Reliability analysis for the fatigue limit state of the ASTRID offshore platform

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1 Summary

A reliability analysis with respect to fatigue failure was performed for a concrete gravity platform designed for the Troll field. The reliability analysis was incorporated in the practical design-loop to gain more insight into the complex fatigue problem. In the analysis several parameters relating to wave excitation, structural response and material properties of the concrete were considered as being stochastic. From the work done it is concluded that designing a concrete platform for the Troll field with respect to fatigue is indeed feasible despite the extremely severe boundary conditions and the many uncertainties associated with the fatigue problem. Furthermore it is concluded that introducing the reliability concept in the design stage is both feasible and useful.

2 Introduction

The design of offshore structures sometimes demands an extrapolation of experience and may involve as a consequence considerable uncertainties. In addition to this, offshore structures involve high construction costs, and failure may lead to very serious damage. The best way to cope with these difficulties is to perform a probability-based reliability analysis. Thus the safety problem is approached in a logical and consistent way and a good insight can be obtained into the importance of the various uncertainties. A reliability analysis, indeed, can be conceived as an extended type of sensitivity analysis.

In the past ten years the reliability concept as a tool for designing and analysing structures has received more and more attention. In the field of offshore structures, for instance, this subject has been a substantial part of the STUPOC and MaTS research programs in the Netherlands (Bouma et al., 1979, Karadeniz et al., 1982). Until now, however, the methods have seldom been used in the practical "design loop" of actual structures. The study presented here aims to show that incorporation in the design loop of a specific structure is practicable and leads to valuable results.

The structure under investigation is a concrete gravity platform from NOCS (Norwegian Offshore Concrete Structures) called "ASTRID". It consists of a central shaft

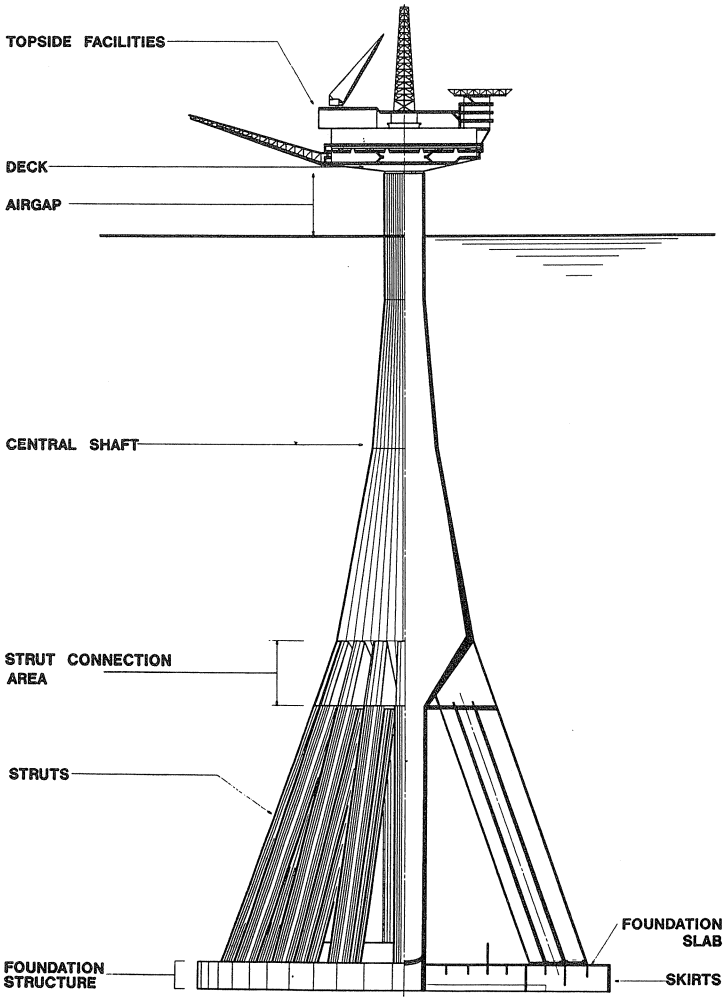


Fig. 1. The ASTRID concept.

supported by sixteen so-called "struts" which transfer loads to the foundation structure and provide bouyancy during construction. The design is represented in Fig. 1. The platform has been designed for the Troll field, which is located in the Norwegian Trench in a water depth of approximately 340 meter. The sea bed at Troll consists of extremely soft clay, the sea climate is very severe, and last but not least the design mass of the deck structures is 60 000 tonnes. This set of severe boundary conditions takes the design into a somewhat unknown area. Major uncertainties concerning soil characteristics, damping, long-term sea behaviour and fatigue properties of the concrete justify the use of a reliability analysis.

