HERON is jointly edited by:
STEVIN-LABORATORY of the
department of Civil Engineering,
Delft University of Technology,
Delft, The Netherlands.
and
TNO-INSTITUTE
FOR BUILDING MATERIALS
AND STRUCTURES.
Rijswijk (ZH), The Netherlands.
HERON contains contributions
based mainly on research work
performed in these laboratories
on strength of materials, structures
and materials science.

ISSN 0046-7316

Secretary:
G. J. van Alphen
Stevinweg 1
P.O. Box 5048
2600 GA Delft, The Netherlands
Tel. 0031-15-785919
Telex 38070 BITHD

HERON vol. 32 1987 no. 2

Contents

STRUCTURAL CONCRETE: SCIENCE INTO PRACTICE

A. S. G. Bruggeling
Delft University of Technology
Faculty of Civil Engineering

Publication in HERON since 1970

Abstract

There is a need for a more rational and unified approach to all types of concrete structure, reinforced of prestressed.

The first chapter explains in a historical review why the approach of reinforced concrete and that of prestressed concrete have hitherto been very different. In outlining the historical background it is also shown why there is so much confusion about partial prestressing. In the second chapter it is explained with the aid of several examples that also in prestressed concrete structures it is of prime importance to provide some basic normal reinforcement. It is also pointed out that, in general, it is not possible sufficiently to control the effects of imposed deformations and that normal reinforcement is always needed for the purpose.

The avoid any misunderstanding, a new designation of reinforced and prestressed concrete structures is introduced, namely, "Structural concrete". In chapter three this designation is defined. In addition, it is explained how important the role of the reinforcement in these structures is and how it will be possible to introduce prestressing as an artificial loading. The five main steps of design of structural concretes are discussed. In chapter four some components of the "structural concrete" model are discussed, namely: Prestress as an artificial load; the tension member in structural concrete (control of cracking); the beam model with the incorporated tension member; and time-dependent effects. In two appendices the engineering models of the tension member and the beam, respectively, are described in greater detail for direct use in practice.

Structural concrete: Science into practice

Introduction

In the last two decades there has existed a tendency to "split up" concrete structures into an increasingly large number of categories or so-called "classes".
In the fifties, besides reinforced concrete, there was a revolutionary development of prestressed concrete. This new construction method seems now to be subjected to an "erosion process". In Germany *limited prestressing* (beschränkte Vorspannung) was introduced. From the beginning there were already some ideas about *partial prestressing.* And in the field of this partial prestressing there is in turn a development towards new categories, characterised by several "degrees of prestressing". Other categories of prestress level have also been introduced. One can really speak of confusion, especially if one also realises that structures belonging to these categories can be designed with different combinations of reinforcing and prestressing steel. For each combination, specific research programs are carried out and new theories for design and dimensioning are developed.
In this paper it will be endeavoured to present a new, clear view of concrete structures. In the model that will be presented, and which is designated as "structural concrete", there exist many structural design possibilities without having recourse to any categories, classes, etc. Therefore, instead of reinforced concrete or fully prestressed concrete or limited or partially prestressed concrete, only *one* designation is chosen, namely "structural concrete".
In this paper it will be shown why such a model is necessary, what the components are and how this model can be used in the design and dimensioning of concrete structures. The author believes that this new approach will present fresh challenges to concrete designers and lead to better concrete structures in some cases, especially with regard to economy and durability. He already has the experience that design based on this new approach opens new possibilities for concrete as a structural material. He hopes in this way to contribute to a forward-looking promotion of concrete, and he will therefore be grateful for any support for, and critical comment on, these proposals. Only in thorough discussion between specialists can a new approach be developed.
The discussion can start now!

1 A brief historical review

1.1 *Reinforced concrete*

In the last decade of the 19th century reinforced concrete came into widespread use as a structural material.
In those days the calculation of the bearing capacity of reinforced concrete beams was

based on the so-called modulus method (n-method). This method has, for very nearly a century now, proved to be a very suitable one.
In 1903 the first Swiss "Provisional Standards" for concrete structures were published by S.I.A. Around the same time Standards for reinforced concrete structures were issued in Austria and in Germany. In the Netherlands the first Standard was published bij the Royal Institute of Engineers in 1912 [1]. The modulus method assumes that in bending plane sections remain plane (Bernoulli's assumption), that the reinforcement resists all the tensile force, and that there is a certain ratio (n) between the moduli of elasticity of steel and concrete. This method been used up till now in many countries. And in countries which abandoned this method there is already a discussion going on about reintroducing it [4]. It is interesting to note that in the Standards for reinforced concrete little, if anything, is said concerning the control of crack width and of deflection. By adopting in the Serviceability Limit State (S.L.S.) "permissible" stresses for concrete in compression and for steel in tension and compression, with stress levels dependent on material properties, sound behaviour of the reinforced structure in service was assured. And many still existing reinforced concrete structures, designed in accordance with these rules, have shown that this approach really was satisfactory. Of course the calculation model used in the modulus method does not predict the stresses in the structures very well, but it is very practical and useful. In the early sixties the "Modern Approach" developed by C.E.B. was introduced [26]. This approach was used as a basis for newly published Standards and Codes. The Ultimate Limit State (U.L.S.) was introduced with a great flourish. And every specialist realised that, using this approach, the overall safety of every structure was the same, whereas that was not the case with the modulus method. In using the new approach drawbacks could not be avoided! At the time of the introduction of reinforced concrete and its calculation method the number of concrete structures to be built was small and the design of these structures was in the hands of a few specialists. So the modulus method could be tried out in practice and was found to guarantee sound concrete structures. In the sixties, concrete structures were widely used. So, if the new approach had its limits, which were not very well defined, overstepping these limits would cause problems.

The main problems encountered were unpredicted heavy cracking of the concrete and unforeseen large deflections of beams and slabs, also increasing over longer periods. Accordingly in those years there was a "boom" in crack width formulas and an ongoing discussion about deflection and its method of calculation. In fact, these discussions are not over yet. As a result crack formulas are regularly revised. Besides one is often faced with problems of whether the concrete tensile zone is really cracked or not.
Especially in the case of slabs (low reinforcement ratio) this phenomenon can cause considerable differences in deflection, also in course of time causing unforeseen damage to brick walls, etc. supported on these slabs.
In the codes of practice one cannot find any relationship between serviceability limit state, crack width, crack spacing and deflection. Yet in actual fact this relationship is very close!

1.2 *Prestressed concrete*

During and especially after the Second World War prestressed concrete was developed and brought into practical use. Freyssinet annouced it in 1942 as a "revolution in the art of construction" because, due to this revolution,...

"Artificial loads and forces are introduced into structures for an unlimited time in such a way that, in collaboration with all loads exerted on the structures, no stresses will develop which cannot be resisted by the materials with complete safety for an unlimited time" [5].

After more than 40 years of use of prestressed concrete it must be agreed that the introduction of the prestressing technique really was a "revolution", especially if the full impact of this new construction method on the development of concrete structures as a whole, as compared with the development of reinforced concrete in those days, is duly realised. As Magnel, in 1948, wrote in his book [6], all the shortcomings of reinforced concrete are overcome by prestressed concrete, such as cracking of the tensile zone, problems with shear forces, cracking of concrete due to shrinkage and the impossibility of taking advantage of improvement (increase) in the strength of concrete and reinforcement.

In those days it emerged very clearly that, by prestressing, the whole cross-section is activated to carry the loads. The stress distribution over the section, both due to bending and due to shear, is well known. Prestressed concrete structures were designed for the serviceability limit state with permissible stresses in the concrete and steel.

Because uncracked concrete structures had been developed the role of the normal reinforcement was greatly reduced. In fact, it had a significant function to perform only in the anchorage zone. The reinforcement used in prestressed concrete was almost non-existent in many structures. On studying those old design drawings it is remarkable that hardly any reinforcement was used. From my own experience I know that this statement is true. Omission of reinforcement in prestressed concrete structures has in general been practised up till now. In several modern Codes of Standards for prestressed concrete no or virtually no rules are given for the dimensioning of the reinforcement. It must be admitted that many "old" prestressed concrete structures have behaved very well over long periods, but it is obvious that this omission of reinforcement has also caused severe drawbacks. They will be discussed later in this paper.

In this review it must be mentioned that already at the start of prestressed concrete construction it was realised that not only the behaviour under service conditions, but also the safety against failure was important. In the Netherlands "Recommendations for prestressed concrete" issued by the first national study group on prestressed concrete "the STUVO" (founded in 1949) it is prescribed that the safety against failure must be checked. In these recommendations the so-called "Stuvo formula" is given [10]:

$$M_u = (d - 0.1h) \cdot A_p \cdot f_{pu} \cdot c$$

c $= 1$ in the case of full bond of prestressing steel
c $= 0.65$ in the case of no bond of prestressing steel

This formula is still used in the Netherlands Standard NEN 3880 [22]. Perhaps the growing interest in the need to check the factor of safety was, in the fifties, the reason for the C.E.B.'s approach to reinforced concrete design on the basis of U.L.S. with check for S.L.S. On the other hand, up till now prestressed concrete structures are primarily designed on the basis of S.L.S. with check for U.L.S. This again illustrates the confusing approach to the design of concrete structures!

- Reinforced concrete since 1960:
  U.L.S. check S.L.S.
- Prestressed concrete:
  S.L.S. check U.L.S.

Why use these different approaches? Should not the same approach be adopted for both constructions methods? This is also a question to which an answer must be given.

1.3. *Partially prestressed concrete*

It is obvious that though it was considered a revolution that concrete structures could be designed in such a way that most of the concrete is under compression, so that no cracks can occur, there was no general agreement with the ideas of some specialists to allow cracks in prestressed concrete structures.

As far back as 1948 Magnel suggested in his book [6] that "perhaps a solution with only half the prestressing force is advisable, because, if under a certain load, a crack appears in the tensile zone, it will close if the load is decreased. Such a beam will be more economical than a fully prestressed concrete beam".

In fact, it was Abeles in the U.K. who tried to introduce partial prestressing. He proposed reducing the steel stresses - or using a number of unstressed wires - in pretensioned prestressed concrete in order to limit the initial compressive stresses in the precompressed tensile zones, without reducing the failure load of the structure. The elements produced in this way were roof girders, bridge beams and electrification poles [11].

Results of tests carried out on these elements show that under service load hardly any cracking occured in the tensile zone.

Freyssinet strongly deprecated this approach and - speaking to Abeles - he said in a lecture presented at the second F.I.P. congress in Amsterdam in 1955:

"The bolts connecting the wheels to the axles of racing cars are always turned as tightly as possible and not 'partly tight'. This must also be the case with prestressing!"

In 1956, Swiss engineers introduced partial prestressing into bridge design.

From the book published to mark the inauguration of the Weinland-Brücke (May 1958) it appears that in the transverse direction the upper flange of the box girder is partially prestressed by combining normal reinforcement and prestressing technique.

Under service conditions the stresses are checked on the assumption of uncracked sections and the stresses are also checked, on the assumption of a reinforced concrete section with a cracked tensile zone [43].

These new developments were introduced by M. Birkenmaier, among others. Since that time the use of partially prestressed concrete has been developed by Swiss engineers. Therefore already in 1968 it was possible for the S.I.A. to publish a Standard which officially opened the way to the use of partial prestressing [15], and partial prestressing has been the normal form of prestressed concrete construction in that country ever since. This means that an impressive body of know-how on the behaviour of these structures is already available. In general, "partial prestressing" has hitherto been a matter of debate, and no commonly accepted approach has as yet emerged.

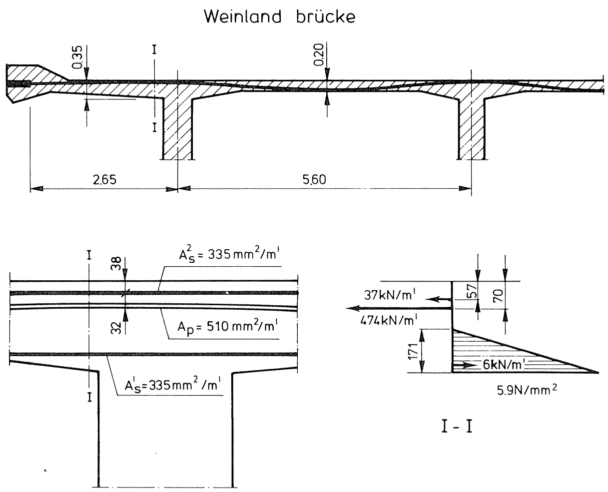

Fig. 1. Weinland bridge, Switzerland. Reinforcement, prestressing and internal forces. (Reproduced from [43] pages 132, 133).

There are several reasons for this situation.

1. There is no generally accepted approach to partial prestressing. As long ago as 1968 the present author published a "staircase" of possibilities [12, 13].

   That staircase still exists, but it is hardly understood how the steps really belong together.

	staircase
7	over-fully prestressed concrete (minimum compressive stress)
6	fully prestressed concrete (no tensile stresses allowed)
5	limited prestressing of concrete (some tensile stresses allowed)
4	prestressed concrete with cracks allowed
3	combination of prestressing and reinforcement
2	reinforced concrete with some artificial forces introduced by prestressing
1	reinforced concrete with "prestressed" reinforcement
0	reinforced concrete

It is evident that in these possibilities one uses:

- only prestressing steel
- only reinforcing steel
- or a combination of both types of steel

In the U.S.S.R. and other countries of Eastern Europe, slabs and beams assignable to "step 1" were extensively used for many years. In fact, with the exception of Switzerland and Eastern Europe, there cannot be said to have been any widespread acceptance of partial prestressing in other countries.

2. Inomata, reporting the results of the F.I.P. symposium held at Bucharest in 1980, speaks of "confusion" with regard to the definition of partial prestressing (18, 19). This confusion exists also with regard to defining the "level" of prestressing. Some authors define serviceability limit state conditions and use the concept "degree of prestressing":

$$\frac{M_{20}}{M_{\max}}$$

Others define ultimate limit state conditions and use concepts such as:

- "mechanical reinforcement ratio":

$$\frac{A_s \cdot f_{sy} + A_p \cdot f_{pk}}{A_c \cdot f_{cc}}$$

- "ratio of prestressing"

$$\frac{A_p \cdot f_{pk}}{A_s \cdot f_{sy} + A_p \cdot f_{pk}}$$

Also of importance is the magnitude of the mean compressive stress which is artificially introduced, and/or the magnitude of the bond between the tendons and the concrete.

3. In general, it is not clearly explained to practical designers what advantages are to be derived from the use of partial prestressing. There is hardly any information on the range of application of partial prestressing, its economic possibilities and its structural advantages.

An international symposium on partial prestressing was organized in Brussels as long ago as 1966 [12]. For the first time an exchange of experience took place, but it had only a very limited result.

The precast concrete industry in the U.S.A. is up till now not convinced that the use of partial prestressing will result in economic advantages. They only expect advantages of minor importance such as an increase in the factor of safety, the control of camber and the possibilities of reduction of initial compressive stresses [23].

In Switzerland partial prestressing has been used on a large scale since its introduction in 1968 with the S.I.A. Standard. Almost every prestressed concrete structure in that country is partially prestressed because this technique has been found to result in more ecomomical solutions.

4. The design and dimensioning of partially prestressed concrete structures are relatively complicated as compared with normal reinforced or prestressed concrete structures.

   The designer is confronted with theoretical problems which are not solved in a very practical manner.

A simple approach to the design is not possible in many cases. The problems mentioned are:

4.1. The influence of time-dependent effects on the stress distrubition in sections and therefore on the load under which cracks are initiated in the tensile zone.

A concrete structure is conceived as consisting of sections in which the stress distribution is calculated independently of other sections. In doing so, one is confronted with the stress distribution in a concrete section with prestressing steel and normal reinforcement. Due to time-dependent effects a redistribution of stresses over the section will take place. This means that compressive stresses develop in the normal reinforcement and that the tensile stresses in the prestressing steel and the compressive stresses in the concrete tensile zone are reduced [14].

In statically indeterminate structures it is almost impossible to calculate these effects and, if possible at all, the calculations are very complicated and time-consuming. The results cannot be interpreted very clearly, because loads normally change with time and therefore time-dependent effects are involved. Under uncreasing or decreasing load the section acts as a composite section (concrete and steel) and must be treated accordingly.

In the Swiss approach these complications are neglected. The losses of prestress are determined in the same way as in normal prestressed concrete structures. This "simple approach" has contributed greatly to the use of partial prestressing in that country.

In fact, it is the "sectional approach" to the structure that complicates the calculation. On the other hand the "structural approach", considers the structure as a whole and not as an assembly of substantially independent sections.

4.2. Control of crack width.

No suitable methods for crack width control have yet been developed. The "crack formulas" used in reinforced concrete design are often adopted for the purpose, and/or bar spacing is specified in relation to a limitation of the diameter of reinforcing bars [16]. In some cases hypothetical tensile stresses are introduced as a basis for crack width control [17].

In Switzerland the reinforced concrete section of partially prestressed concrete is treated as an eccentrically loaded section, concrete in tension being neglected [15]. This approach has proved to be very simple and practical.

4.3. The calculation of deformations (deflection, camber) is not at all simple.

4.4. The requirements with respect to the durability of the structure:

The conditions under which cracks are allowed in the concrete tensile zone are not well defined.

This also relates to the minimum requisite cover to the tendons.

There are no requirements with respect to the maximum permissible stresses in the prestressing steel crossing cracks and with respect to the permissible amplitude in steel stresses in dynamically loaded structures.

4.5. The generally accepted method for the design of reinforced concrete structures in the ultimate limited state cannot simply be applied to partially prestressed concrete structures [25].
Therefore it is impossible to give adequate rules for the detailing of the reinforcement, because control of crack width and deflection is always necessary. This means that there is a need for clear information on how to design partially prestressed concrete.

1.4 *Structural concrete*

At the "Advance Research Shop on Partial Prestressing", held in Paris in 1984 [21], the author made some statements on partial prestressing.
The first statement was as follows:
"Prestressed concrete has since about 1940 developed as a completely new method of construction in concrete side by side with reinforced concrete. Although this separate development has vastly increased the application of prestressed concrete and of concrete construction in general, it is now time to integrate the two construction methods (reinforced and prestressed concrete) into one common and general "structural concrete" method.
This can lead to:

- new possibilities of concrete construction by utilizing the whole range between fully prestressed and normally reinforced concrete;
- improvement of long-term behaviour of concrete structures (deformations, durability,) both in the serviceability and in the failure states.

Integration of reinforced and prestressed concrete does not lead to abolition, but to extension and improvement, of building in concrete".

In the present paper the features of the "Structural Concrete""model will be described in detail in order to initiate discussion also at international level.

2 Structural problems illustrating the need for adequate normal reinforcement

2.1 *Statically indeterminate prestressed concrete bridge beams*

The longitudinal shape of the tendon profiles in statically indeterminate structures is shown in Fig. 2.
This figure shows that the prestressing tendons are mainly concentrated in the bottom flange of the beams at midspan and in the top flange above the supports. This tendon profile can be determined bij means of the graphical method introduced bij Guyon and shown in Fig. 3. This approach can be assumed to be well known to every designer of

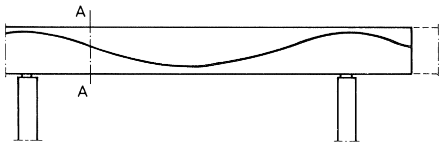

Fig. 2. Tendon profile - statically indeterminate beam.

