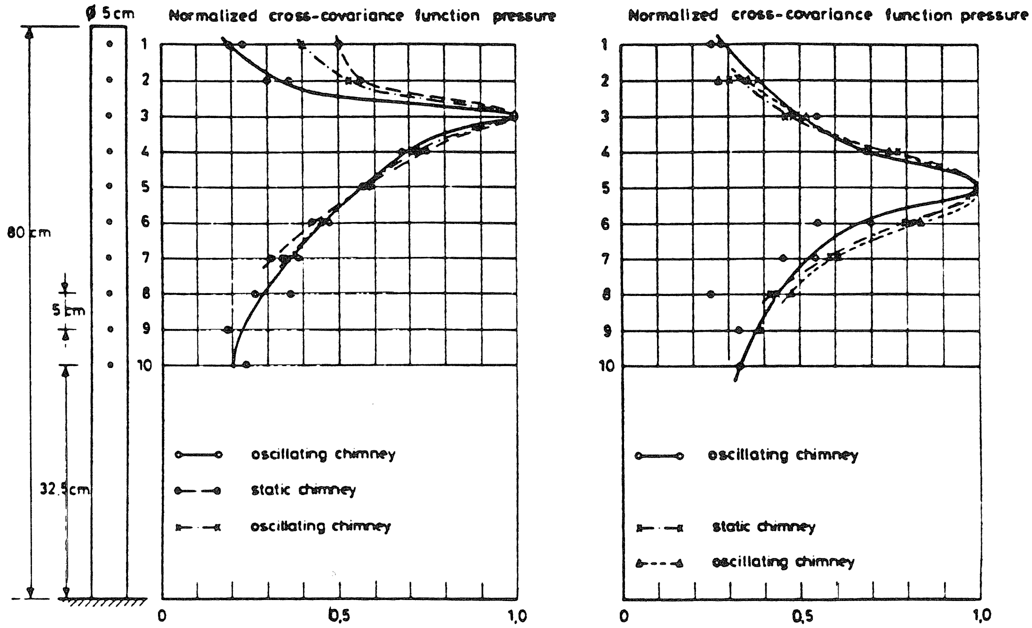


Fig. 3. Normalised cross covariance function pressure, measured at intervals of 5 cm over top 60% of chimney model.

and

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{0.25mL}}$$

Equation (1) can be rewritten in the following dimensionless form:

$$\frac{w}{D} = \frac{C_2}{\lambda Sc} \tag{1a}$$

where:

St = Strouhal number
f_1 = fundamental natural frequency
λ = aspect ratio (L/D)
L = height of chimney
Sc = Scruton number $\left(=\frac{4\pi m\zeta}{\varrho D^2}\right)$
m = mass/unit length
C_2 = coefficient related to lift and Strouhal number

In the case of chimneys with non-uniform diameter, a reasonably accurate approximation to the fundamental natural frequency can be obtained by substituting "effective length" (L_e) for length L in the expressions for f_1 and λ above. "Effective length" is calculated from the following expression (see Fig. 4):

$$L_e = L_1 + \sum L_n \left(\frac{D_1}{D_n}\right)^3 \tag{2}$$

The wind-tunnel tests described above showed that there is a non-linearity in the accuracy of predictions using formula (1) or its derivative (1a). This is due to the so-called self-excitation effect. This effect can be caused by:

- A force induced by the movement itself. This is related to the interaction of the wind-flow and the chimney's motion. It can be expressed as a function of:
  - either $\frac{1}{2}\varrho V^2 \frac{dw}{dt}$ (see [3])
  - or $w \frac{dw}{dt}$ (see [4])
- A non-linear damping effect related to the movement:

  $c = c_1 + wc_2$

  where:

  c, c_1 and c_2 are damping terms

The effect of these non-linearities can be found by their substitution into the basic equation of dynamic equilibrium. For instance, considering the effect due to a force increase related to $w(dw/dt)$:

$$m\frac{d^2w}{dt^2} + c\frac{dw}{dt} + kw = q + F\left(w, \frac{dw}{dt}\right) \tag{3}$$

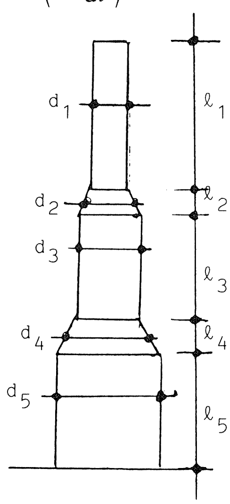


Fig. 4. Dimensions for deriving "Equivalent Height" of chimney with varying diameter.

where:

$q=\frac{1}{2}\varrho V^2$
$F=$ an arbitrary function
t = time

If the motion is assumed to be harmonic:

$$w=\bar{w}\sin\omega t \tag{4}$$

and

$$F=\alpha\frac{dw}{dt}+\beta w\frac{dw}{dt} \tag{5}$$

where:

ω = angular natural frequency
$\bar{w}$ = maximum value of movement amplitude
α and β are constants, depending on Reynolds number

Substitution of equations (4) and (5) into equation (3) leads to:

$$\omega c\bar{w}=q+\alpha\omega\bar{w}+\beta\omega\bar{w}^2 \tag{6}$$

This equation is satisfied if:

$$\bar{w}=\frac{\omega(c-\alpha)}{2\beta\omega}-\sqrt{\left(\frac{\omega(c-\alpha)}{2\beta\omega}\right)^2-\frac{1}{\beta\omega}} \tag{7}$$

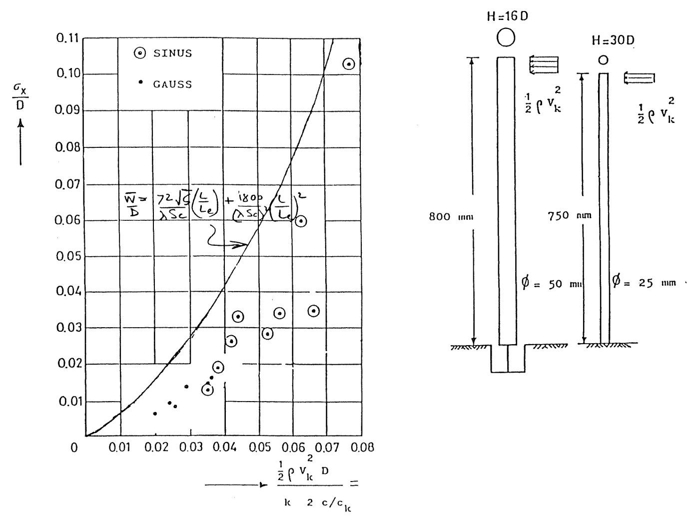


Fig. 5. Use of observed model movements to derive values of "A" and "B" at low Reynolds numbers.

As an approximation, this solution is developed in a series:

$$\bar{w} = \frac{c-\alpha}{2\beta}\left|1-\sqrt{1-\frac{4q\beta}{(c-\alpha)^2\omega}}\right| =$$

$$= \frac{c-\alpha}{2\beta}\left|\frac{2q\beta}{(c-\alpha)^2\omega}+\frac{1}{2}\left(\frac{q\beta}{(c-\alpha)^2\omega}\right)^2+\ldots\right| =$$

$$= \frac{q}{(c-\alpha)\omega}+\frac{q^2\beta}{4(c-\alpha)^3\omega^2}+\ldots$$

$$\approx \frac{q}{c\omega}\left(1+\frac{\alpha}{c}+\frac{\alpha^2}{c^2}+\ldots\right)+\frac{q^2}{c^2\omega^2}\left(\frac{\beta}{4}+\frac{3\alpha\beta}{c}+\frac{3\alpha^2\beta}{2c^2}+\ldots\right)$$

$$= A\frac{q}{c\omega}+B\frac{q^2}{c^2\omega^2} \tag{8}$$

This expression is equally valid if the self-excitation is related to the disturbance of the flow by the chimney's movement, i.e. if $\alpha = \alpha' q$.

It is necessary to determine the values of A and B by tests. Full scale tests are preferred because the damping is non-linear, depending upon the level of stress, i.e.:

$$c = c_1 + c_2 w$$

introducing this complication has little effect on equation (8), but does affect the values of A and B.

Equation (8) can be rewritten in dimensionless form, involving the Scruton number and aspect ratio as follows:

$$\frac{\bar{w}}{D} = \frac{A}{\lambda Sc}+\frac{B}{(\lambda Sc)^2} \tag{9}$$

Note the similarity to equation (1a).

Further modification to equation (9) is required to compensate for two errors in the original assumptions:

- The loading was assumed to be harmonic, whereas the tests showed the response to be random at low amplitudes.
- Use of the effective length (L_e) in the equations underestimates the movement in the lower parts of the chimney.