3 Wave climate and wave force analysis

The time variability of the sea waves has been dealt with in accordance with the well known concept of piecewise stationary sea states. Within every sea state the water elevation is assumed to be described by a zero mean Gaussian distribution and a multi-directional (cosine square) Pierson-Moskowitz spectrum $S_{\eta\eta}$ (Fig. 2). As far as the long-term statistics are concerned, measured data from the northern part of the North Sea were available, for the main wave direction as well as for the significant wave height H_s and the zero crossing period T_Z. It was considered convenient to replace the scatter diagram for H_s and T_Z by a continuous representation (see Fig. 3). This leads to a joint two-dimensional density function, which can be written as:

$$f_{H_S T_Z}(h,t) = f_{H_S}(h)\, f_{T_Z|H_S}(t;h) \tag{1}$$

where $f_{H_s}(h)$ is the marginal density function of H_s, for which a three-parameter Weibull distribution has been adopted. The function $f_{T_Z|H_S}(t;h)$ is the conditional function of T_Z, which could be described by a shifted lognormal (three-parameter) distribution. In the reliability analysis presented in Section 6 the shift parameter for the zero crossing period T_Z has been introduced as a random basis variable. This has been done in order to simulate the uncertainty involved in the derivation of the joint density function (1).

$$S_{\eta\eta} = \frac{4 H_s^2 \pi^3}{\omega^5 T_z^4} \cdot \frac{2}{\pi} \cos^2\theta \,.\exp\left[- \frac{16\pi^3}{T_z^4 \omega^4} \right]$$

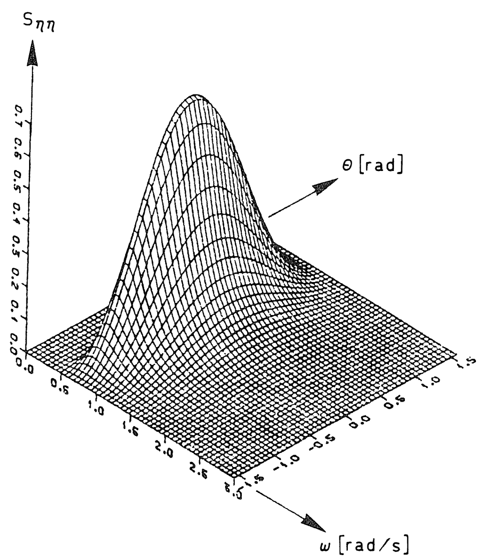
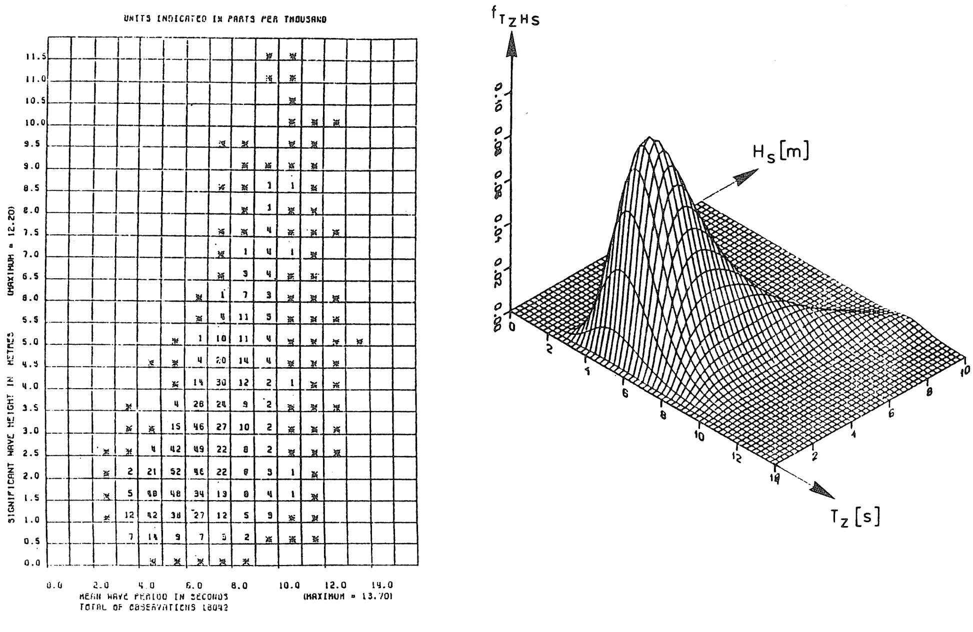


Fig. 2. Multidirectional Pierson-Moskowitz spectrum.



a. scatter diagram

b. probability density function

Fig. 3. Statistical data for H_S and T_Z.