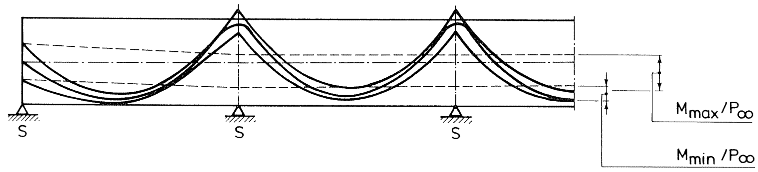

Fig. 3. Determination of the longitudinal shape of the tendon profile in a statically indeterminate beam.

prestressed concrete structures. It must be realised that the basis of the approach is *to avoid tensile bending stresses at every section of the beam.*

Of course this graphical method can also be used in a case where limited tensile stresses due to bending moments are allowed at every section. But due to the need to avoid (or to limit) tensile stresses, the tendon profile in the longitudinal direction is more or less *imposed.* The designer is very limited in his scope for choosing a simple tendon profile.

This imposed approach is the cause of many practical and structural problems such as:

- construction problems;
- problems caused by imposed deformations;
- rotational capacity;
- shear resistance.

All of these problems can be reasonably solved by using adequate reinforcement, as will be shown here.

2.1.1 A simple tendon profile in view of construction problems

As already mentioned, the tendon profile in the longitudinal direction can be complicated if it is based on the assumption that no (or limited) tensile stresses are allowed at every section of a beam. Fig. 4 clearly illustrates the effect of this approach in the case of a multispan box girder structure for a bridge.

Tendons are distributed in the bottom flange of the box girder at midspan and in the upper flange over the supports. This means that the tendons are curved in several planes (the webs and the flanges). The tendon profile is complicated and this awkward to achieve in practice. But it can also cause secondary effects, affecting the durability of the structures.

Because of the complex profile of a tendon it may occur that in certain - unpredictable - parts of the tendon considerable transverse (radial) forces are exerted by it on the concrete and that the latter is unable to resist these forces.

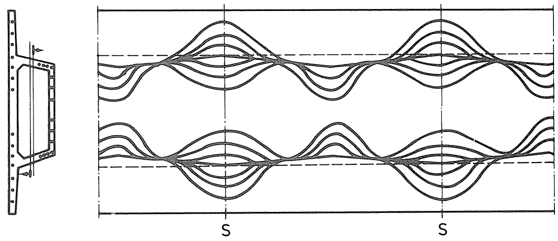

Fig. 4. Horizontal plan of tendons in a box girder (width exaggerated).

In French this effect is called "poussée à vide" (see Fig. 5). It sometimes causes considerable splitting or spalling of concrete structures on exposed surfaces which will subsequently play a major part in determining the appearance of these structures [48].
In the case of curved tendons lying side by side there is also the possibility that, on stressing the cable in the other duct, this cable will be pressed into the inner duct due to insufficient spacing of the ducts in relation to the magnitude of the radial force in the curved parts.
Besides these effects, in multicurved tendons there is likely to be a high risk that the tendons are not properly grouted.
Recent research in the Netherlands has shown that due to bleeding in the grout, also in cases of secondary grouting of tendons, the ducts are not completely filled with hardened cement paste [47].
Because this effect can be observed primarily in the upper part of curved tendons, there is a possibility of severe corrosion of the prestressing steel in these – not correctly grouted – ducts. This also shows that a simple tendon profile (in one plane) is very much to be preferred because the undesirable phenomenon of insufficient filling of ducts with grout will occur less in the case of such a tendon profile.
A simple profile means that there are no sharp curvatures over the supports. The tendons run smoothly through the concrete structure. Therefore they are not confined to the zone shown in fig.3. As a result, tensile stresses or perhaps cracks may develop in certain parts of the structure where the centre of gravity of the tendons is situated outside the zone of confinement in the case of full prestressing. Of course, in these zones normal reinforcement is now needed to control crack width. In a case where the number of tendons is to be reduced to obtain a simpler tendon profile as compared with full prestressing, normal reinforcement must be used to guarantee structural safety and serviceability.

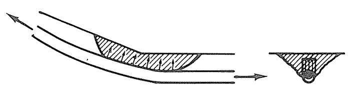

Fig. 5. Radial forces causing damage.

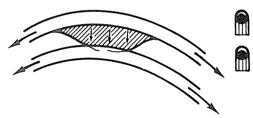

Fig. 6. Tendon pressed into an adjacent tendon.

2.1.2. Imposed deformations

In 1980 some damage was discovered in a number of bridges and viaducts in the Federal Republic of Germany (West Germany) and Austria. Wittfoht published a very interesting paper [27] wich will here be referred to. It deserves careful study. The problems will be discussed here very briefly.

The cross-sectional area of normal reinforcement in prestressed concrete bridges is, generally speaking, very limited. This means that there are zones in statically indeterminate structures in which the tendons "undulating in the longitudinal direction" are more or less confined to the centre of the concrete structure (box girder, for example). These zones are in the vicinity of the "zero moment" zones of statically indeterminate beams (see Fig. 2).

In general the cross-sectional area of the reinforcement in these zones is also very small in relation to the concrete section. In certain bridge construction techniques the construction joints are situated in these zones.

This means that the anchorage of most of the tendons are located at these joints and that couplers are used in the tendons. Also, to fulfil practical requirements of execution, hardly any reinforcement is continued across these construction joints. In this zone only the anchorages of the prestressing tendons are situated, with the transverse reinforcement needed to transfer the prestressing forces to the concrete.

In the case of imposed deformations (e.g., solar radiation causing a temperature gradient over the depth of the structure) bending moments will be generated due to restrained curvature. The magnitude of these bending moments depends also on the stiffness of the structure (e.g., depth-span ratio). In the case of solar radiation these bending moments will be positive over the whole length of the statically indeterminate structure. They are constant over the whole length, with the exception of the end spans.

In the zero moment zones of the structure the tensile stresses resulting from these bending moments can, up to a certain magnitude, be resisted by centrically compressed concrete sections. However, due to the presence of relatively large anchorage elements a part of the compressive forces is redistributed by time-dependent effects and therefore more or less "flow" through the steel components and not through the concrete interfaces of the construction joint. This means that the compressive stresses are often considerably reduced. At the construction joints the tensile strenght of the interface will be low. Therefore imposed bending moments can, in many cases, exceed the cracking moment of these zones, so that large cracks will appear in the vicinity of the construction joints. See Fig. 7, taken from [28].

This means that, after cracking, this joint will act more or less as hinge, this reducing the effect of solar radiation, but adversely affecting the safety of the structure because the anchorages and their couplers are subjected to hinge effects due to rotations.

This can also be the case in zero moment zones if the solar radiation is so intense that the resulting bending moments exceed the cracking resistance of solid concrete sections.

In some cases the radiation effects are increased by effects due to too much friction in

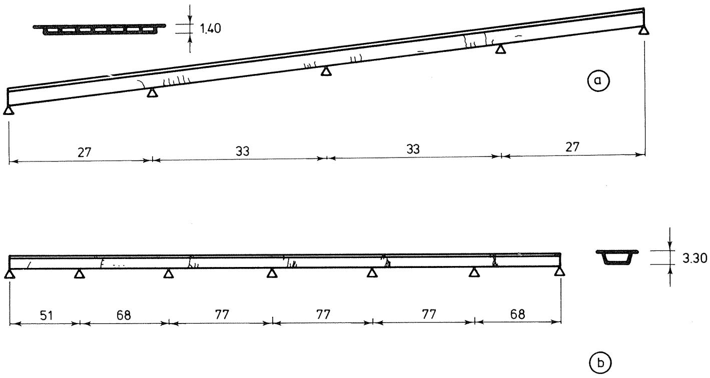

Fig. 7. Cracks in bridge structures:
a. Bridge slab.
b. Box girder (span-by-span construction).

the bridge bearings, which can result in large tensile forces if the temperature of the structure falls (at night, in winter, etc.)
If sufficient longitudinal reinforcement is present in these zones, only controlled cracking will develop in consequence of these imposed effects. Due to this cracking the structure will "soften" over a certain length, this reducing the imposed bending moments or imposed tensile forces.
Fig. 8 shows the distribution of bending moments over the length of a bridge structure:
1. due to normal loads;
2. with the influence of solar radiation, assuming constant stiffness over the length of the structure (no cracked zones) - linear elastic behaviour;
3. with the influence of solar radiation and the effect of "softening" due to controlled cracking of certain zones.

Because the effect of solar radiation is, generally speaking, a stochastic effect, a structure should always be so designed that its sensitivity to these effects is fairly low. In

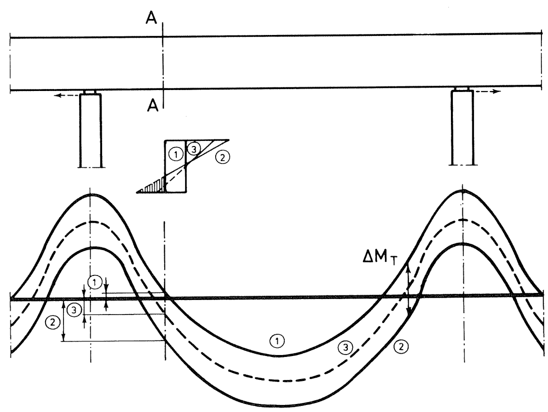

Fig. 8. Bending moments in a statically indeterminate beam.

other words, one has to design in such a way that, due to softening of some zones with accepted controlled cracking, the structure will respond within accepted limits of crack widths, steel stresses, etc.
The effectiveness of this approach is also demonstrated by research carried out by Cooke, Priestly and Thurston in the U.S.A. [29].

It will be stated here that, generally speaking, *"ONE CANNOT SUFFICIENTLY CONTROL* the effect of restrained imposed deformations by prestressing".
This statement will be explained as follows:

1. In most cases the magnitude of an imposed deformation is not known very accurately. For example, the magnitude of heat flux into a concrete structure by solar radiation depends on several factors such as time, season, duration of the heat flux, wind, orientation of a structure with respect to the sun, etc. In some cases measurements are made to determine this influence, but it is never certain that the measurements have been performed on a "normal" day or on a day with very exceptional conditions. So the distribution of this imposed deformation over a very long period is unknown. It means that one cannot determine the probability that a certain imposed deformation will occur (once a year, once in a hundred years, etc.). So it is impossible "to take account of the sun with sufficient accuracy in design calculations." It means that a particular assumption may be very greatly exceeded. The same considerations apply to settlement, shrinkage, creep, etc.
   Therefore it is necessary so to design a structure that it is less sensitive to the effects of restrained imposed deformations.
2. The magnitude of an imposed deformation often makes it necessary to introduce very high initial compressive stresses. In the case of a temperature change of short duration or an one day/one night effect the response of a structure is almost linearly elastic. So every change of temperature of one degree Celsius changes the stress by an amount $E_c \cdot \alpha$, i.e. means 0.3–0.5 N/mm^2/°C. A daily change in temperature of a restrained structural concrete element results in changes of stress of the order of 3–5 N/mm^2. This value is already far beyond the tensile strength of the concrete. But in many cases the imposed deformation is much more important, e.g. -100×10^{-6} and perhaps -300 to -500×10^{-6}. Then the extra compressive forces which have to be introduced by prestressing in order to prevent cracking of the magnitude of 10–15 N/mm^2. This will be impractical, uneconomical and in many cases impossible, because the initial compressive stresses are too high and decrease considerably in time due to creep effects.

Combining the two observations, it can be concluded that an uncracked prestressed concrete structure is not well suited to resist imposed deformations from a natural source. The structure is too stiff to respond adequately to imposed deformations, and it has not enough sensitivity with respect to the stochastic character of imposed deformations of natural origin. As will be shown in this case, the acceptance of controlled cracking can in many cases provide a solution for dealing with imposed deformations. In this respect prestressing plays a role of minor importance. This statement will be illustrated with some examples.

Underpasses under major roads

For the convenience of pedestrians and cyclists small tunnels are often constructed under main roads. They may have a length of 40 m or more. The same construction is used for culverts in which canals or streams are routed across main roads. To prevent cracking perpendicular to the axis of these small tunnels, prestressing is sometimes used, thus introducing compressive stresses into the concrete. In fact, the structure is thereby *pre*deformed longitudinally.

Because, after construction, these tunnels are embedded in the dams of the road, substantial friction between the soil and the concrete structure can develop in course of time. Due to temperature drop, combined with the effect of shrinkage of the concrete and relaxation of the compressive stresses, the ultimate tensile strain of the concrete, though somewhat improved by the predeformation, is too low to resist this imposed effect without cracking. Since insufficient or indeed hardly any longitudinal reinforcement is used, some large cracks will develop. Of course the same effect will also cause cracks in underreinforced underpasses built of normal reinforced concrete [46].

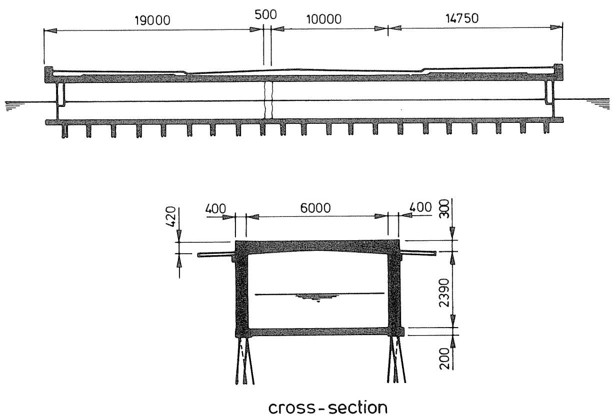

Fig. 9. Large cracks in an underreinforced underpass (measures in mm).

Box girders for bridges

The box girder is a well known and frequently used structural element in long-span bridges.

In view of imposed deformations, the cantilevered parts of the top flange of the bridge deck are in a much more unfavourable condition than the "spine beam" which is formed by the actual box section.

In this box girder the daily change in temperature is small, due to the mass of the structure and the climate inside the "box". The cantilevered parts, however, are exposed to the open air both on the upper and on the lower face.

Their dimensions are mostly kept to a minimum, in order to reduce the dead load of the bridge structures. Therefore these cantilevered slabs are prestressed transversely and are uncracked. It is assumed that in prestressing the box girder its cantilevered parts are also prestressed in the longitudinal direction. This would be true if the temperature of the whole structure were constant with time. But this is not the case!
The cantilevered parts subjected to imposed deformations by day and night. This means not only a continuous interchange of forces in the box girder beam between the spine and cantilevers - temperature rise causes increase in compressive stresses and some creep effects in the flanges, this relieving the spine; lowering of the temperature produces the opposite effect but will also induce tensile stresses in the cantilevers, resulting in cracks. If not enough longitudinal reinforcement is provided in these cantilevered slabs, large cracks may appear in the transverse direction, especially over the supports of the box girders if they are statically indeterminate.

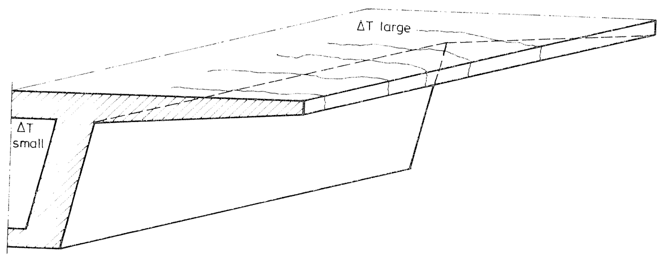

Fig. 10. Box girder with flanges.

2.1.3. Rotational capacity of support zones

In fully prestressed concrete structures the tendons are mostly curved sharply over the supports with the maximum permitted curvature, as already mentioned. This means that in the zones near the supports they „dive" into the concrete structure, this leaving its top zone without adequate reinforcement.
It is well known that in normally reinforced concrete structures the reinforcement must also be installed in the top zone near the supports. Due to oblique cracks in these zones the cracks in the top part of the structure will develop over a considerable part of the zone near the supports.
The bending moment distribution in these parts is "shifted" towards the span over a distance equal to the depth of the structure. But in fully prestressed concrete structures the "reinforcing tendons" are sharply curved downwards in the immediate vicinity of the support! Desk research carried out by Stuvo (30) has shown that fully prestressed concrete bridge structures actually display brittle behaviour near failure.
The following has emerged from that research:

a. The so-called "parasitic" bending moments still exist at failure. This means that at failure the distribution of bending moments over the length of the structure is almost the same as under linear-elastic conditions.

b. In calculating the failure load of fully prestressed concrete structures it is not permissible to use full plasticity because that will result in a much overestimated failure load.

In fact the conclusion of this Stuvo report is that at failure fully prestressed concrete structures do not behave with a fully plastic failure mechanisme. This means that, in general, the rotational capacity of the support zone is insufficient. At failure only a few large cracks will develop over the support and no oblique cracks. Because oblique cracks increase the size of the elongation region of the upper zone, they contribute considerably to the rotational capacity of the support zone (see fig. 11). But these cracks are possible only if sufficient tensile reinforcement is provided in this zone. And of course this reinforcement must be anchored in the concrete outside the support zone, in due conformity with the rules widely accepted for reinforced concrete structures.

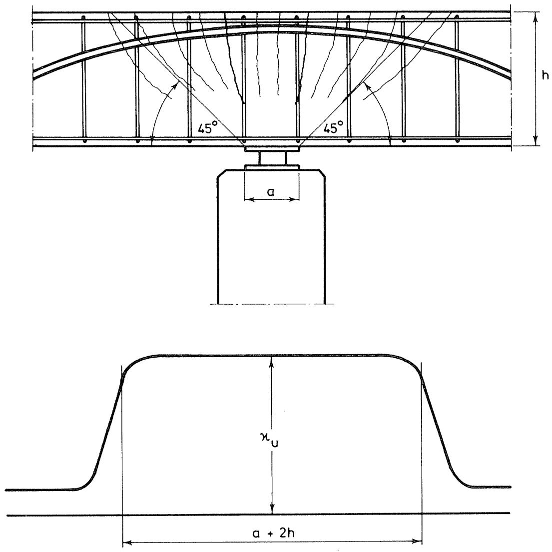

Fig. 11. Cracking near the plastic hinge over the support.

If the support zone is adequately reinforced it will possess sufficient rotational capacity; in that case it will be possible to assume full plasticity of a hinge over the support without overestimating the failure load.

Of course the amount of reinforcement in this zone must be in accordance with the laws of sound reinforcement of the tensile zone. This requirement is fulfilled only with a relatively large area of bonded reinforcement!

Fig. 11 shows the crack pattern of a support zone. The length of the plastic hinge over the support is assumed to be $(a + 2h)$. The maximum rotation of this hinge is (on both sides of the support) $\frac{1}{2}\varkappa_u(a + 2h)$, if $\varkappa_u$ is the maximum curvature at failure.

2.1.4. Shear resistance

The shear resistance of the support zone can be considered in close relationship with the rotational capacity of this zone.
In fig. 12 it is shown how in a fully prestressed concrete girder the failure load is transmitted to the supports by the interaction of the concrete structure and prestress.
The load on the structure is carried in the span by:

a. The upward load exerted by the curved tendons as such (radial forces).
   The curvature of these tendons is increased by the deflection of the girder. The tensile forces carrying this part of the load are assumed to have the same magnitude as the prestressing force before cracking of the tensille zone.
b. The upward load exerted by the (upward) curved compressive zone (arch action).
   It can, however, be shown that in many cases the compression line (line of thrust) is almost linear. This means that at failure the compressive force does not transmit any load to the support zone.
   This compressive force is assumed to have the magnitude of the prestressing force at the ends of the statically indeterminate structure.
c. The load carried by a "reinforced" concrete structure composed of bonded tendons, with increased tensile stresses due to partial cracking of the bottom flange.