To account for these errors, the following modification is proposed to equation (9):

$$\frac{\bar{w}}{D} = \frac{A\sqrt{\zeta}}{\lambda Sc}\left(\frac{L}{L_e}\right)+\frac{B}{(\lambda Sc)^2}\left(\frac{L}{L_e}\right)^2 \tag{10}$$

The damping term in equation (10), quoted both directly and as part of the Scruton number, contains both the structural damping and an aerodynamic component. Struc-

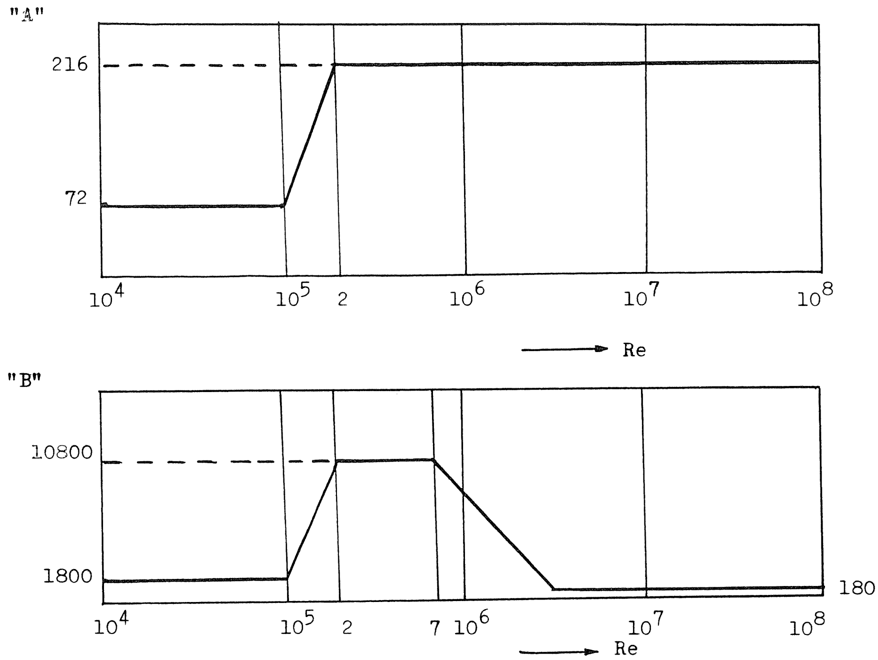


Fig. 6. Values of "A" and "B" at varying Reynolds numbers.

tural damping depends on the construction of the chimney. Aerodynamic damping (ζ_A) is calculated from the following expression (see [5]):

$$\zeta_A = \frac{1.22 V_{cr}}{f_1 t} \times 10^{-6}$$

where:

t = equivalent thickness of top 30% of chimney shell, at density = 7800 kg/m^3

4 Determination of the coefficients A and B

The results of the model tests are reasonably well and safely approximated if the following values are assumed: $A = 72$ and $B = 1800$, with $\bar{w} = 3 \times$ R.M.S. of the measured values (see Fig. 5). These values are, however, only valid for the low Reynolds numbers at which the tests were carried out.

Values of "A" and "B" relevant to higher Reynolds numbers were obtained by reference to the full scale data collected by Pritchard (see [2]). This data provides performance and construction records for 64 full scale chimneys varying in height from 18 m to 140 m. For 36 of the chimneys, the data includes measured fundamental natural frequencies, allowing accurate calculation of the aerodynamic component of damping. Of these chimneys, 13 were members of a closely spaced group and were ignored, because the along- and cross-wind load can be increased by turbulence effects from other chimneys (see the magnification factors of Fig. 8). Using the data related to the remaining 23, values of A and B were developed as a function of Reynolds number (see Fig. 6). It appears that the relationships exhibit a disturbance (similar to that of the Strouhal number) in the range $10^5 < Re < 10^6$.

Using values of A and B from Fig. 6, structural damping from Table 1 and calculated aerodynamic damping, the top deflections of the 25 chimneys were calculated using equation (10). These deflections were compared with the observed deflection (see Fig. 7). It appears that some scatter exists in the results, but the predictions are generally safe. In the case of chimney No. 52, there is some doubt whether peak to peak or single amplitudes were reported.