The transfer function for the wave force at each point of the central shaft has been found by applying linear wave theory and Morison's formula (Fig. 4a). Morison's formula consists of a linear part (inertia forces) and of a non-linear part (dragforce). Because of the comparatively large external dimensions of the structure (minimum external diameter of central shaft is 19 m), the drag force can be neglected for wave heights less than 38 m. Hence only inertia forces have been taken into account. For the high wave frequencies a correction based on diffraction theory has been applied (Chakrabarti, 1973). This correction does not only affect the wave force maximum at a specific point of the structure but also the phase shift of wave maxima over the height of the structure. This phase shift further reduces the maximum wave force.

4 Structural analysis

The results of a feasibility study carried out earlier indicated that the possibility of failure due to fatigue damage in the cross-section at 310 m from the bottom (at an arbitrary point P) should be investigated in more detail. Therefore the analysis of the dynamic behaviour of the structure concentrated on obtaining the transfer function from water elevation to maximum stress at the point P.

In general, detailed finite element modelling is often used for describing the dynamic behaviour of a complex structure such as the one here under discussion. Since a reliability analysis demands several repetitions of the evaluation of the dynamic behaviour, however, the use of a finite element model could greatly increase the total computation costs. Therefore the structure has been modelled in a very simple way, introducing only three degrees of freedom. This approach leads to closed-form solutions for the response of the structure under harmonic wave loading.

The dynamic model is shown in Fig. 4b and consists of a non-uniform element with two nodes and three degrees of freedom (v_1, v_2, v_3). The linear soil springs K_f and K_m represent the stiffness of the foundation against a horizontal displacement v_2 and a rotation v_3 respectively. In a first attempt to solve this problem a stiffness matrix was derived by applying unit loads and the mass matrix was found by a simple lumping procedure. However, due to the very small number of degrees of freedom and because of the large mass and stiffness variations between the nodes, lumping of the masses resulted in a very unsatisfactory description of the dynamic behaviour of the structure. Therefore a different approach has been adopted. Stiffness and mass characteristics of the non-uniform element from Fig. 4b have been represented in the three degrees of freedom by applying the basic principles of finite element theory. Using interpolation functions for the displacement field, both a stiffness matrix and a consistent mass matrix have been derived, including added water and soil masses. At this stage the three eigenfrequencies and the three eigenvectors (in the three degrees of freedom) are obtained in closed form. The three oscillation forms are then derived using the eigenvectors and the interpolation functions, again in the same manner as is usual in finite element analysis. Response of the structure is finally obtained by standard modal analysis.

Due to the small number of oscillation forms, it can be questioned whether the quasi-static part of the response is calculated sufficiently accurately by describing the total response by means of modal analysis. To avoid an inadequate description of the quasi-static part of the response the total response has been divided into a static and a dynamic part:

$$M = M_{\text{static}} + M_{\text{dynamic}} \tag{2}$$

where M is the total bending moment at the cross-section under consideration. The

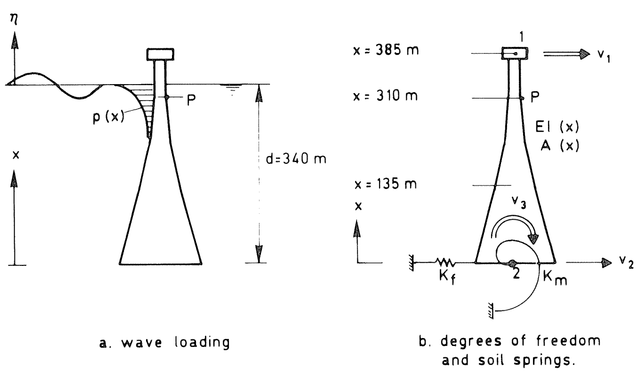


Fig. 4. Schematization for loading and structure.

static part of the bending moment response M_{static} is easily obtained from the wave load $p(x)$:

$$M_{static} = \int_{x_p}^{d} \{x - x_p\}\, p(x)\, dx \tag{3}$$

where $d = 340$ m is the water depth and $x_p = 310$ m is the x-coordinate of the point P. The dynamic part $M_{dynamic}$ is given by:

$$M_{dynamic} = \int_{x_p}^{385\,m} (x - x_p) m(x) u(x)\, dx \tag{4}$$

where $u(x)$ is the dynamic horizontal displacement field of the structure (obtained from the analysis described before) and $m(x)$ is the mass distribution over the height of the structure. In order to check the results from the one-element model, transfer functions from horizontal force at the waterline ($x = 340$ m) to the bending moment at level $x = 310$ m have been compared with ICES-STRUDL runs on a finite element model (Fig. 5). From Fig. 5 it can be concluded that in the frequency region of interest the results are in excellent agreement with each other for a wide range of the soil stiffness. Fig. 6 represents the transfer function from water elevation to stress at point P ($x = x_p = 310$ m), as obtained by the simple model for the mean values of all the relevant parameters.