Remark: Only the extra tensile stresses besides those introduced by prestressing are taken into account here!

This part of the load is transferred to the support zone by arch action of the compressive zone and curvature of the "reinforcement".
The concrete arch and the "reinforcement" are connected to each other in the non-cracked parts of the girder. Fig. 12 clearly shows that this "reinforced" concrete structural element is not adequately reinforced.
The arch cannot develop properly and its anchorage in the "reinforcement" is poorly detailed. This is also the case because stirrups cannot play a role in this anchorage zone!

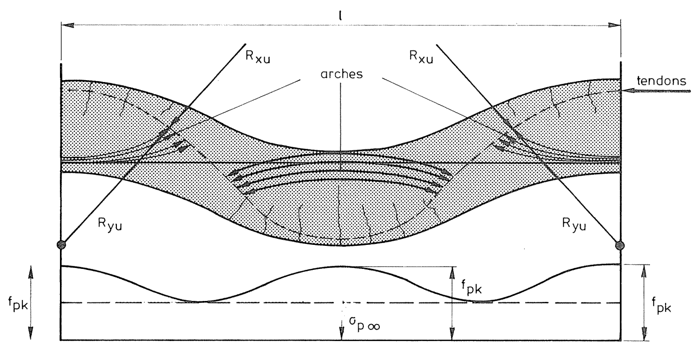

Fig. 12. Transfer of load to support (no longitudinal reinforcement).

In the same way - but upside down! - the load is transferred from the span to the support in the support zone. Here, too, there is seen to be a poorly detailed "reinforced" concrete structure.
Fig. 13 shows the same girder, but now also provided with sufficient normal reinforcement, mostly combined with reduction of the amount of prestressing. In this case the reinforced concrete structure is satisfactorily detailed. The arch (or truss system) is anchored in the tensile zone. The arch action of the compressive zone is therefore well developed.
The stirrups reinforce this anchorage and "lift" the forces anchored at the bottom of the beam towards the top of the beam. At the top of the beam these forces are in equilibrium with those of the support zone and are transferred to the supports in the same way as in the span. Comparison of the two diagrams clearly shows that without suitably detailed longitudinal reinforcement a prestressed concrete structure cannot behave well at failure. There is no clear "flow" of forces through the structure.

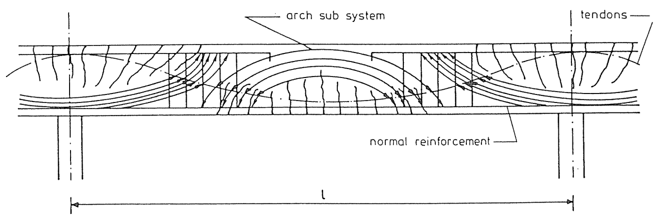

Fig. 13. Transfer of load to support (sufficient longitudinal reinforcement).

From these four observations discussed in 2.1, it will be apparent that in every statically indeterminate beam system, including prestressed concrete, adequate reinforcement is necessary. This means that as a basis for such a concrete structure a certain minimum amount of reinforcement is necessary to ensure sound structural behaviour under service conditions and at failure.

2.2 *Flat slabs and beam-and-slabs systems*

Prestressing is often used, mainly with unbonded tendons, in the construction of car parks, residential and other buildings. Here again the application of prestressing techniques has caused many problems.

2.2.1 Flat slabs

In the Netherlands the use of unbonded tendons was introduced in the early seventies. The field of application was slabs and flat slabs. Only the last-mentioned structures will be discussed here.
Introduction of a prestressing technique into flat slab construction means simply that the Standards for prestressing (the so-called part F of the Netherlands Standard) are

applied to flat slabs. This Standard was not written with flat slabs in mind, but they were not excluded either!
As a result, flat slabs were designed and constructed according to these parts of the Standard. This resulted in the absence of normal reinforcement, with the exception of the zones around the supporting columns. It is well known that the reinforcement also plays a very important part during construction. Due to hardening of the concrete (shrinkage, temperature) tensile stresses will already develop a few hours after concreting. At this stage the role of the reinforcement is very important in avoiding cracks or in limiting crack widths. Due to lack of any reinforcement, large cracks developed in the slabs in several directions, sometimes causing parts of slabs to become completely detached - except for a few unbonded tendons - and collapsing after a time.
Of course very soon measures were taken to prevent these effects. Up till now, no adequate solution for this problem has been found. This is only possible by adopting a different approach in which basic reinforcement is required both in reinforced concrete and in prestressed concrete structures. Now some "fudging" has been done by issuing an extra Standard (part H of the Netherlands Standard) stating requirements for normal reinforcement to be provided in slabs. But it is evident that problems such as those mentioned here can in future be avoided only by adopting an entirely new conception of concrete structures.

2.2.2 Beam-and-slab systems

In large office buildings or industrial buildings the concrete structure often consists of slabs monolithically connected to the supporting beams. These statically indeterminate beams are supported by columns. Normally such forms of construction are used in multi-storey buildings.
The beam and slabs jointly transfer the load to the supporting columns. The beam is in fact converted into a T-Beam by this combined action of slab and beam (see Fig. 14).

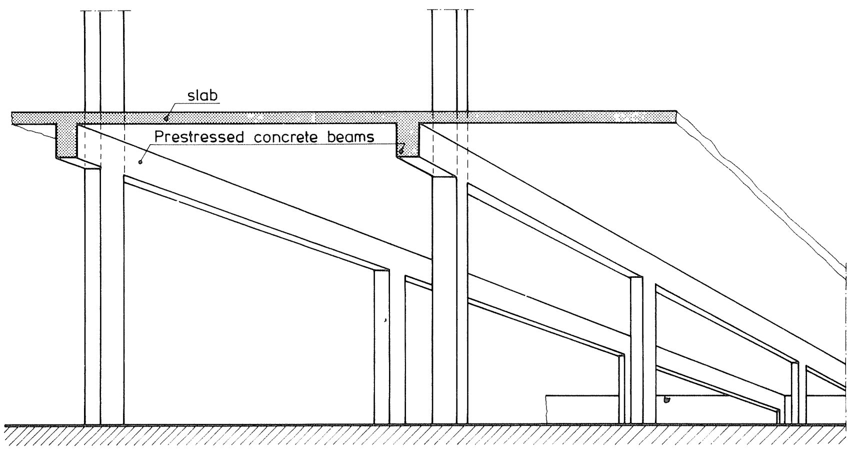

Fig. 14. Beam-and-slab system.

If the beams are prestressed longitudinally, there arises a problem concerning the effective flange width to take into account for calculating the section properties (moment of inertia, section modulus, location of centre of gravity) of the T-beam in the span and over the supports, because the effective flange width affects these properties. In fact this problems cannot be adequately resolved in this way because the prestressing force exerted by the tendons at the end supports is carried – after a zone of distortion – by the entire cross-section comprising beams and slabs, but the upward or downward load due to the curved tendons and the whole load of the structure is carried by part of the beams and slabs acting together as T-beams.
It can be shown that in this case a good design can be obtained only by considering the combined action of a reinforced concrete structure artificially loaded by prestressing. In this respect the problems already referred to in 2.1 are of importance [50].

3 The model of structural concrete

3.1 *Why structural concrete?*

In this paper it has already been explained that in practice a very important distinction has been introduced between reinforced concrete, partially prestressed concrete, prestressed concrete with limited prestressing, fully prestressed concrete, etc.
Standards or codes of practice are issued in many countries for several of the abovementioned categories of concrete structures. The designation "structural concrete" combines all these categories into one construction material called "structural concrete". This means that there no longer exists any distinction in structural approach to the categories mentioned.
The second reason for this designation lies in the diagnosis made in the foregoing chapters that there is, generally speaking, always a need for a certain minimum amount of reinforcement in every concrete structure, not just in reinforced concrete. In contrast with common practice the basis of every structure – reinforced or prestressed – will be, in this approach, a *reinforced concrete structure* which is not, or is somewhat, or is substantially strengthened by the introduction of artificial loads (or forces) by means of prestressing systems.
As requisite minimum reinforcement in a structure can also be provided in pretensioned prestressed concrete, the prestressing steel in such structural elements is fully bonded to the concrete and well distributed over the cross-section.
So in this case it acts as bonded reinforcement even already in the early stage after concreting and during hardening of the cement paste. Of course one has to be careful because the sensitivity of the prestressing steel to corrosion is much greater than that of normal reinforcement. The prestressing steel is directly embedded in the concrete with normal cover, so one must be very cautious about allowing cracks in the tensile zone of pretensioned prestressed concrete beams. Because such structural elements are factory-produced, their shape, cross-section, etc. are developed for economical factory production. Therefore the design of these elements is quite different from the design principles applied to normal concrete structures, mainly concreted in situ.

Because the designation "structural concrete" is now used for all concrete structures containing basic reinforcement, unreinforced concrete structures must now be designated as "unreinforced structural concrete" (dams, walls of housing structures etc.).
This type of concrete structure has to be treated differently from the structural concrete discussed in this paper.
Structural concrete covers the whole field of application of (normally) reinforced concrete structures with or without artifcial loading by prestressing. The following definition of structural concrete can therefore be given:
"Structural concrete" structures are built of reinforced concrete which can - optionally in combination with artificial loading, introduced by prestressing techniques - resist, in a controlled way, all the actions exercised on these structures by loads, imposed deformations and other influences (earthquakes, explosions, etc.) Moreover, these structures must be constructed in a safe and economical way.
In this definition "controlled way" indicates control of deformations, cracking, durability, structural safety, etc.
It must be clear stated that there are perhaps other possibilities which can be regarded as partially prestressed concrete. In view of its importance in the field of developments in concrete construction, "structural concrete" as has already been defined here will be mainly considered in this paper.

3.2 *The role of the reinforcement*

In structural concrete the role of the reinforcement is very important. This means that - in general - every concrete structure needs *basic reinforcement.* In combination with added reinforcement and/or prestressing the structure will resist all the influences acting on it (loads, imposed deformations, etc.) In general terms the role of this reinforcement can be described as follows.

3.2.1 During the period of construction

Limiting cracking of the "green" concrete due to shrinkage, temperature gradients, etc.
In this part of the life of a concrete structure the role of the reinforcement is already very important and cannot be neglected.

3.2.2 Serviceability limit state

Control of crack width due to the influence of loads and imposed deformations: Especially in the cases where free deformation of a concrete structure subjected to imposed deformations is resisted by other structural components or by the shape of the structure itself (circular tanks) the role of the reinforcement in controlling crack widht is predominant. In these cases it is hardly possible to control these effects by prestressing as such, as has already been explained in Section 2.
Normal reinforcement not only contributes to controlled cracking but - in the case of

concrete structures with a cracked tensile zone - also affects the stiffness of the structure. In the case of imposed deformations it therefore reduces the internal forces introduced by these deformations.
In concrete structures the normal reinforcement is very important to control crack width under the influence of several actions. Therefore it is necessary to provide the practical designer with appropriate "design tools" for the calculation of the correct reinforcement.
The control of crack width of concrete depends on a number of factors, such as:
1. the tensile strength of the concrete;
2. the bond behaviour of the reinforcement.
   With regard to this behaviour the surface of the bar (plain or ribbed), the concrete strength and the cover to the reinforcement are of importance;
3. the diameter of the reinforcing bars;
4. the detailing of the reinforcement - bar spacing and quantity of reinforcement;
5. the magnitude of the imposed deformations (shrinkage, temperature);
6. the crack widths which are permitted in view of durability.

Only a simple but adequate model, to be used in several types of structures, can fulfil the designers' requirements. Such a model will be described in Chapter 4.

3.2.3 Abnormal conditions, calamities

Normal reinforcement contributes to the fire resistance of concrete structures. In the case of earthquakes, adequate reinforcement can provide sufficient ductility of a concrete structure, especially of its connections, and can therefore prevent premature failure. Under impact loading the reinforcement also provides sufficient resistance of the structure thus affected. In the case of impact by (very) cold liquids (so-called "cold spot" problems) adequate reinforcement can provide sufficient structural safety.

3.2.4 Ultimate limit state

Normal reinforcement can ensure overall structural safety. With adequate reinforcement a concrete structure can behave in a ductile manner. This reinforcement can make sure that there is sufficient plasticity of the structure on approaching failure. In this context the rotational capacity of statically indeterminate beams over supports, already discussed earlier, calls for mention. It is very important that concrete structures should "give warning" of overloading before they collapse due to excessive cracking and deflection. Brittle failure of an overloaded concrete structure can be permitted only under very severe restrictions.

3.3 *The introduction of prestressing*

In general the prestress which is introduced into concrete structures can be primarily regarded as an artficial load (see Fig.19).

This artificial load normally consists of two components:
- compressive force introduced into the structure in the anchorage zone;
- transverse forces perpendicular to the axis of the prestressing tendons in the curved part of these tendons.

  Particularly this transverse force can, more or less, "balance" the load on the structure. Already in the sixties this approach was developed by several designers, e.g., T.Y. Lin in the U.S.A.

Of course this behaviour of the prestress is very important under normal conditions. If the prestressing steel is bonded to the concrete it acts - more or less - as reinforcement in the case of cracking of the tensile zone.

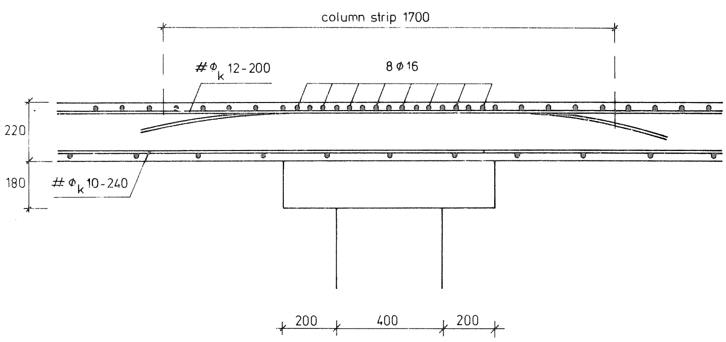

Fig. 15. Prestressing and normal reinforcement in slab to column connection.

It depends on the strength of the bond, as compared with the bond of normal reinforcement, whether this contribution of the prestressing steel to controlling the crack width is of interest or not. The fact that - in the case of a cracked tensile zone - the stress in the prestressing steel may increase shows that this phenomenon must not be ignored. There is a possibility that the magnitude of the increase in stress in the prestressing steel across cracks is unacceptable under static conditions (durability) as well as under dynamic loading. Bonded prestressing steel also contributes to increasing the failure load of concrete structures.

The introduction of prestressing as a very real "artificial loading" can be illustrated by prestressing with unbonded tendons, widely used in flat slabs (see fig.15),and by the use of external tendons in bridges built in France and in the U.S.A. (see Fig. 16). In both cases the function of the prestressing is mainly that of an artificial loading. The prestressing steel does not play a role in controlling crack width (if cracks are permitted). Also, at failure the increase in the tensile force in the external (or unbonded internal) tendons is small because it is caused only by the deflection of the whole structure. This shows the importance of the model of structural concrete, a model with always a certain minimum of reinforcement. In this respect it must be mentioned that several of the bridges built with external described here because they are composed of prefabricated (precast) elements without longitudinal reinforcement across the joints.

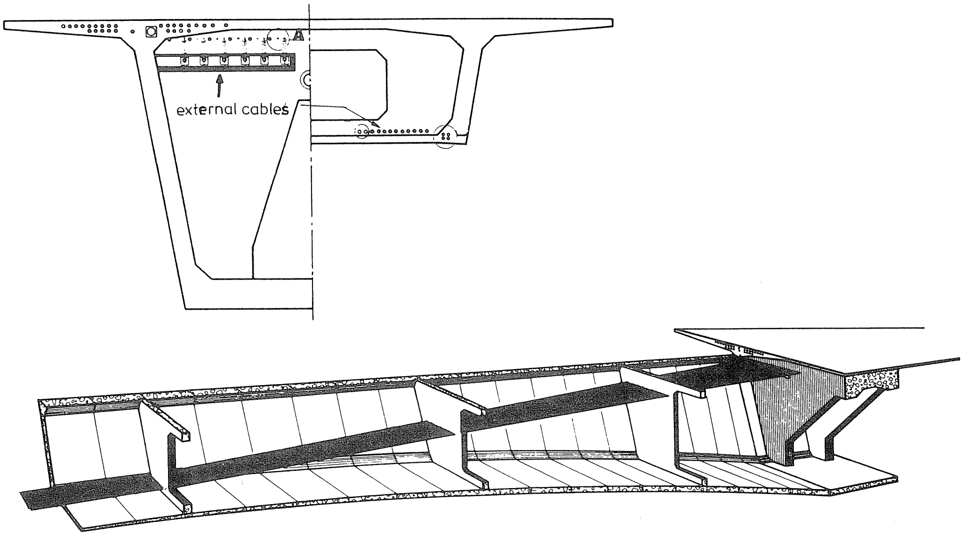

Fig. 16. Box girder prestressed with external tendons; taken from [49].

3.4 *The design procedure*

The designe procedure for "structural concrete" will be described here in more detail. The procedure may comprice several steps (see fig. 17).

3.4.1 Step 1: The starting point is a concrete *structure* or a concrete structural element of which are known:

- The shape and dimensions of the cross-section, e.g. rectangular beam, T-beam, box girder, slab;
- the span, distance between columns, supporting slabs;
- the type of support;
- the construction method (concreted in situ, precast construction);
- the quality of the materials (concrete and steel) to be used in the structure.

This concrete structure, presumed to be of reinforced structural concrete, is provided with basic reinforcement, e.g. a certain minimum cross-sectional area of reinforcement in beams, slabs and columns. The reinforcement is also installed in the cross-section and in the transverse direction of box girders. This basic reinforcement is, in general, specified in the national standards for reinforced concrete structures.
It is assumed that the loads, imposed deformations, etc. are also known.
This basic reinforcement can be determined with the rules, given in C.E.B. model code and National Standards.
It is assumed that in step 1 the loads and the expected imposed deformations for which the structure must be designed are also known.

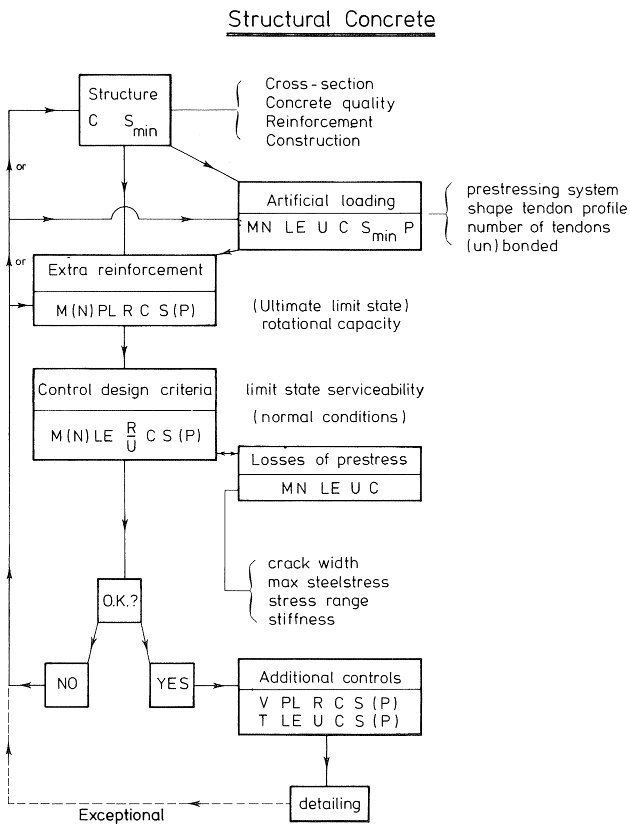

Fig. 17. The model of structural concrete. Notation is explained in Chapter 8.