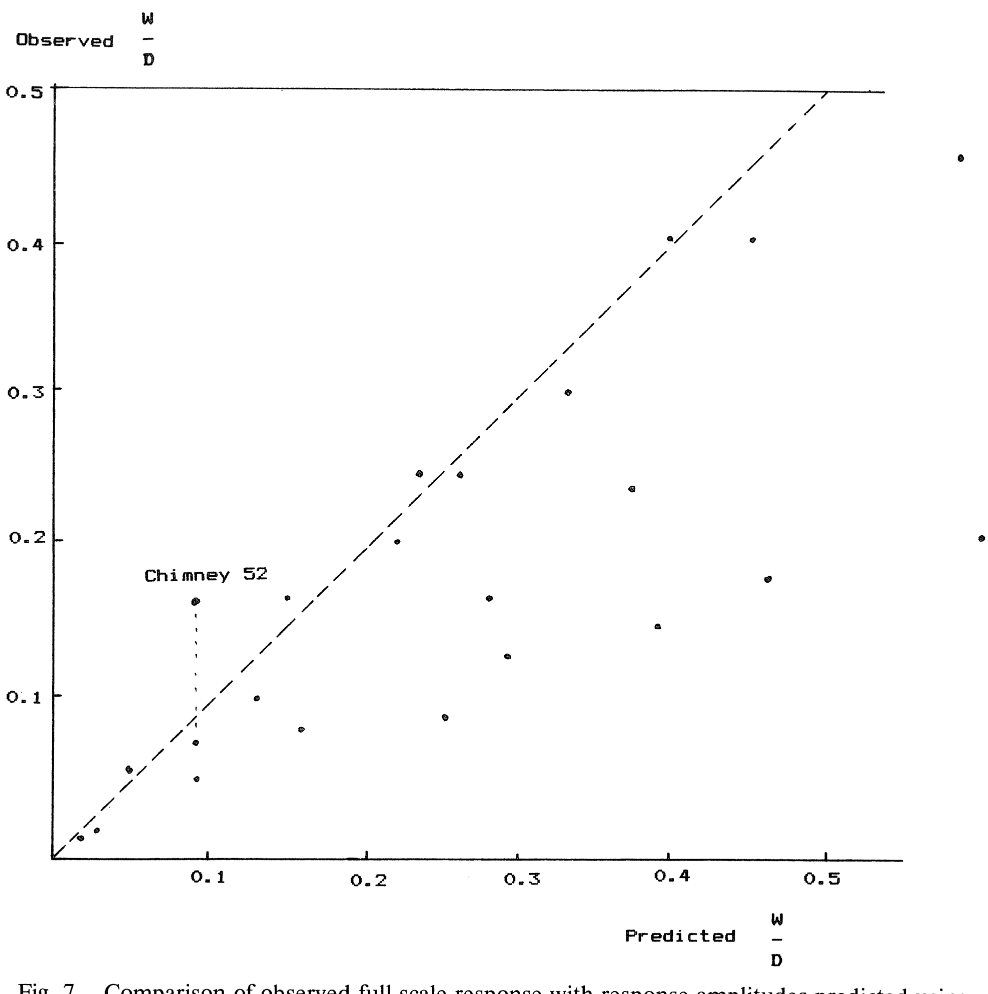
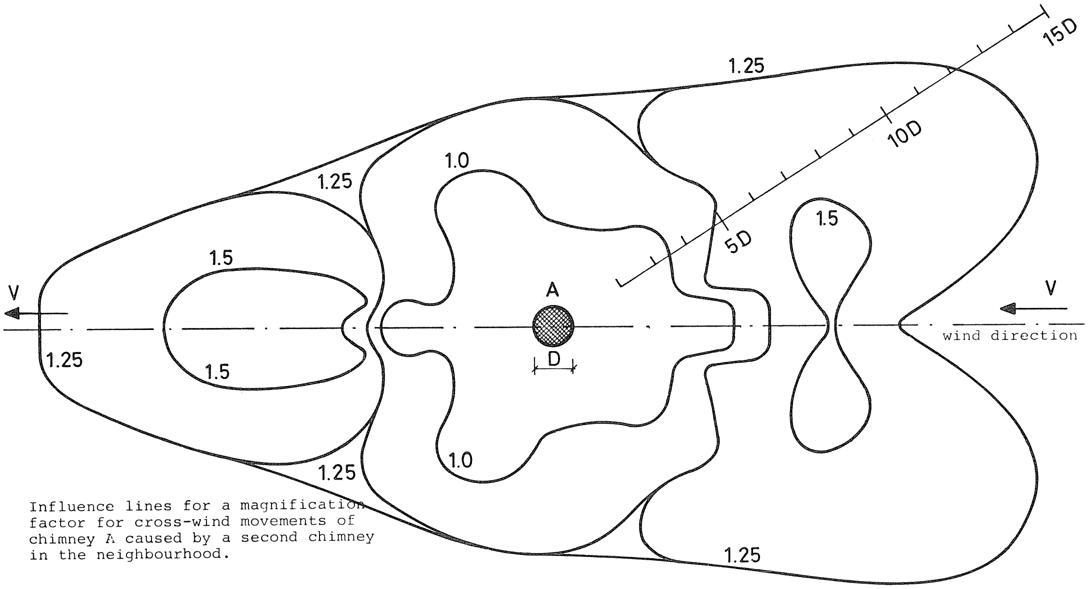


Fig. 7. Comparison of observed full scale response with response amplitudes predicted using expression (7).