When the transfer function is known, the resultant stress spectrum at point P originating from wave excitation can easily be computed. For the further fatigue analysis only two characteristics of the response spectrum are of interest, namely, the area or variance (m_0) and the second moment (m_2).

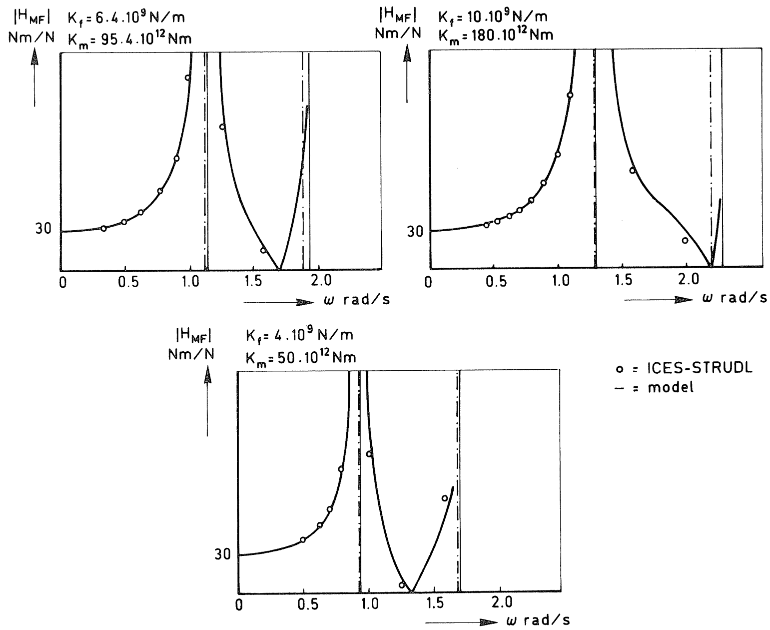


Fig. 5. Comparison of transferfunction H_{MF} of the model with ICES-STRUDL runs for various values of the soil stiffness.

5 Fatigue model

The fatigue model establishes the relationship between the stresses and the damage of the material. Starting point of this relationship is Miner's rule, which states that the damage after n_i cycles of stress range $s = 2s_i$ (twice the stress amplitude) equals:

$$D_i = \sum \frac{n_i}{N_i} \tag{5}$$

where N_i is the total number of cycles to failure with respect to a stress range $2s_i$. This number N_i can be expressed as a function of the stress range S by a so-called $S-N$ curve. In this study straight $S-N$ lines on log-log scale have been adopted:

$$N_i = \left[\frac{2s_i}{S_F}\right]^{-k} \tag{6}$$

In (6), S_F and k are constants which can be determined by constant-amplitude tests. Supposing the response spectrum to be narrow-banded, which in view of the transfer function of Fig. 6 is a reasonable assumption, the stress maxima are Rayleigh distributed:

$$f_s(s) = \frac{s}{\sigma^2} \exp\left[-\frac{s^2}{2\sigma^2}\right] \tag{7}$$

where σ^2 is the variance of the stresses and is obtained from the response spectrum. Applying the Rayleigh distribution of the stress maxima, the damage after the total lifetime T_L can be written as:

$$D_{tot} = \int_0^\infty \int_0^\infty \left[f_s(s) \cdot \left(\frac{2s}{S_F}\right)^k \mathrm{d}s\right] \frac{T_L}{T_m} \cdot f_{H_sT_Z} \, \mathrm{d}h \, \mathrm{d}t \tag{8}$$