3.4.2 Step 2: Artificial loading (see fig. 18)

The structure is assumed to be prestressed. (This is not essential per se! In some cases this step can be omitted. It is then a case of designing an "ordinary" reinforced concrete structure!).

There are a number of reasons for considering the introduction of prestressing in a basically reinforced structure, e.g.:

- Limiting the deflection of the structure:
  By prestressing, a part of the total load acting on a concrete structure can be balanced, no time-dependent deflection of the structure will occur (only axial shortening);
- Influencing the live load causing tensile flexural stresses:
  In this case it is possible to influence the live load which will cause cracks in the tensile zone and therefore also the live load at which existing cracks will close on unloading. The choice of the magnitude of the live load at which cracks start to re-open can be based on the need for sufficient stiffness of the structure under certain

load conditions and the need to avoid open cracks in the tensile zone under sustained load, e.g. with a view to ensuring the durability of the structure in an aggressive environment;

- Assurance of liquid-tightness - depth of the compressive zone, limitation of crack width in the tensile zone - in walls of liquid-containing tanks;
  This can be especially of importance in the case of temperature gradients in the wall. In circular tanks the circumferential prestress is, in fact, a preload of the wall in the opposite direction to that of the liquid load;
- Balancing of a part of the live load on heavily loaded concrete structures:
  The aim of this balancing may be to reduce the depth of the structure or to obtain a simple cross-sectional shape, e.g. T-section instead of I-section. The shape can this be made very suitable for constructing the mould and therefore contribute to greater economy of construction;
- Influencing the behaviour of a restrained concrete structure in such a way that above a certain level of imposed deformations the structure will "soften" due to controlled cracking, this avoiding an unacceptable increase in internal force caused by these imposed influences, such as temperature gradient, shrinkage gradients, settlements.

To determine the mode of prestressing, it is necessary primarily to choose:

1. The prestressing system; the type and tensile capacity of the tendons to be used; bonded or unbonded tendons.
2. A simple shape of the tendon profile over the length of the structure. The curvature of the tendons must, preferably, lie within one plane (no curvature outside this plane!). This is very important with view to economical construction, effective grouting of the ducts, and the possible occurrence of unforeseen transverse forces in curves of tendons near the surface of the structure, causing spalling of the concrete cover, as already discussed.
3. The magnitude of the initial prestressing force including the effects of friction of tendons in the ducts during the prestressing operations.
4. The magnitude of the working (effective) prestressing force, i.e. taking account of the losses of prestress due to shrinkage and creep of the concrete and relaxation of the steel.

In this step the bending moments, the shear forces and the deformations can be calculated assuming a linear-elastic state of stress in the structure.
The result of this design step will be a concrete structure with basic reinforcement and a number of tendons with simple profiles over the length of the structure.

3.4.3 Step 3: Checking the factor of safety - Ultimate Limit State (U.L.S.)

In this step the failure load of the structure will be calculated. If the factor of safety is acceptable and in conformity with the requirements, no further investigations are necessary in this step. If the factor of safety is insufficient, normal reinforcement can be added in the tensile zone to fulfil the requirements. In the calculation of the failure load of the structure the rotational capacity of critical zones (especially over the supports) must be taken into account. If this capacity is insufficient, adequate measures must be taken.

3.4.4 Step 4: Checking the structure in the Serviceability Limit State (S.L.S.)

In this step the following effects must be checked:
- Deformations, including time-dependent ones; stiffness of the structure;
- Maximum steel stresses in the normal reinforcement as well in the prestressing steel;
- The amplitude of the steel stresses in the case of cyclic loading;
- The crack width (and its development with time), especially with regard to requirements of durability;
- The safety of the structure in various stages of the construction process.

If this check reveals that some requirements are not fulfilled, it will be necessary to take a step back in the design process either by reshaping the cross-section (for example, altering the depth) or by changing the artificial loading by prestressing and/or normal reinforcement.
The normal reinforcement must be so detailed as regards bar diameter and bar spacing that the requirements of crack width and crack distribution can be fulfilled.
In Chapter 4 some components of the model of structural concrete are described which can be used as design tools.
The relationship load (bending moment) - crack width - stiffness can be determined with these components. In the case of alternating (dynamic) loads the amplitude of the changing steel stress can also be calculated.
If it is not possible to achieve a sound structure - with respect to crack width, stiffness, variation in steel stresses, etc. - with practical values for bar diameter and bar spacing, then it will be necessary to go back into the design procedure.
The following possibilities must be considered:
- enlarging the cross-section of the longitudinal reinforcement (in most cases this is neither economical nor practical);
- increasing the artificial load due to prestressing;
- adapting the cross-sectional dimensions of the concrete with respect to depth and other dimensions or shape.

In the case of imposed deformations it must be checked that:
- cracking of the concrete tensile zone does not start below a certain agreed level of bending (load + imposed deformations);
- the stiffness of the structure is so reduced that the reduction of bending moments and forces which is due to imposed deformation is sufficient. If this reduction is not sufficient, the decompression bending moment must be decreased by reduction of the prestressing forces.

In this step it must also be checked that the other requirements are fulfilled in choosing a certain level of artificial loading (prestressing).

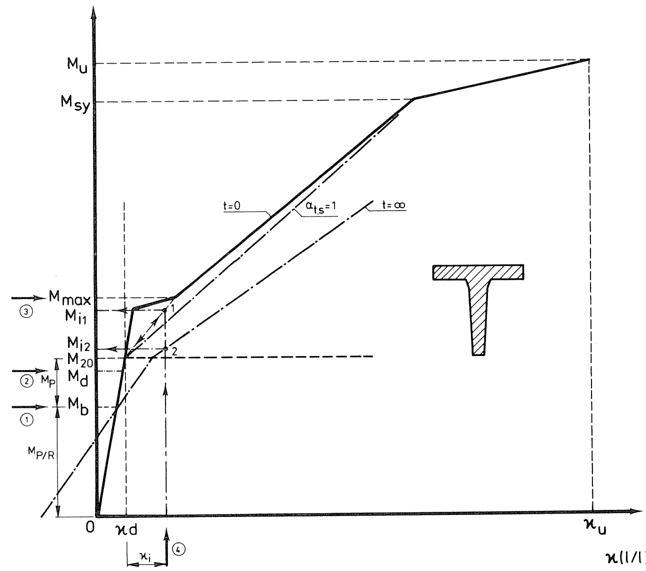

Fig. 18. Moment-curvature relationship of structural concrete.

Fig. 18 illustrates the approach with a moment-curvature diagram. In this diagram the following bending moments are indicated:

M_b = due to balanced load
M_d = due to dead load
M_{20} = M_{dec} = due to decompression of tensile zone (in fact, the "start" of the behaviour of the structure in the manner of normal reinforced concrete)
M_{max} = due to total load
M_{i1} = crack pattern fully developed under rapidly imposed loading
M_{i2} = crack pattern fully developed under slowly imposed loading

In this diagram the dot-dash line shows the moment-curvature relationship for the case where M is held constant at every level.
The time-dependent deformations have taken place at the end of the period $t=0 \rightarrow t=\infty$.
The diagram shows that, due to these time-dependent effects, the bending moment M_{i2} is much smaller than M_{i1}. This means that, for example in the case of slowly developing settlements, the "softening" of the structure already starts at lower bending moments than in the case of rapidly imposed loading (e.g. daily changes in temperature).
There are several considerations on which the choice of magnitude of the artificial load introduced by prestressing may have to be based:

Arrow 1 Radial forces due to curved prestressing tendons balance the dead load and normal sustained load.
In this case no important time-dependent deflection will occur.

Arrow 2 The bending moment M_d due to dead load (and sometimes in combination with normal sustained load) is smaller than the decompression moment M_{20}. This means that under normal conditions there are no open cracks in the tensile zone because there are now active compressive stresses in this tensile zone.

Arrow 3 In the case of the maximum bending moment M_{max} the crack distribution in the tensile zone is not fully developed. It means that the crack width is controlled and some unforeseen extra imposed deformations will not cause excessive cracking.

Arrow 4 Choice of the magnitude of the artificial loading by prestressing to such a level that under rapidly imposed deformations the cracks in the tensile zone are under control also in the case of repeated imposed effects (solar radiation). As already explained, the bending moment M_{i2} due to slowly increasing imposed deformation settlements will be smaller than M_{i1} due to rapidly increasing deformations.

3.4.5 Step 5: Detailing of the reinforcement - shear - torsion

Step 5 will comprise other checks, such as: shear resistance, effect of torsion and, last but not least, detailing the reinforcement and the prestressing. This means, for example, determining the cross-sectional area of stirrups.
The given design procedure is a well-known one. If step 2 in this procedure is omitted, one is left with the normal procedure used for reinforced concrete structures.
The set-up of the procedure also shows the smooth transition from "normally" reinforced concrete due to the introduction of prestressing techniques towards partially and fully prestressed concrete. It must be realized that in the latter case a suitable quantity of basic - normal - reinforcement is also required!

3.5 *Possibilities for the introduction of prestressing in reinforced concrete structures*

Introduction of prestressing offers many (new) possibilities in the design of concrete structures, such as: .

1. Possibility of simple and more economical methods of construction;
   Simple tendon profile over the length of the structure;
   Limiting the number of tendons by taking full account of the normal reinforcement [24];
   Simple shape of moulds [24];
   Limiting initial compressive stresses - after prestressing - in precast elements, thus increasing the loadbearing capacity in some cases (e.g. short span - large live loads) [24];
   Simple way of reinforcing flat slabs and beam-slab systems, for example;
   Simple wall-to-base connections in liquid-containing tanks [24; 41];

Simple methods of construction, e.g. in bridge construction, by limiting steps in the construction procedure.

2. Increasing the loadbearing capacity of heavily loaded structures, such as the roof and invert of submerged tunnels, loadbearing walls, etc. In several cases the depth of a structure can be reduced [24].
3. Control of crack width:
   - due to imposed deformations (temperature gradient - settlement);
   - to fulfil requirements of liquid-tightness [41];
   - heavily loaded structures of limited depth.
4. Limiting the amplitude of steel stresses under cyclic loading.
5. Improving the durability of concrete structures by avoiding cracking or allowing cracks only under incidential heavy loading.
6. Control of deformation of concrete structures under long-term loading (dead load), e.g. in the case of bridges, flat slabs, roofs. In this way the stiffness of the structure can be improved. This means that under certain conditions more slender structures can be used.
7. Limiting the influence of imposed deformations by allowing controlled cracking (controlled "softening").
8. Limiting bending moments and shear forces by the introduction of load-balancing prestressing forces.
9. Improving the behaviour of concrete structures, such as their rotational capacity, torsional stiffness, shear resistance.

3.6 *Conclusion*

The possibilities of a new approach - structural concrete - have been indicated in this chapter. It will be clear that in the model presented here there is no significant difference in the design procedure between the case where no prestressing is employed and the case where prestressing techniques are introduced.
For both cases the model is consistent and can be used in the same way.
Of course for checking the behaviour of the structure or for use in the design stage the designer must have various resources at his disposal. In the next chapter some components of this model will therefore be described.

4 Components of the "structural concrete" model

4.1 *Prestressing used as artificial loading*

The determination of the internal forces in a reinforced or prestressed concrete structure is always based on the calculation of stresses in the concrete and steel at a section or several sections.
In prestressing concrete structures the stresses due only to prestressing are calculated with an eccentric force acting at the section. In statically determinate structures the

eccentricity of this force is equal to the eccentricity of the centroid of the prestressing steel. In statically indeterminate structures the prestressing force only acting at a section will, in general, not coincide with the centroid of the prestressing steel.
In this case the so-called secondary (or "parasitic") bending moments must be taken into account. In fact, these bending moments are *not "parasitic"* but are caused only by statically indeterminate effects! In 1957 Mehmel already introduced prestressing as an artificial load acting on a statically indeterminate structure. This approach was also introduced by T. Y. Lin in the U.S.A. and by the present author in the Netherlands [31 - first edition 1963].
As is generally known, the influence of the prestressing can be separated into three parts:

1. A centric compressive force, acting in the end zone of a concrete structure (the anchorage zone).
2. A radial load perpendicular to the centre-line of curved tendons, acting on the concrete structure. This load can act upwards or downwards, depending of the direction of curvature of the centre-line in question.

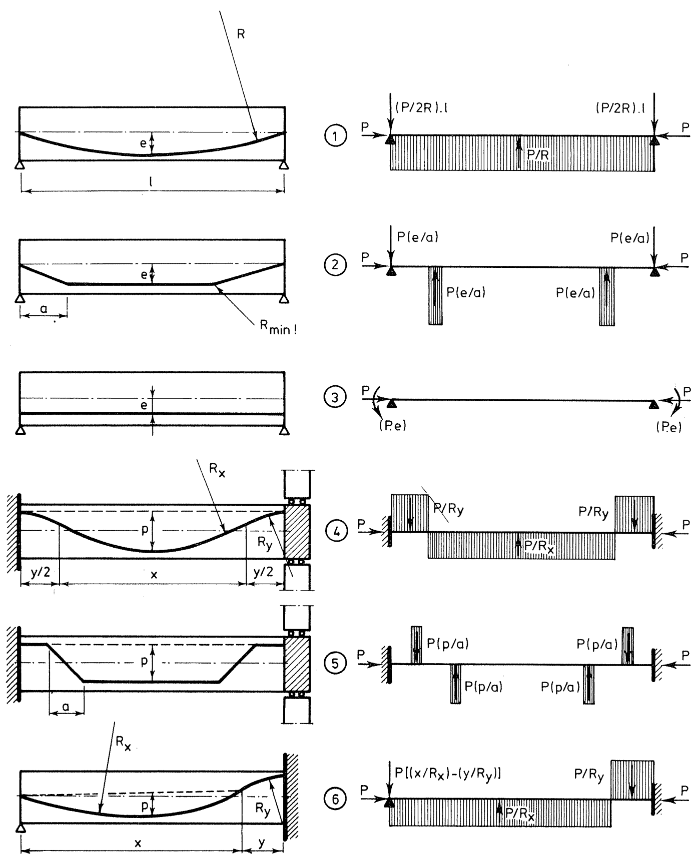

Fig. 19. Artificial loading by prestressing.

3. Bending moment in the end zone acting on the concrete structure if the centroid of the prestressing tendons (with anchorages) does not coincide with the centroid of the end zone.

By introducing the prestress as an "externally" acting artificial load the structure can be considered as a total system *(structural approach)* and no longer as an assembly of a number of sections *(sectional approach).*

This structural approach is of major importance for the "structural concrete" model because now the artificial load, due to prestressing, acts on a - reinforced - concrete structure.

Some examples of this approach are given in Fig. 19.

4.2 *The tension member of structural concrete*

The model of the tension member will be briefly described. Background information and applications are presented in Appendix 1.

4.2.1 The tension member of reinforced concrete

4.2.1.1 The "spring" in the distortion zone of a crack

The distortion zone of a crack is that part of a reinforced concrete tension member in which the stresses in the concrete and reinforcing steel are influenced by the initiation of the crack. Outside the distortion zone, the stresses in the concrete and steel are constant and there are no bond stresses between the reinforcing bars and surrounding concrete.

If an uncracked reinforced concrete tension member is loaded in tension, the tensile force is thus resisted by concrete and reinforcement as well.

At a certain magnitude of the tensile force the tensile strength σ_{cr} of the concrete and its elongation capacity ε_{cr} will be reached in the weakest part of the tendon. The tensile strength of the concrete is in general not constant over its length but varies randomly from section to section.

After initiation of the first tensile crack the tensile force in the crack is resisted only by the reinforcement. The tensile force in the reinforcement in the crack is transferred to the concrete in the distortion zone on both sides of the crack. This means that the stresses in the concrete increase from zero (near to crack) to a value which is again constant outside the distortion zone. Because the tension member is cracked at its weakest place the concrete can resist - without cracking - the tensile stresses σ_{cr} outside the distortion zone.

For reasons of equilibrium at every section of the distortion zone the total tensile force resisted by concrete and reinforcement is constant equal to the externally acting tensile force.

Due to the initiation of a crack the reinforced concrete tension member undergoes an extra elongation caused by the increasing deformations of the reinforcement over the distortion zone.

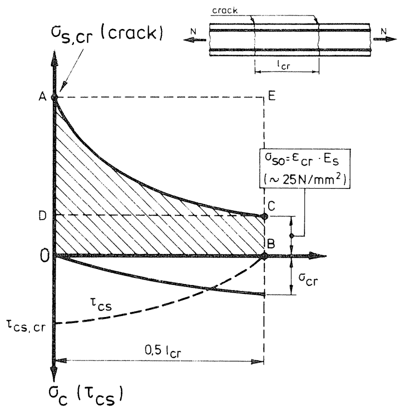

Fig. 20. Stresses in steel and concrete and bond stresses over one half of the distortion zone.

This part of the reinforced concrete tension member acts more or less like a "spring". The external tensile force is, however, limited by the tensile strength σ_{cr} of the concrete in the next weakest part of this member!
The crack width is determined by the extra elongation of the reinforcement over the distortion zone and the shortening of the concrete over this zone.
If the tensile force on the tension member is gradually increased, more cracks will be initiated and a crack pattern will develop over the length of this member.

4.2.1.2 Behaviour under restrained imposed deformations

To illustrate the behaviour of the reinforced concrete tension member under increasing load the case of imposed deformation will be described here. In this case the length of the tension member is held constant if, for example, the tendon is subjected to a gradual lowering of temperature.
This means that the tension member is assumed to be fully restrained at the ends. In the first phase this member behaves in the manner described in 4.2.1.1. Now the external force is initiated by the restraint of the tension member.
Due to the initiation of the first crack, however, the external force decreases suddenly by "spring action" of the zone around the first crack. As a result, the tendon undergoes an overall reduction of stiffness; so the external force decreases. The magnitude of this force depends also on the stiffness and the length of the tension member.
If the drop in temperature takes place gradually, the tensile force in the reinforcement concrete tension member will increase again till in the next weakest zone a crack occurs and the same effect as that associated with the first crack will be repeated.
Fig. 21 shows the serrated curve of the reinforced concrete tension member under imposed deformation. This diagram shows also that the decrease in the tensile force is smaller in relation to the previous decrease upon initiation of a new crack.
The distortion zone of the first cracks will generally *not* overlap. However, if more

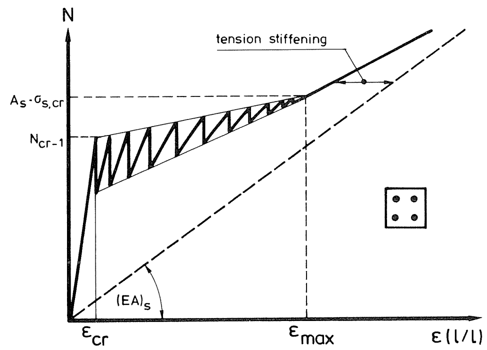

Fig. 21. The $N-\varepsilon^{1}/1$ relationship for a reinforced concrete tension member under imposed deformation.

cracks develop, the distortion zones of some cracks may overlap. It can be shown that partial overlapping of distortion zones does not greatly influence the "spring action" of each crack. After a certain drop in temperature the crack pattern in the reinforced concrete tension member is *fully developed.* This means that no new cracks will be initiated if a further drop in temperature occurs, because the tensile stresses in the remaining uncracked concrete zones are so small that new cracks cannot be formed. Any further drop in temperature will then only result in widening of the already existing cracks and only exceptionally in the initiation of a new crack.