Influence lines for a magnification factor for cross-wind movements of chimney A caused by a second chimney in the neighbourhood.

Fig. 8.

5 The number of load cycles

The number of load cycles has been determined from the results of a model test. This test was made in a boundary layer wind-tunnel and the movement of the top of the chimney was observed. The standard deviation of the top amplitude is plotted as a function of the wind velocity in Fig. 9. The standard deviation has its maximum at a wind-speed of 6 m/s (critical wind-speed).

The probability of occurrence of the wind-speed in the interval $0.95\,V_{cr}$ to $1.05\,V_{cr}$ is the proportion of the time in which cross-wind movements rake place.

The number of load cycles in T years is about:

$$N = f \times T \times 365 \times 24 \times 3600 \times 0.1\,V_{cr} \times p(V) =$$

$$= 0.64 \times 10^{7} \times f \times T \times \left(\frac{V_{cr}}{V_0}\right)^{2} e^{-\left(\frac{V_{cr}}{V_0}\right)^{2}}$$

where:

f = the resonance frequency
$p(V)$ = the distribution of the wind-speed (Rayleigh distribution)
V_0 = the yearly mean wind-speed at the top of the chimney ($V_0 \approx 0.3\,V_{max}$)

The value of N as a function of V_{cr}/V_0 is given in Fig. 10. The top movement of the model has also been observed when helical strakes had been fitted. The three helical strakes had a width of $0.1D$ and were fitted over $7.5D$ and over $10D$ along the top of the model.

The standard deviation of the top movements have been plotted in Fig. 9.

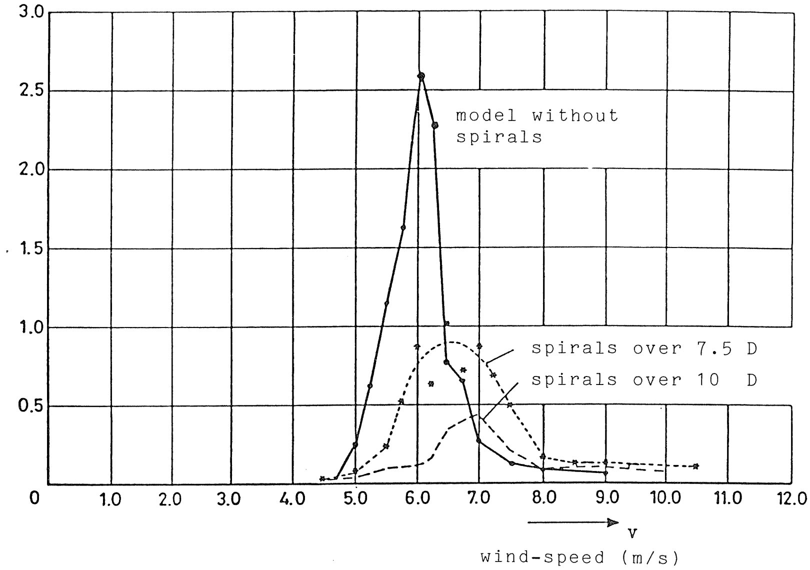


Fig. 9. The standard deviation of the top movement (mm).

The stresses are reduced by a factor of about:

$$0.5\left(\frac{s}{L}\right)^2\left(3-\frac{s}{L}\right)$$

where:

s = the length of the bottom part of the chimney without strakes

6 Conclusions

By applying the results of a series of wind-tunnel tests on models of variable stiffness and damping to the basic aerodynamic equations governing the cross-wind response to Von Karman vortex excitation, a simple non-dimensional expression has been derived.

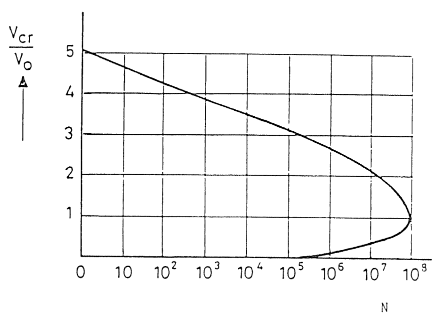


Fig. 10. The number of load cycles.

This relates the predicted maximum movement at the top of the chimney to its effective length, aspect ratio, Scruton number, damping and coefficients which were derived from wind-tunnel and full scale data. The expression gives generally safe predictions when compared with the full scale data from which the coefficients were derived. It is the author's intention to test this relationship against the performance of other full scale chimneys as data becomes available.

7 Literature

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