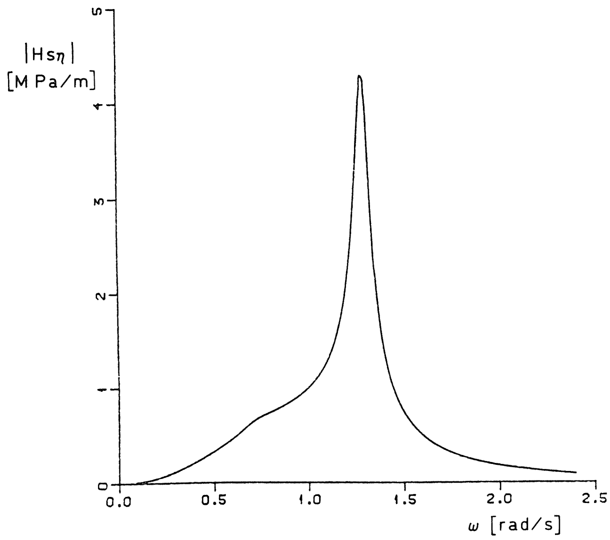


Fig. 6. Transferfunction from water elevation to stress.

where T_m is the mean stress period within a sea-state which is derived from the response spectrum.

By definition failure due to a constant amplitude loading occurs when the total damage equals one. Applying random variable-amplitude loading, however, failure may take place at other values of the total damage due to influences associated with the "loading history". The value of D_{tot} at which failure occurs will be further designated as D_F.

For concrete the $S-N$ curve is dependent not only on the concrete strength but also on the mean stress. At the cross-section at 310 m here under discussion the mean stress due to deck weight and prestress is 12 MPa. The concrete cube strength is 55 MPa. For these values of mean stress and cube strength two $S-N$ curves have been adopted to indicate the model uncertainty related to the $S-N$ curve. One is Wagaard's curve (Wagaard, 1981), which forms the basis for the $S-N$ curve as prescribed in the DNV rules. The other curve adopted is provided by Van Leeuwen and Siemens (Van Leeuwen et al., 1980) and resembles the curve used in the NPD rules. Both curves are shown drawn to a log-log scale in Fig. 7. For values of S higher than 24 MPa ($\log S = 1.38$) tension will occur, which prevents the use of the above mentioned $S-N$ curves. For values of log N greater than approximately 6.5 the curves are not based on measurements but solely on theoretical extrapolations.

From this it may be concluded that the curves are based on a rather small region of

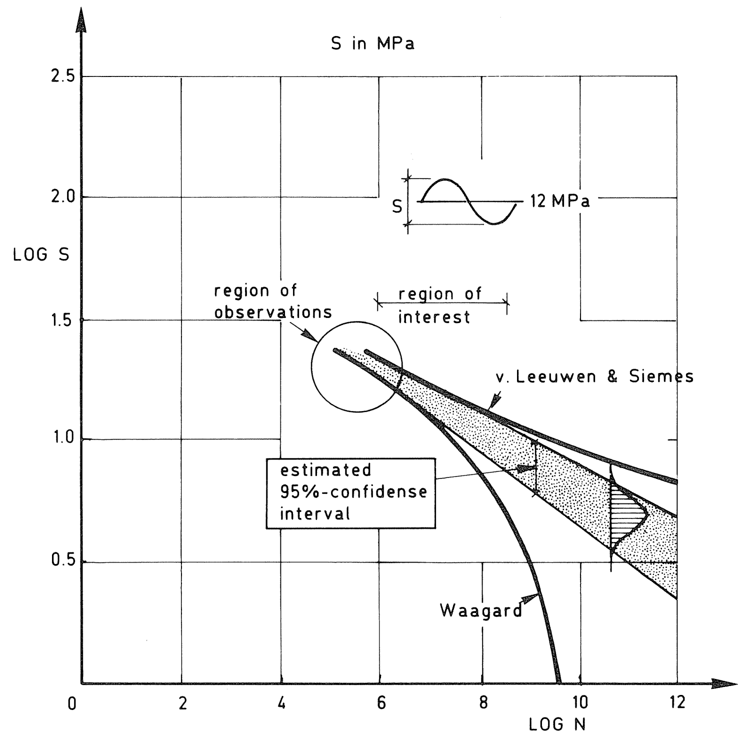


Fig. 7. Fatigue models for plain concrete (cube strength 55 MPa, average stress level 12 MPa).

observations, which explains the large differences between them. Hence considerable model uncertainty is associated with the $S-N$ curve. To incorporate this uncertainty in the reliability analysis, mean values and standard deviations for S_F and k have been estimated, resulting in a coefficient of variation (standard deviation divided by mean value) of 0.2 for S_F and 0.1 for k. Furthermore a strong negative correlation between S_F and k has been assumed. This results in a 95% confidence interval for the $S-N$ curve as shown in Fig. 7. It is felt by the authors of this paper that this confidence interval more or less represents the uncertainty related to the $S-N$ curve.