The following assumptions are made in this engineering model:

1. The external tensile force causing a fully developed crack pattern is 20% larger than the external tensile force causing the first crack.
2. The mean spacing of cracks is $0.75 l_{cr}$ if l_{cr} is the total length - on both sides of the crack - of the distortion zone.

With these assumption the following can be calculated:

1. The crack width under the maximum tensile force causing a fully developed crack pattern. It can be shown that the model is valid only if the distance between two cracks is at least eight times the bar diameter ($\geq 8\,\varnothing_k$).
2. The elongation ε_{max} of the reinforced concrete tension member under this force.

In Fig. 20 this elongation can be calculated from the hatched area of OABC.
Since $\sigma_{s,cr}$ is related to the tensile stress σ_{cr} in the concrete just before cracking and also σ_{so} outside the distortion zone, this hatched area is dependent only on the magnitude of $\sigma_{s,cr}$ and l_{cr}. Because the mean crack spacing is assumed to have the magnitude $0.75 l_{cr}$, the maximum elongation of the tension member is dependent only on $\sigma_{s,cr}$ and on the shape of the curve AC. This shape depends on the bond behaviour of the reinforcement.

4.2.1.3 The designer's resources

In Fig. 22 the relationship is shown, for given bond-slip behaviour, between:

- the steel stress $\sigma_{s,cr}$ in the reinforcement in a crack if $\varepsilon < \varepsilon_{max}$;
- the bar diameter as a function of the permissible mean value of the crack width;
- the maximum elongation ε_{max} of the reinforced concrete tension member.

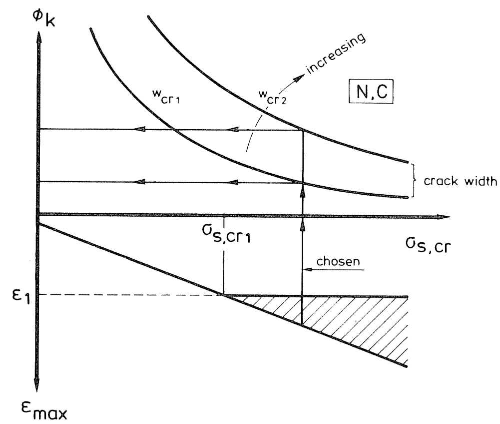

Fig. 22. Relationship $\sigma_{s,cr} - \varepsilon_{max}$; $\sigma_{s,cr} - \varnothing_k$ for a given crack width w_{cr}.

Fig. 23 shows the relationship between the steel stress $\sigma_{s,cr}$ in a crack, two values of the concrete tensile stress σ_{cr} causing the next crack if $\varepsilon < \varepsilon_{max}$ and the relative area ϱ of the reinforcement.

If a value ε_1 of the imposed deformation as given in Fig. 22 shows that $\sigma_{s,cr} > \sigma_{s,cr1}$, it is advisable to choose a value of the steel stress which is higher than $\sigma_{s,cr1}$ because in that case the sensitivity of the crack width with respect to ε_1 is small.

For a given value of $\sigma_{s,cr}$ the crack width w_{cr} depends on the bar diameter $\varnothing_k$. The larger the bar diameter, the larger the crack width. Because the relative area of the reinforcement is constant for a certain value of $\sigma_{s,cr}$ it also means that reduction of bar diameter *and* therefore of bar spacing will result in a smaller crack width.

Fig. 23 shows that the area $A_s (= \varrho \cdot A_c)$ of the longitudinal reinforcement depends on the tensile strength σ_{cr}. With increasing tensile strength the required area A_s increases considerably. In this respect one has to consider carefully the magnitude of the tensile strength σ_{cr}. As is generally known, this strength is for example reduced under sustained load. This favourable effect can, if relevant, be taken into account.

Some graphs with the above-mentioned relationships are reproduced in Appendix 1.

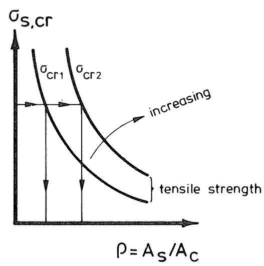

Fig. 23. Relationship $\sigma_{s,cr} - \varrho$ for two values of the tensile strength σ_{cr}.

4.2.1.4 The N-ε relationship for a reinforced concrete tension member

In Fig. 24 this N-ε relationship is given for static loading and gradually increasing imposed deformation.

The following parts can be distinguished in the N-ε diagram:

a. $N < N_{cr-1}$; $\varepsilon < \varepsilon_{cr}$.
   Uncracked concrete

b. $N_{cr-1} < N < 1.2N_{cr-1}$; $\varepsilon_{cr} < \varepsilon < \varepsilon_{max}$.
   Development of the crack pattern.
   Softening effect due to increasing number of cracks.
   If $N = 1.2N_{cr-1}$ the crack pattern is assumed to be fully developed.

c. $1.2N_{cr-1} < N < N_{sy}$; $\varepsilon_{max} < \varepsilon < \varepsilon_{sy}$.

Note: In the model is assumed that there is no tension stiffening left if the reinforcement starts to yield. Depending on the amount of reinforcement, some tension stiffening can, however, exist when yielding of the reinforcement occurs. In this engineering model this effect is neglected because it is only of minor importance with regard to crack width, etc.

Fig. 24 clearly shows that in the engineering model of a reinforced concrete tension member, as presented here, there exists a close relationship between the external force, the steel stresses in a crack, the crack width (and mean distance of cracks) and the elongation.

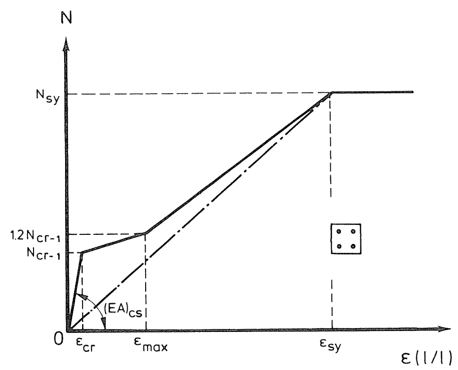

Fig. 24. N-ε diagram for a reinforced concrete tensions member.

Cyclic loading

Cyclic loading influences the concrete tensile strength as well as the bond behaviour between reinforcement and concrete.

Fig. 24 clearly shows that in the case where $N_{max} > 1.2N_{cr-1}$ under cyclic loading the deformations of the tension member are limited. The limit is given by the N-ε relationship of the reinforcement as such, assuming no tension stiffening of the concrete. Because no increase of the number of cracks is expected, the maximum crack width can easily be determined.

In the case of cyclic imposed deformations with $\varepsilon > \varepsilon_{max}$ a limit is also very clearly given by the N-ε relationship of the reinforcement only.
However, one has to be careful with the effect of cyclic loading if the crack pattern in the reinforced concrete tension member is *not* fully developed. In that case cyclic loading as well as cyclic imposed deformations can result in widening of some existing cracks due to decrease in the effectiveness of the bond-slip behaviour.

Time-dependent effects
The same comments as those relating to cyclic loading can be made here. If $N > 1.2 N_{cr-1}$ the limits of deformation and crack width are well known. If $N < 1.2 N_{cr-1}$ one has to be careful in estimating the maximum crack width. Perusal of the literature shows that in this case the maximum crack width may increase by 40%.

4.2.2 The reinforced concrete tension member with artificial loading by prestressing

A reinforced concrete tension member can be artificially loaded by a centric prestress.

Unbonded tendons
If this prestress is obtained with unbonded tendons, the prestressing steel only applies a normal compressive force to the section. Due to time-dependent effects (shrinkage, creep, relaxation) this compressive force will decrease with time.
The deformations of the tension member under tensile forces are relatively small with respect to the elongation of the prestressing steel. Therefore it can - provisionally - be assumed that the prestressing force P will remain constant during loading of the member in tension. On the assumption the concrete tension member will crack under a tensile force $(N_{cr-1} + P)$. The crack pattern will be fully developed at the tensile force $(1.2\ N_{cr-1} + P)$.
This shows that in fact the prestressing force P acts only as an artificial load. Of course one can also calculate more accurately and take into consideration some increase of P. In doing so one finds a fully developed crack pattern at a tensile force of:

$$1.2 N_{cr-1} + P + A_p \cdot \varepsilon_{max} \cdot E_p$$

In general this small increase of P can be neglected.

Bonded tendons
In the case of bonded tendons (in grouted ducts) these tension members will, more or less, act as reinforcement if $N > N_{cr-1} + P$.
It must be realized that, in general, the bond behaviour of prestressing tendons is different from (less effective than) the behaviour of the normal reinforcement. It means that one is confronted with a complex behaviour of the cracked structural concrete tension member. The bond between the prestressing tendon and the concrete is favourable in some respects but unfavourable in others.

Favourable:
- reduction of steel stresses in the normal reinforcement;
- reduction of crack width;
- possibility of reduction of normal reinforcement in controlling crack width;
- reduction of the amplitude of steel stresses in normal reinforcement under cyclic loading.

Unfavourable:
- increase of steel stresses in the prestressing steel across cracks;
- increase of amplitude of steel stresses in prestressing steel under cyclic loading in relation to the assumption of no bond.

It is therefore necessary to introduce low bond characteristics of the prestressing steel in the case where this bond is favourable and high bond characteristics in the case where this bond is unfavourable.
As will be shown in more detail in Appendix 1, the present author has introduced a reduction factor c for the calculation of the contribution of the prestressing tendons to the behaviour of the structural concrete member.
The area A_p of the prestressing steel is reduced by this factor in the calculation. Thus:

$$A_{ps} = A_s + c \cdot A_p \qquad \text{(with } c \leq 1\text{)}$$

For c a very simple formula is given to calculate a low value in the case where bond is favourable:

$$c = \frac{1}{\sqrt{n}} \cdot \frac{\varnothing_k}{\varnothing_p} \qquad \text{(see Appendix 1)}$$

In the case of unfavourable effect of bond it is proposed to calculate with $c = 1$.
It is of great importance to carry out more research on the important aspect of the bond behaviour of prestressing tendons in cracked tensile zones.

Note: In the case of unbonded tendons c can always be assumed to be zero.
To avoid longitudinal cracking under prestressing tendons the cover to these tendons must be at least 1.5 times the diameter (of the sheath) of the tendon.

4.3 *The structural concrete beam*

Background information is given in Appendix 2.

4.3.1 From tension member to beam in structural concrete

Fig. 25 shows, in general, how the model of the structural concrete beam is conceived. The structural concrete tension member is assumed to be the "reinforcement" of the beam.
Of course in Fig. 25 the model has been simplified, assuming a long zone of constant

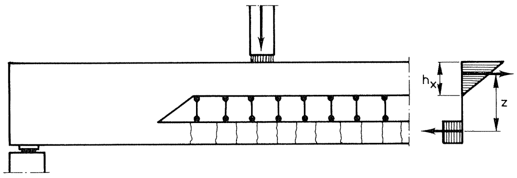

Fig. 25. Simplified model of a structural concrete beam with a cracked tensile zone.

bending moment in the beam. In most cases this zone will be of limited length, but the model can therefore also be used as a practical assumption and thus as an engineering model.
The link between the behaviour of the beam without cracked tensile zone and the model shown in Fig. 25 is given by the bending moment at which the first flexural cracks will occur.

4.3.2 Flexural tensile strength

The bending moment causing cracks in the tensile zone can be calculated from:

$$M_{cr} = W_{cs} \cdot \sigma_{cr,fl}$$

In this formula W_{cs} is the section modulus with respect to the tensile zone and $\sigma_{cr,fl}$ is the so-called flexural tensile strength. In fact, this flexural tensile strength is *not* really a strength characteristic of the material concrete but is a notational strength introduced for simplicity of calculation. The magnitude of $\sigma_{cr,fl}$ depends on the direct tensile strength of the concrete and the strain *gradient* in the tensile zone.
Fig. 26 shows the influence of the strain gradient on the magnitude of the flexural tensile strength.
In the case of a steep gradient (large strain decrease with respect to the depth) the flexural tensile strength will be high in relation to the direct tensile strength. In the case of a low gradient this flexural tensile strength will tend to the value of the direct tensile strength. In the case of centric tension (strain gradient is zero) only the direct tensile strength has to be considered in the calculation.
A formula for the relationship $\sigma_{cr,fl} - \sigma_{cr}$ is given in Appendix 2. In this formula the depth of the tensile zone is of primary importance.

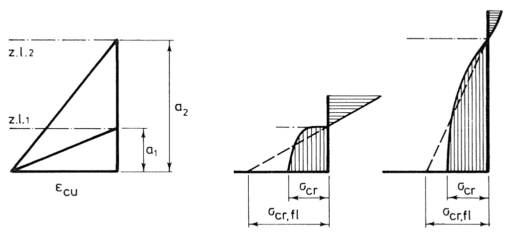

Fig. 26. Flexural tensile strength versus direct tensile strength.

The flexural tensile strength is increased by:
- decreasing depth of reinforced concrete elements, e.g. beams in relation to slabs;
- increasing compressive forces (due to artificial loading) in structural concrete.

Due to increase of the compressive forces acting on a section the depth of the tensile zone will decrease. Introduction of compressive forces into flexural members will cause the "flexural tensile strength" to increase!

4.3.3 The principle of determining the cross-section of the structural concrete tension member in the engineering beam model

To determine the relationship between bending moment and curvature (deflection), crack spacing, crack width, etc. a beam model composed of a compressive zone and a structural concrete tension member can be used.

The transition from the uncracked phase to the phase with cracked tensile zone will be described here.

Fig. 27 shows the stress distribution in a structural concrete beam section just before and just after initiation of the first crack in the tensile zone.

$M < M_{cr}$

The flexural tensile strength $\sigma_{cr,fl}$ is calculated as a function of the depth of the tensile zone. The force in the prestressing tendon has the magnitude P. The bending moment M_{cr} is transferred by the section due to the combined action of the prestressing force, the compressive force on the section and the internal lever arm z_{cr}.

$$M_{cr} = P \cdot z_{cr}$$

$M > M_{cr}$

Directly after initiation of the first crack the internal lever arm will be z, which is calculated on the assumption that there is no tension stiffening of the concrete in the tensile zone. This assumption is satisfactory because this calculation is related to a cross-section.

After the tensile zone has cracked, the bending moment M_{cr} is resisted by further combined action of the compressive zone and the structural concrete member because:

$$M_{cr} = (P + N_{cr-1}) \cdot z$$

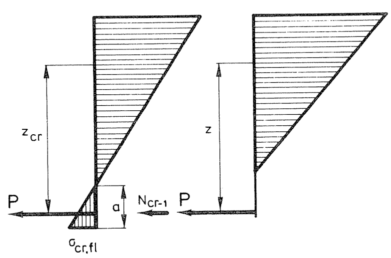

Fig. 27. Internal forces in a beam section.

Thus:

$$N_{cr-1} = P \cdot \left(\frac{z_{cr}}{z} - 1 \right)$$

The cross-sectional area "A_c" of the "structural concrete tension member" can now be calculated, if necessary, assuming that the first crack in the tendon is initiated at a tensile force N_{cr-1} because the tensile strength σ_{cr} reached.
So:

$$\text{“}A_c\text{”} \cdot \sigma_{cr} + (A_s + c \cdot A_p) \cdot \varepsilon_{cr} \cdot E_p = N_{cr-1}$$

Because ε_{cr} has a value between 100×10^{-6} and 150×10^{-6} and $E_p = 205{,}000$ N/mm^2, the value of $\varepsilon_{cr} \cdot E_p$ can be approximated with sufficient accuracy by adopting 25 N/mm^2.
Therefore:

$$\text{“}A_c\text{”} = \frac{P \cdot \left(\frac{z_{cr}}{z} - 1 \right) - 25 \cdot (A_s + c \cdot A_p)}{\sigma_{cr}}$$

Note: If it is desired to determine only the crack width and the crack spacing, it is not necessary to determine "A_c".
The internal lever arms z_{cr} and z must be calculated accurately because the difference between the ratio z_{cr}/z and the unity (1) is small. It means that the magnitude of the tensile force N_{cr-1} is very sensitive because:

$$N_{cr-1} = P \cdot \left(\frac{z_{cr}}{z} - 1 \right)$$

If z_{cr}/z increases by 10%, N_{cr-1} may increase by 100%.

In the case of reinforced concrete the same approach is possible. Now z must be calculated for the stress distribution, calculated with the so-called modulus method. This means that no tension stiffening of the tensile zone must be taken into account. In general, introduction of this tension stiffening does not affect the magnitude of z considerably.
If the relation is established between the beam and the structural concrete tension member, the N-ε relationship for the latter can be determined. The tension member to which this N-ε relationship applies is used to calculate the moment-curvature relationship of the beam, with a cracked tensile zone.
Fig. 28 shows this M-$\varkappa$ relationship for a structural concrete element. Due to the artificial loading by the prestress P the bending moment M_{20} is increased from zero (reinforced concrete) to:

$$M_{20} = P(e + k_1)$$

This bending moment M_{20} is the "zero point" of the behaviour of this beam, assuming normal reinforced concrete.

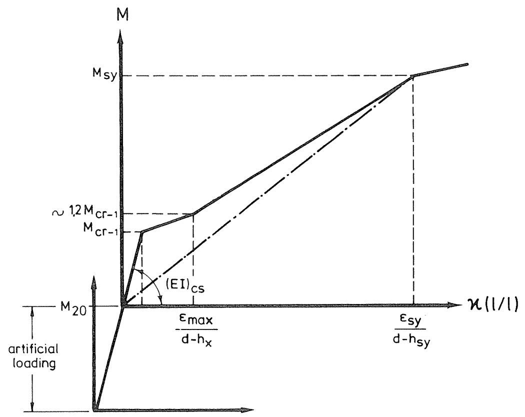

Fig. 28. Moment-curvature relationship for a structural concrete element.

Fig. 28 illustrates very clearly that, from the point of view of load bearing capacity, prestressing can simply be regarded as an artificial loading!

4.4 *Simplified calculation of time-dependent effects on the magnitude of artificial loading by prestressing*

In [38] the possibilities of a simplified calculation of time-dependent effects are discussed. In that paper it is shown that in fact a "simple" calculation of the losses of prestress due to shrinkage and creep of concrete is possible and that, in general, no complete calculation of redistribution of stresses due to time-dependent effects is necessary to determine the load at which the concrete section of the extreme fibres of a section are decompressed.