6 Reliability analysis

6.1 *The level II procedure*

To start this chapter, a short description of the First Order Second Moment (Level II) reliability analysis used in this study will be presented. For more elaborate treatments the reader is referred to the literature (e.g., Baker and Thoft-Christensen, 1980, Schueller, 1981). The first step of the procedure is to choose a reliability function Z in such a way that negative values of Z correspond to failure and positive values of Z correspond to non-failure. Values of Z that equal zero correspond to the so-called failure boundary. In general, Z is a function comprising a number of stochastic variables, $X_1, \dots X_n$. In the level II approximations the reliability function is linearized at a point $X_1^0, \dots X_n^0$. Suppose that the stochastic variables X_i are mutually independent and that their mean values $\mu(X_i)$ and their standard deviations $\sigma(X_i)$ are known. Linearization at $X_1^0, \dots X_n^0$ then results in the following approximation of the mean value and standard deviation of Z:

$$\mu(Z) = Z(X_1^0, \dots X_n^0) + \sum_{i=1}^{n} \{\mu(X_i) - X_i^0\} \cdot \frac{\partial Z}{\partial X_i} \qquad (9)$$

$$\sigma^2(Z) = \sum_{i=1}^{n} \left\{\sigma(X_i) \cdot \frac{\partial Z}{\partial X_i}\right\}^2 \qquad (10)$$

The derivate $\partial Z/\partial X_i$ is evaluated at the point X_i^0. Assuming a normal distribution for Z we can then determine the failure probability from:

$$P\{Z<0\} = \int_{-\infty}^{0} f_Z(z)\,\mathrm{d}z = \phi_n(-\beta) \qquad (11)$$

$$\beta = \mu(Z)/\sigma(Z) \qquad (12)$$

where β is the reliability index, f_Z the normal distribution for Z and ϕ_n the distribution function for a normally distributed variable with zero mean value and standard deviation equal to one. If the values for X_i^0 are equal to the mean values $\mu(X_i)$ the procedure described above is called a level II mean value analysis. In the actual calculations, however, also the more accurate AFDA (Approximate Full Distribution Approach) has been employed. In this approach the linearization is performed at the so-called design point, which is defined as the point on the failure boundary corresponding to the highest probability density. This design point has to be evaluated iteratively.

According to equation (12) the total variance $\sigma^2(Z)$ is the sum of a number of individual contributions, each resulting from one of the basic variables. This opens the possibility to quantify the relative importance of the various variables as the ratio α_i^2 of the individual contribution to the sum:

$$\alpha_i^2 = \frac{\left[\frac{\partial Z}{\partial X_i}\sigma(X_i)\right]^2}{\sum_j \left[\frac{\partial Z}{\partial X_j}\sigma(X_j)\right]^2} \tag{13}$$

The α_i are often referred to as the probabilistic influence factors. In order to have a high influence, a variable must be highly uncertain (large $\sigma(X_i)$) and the structure must be sensitive to the variable (large $\partial Z/\partial X_i$). This confirms the validity of the statement made in the introduction that the reliability analysis can be regarded as an extended sensitivity analysis.

6.2 *Reliability function*

The most obvious way to define the reliability function in this case would be:

$$Z = D_F - D_{tot} \tag{14}$$

Because of the logarithmic character of Z, however, this reliability function may lead to numerical problems in the iteration proces. Therefore we follow (Karadeniz 1982) and define the reliability function as:

$$Z = -\ln\left[\frac{D_{tot}}{D_F}\right] \tag{15}$$

where D_{tot} is presented by (8).

6.3 *Numerical data and results*

The stochastic variables together with their types of distribution, their mean values and their variances (standard deviation divided by mean value) are presented in Table 1. The last column of this table shows the relative contribution α_i^2 of each parameter to the variance of Z (see 13).

The stochastic parameters can be grouped into "wave parameters", "structural parameters" and "fatigue parameters", corresponding to the set-up of the analysis. The respective parameters will now be briefly examined in that order.

The first stochastic parameter in the *wave force model* is concerned with the uncertainty in the long-term description of the zero crossing period T_Z (see Section 2). The coefficient of variation of a parameter P of T_Z has been evaluated as 0.05. This corresponds to a coefficient of variation in the wave steepness of approximately 0.10.

Table 1. Statistical properties for the various random variables and their contribution $\alpha^2(x)$ to the variance of Z. (Advanced level II analysis, resulting in $\beta = 4$). The parameter S_f and k are correlated ($\varrho = -0.95$).

x	designation	$\mu(x)$	$V(x)$	$\alpha^2(x)$
C_T	parameter T_Z distribution	1.6688	0.05	8%
d	waterdepth	340 m	0.02	5%
C_M	inertia coefficient	2.0	0.15	15%
m_d	deck mass	60.000 ton	0.05	3%
c_u	soil strength	70 kPa	0.25	8%
E	Youngs modulus	40 GPa	0.05	2%
t	wall thickness	1.1 m	0.05	2%
M_0	rotation inertia subsoil	1.0	0.25	0%
ζ_1	damping	0.03	0.35	17%
S_f	intersection SN-line	136 MPa	0.2	40%
k	slope SN-line	7.5	0.1	
D_F	critical Miner sum	1.5	0.25	1%

The effect on the total result is not inconsiderable ($\alpha^2 = 8\%$).

Because the cross-section at 310 m lies only a small distance below the water surface, small variations of the water depth could result in large variations of the damage at 310 m. Additional uncertainty is related to the "water depth" caused by sea-bed unevenness, tidal range uncertainty and uncertainty concerning the settlements of the structure. Therefore is was felt that the water depth should be treated as a stochastic parameter. The very small coefficient of variation of 0.02 that has been assigned to the water depth, however, still has a non-negligible effect ($\alpha^2 = 5\%$) on the safety.

A frequency-independent coefficient of variation of 0.15 has been assigned to C_M, quantifying in a intuitive manner the rather large uncertainties associated with this parameter. The effect of the inertia coefficients turns out to be important ($\alpha^2 = 15\%$).

Stochastic parameters relating to the *structural model* are the deck mass, the soil shear strength c_u (related to the stiffness of the foundation), the modulus of elasticity E, the wall thickness t at 310 m, the added soil masses M_0 and the damping ratio ζ of the first mode. It should be noted that all coefficients of variation assigned to these parameters are based on intuitive considerations, which could be updated if more data were available. Some of the above-mentioned parameters will be briefly discussed.

The stiffness of the soil can be considered as a very highly uncertain quantity. Estimates from various experts and laboratories may differ by a factor 3 or more. In the analysis this has been modelled by using a coefficient of variation equal to 0.25, which indeed represents a high uncertainty. Fortunately it turns out that this high uncertainty does not have a major effect on the variance of Z ($\alpha^2 = 8\%$). Also interesting is the negligible influence of the large model uncertainty M_0 involved in the added soil masses (rotational as well as translational were taken into account, assuming full correlation between them). The largest contribution to the variance of Z originates from the damping ratio ($\alpha^2 = 17\%$). This is caused partly by the large coefficient of variation of 0.35, quantifying the great uncertainty associated more particularly with viscous soil damping. Another cause for this large contribution is apparently the strongly dynamic behaviour of the structure, which will be further discussed later on.

The last parameters to be reviewed are those belonging to the *fatigue model.* As has already been discussed in Section 4, the fatigue model consists of Miner's rule in combination with straight $S-N$ lines in the log N- log S plane. The parameters S_F and k have coefficients of variation equal to 0.2 and 0.1 respectively, with a correlation of -0.95 between them. It should be noted that the mean stress and concrete cube strength, which both affect the $S-N$ curve, have been treated as deterministic variables.

Using level II procedures the stochastic variables need to be mutually independent. Therefore S_F and k have been transformed into two independent stochastic parameters. Because of this transformation it is not possible to separate the individual contributions of S_F and k to the variance of Z.

The parameters describing the $S-N$ curve have a very important impact on the result. Approximately 40% of the total variance of Z originates from uncertainty in these parameters.

The main result of the analysis of course is the *reliability index* β, which has been found to be 4.0, corresponding to a failure probability of approximately 3×10^{-5}. Having regard to the high uncertainties as reviewed earlier, this result is surprisingly good and shows that the concept of the structure is very sound. Approaching the fatigue problem in a deterministic manner, using DNV regulations, it has been found that the total damage at 310 m just reaches the upper limit of 0.2 as prescribed in the DNV rules. It could be concluded that a failure probability of roughly 3×10^{-5} is embodied in those rules. This also corresponds to structures designed by the Netherlands rules where a mean value of the reliability index β of 3.8 has been adopted.