In the so-called simple calculation the following are determined:

- the effective shrinkage shortening of the concrete $\varepsilon_{cs,\infty}$;
- the concrete stress σ_{cpo} at $t = 0$ under sustained load in the section at the level of the centroid of the prestressing steel;
- the creep factor $\varnothing_{\infty}$.

The simplified calculation of time-dependent effects is a safe method because it neglects the reducing influence of the normal reinforcement on the loss of prestress. The shortening of the concrete due to shrinkage and creep can be calculated from the formula:

$$\varepsilon_{c\infty} = \varepsilon_{cs,\infty} + \frac{\sigma_{cpo}}{E_c} \cdot \varnothing_{\infty}$$

The loss of prestress due to shrinkage and creep can be calculated by multiplying $\varepsilon_{c\infty}$ by the modulus of elasticity E_p and the cross-sectional area A_p of the prestressing steel. The relaxation loss $\Delta\sigma_{p,\infty}$ can than be calculated from the CEB-FIP formula:

$$\Delta\sigma_{p,\infty} = 3\,\Delta\sigma_{p,1000}\left(1 - \frac{2\varepsilon_{c,\infty}\cdot E_p}{\sigma_{po}}\right)$$

In this formula $\Delta\sigma_{p,1000}$ is the relaxation loss over a period of 1000 hrs and σ_{po} the initial stress in the prestressing steel. For $t = \infty$ the effective steel stress $\sigma_{p,\infty}$ can be calculated as follows:

$$\sigma_{p,\infty} = \sigma_{po} - \Delta\sigma_{p,\infty}\cdot E_p$$

This simple method can also be used in the case where there is a relatively large cross-sectional area of normal reinforcement. Calculations with sophisticated models have shown that the simple method gives values of the decompression load which are a good approximation of the values obtained from a more complex calculation. For an explanation of this conclusion see [38].
Fig. 29 is reproduced from this report. It shows the calculated decompressive bending moment M_{20} for double-T-beams as a function of several degrees of prestressing K. In Fig. 29 the solid line gives the relationship between M_{20} and K calculated by the simple method, and the other lines give this relationship calculated with Dischinger's approach. Three values of shrinkage and creep are taken into consideration to check the sensitivity of the calculations.
The diagram shows a slight increase of M_{20} for the simple approach as compared with the calculation according to Dischinger. In this case the simple calculation will also result in a slight underestimation of the crack width.
It can be stated that in most cases encountered in practice the simple approach to the decompression load results in acceptable values for this load and for the crack width. It must be realised that mostly the crack widths are slightly underestimated if the bending moment M_{20} is calculated by the simple method. In this context it must be mentioned that a realistic and economic calculation method has hitherto not been developed, especially not for statically indeterminate structures.
In the author's opinion there is in fact no need for a very sophisticated calculation method of time-dependent effects, because using such a method will spuriously suggest that the result of the calculation must possess high accuracy.

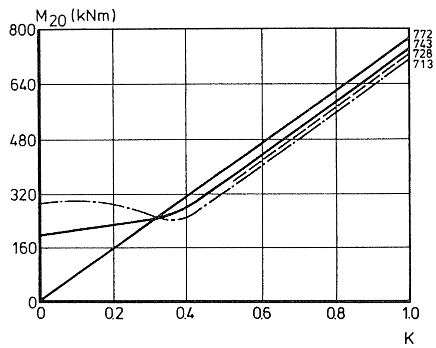

Fig. 29. Decompression moment on a double-T-beam

The creep and shrinkage behaviour of concrete cannot be predicted within a small range of deviation. In reality the deviation in this behaviour – and therefore in the actual deformations – is large. We must at present accept these large deviations because on the site, but also in the factory, we have not yet enough knowledge and resources to produce concrete with accurately known time-dependent behaviour.
If the deviations are so considerable, why then use a complex method of calculation which gives supposedly very "accurate" results?
So we must be realistic and practical. It means that we must detail the reinforcement in our structures in such a way that deviations in time-dependent effects will not greatly influence the cracking of our concrete.
If we design our structures on the basis of this approach, a simple calculation of time-dependent effects will be sufficient to result in a sound concrete structure.

5 Conclusion

The "structural concrete" has been described in this paper. It is explained why, in in the author's opinion, there is a need for a new approach of the design of concrete structures. The model also offers the possibility of considering the behaviour of rather complex concrete structures in a different way.
By introducing the prestress as a mode of artificial loading of (reinforced) concrete structures, new applications in concrete structures, including ordinary reinforced ones, become possible.
The designer, however, will have to be "released" from the "shackles" of thinking in categories such as reinforced concrete, prestressed concrete, etc. This change-over is not as simple as it may appear. It demands a new and fresh look at the behaviour of concrete structures and it also demands some imagination to device new possibilities.
But experience has shown the author that many designers are willing to do this. Interesting solutions have already been implemented. The new generation of engineers – the students – is adapting very rapidly and easily to this new approach.
The possibilities offered by this approach will therefore certainly be tried out in the future.
This paper has been written with the aim to invite more designers to prove that this approach is a suitable one and opens up the way to other applications in the technology of concrete structures.
Of course this paper constitutes only a first step in this direction. Components of the model are described, but one must take care not to assume that the model and the engineering models described in the appendices are complete. Structural concrete opens new fields and new applications, but it also raises fresh problems.
In conclusion some of these problems will be mentioned:

x *The behaviour of the reinforced concrete tension member in structural concrete:*
  - the "bond" of prestressing tendons in ducts;
    interaction between these tendons and the normal reinforcement;

- aspects of cyclic loading and time-dependent effects and its influence on the width of cracks;
- improvement of bond behaviour.

x *The behaviour of the beam:*
- improvement of the approach to the concrete tension member in the beam;
- improvement of a simple approach to the calculation of time-dependent effects.

x *Design of the concrete structure and its detailing:*
- development of simplified methods for the detailing of concrete structures such as anchorage zones, support zones, etc.

6 Acknowledgements

This article is based on studies and experiments by the author during the last twenty years.
In the final model very interesting discussions with Dr. Ing. M. Birkemaier, during several "Davos" meetings, played a very important part, as also did the collaboration with Prof. Dr. Ing. H. Bachman of the Swiss federal University of Technology, Zürich, in preparing papers for the International Symposium on "Non-linearity and continuity in prestressed concrete" of Waterloo, Ontario, in 1983.
Especially our joint paper in the P.C.I. Journal in 1984/1985 can be mentioned in this context.
The work presented here is of course the result of a perusal of many interesting papers published by a number of experts all over the world, and also of many discussions at committee meetings, symposia and congresses. Also the stimulating discussions in the Dutch group "Concrete mechanics" under the leadership of Blaauwendraad, Monnier, Kusters and Reinhardt has influenced my look on science into practice in structural concrete. I accordingly wish to thank all those with whom I had opportunities to discuss the model described in this article.
Van der Veen had a major share in the development of the engineering model of the reinforced concrete tension member. Pruijssers gave valuable help in developing the beam model. Both engineers are on the staff of the Stevin Laboratory of the Delft University of Technology.
Van Amerongen assisted me in editing my manuscript into correct English.
Mr. Spiewakowski very carefully prepared the drawing and Mrs. Polderman excellently typed the final version of the report wich formed the basis of this publication.

7 Notations

A_c	cross-sectional area of concrete
A_p	cross-sectional area of prestressing steel
A_s	cross-sectional area of normal reinforcement
A_{ss}	cross-sectional area of stirrups
B	indication of concrete quality (characteristic cube strength), e.g. B 30
C	factor in the bond-slip relationship $\tau_{cs}=C\cdot\delta^N$
E_c	modulus of elasticity of concrete
$E_p=E_s$	modulus of elasticity of steel (reinforcement or prestress)
F_{cr}	external centric force at which the first crack is initiated
F_d	external centric sustained force
$F_{R,\infty}$	external centric decompression force at $t=\infty$
I_c	moment of inertia of a concrete section
K	degree of prestressing, $K=\frac{F_{R,\infty}}{F_{max}}$; $K=\frac{M_{20}}{M_{max}}$
M_b	bending moment due to load exerted by curved tendons
M_{cr}	bending moment at which the first crack in the tensile zone is initiated
M_d	bending moment due to dead load
M_l	bending moment due to live load
M_{max}	$M_{max}=M_d+M_l$
$M_{20}=M_{dec}$	bending moment of decompression; concrete stress at top (or bottom) fibre (tension zone) is zero
M_u	ultimate bending moment
N	normal force
N	factor in the bond-slip relationship $\tau_{cs}=C\cdot\delta^N$
N_{co}	normal force of decompression (tension member)
N_{cr-1}	normal force at which the first crack is initiated (tension member)
N_{cr-2}	normal force at which the crack pattern is fully developed
N_T	tensile force caused by an imposed deformation due to drop in temperature ΔT
P_0	initial prestressing force
P_∞	effective prestressing force at $t=\infty$, taking all losses of prestress into account
R	radius of the centre line of curved tendons
T	temperature
W_c	section modulus (concrete section only)
W_{cs}	section modulus (composite section of concrete and steel)
W_{cs1}	with respect to upper fibre (compression)
W_{cs2}	with respect to bottom fibre (tension)
a	length of support zone
	depth of the tensile zone in flexure
b_w	width of web; I-beam, T-beam, box girder
c	factor associated with bond - Stuvo formula
c	factor associated with bond of tendons; $c=\frac{1}{\sqrt{n}}\cdot\frac{\varnothing_k}{\varnothing_p}$
d	effective depth
	distance between the centroid of the tensile reinforcement and the top fibre of the compressive zone
e	distance between the centroid of a group of tendons and the centre of gravity of the concrete cross-section
f	strength
f_{cc}	characteristic compressive strength of concrete
f_{ccm}	mean value of compressive strength of concrete
f_{ct}	characteristic tensile strength of concrete
f_{pk}	characteristic tensile strength of prestressing steel
f_{pu}	"guaranteed" tensile strength of prestressing steel (designation used in the fifties)

f_{sy}	characteristic yield strength of reinforcement
h	depth of a cross-section
k	distance between upper (k_1) or lower (k_2) kern point and the centre of gravity of the cross-section
l	length, span
l_{cr}	length of distortion zone on both sides of a crack
l_{st}	length of distortion zone on one side of a crack
n	modular ratio $\frac{E_s}{E_c}$
n	number of wires in a prestressing tendon; every wire in a strand counts
q	equally distributed load
t	time; $t=0$; $t=\infty$
y	distance from centroid of concrete section to extreme fibre; y_1 top fibre; y_2 bottom fibre
w	crack width (general)
w_{cr}	crack width in the case of a developing crack pattern
z	lever arm
z_{cr}	lever arm just before the initiation of the first crack
α	coefficient of thermal expansion
$\alpha_{t,s}$	factor of tension stiffening
δ_x	displacement of a section of a bar in relation to the concrete section outside the influence zone of bond
δ_{cr}	δ_x in a crack ("slip")
$\varepsilon_c \cdot {}^1/_1$	mean value of elongation (shortening) of an element
$\varepsilon_{c\infty}$	effective shortening of concrete due to shrinkage and creep
$\varepsilon_{cs,\infty}$	effective shrinkage of concrete - at $t=\infty$
ε_{cr}	ultimate concrete strain in tension; elongation capacity
ε_{max}	the maximum elongation of a structural concrete tension member if the crack pattern is just fully developed
$\varkappa$	curvature of a section
$\varkappa_u$	curvature of a section just before failure
ϱ	reinforcement ratio; $\frac{A_s}{A_c}$ respectively $\frac{A_s + cA_p}{A_c}$
σ_c	concrete stress (general)
$\sigma_{c,\infty}$	concrete stress $t=\infty$, all losses of prestress having taken place
σ_{cpo}	compressive stress in concrete at the centroid of the prestressing steel
σ_{cr}	tensile strength and also the tensile strress in the concrete outside the distortion zone of a crack
$\sigma_{cr,fl}$	flexural tensile strength
σ_p	stress in prestressing steel (general)
σ_{po}	initial stress
$\sigma_{p,\infty}$	effective stress
σ_s	stress in reinforcement (general)
σ_{so}	steel stress in a tension member outside the distortion zone of a crack, just before initiation of a crack
$\sigma_{s,cr}$	steel stress in a crtack just after initiation of a crack
τ_{cs}	bond stress
$\tau_{cs,x}$	bond stress in a section x
$\tau_{cs,cr}$	bond stress in the vicinity of a crack
$\varnothing_\infty$	factor over the period $t=0$ to $t=\infty$
$\varnothing_k$	diamneter of a reinforcing bar
$\varnothing_p$	diamneter of a wire (e.g. one wire of a strand) of prestressing steel
$\Delta\sigma_{p,\infty}$	effective relaxation loss of prestressing steel, dependent on σ_{po}
$\Delta\sigma_{p,1000}$	relaxation loss of prestrressing steel over a period of 1000 hours

8 Notation – concrete mechanics

1. *Type of external load*	*Code*
Centric normal force	N
Bending moment	M
Shear	V
Torsion	T
Eccentric normal force (N + M)	MN
Bending moment and shear	MV

2. *Material behaviour*	*Code*
Linearly elastic	LE
Non-linearly elastic	NLE
Plasticity (in failure)	PL

3. *Cracking of the tensile zone*	*Code*
Uncracked	U
Cracked	R

4. *Parts of the cross-section taken into consideration*	*Code*
Concrete	C
Normal steel reinforcement	S
Prestressing steel	P

Examples

N.LE.U.C	Unreinforced section, centric normal force, linearly elastic material behaviour.
M.LE.R.CS	Reinforced concrete beam section in bending with cracked tensile zone (modulus method).
MN.LE.U.C	Beam section with eccentric prestressing force. Only the concrete section is taken into consideration in the calculation of stress distribution.
MN.PL.R.CSP	Beam section in ultimate limit state. Normal reinforcement and prestressing steel are taken into consideration in the calculation of failure.

9 References

1. Koninklijk Instituut van Ingenieurs, Gewapend-betonvoorschriften 1912, Bijlage U van het blijvend gedeelte van het jaarboekje van het K.I.v.I.
2. Scharroo, P. W., Cement en beton – oud en nieuw – 1946, Publ. L. J. Veen's uitgeversmij. N.V.
3. Bergsma, P., Beton en Gewapend beton – 1934, Publ. "Kosmos".
4. Mortelmans, F., Vergelijking tussen de elastische en de breukmethode bij de berekening van gewapend beton. Tijdschrift der Openbare Werken in België, No. 6, 1984.
5. Freyssinet, E., Une révolution dans l'art de bâtir: les constructions précontraintes. Travaux, November 1941.
6. Magnel, G., Le béton précontraint. Editions Fecheyr, 1948, Ghent.
7. Dischinger, F., Elastische und plastische Verformungen der Eisenbetontragwerke. Bauingenieur, Vol. 18, 1937 and Vol. 20, 1939.
8. Strijcker, R. de, Verslagen over Navorsingen. I.R.S.I.A.-I.W.O.N.L., No. 11, July 1953. Rechches du Centre du Louvain. See also: Stuvo's "Introduction" No. 9 to the paper presented on 10 January 1950: "Onderzoek naar de grootte van de 'ontspanning' bij hoogwaardig staal."

9. Duijster, H. C., Hangars in voorgespannen beton - Luchthaven Melsbroeck. Cement 1949, pp. 117-124. De Ingenieur 1948, No. 18, Beton 4.
10. Bruggeling, A. S. G., Reflections on the action of prestressed concrete beams subjected to overloading. Paper, First FIP-Congress, Session II, London 1953.
11. Abeles, P. W., Partially Prestressed Concrete Constructions built in the Eastern Region of British Railways. IABSE publication, Vol. 12, 1952, pp. 1-14.
12. Bruggeling, A. S. G., Gedeeltelijk voorgespannen beton. Cement 1966, No. 12.
13. Bruggeling, A. S. G., Wapenen of voorspannen. Cement 1968, No. 3.
14. CUR-rapport 87, Gedeeltelijk voorgespannen beton, 1977.
15. SIA-Norm 162, Bruchwiderstand und Bemessung von Stahlbeton- und Spannbetontragwerken, 1968.
16. DIN-Norm 4227 - Teil 2, Spannbeton-Bauteile mit teilweiser Vorspannung. (Vornorm Mai 1984).
17. ACI 318-77, Building Code Requirements for Reinforced Concrete. ACI, 1977.
18. Shun Inomata, Conclusions of the FIP symposium on partial prestressing: 15 and 16 September 1980, Bucharest, FIP notes 90, January-February 1981.
19. Hill, A. W., Partial prestressing - A report on FIP symposium Bucharest 1980 and recommendations. FIP notes 91, March-April 1981.
20. Non-linearity and continuity in prestressed concrete. Proceedings International Symposium, University of Waterloo, Ontario, Canada, July 4-6, 1983.
21. Advanced Research Workshop on Partial Prestressing. June, Paris 1984.
22. NEN 3880, Voorschriften beton. VB 1974/1984.
23. Jenny, D. P., Current status of partial prestressing for pretensioned products in North America. PCI Journal, January-February 1985.
24. Bruggeling, A. S. G., Partially prestressed concrete structures - A design challenge. PCI Journal, January-February 1985.
25. Bruggeling, A. S. G., Het gedrag van betonconstructies - 3 parts. Publ. Stichting PBF, 1976.
26. CEB, Praktische richtlijnen voor de berekening en uitvoering van gewapend-betonconstructies. Betonvereniging, 1966.
27. Wittfoht, I., Reflections on the theory and application of prestressing in concrete bridge constructions. FIP notes 93, July-August 1981.
28. Wicke, M., Folgerungen für das Bemessungskonzept aus den Beobachtungen von Bauwerken. Österreichischer Betonverein. Aktuelle Fragen des Spannbetons, March 1984.
29. Cooke, N., M. N. J. Priestly and S. J. Thurston, Analysis and design of partially prestressed bridges under thermal loading. PCI Journal, May-June 1984.
30. Blaauwendraad, J., M. van Poelgeest and L. Koornneef, Stuvo Report No. 37, Discussienota "Parasitaire" moment.
31. Bruggeling, A. S. G., Theorie en praktijk van het voorgespannen beton (2 volumes). Publ. Stichting PBF, 1982 (in Dutch).
32. Rehm, G., Über die Grundlagen des Verbundes zwischen Stahl und Beton. Deutscher Ausschuss für Stahlbeton, No. 138, 1961.
33. Bruggeling, A. S. G., Imposed deformation and crack width. Research report 5-80-D13, Univ. of Techn., Delft, 1980.
34. Noakowski, P., Verbund im Stahlbetonbau. SE-publikatie, No. 8, Betonvereniging, 1983. Titel: "Betonstaal, voorspanstaal".
35. Bruggeling, A. S. G., in collaboration with W. A. de Bruijn, Theorie en praktijk van het gewapend beton (2 volumes). Publ. Stichting PBF, 1986 (in Dutch).
36. Bruggeling, A. S. G., Gedeeltelijk voorgespannen beton. Van theorie naar praktijk. PT/Civiele Techniek 1984, Vol. 39, No. 9.
37. Bruggeling, A. S. G., S. H. Brunekreef and J. C. Walraven, Partially prestressed concrete; Theory and experiments. Heron, Vol. 23, No. 1, 1978.
38. Bruggeling, A. S. G., Towards a simple method of analysis for partially prestressed concrete. Research Report 5-82-D16, Univ. of Techn., Delft, 1982.
39. Bachmann, H., Design of partially prestressed concrete structures, based on Swiss experiences. PCI Journal, July-August 1984.

40. FIP recommendations 1984, Practical design of reinforced and prestressed concrete structures.
41. Proceedings of International Symposium: "Design, construction and maintenance of concrete storage structures". New Castle upon Tyne, 1984.
A. S. G. Bruggeling, Why partial prestressing?
K. van Breugel, Problems associated with wall-to-base connections.
42. Bruggeling, A. S. G., Betonconstructies in de Civiele Gezondheidstechniek. Publ. Stichting PBF, 1986 (in Dutch).
43. Die Weinlandbrücke in der Umfahrungsstrasse von Andelfingen. Gedenkschrift zur Einweihung und Verkehrsübergabe am 17 Mai 1958.
44. Bruggeling, A. S. G., Constructief beton. Van wetenschap naar praktijk. Publ. Delft Univ. of Techn.
45. Bruggeling, A. S. G., Partial prestressing. Teach-in session 10, 10th FIP Congress, New Delhi.
46. Schadegevallen aan bruggen - case 1. Cement 1986, No. 2.
47. Zielinski, A. J., Injecteren van voorspankanalen. Cement 1986, No. 4.
48. Podolny, W., The cause of cracking in post-tensioned concrete box girder bridges and retrofit procedures. PCI Journal, March-April 1985.
49. Information "A 40 Mâcon–Genève Tunnel du Mont Blanc". Viaduc de Poncin. From: Scetauroute, France.
50. Bruggeling, A. S. G., Constructief beton (Problem of beam-slab system, Fig. 14). Cement 1986, No. 12; Cement 1987, Nos. 1, 2, 3, 4.

APPENDIX 1

Engineering model of the tension member in structural concrete

1 Bond-slip relationship

The bond-slip relationship of reinforcing bars and prestressing steel can experimentally be determined by the RC 8 test devised by RILEM-CEB-FIP and called "Bond test reinforcing steel; 2. Pull-out test".
This test was first proposed by Rehm [32].
Such a relationship is presented in Fig. A1-1.
With curve-fitting the experimental data can be written as an exponential curve:

$$\tau_{cs,x} = C \cdot \delta_x^N$$

In this curve the two factors are related as follows:

N with the shape of the τ-δ diagram
C with the bond strength

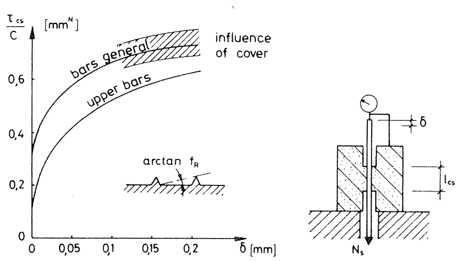

Fig. A1-1. Bond-slip relationship of reinforcing bars.

In the subjoined table some values of N and C are given in the case of bars with sufficient concrete cover (at least $2-3\,\varnothing_k$).

type	C/f_{ccm}	N
normal reinforcing bars		
$f_R \sim 0.065$ (Fig. A1-1)		
general case	0.38	0.18
upper bars	0.32	0.28
prestressing steel complying		
with Standard EU 138		
wires	0.17	0.32
strands	0.12	0.27

Note: f_{ccm} can be approximated as $f_{ccm}=f_{cc}+4$ N/mm².

2 The stresses in the distortion zone of a crack

l_{st} is the length of the distortion zone on one side of a crack.

2.1 *Shape factors*

The shape factor $S\sigma$ is defined as the ratio:

$$\frac{\text{hatched area ADC}}{\text{area ADEC}} = S\sigma \qquad \text{(see Fig. 20)}$$

The shape factor $S\sigma$ is defined in the same way.
Therefore:

$$S\tau = \frac{\int_0^{l_{st}} \tau_{cs,x} \cdot dx}{\tau_{cs,cr} \cdot l_{st}}$$

It can be shown analytically that in the distortion zone of a crack both $S\sigma$ and $S\tau$ are dependent of the magnitude of $\sigma_{s,cr}$ and $\tau_{cs,cr}$.
The magnitude of the shape factors is dependent only on the factor N in the exponential curve of the bond-slip relationship [34 and 35]:

$$S\sigma = \frac{1-N}{2}; \quad S\tau = \frac{1-N}{1+N}; \quad \frac{S\sigma}{S\tau} = \frac{1+N}{2}$$

2.2 *The relationship between $\sigma_{s,cr}$, σ_{so} and σ_{cr}*

In Fig. A1-2 this relationship is given for the case where the distortion in the end zone ("Goto effect") is neglected.

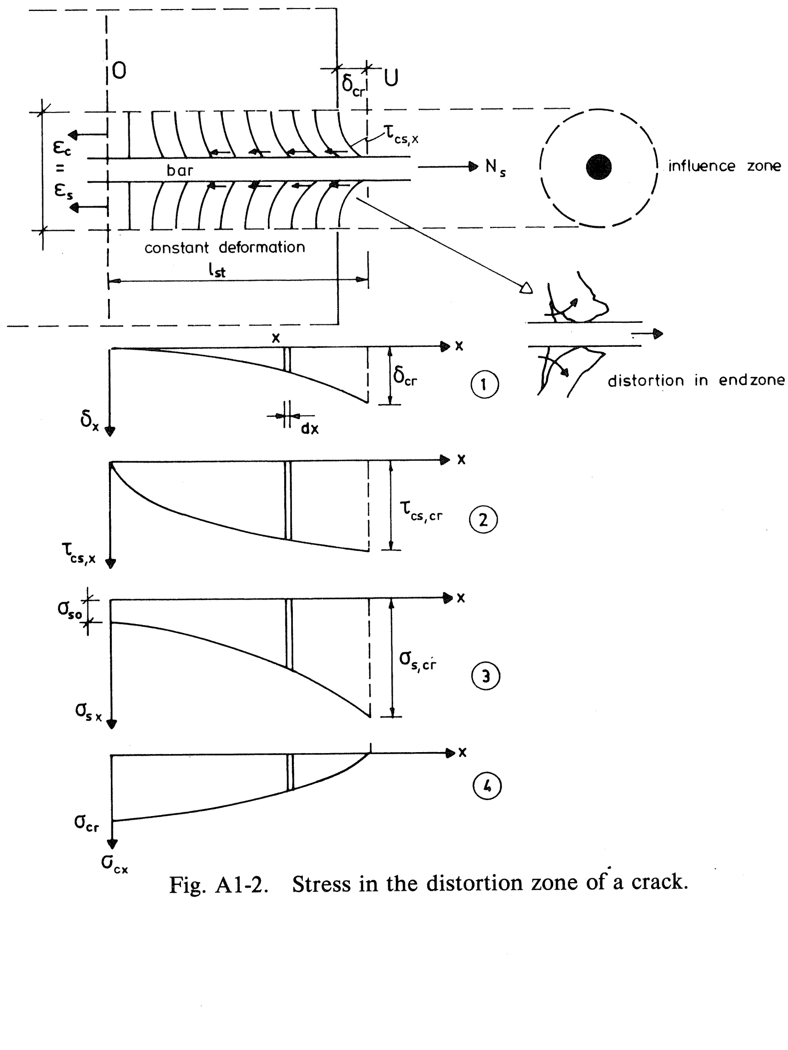

Fig. A1-2. Stress in the distortion zone of a crack.

$$\sigma_{s,cr} = \sigma_{so} + \sigma_{cr} \cdot \frac{A_c}{A_s}$$

$\sigma_{so} = n \cdot \sigma_{cr}$ (assumption of linearly elastic behaviour of concrete and steel)

This relationship is chosen because it simplifies the formulas

$$\sigma_{s,cr} = \sigma_{cr} \left(n - \frac{1}{\varrho}\right)$$

$$\sigma_{s,cr} - \sigma_{so} = \frac{\sigma_{s,cr}}{1 + n \cdot \varrho}$$

3 The length l_{st} of the distortion zone and the relative displacement δ_{cr} ("slip") in a crack between a bar and the concrete surface

The tensile force $A_c \cdot \sigma_{cr} + A_s \cdot \sigma_{so}$ outside the distortion zone is resisted in the crack by the tensile force $A_s \cdot \sigma_{s,cr}$ in the reinforcement.
Therefore the tensile force $A_s(\sigma_{s,cr} - \sigma_{so})$ is transferred by bond to the concrete in the distortion zone.
If the minor influence of the lack of bond in the end zone just near a crack is neglected, the equilibrium of tensile force and bond force can be written as follows:

$$\frac{\sigma_{s,cr}}{1 + n \cdot \varrho} \cdot \tfrac{1}{4}\pi \cdot \varnothing_k^2 = S\tau \cdot l_{st} \cdot \tau_{cs,cr} \cdot \pi \cdot \varnothing_k$$

The length l_{st} can be calculated with the exponential bond-slip relationship:

$$l_{st} = \frac{\sigma_{s,cr} \cdot \varnothing_k}{(1 + n \cdot \varrho) \cdot 4 \cdot S\tau \cdot C \cdot \delta_{cr}^N}$$

The relative displacement δ_{cr} is determined by the extra elongation of the reinforcing bar and the shortening of the concrete over the distortion zone with a length l_{st}.

Therefore:

$$\delta_{cr} = S\sigma \left(\frac{\sigma_{s,cr} - \sigma_{so}}{E_s} + \frac{\sigma_{cr}}{E_c}\right) \cdot l_{st}$$

$$\delta_{cr} = S\sigma \cdot \frac{\sigma_{s,cr}}{E_s} \cdot l_{st}$$

On combining the formulas for l_{st} and δ_{cr} the crack width and the length of the distortion zone can be expressed in known parameters:

$$\delta_{cr} = \left(\frac{1+N}{2} \cdot \frac{\varnothing_k}{4} \cdot \frac{1}{C \cdot E_s} \cdot \frac{\sigma_{s,cr}^2}{(1 + n \cdot \varrho)}\right)^{\frac{1}{1+N}}$$

$$l_{st} = 2\,\frac{\delta_{cr} \cdot E_s}{(1-N) \cdot \sigma_{s,cr}}$$

with:

$$\sigma_{s,cr} = \sigma_{cr} \left(n + \frac{1}{\varrho}\right)$$

or with:

$$\sigma_{s,cr} = \frac{\sigma_{cr}}{\varrho} + 25\ \text{N/mm}^2$$

4 The elongation of a tension element of reinforced concrete over the distortion zone l_{st}

This elongation can be calculated by using the distribution of steel stresses over the distortion zone:

$$\varepsilon_{mean} = \frac{1}{E_s} \{S\sigma(\sigma_{s,cr} - \sigma_{so}) + \sigma_{so}\}$$

$$\varepsilon_{mean} = \frac{\sigma_{s,cr}}{E_s} \cdot \frac{0.5(1-N) + n \cdot \varrho}{1 + n \cdot \varrho}$$

The total elongation of this tension element - after initiation of the crack - is:

$$\varepsilon_{mean} \cdot l_{st}$$

Fig. A1-3 gives the relationship between ϱ; $\varnothing_k$; δ_{cr}, $\sigma_{s,cr}$ and ε_{mean}.

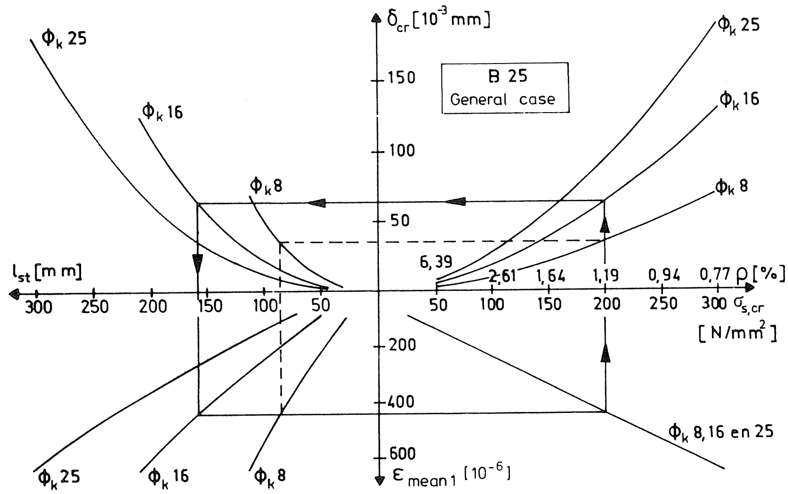

Fig. A1-3. Relationship between: ϱ; $\varnothing_k$; δ_{cr}; $\sigma_{s,cr}$ and ε_{mean}; f_{cc}= 25 N/mm²; reinforcing bars - general case.

5 Reinforced concrete tension member

5.1 *Tensile strength σ_{cr} to be taken into account*

Relationship between the tensile strength and the compressive strength, used in these calculations:

- characteristic lower strength:

$$f_{ct} = 0.87(1 + 0.05 f_{cc})$$

- mean strength:

$$f_{ctm} = 1.45 f_{ct}$$

- characteristic upper strength:

$$f_{ctk,0.95} = 1.9 f_{ct}$$

- the tensile strength is reduced by 40% under sustained load:

$$f_{ct,\infty} = 0.6 f_{ct}$$

Example (values in N/mm²)

f_{cc}	f_{ct}	f_{ctm}	$f_{ctk,0.95}$	$f_{ct,\infty}$
25	2.0	2.9	3.8	1.2
35	2.4	3.5	4.6	1.4

In the calculation the tensile strength σ_{cr-1} - initiation of the first crack - is taken into account as follows:

Slowly imposed deformation σ_{cr-1}: $0.62 f_{ctm}$
(settlement; shrinkage)

Sustained load σ_{cr-1}: $0.5\ f_{ctm}$

Rapidly imposed deformation σ_{cr-1}: $0.75 f_{ctm}$
(solar radiation)

The tensile strength σ_{cr-2} - crack pattern just fully developed - is taken as $1.2\sigma_{cr-1}$. This assumption is based on investigations.

5.2 *Maximum elongation ε_{max} - crack pattern just fully developed*

Investigations indicate that the mean distance of cracks can be taken $1.5 l_{st} = 0.75 l_{cr}$, enabling ε_{max} to be calculated as follows:

$$\varepsilon_{max} = \frac{\sigma_{s,cr}}{E_s} \cdot \frac{0.67(1-N) + n \cdot \varrho}{1 + n \cdot \varrho}$$

This calculation is based on the assumption that partial overlapping of distortion zones does not greatly influence the addition of the elongation of two distortion zones.

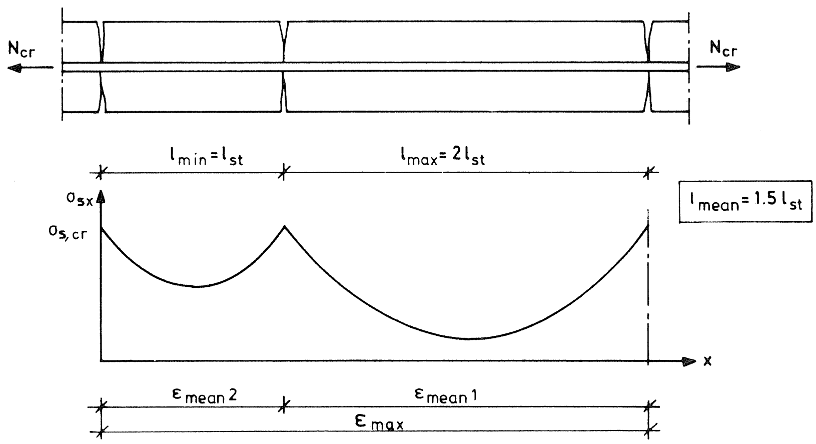

Fig. A1-4. Overlap of the distortion zones.

In the general case of bars ($N = 0.18$) ε_{max} can be estimated:

$$\varepsilon_{max} = (35 + 2.7\sigma_{s,cr}) \times 10^{-6} \quad \text{or} \quad \varepsilon_{max} = \frac{\sigma_{s,cr}}{1.7 E_s}$$

5.3 *Crack width*

a. $\varepsilon \leq \varepsilon_{max}$
In this case the mean crack width $w_{cr-mean} = 2\delta_{cr}$.
δ_{cr} can be calculated as follows:

$$\sigma_{s,cr} = \sigma_{cr-2}\left(n + \frac{1}{\varrho}\right) \quad \text{or} \quad \frac{\sigma_{cr-2}}{\varrho} + 25 \text{ N/mm}^2$$

Investigations have shown that the maximum crack width must be determined, taking into account dispersion and the effect of the distortion in the end zone ("Goto effect") as shown in Fig. A1-2.
Therefore:

$$w_{cr-0.95} = 1.5 w_{cr-mean}$$

b. $\varepsilon > \varepsilon_{max}$
In this case the maximum crack width w can be approximated by:

$$w = \frac{1.5 l_{st} \cdot \sigma_s}{E_s}; \quad \sigma_s = \frac{N}{A_s}$$

This value of this crack width is an upper bound.

5.4 *N-ε diagram - static loading*

Because the magnitude of N and of ε is known, the N-ε diagram can be drawn (see Fig. 24).

$$N_{cr-1} = A_c \cdot \sigma_{cr-1} + A_s \cdot \sigma_{so}; \quad \varepsilon_{cr-1} = \frac{\sigma_{cr-1}}{E_c}$$

$$N_{cr-2} = 1.2 N_{cr-1} \quad ; \quad \varepsilon_{cr-2} = \varepsilon_{max}$$

$$N_{sy} = A_s \cdot f_{sy} \quad ; \quad \varepsilon_{sy}$$

5.5 *Cyclic loading*

Experiments have shown that cyclic loading can increase the crack width by 20–50% if the crack pattern is not fully developed ($\varepsilon < \varepsilon_{max}$). In that case a single crack can widen considerably as compared with the case of a dense crack pattern (small distances between cracks).
In all cases with $\varepsilon > \varepsilon_{max}$ the crack width is limited.
The most unfavourable assumption (resulting in a maximum crack width) will be to neglect tension stiffening and to assume unrestrained shrinkage of the concrete between two cracks. The maximum crack distance can therefore be assumed to be $2 l_{st}$.
High steel stresses in the cracks, e.g. 300 N/mm^2–400 N/mm^2, will more particularly cause important crack growth due to cyclic loading.

5.6 *Sustained load*

In the case of a sustained tensile force similar effects in the behaviour of tension members have been observed as in that of cyclic loading. This effect is, however, less important with higher reinforcement ratios.
It is therefore recommended to assume a 40% increase in crack width due to time-dependent effects. This is a safe value because it is associated with low reinforcement ratios.
The design must therefore be based on a mean crack width which is only 70% of that calculated in 5.3 with the model of the reinforced concrete tension member. The maximum crack width depends on how much elongation occurs (see 5.3).