6.4 *Dynamic behaviour*

In designing for very deep water, soft soil conditions and high deck masses, the first eigenfrequency will decrease and the dynamics of the structure will make an increasingly large contribution to the stresses that occur. For the structure under discussion here, the lowest eigenfrequency is 1.28 rad/s, which indeed is much lower than the first eigenfrequency of most fixed offshore structures. Therefore structural dynamics can be expected strongly to influence the stresses, which explains the rather large contribution made by damping and zero crossing period of the waves to the variance of Z. Because of the above-mentioned, the relation between the first eigenfrequency and the reliability index has been investigated and is presented in Fig. 8. From this diagram it is obvious that the dynamics of the structure play an important role in the safety against failure due to fatigue. It is also clear that a decrease of the first eigenfrequency induced by for example larger water depths will result in a high failure probability. Additionally it should be noted that the total damage varies strongly with variations of the eigenfrequency: between a first eigenfrequency of 1.28 rad/s and 11 rad/s the total fatigue damage varies by a factor of 2×10^{-8}. The reliability index on the other hand varies in this frequency region by a factor 1.8.

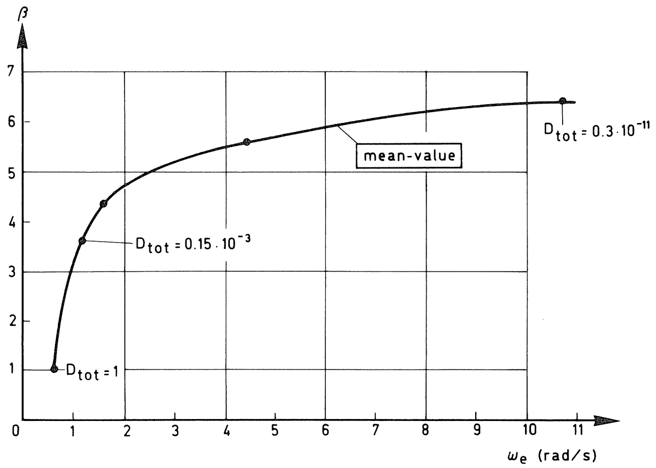


Fig. 8. Reliability index β as a function of the structural natural frequency.

6.5 *Reliability as a function of wall thickness*

A next step has been to investigate the possibilities of enlarging the reliability index assuming all overall dimensions to be fixed. One obvious possibility is to increase locally the wall thickness while maintaining the same external diameter. The effect of this increase is graphically shown in Fig. 9, which shows that local fatigue problems can be solved by making the wall thicker. This, however, negatively influences the necessary buoyancy during tow-out of the structure. It has been found that the best way to reduce fatigue damage and to maintain buoyancy is by means of a small increase in the external and the internal diameter as well as in the wall thickness.

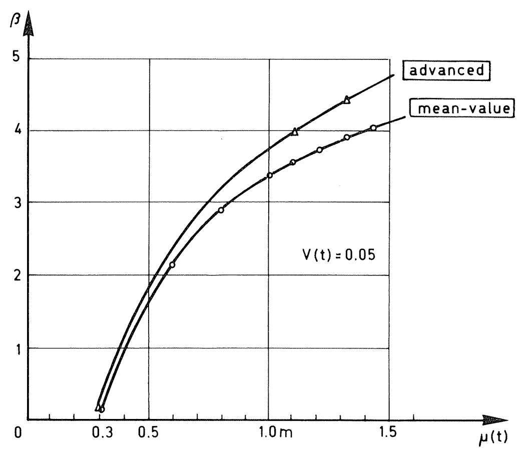


Fig. 9. Reliability index β as a function of the wall thickness.

7 Conclusions

The probability of failure due to fatigue at the most critical cross-section of the structure under investigation is 3×10^{-5}, which is an acceptable value. Hence it can be concluded that it is quite feasible to design such a structure with regard to fatigue despite great deck mass, great water depth, soft soil, severe sea climate and the considerable uncertainties associated with the fatigue problem. Due to the strong dynamic behaviour of the structure, however, a water depth in the order of 340 m should probably be regarded as the economically attractive upper limit for this design.

Additionally it can be concluded that incorporating the reliability concept in the practical design loop is indeed possible and very valuable as a basis for design decisions.

The great uncertainties associated with the $S-N$ curve for concrete have a major effect on the confidence level of the design. The authors of this paper therefore feel that more data concerning the fatigue behaviour of concrete, especially under small-amplitude stress cycles, should be gathered. In future analysis it is furthermore recommended to take the cube strength of the concrete as well as the mean stress into account as stochastic parameters. Both have an influence on the $S-N$ curve and can therefore be regarded as potentially important parameters.

Finally it should be noted that in the authors' opinion the calculated lifetime or Miner's sum is not a good criterion for judging the safety against failure due to fatigue, since this criterion is liable to vary by several orders of magnitude as a result of relatively small variations in design, boundary conditions or material properties.

8 Acknowledgements

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