6 Tension member of structural concrete, with artificial loading by centric prestress

In this case it is important to consider the influence of the bond of tendons on the behaviour of the tension member after initiation of the first crack.
If the bond behaviour of tendons is assumed to be comparable with that of bars and the bond-slip relationship can also be written as an exponential curve:

$$\bar{\tau}_{cs,x} = \bar{C} \cdot \delta_x^{\bar{N}}$$

then a solution can be given for performing a calculation to take the contribution of tendons into account.
If the increase in the stress in prestressing steel crossing a crack is $c \cdot \sigma_{s,cr}(c \leq 1)$, then the magnitude of c can be estimated: c can be determined in such a way that the crack width calculated with an increase $c \cdot \sigma_{s,cr}$ in the steel stress in a tendon (acting as reinforcement) is equal to the crack width calculated with an increase $\sigma_{s,cr}$ of the steel stress in the reinforcement itself. With this assumption the magnitude of c can be deduced:

$$c = \frac{1}{\sqrt{n}} \cdot \frac{\varnothing_k}{\varnothing_p} \sqrt{x \cdot \frac{\varnothing_d}{\varnothing_k}}$$

with

$$x = \frac{\delta_{cr} 1 + \bar{N}}{\delta_{cr} 1 + \bar{N}} \cdot \frac{1 + N}{1 + \bar{N}} \cdot \frac{\bar{C}}{C}$$

There is not enough information available for determining the magnitude of x accurately.
Moreover a number of variables are of importance in this respect, such as:
- quality and corrugation of the sheath;
- number of wires or strands in a tendon;
- strength of the grout;
- degree of filling of the duct with grout.

A study of the scarce available information indicates a possible fluctuation of x between 0.18 and 0.45.
The diameter $\varnothing_k$ of the duct varies between 40 and 80 mm.
The diameter $\varnothing_k$ of the reinforcing bars will, in general, be 10, 12 or 16 mm.
Combination of these values results in a variation of

$$\sqrt{x \cdot \frac{\varnothing_d}{\varnothing_k}}$$

between 0.8 and 1.2 in the case of wires and occasionally up to 1.5 in the case of strands.
Therefore it is proposed to use – as a low value – in the case of favourable effect of the cooperation of tendons as reinforcement in structural concrete tension elements:

$$c = \frac{1}{\sqrt{n}} \cdot \frac{\varnothing_k}{\varnothing_k} \qquad c \leq 1$$

Instead of with

$$\varrho = \frac{A_s}{A_c}$$

the magnitude of ϱ can then be calculated from:

$$\varrho = \frac{A_s + c \cdot A_p}{A_c}$$

The stress increase $\Delta\sigma_{p,cr}$ in the prestressing steel due to the initiation of cracks is then:

$$\Delta\sigma_{p,cr} = c \cdot \sigma_{s,cr}$$

The N-ε diagram of the structural concrete member can be determined in the same way as described in 5.4. If $\sigma_s = \sigma_{sy}$, then c must be assumed to be 1.

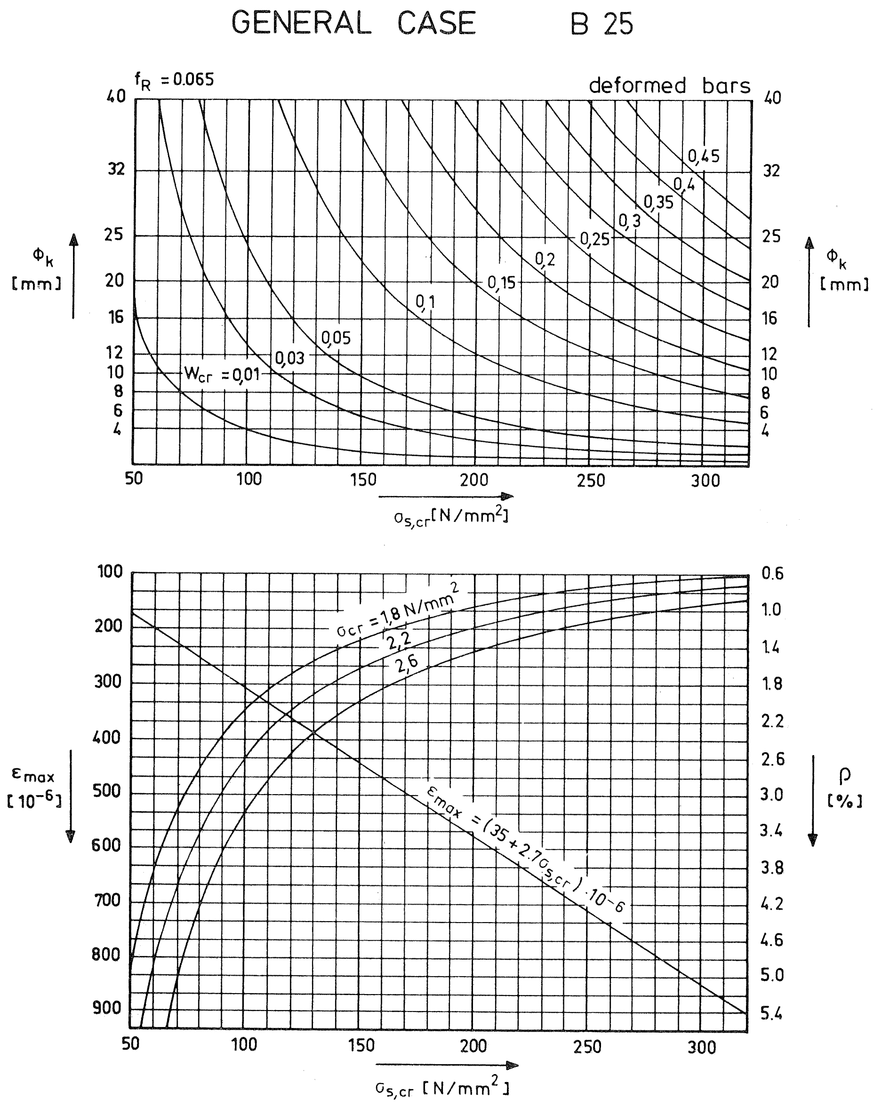

Fig. A1-5. Concrete strength B (f_{cc}) 25.
Bars: general case - $f_R = 0.065$.
Relationship assuming $\varepsilon \leq \varepsilon_{max}$ between w_{cr}; ϕ_k; $\sigma_{s,cr}$; σ_{cr}; ϱ and ε_{max}.

7 Example of a road without expansion joints and provided with continuous reinforcement
(related with Fig. A1-5)

Effective shrinkage $\varepsilon_{cs,\infty} = -300 \times 10^{-6}$.
Maximum temperature drop (seasonal) $-25\,°C$; $\alpha = 12 \times 10^{-6}$.
Maximum crack width 0.30 mm.
Dispersion, cyclic loading and sustained load must be considered in the design, on the assumption of

$$w_{cr} \leq \frac{0.3}{1.4 \times 1.5} = 0.15 \text{ mm}.$$

Tensile strength $\sigma_{cr} = \sigma_{cr-2} = 0.75 f_{ctm} = 2.2 \text{ N/mm}^2$.
Maximum imposed deformation - 600×10^{-6}.
Therefore:

$$\sigma_{s,cr} > \frac{600}{2.7} - 35 = 187 \text{ N/mm}^2 \text{ if } \varepsilon \leq \varepsilon_{max}$$

Because:

$$\frac{\sigma_{cr-2}}{\varrho} + 25 = \sigma_{s,cr}; \quad \varrho \geq 14 \times 10^{-3}$$

The cross-sectional area of the reinforcement is very large!
If bars $\varnothing_k$ 16 are used, $\sigma_{s,cr}$ may increase to 220 N/mm^2 if $w_{cr} \leq 0.15$ mm, as Fig. A1-5 shows.
Then $\varrho \geq 11.3 \times 10^{-3}$. Again a large area of the reinforcement.
If bars $\varnothing_k$ 12 are used, then $\varrho \geq 9.4 \times 10^{-3}$.
In a road slab with a depth of 180 mm therefore 15 $\varnothing_k$ 12 are necessary to fulfil the requirements, e.g. $\varnothing_k$ 12–130 (two layers).
Note that a large quantity of reinforcement has to be used.

Check of crack width:

$$\sigma_{s,cr} = \frac{2.2}{9.7 \times 10^{-3}} + 25 = 253 \text{ N/mm}^2$$

$$1 + n \cdot \varrho = 1.068$$

$$w_{cr} = 2 \left\{ \frac{1.18}{2} \times \frac{12}{4} \times \frac{1}{0.38 \times 29 \times 205000} \times \frac{253^2}{1.068} \right\}^{\frac{1}{1.18}} = 0.15 \text{ mm}$$

$$\varepsilon_{max} = (2.7 \times 253 + 35) \times 10^{-6} = 718 \times 10^{-6} \qquad (\varepsilon < \varepsilon_{max})$$

APPENDIX 2

Engineering model of the beam in structural concrete

1 Flexural tensile strength

As already shown, the magnitude of the flexural tennsile strength $\sigma_{cr,fl}$ depends on the direct tensile strength of the concrete and the strain gradient in the tensile zone.
The magnitude of this strength is, in accordance with investigations published in Heft 269 of Deutscher Ausschuss für Stahlbeton, calculated from:

$$\sigma_{cr,fl} = (0.8 + 0.4(2a)^{-0.6}) \cdot \sigma_{cr} \qquad (a \text{ in m})$$

where

a = the depth of the tensile zone

In reinforced concrete beams $a = 0.5h$.
In structural concrete beams a can be substantially lower than $0.5h$ due to the influence of the prestressing force.
The flexural tensile strength of structural concrete beams will be higher than that of normal reinforced concrete beams of the same depth if the structural concrete beams are also prestressed.
The ratio $\sigma_{cr,fl}/\sigma_{cr}$ is limited to 2.
The flanges of box girders must be conceived as structural concrete tension elements loaded in pure tension.In that case $\sigma_{cr,fl} = \sigma_{cr}$.

2 The area "A_c" of the reinforced concrete tension member in reinforced concrete beams of rectangular cross-section (see Fig. 25)

In the calculation of N_{cr} the magnitude of z is estimated as $0.9d$.
Therefore:

$$N_{cr-1} = \frac{M_{cr}}{0.9d}$$

Because:

$$N_{cr-1} = \text{“}A_c\text{”} \cdot \sigma_{cr-1}(1 + n \cdot \varrho)$$

$$\text{“}A_c\text{”} = \frac{M_{cr}}{0.9d \cdot \sigma_{cr-1}} - n \cdot A_s$$

In this formula:

$$M_{cr} = W_{cs}(0.8 + 0.4h^{-0.6}) \cdot \sigma_{cr-1}$$

If W_{cs} is approximated with an average value $1.13\,W_c = 0.19b \cdot h^2$, the area "$A_c$" of the reinforced concrete tension member can be calculated from:

$$\text{“}A_c\text{”} = (0.14 + 0.092h^{-0.6}) \cdot b \cdot h \qquad (h \text{ in m})$$

Example:

h	$=0.3$ m	0.5 m	1.0 m
$\dfrac{\text{“}A_c\text{”}}{b \cdot h}$	$=0.33$	0.28	0.23

3 Moment-curvature relationship of a part of a reinforced concrete beam

The relationship between bending moment curvature - steel stress in the reinforcement - crack width - crack distance can be determined with the well-known theory of the modulus method (M.LE.R.CS.).

Instead of the stiffness of the tensile reinforcement $A_s \cdot E_s$ now the mean stiffness of the reinforcement of the reinforced concrete tension member has to be taken into account. This mean stiffness is associated with the mean value of the steel stress $\sigma_{s,mean}$ and can be calculated by introducing the tension stiffening factor $\alpha_{t,s}$.
This factor depends on the magnitude of the tensile force N in the tension member:

$$\alpha_{t,s} = \frac{\sigma_s}{\sigma_{s,mean}} = \frac{\sigma_s}{\varepsilon l/l \cdot E_s}$$

In this formula $\varepsilon l/l$ is the mean elongation of the reinforced concrete tension member at a tensile force N (see Fig. 21 and 24).
This force is associated with a bending moment M because $N = M/z$.
By introducing $\alpha_{t,s}$ the mean depth h_x of the compression zone – linearly elastic behaviour – can be calculated:

$$h_x = d\,\{-n \cdot \alpha_{t,s} \cdot \varrho + \sqrt{(n \cdot \alpha_{t,s} \cdot \varrho)^2 + 2n \cdot \alpha_{t,s} \cdot \varrho}\}$$

$$z = d - \frac{1}{3} h_x$$

Investigations have shown that this engineering model gives crack spacings which are of the same magnitude as those measured in experiments. Further information is given in [35].

4 The area "A_c" of the structural concrete tension member

4.1 *Calculation of the internal lever arm z_{cr}*

The stress distribution in a cross-section due to prestressing and the bending moment M_{cr} is shown in Fig. 27.
The magnitude of $\sigma_{cr,fl}$ depends on the depth a of the tensile zone, but in turn the depth a depends on $\sigma_{cr,fl}$.
The stress distribution can be determined and M_{cr} calculated by an iterative procedure:

$$\frac{M_{cr}}{W_{c2}} - \frac{P(e+k_1)}{W_{c2}} = \sigma_{cr,fl} \qquad \text{(MN.LE.U.C.)}$$

$$M_{cr} = \sigma_{cr,fl} \cdot W_{c2} + P(e + k_1)$$

The centroid of the result compressive force on the concrete section can now be determined. The distance between this centroid and the centroid of the prestressing force on the concrete section (in the absence of any combination with bending moments due to dead load, etc.) is the internal lever arm z_{cr}.

Note: In freely supported beams the centroid of the prestressing force coincides with the centroid of the prestressing steel. In statically indeterminate structures this is not necessary the case!

4.2 *Calculation of the internal lever arm z when the first crack in the tensile zone has been initiated*

As already is mentioned in 4.3.3, z can be calculated on the assumption that there is no tension stiffening of the concrete zone, because this calculation relates to a cross-section.
This means that contrtibution to the stiffness of the concrete in the tensile zone is not taken into account.
The factor c, associated with the bond of tendons, can in this calculation be taken as $c = 1$. The sensitivity of z with respect to the magnitude of c can be checked, if desired.
Calculation of z (see Fig. A2-1).
In this case it is assumed that in the tensile zone the centroids of the prestressing steel and of the normal reinforcement coincide.
If this is not the case, the calculation of z is somewhat more complex, but based on the same approach.

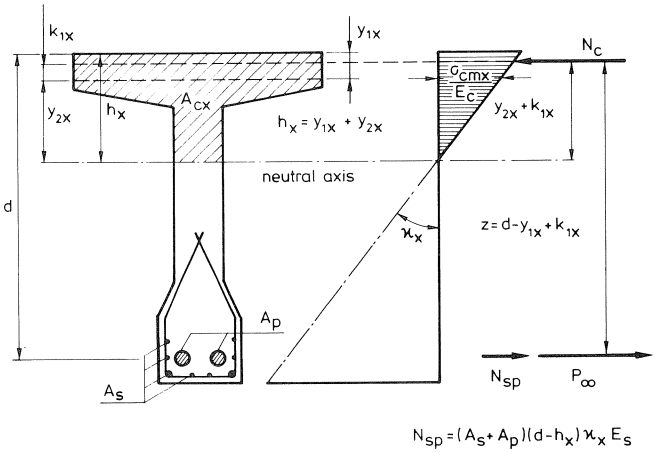

Fig. A2-1. Derivation of z under M_{cr}.

Concrete mechanics model MN.LE.R.C.P.S.
In the calculation the concrete stresses in the compression zone are assumed to be in the linearly elastic part of the stress-strain diagram.
With an assumed magnitude of the depth h_x of the compression zone all the properties of the part of the section under compression can be determined, such as (see Fig. A2-1 - hatched area):

$$I_{cx};\ W_{cx1};\ W_{cx2};\ k_{1x};\ k_{2x};\ y_{1x};\ y_{2x};\ A_{cx};\ \text{etc.}$$

Therefore if h_x is known and the curvature $\varkappa_x$ of the section is known, the mean compressive stress σ_{cmx} of the compression zone can be calculated:

$$\sigma_{cmx} = -\varkappa_x \cdot y_{2x} \cdot E_c$$

Equilibrium of forces and moments in the section:
- Horizontal forces:

$$A_{cx} \cdot \sigma_{cmx} + (A_s + A_p)(d - h_x) \cdot \varkappa_x \cdot E_s + P_\infty = 0$$

- Bending moments:

$$\{P_\infty + (A_s + A_p)(d - h_x) \cdot \varkappa_x \cdot E_s\} \cdot z = M_{cr}$$

On elimination of $\varkappa_x$ from both equations the following expression is obtained:

$$\frac{M_{cr}}{P_\infty} = \frac{A_{cx} \cdot y_{2x}(d - y_{1x} + k_{1x})}{A_{cx} \cdot y_{2x} - (A_s + A_p)(d - h_x) \cdot n}; \quad n = \frac{E_s}{E_c}$$

The depth h_x in the case $M = M_{cr}$ can be determined graphically or by interpolation.

The ratio on the left-hand side can be calculated for several values of h_x because the magnitude of this ratio depends only on the assumed magnitude of h_x (and the associated section properties) and the magnitude of $(A_s + A_p)$ and n.
If h_x is determined, it means that the left-hand ratio is equal to M_{cr}/P_∞.
The the lever arm:

$$z = d - y_{1x} + k_{1x} = d - h_x + y_{2x} + k_{1x}$$

Note: To check the sensitivity of z with respect to c the calculation can be repeated with a cross-sectional area of reinforcing steel and prestressing steel of the magnitude $(A_s + c \cdot A_p)$

5 The moment-curvature relationship of a part of a structural concrete beam

As already shown in 4.3.3, the tensile force N_{cr-1} in a structural concrete tension member can be determined with:

$$N_{cr-1} = P_\infty \left(\frac{z_{cr}}{z} - 1\right)$$

with

$$\text{“}A_c\text{”} = \frac{N_{cr-1} - 25(A_s + c \cdot A_p)}{\sigma_{cr}}$$

The N-ε diagram of the structural concrete tension member can now be determined (see Appendix 1-5.4 and Appendix 1-6).
The tensile force N_{cr-2} for the fully developed crack pattern can be taken as equal to $1.2 N_{cr-1}$ and $\varepsilon_{cr-2} = \varepsilon_{max}$ (from M_{20}).

$$N_{sy} = (A_s + A_p) \cdot f_{sy} + P_\infty \quad \text{with} \quad \varepsilon = \varepsilon_{sy}$$

For every value of the tensile force N_x the elongation ε_x of the structural concrete tension member is known and therefore also the value of the tension stiffening factor $\alpha_{t,s}$:

$$\alpha_{t,s} = \frac{N_x}{\dfrac{(A_s + c_x \cdot A_p) \cdot E_s}{\varepsilon_x}}$$

$$\alpha_{t,s} = \frac{N_x}{\varepsilon_x \cdot E_s (A_s + c_x \cdot A_p)}$$

The factor c_x takes into account the increase in collaboration of reinforcement and prestressing steel with increasing crack width; c_x approximates the value 1 with increasing magnitude of the tensile force.
With the N-ε diagram of the structural concrete tension element the M-$\varkappa$ diagram of the beam can be determined by introducing this element (and its influence on the tension stiffening of the reinforcement) as the “reinforcement” of this beam. See also Fig. 28.
Again the relationship between the external load, the stresses in the reinforcement, the crack width, the crack distance and the curvature can be determined in this way.
With the same aproach, already explained, and assuming $N_{cr-2} = 1.2 N_{cr-1}$ it is possible to calculate M_{cr-2}:

$$\frac{M_{cr-2}}{P_\infty} = \frac{A_{cx} \cdot y_{2x}(d - y_{1x} + k_{1x})}{A_{cx} \cdot y_{2x} - (A_s + c \cdot A_p)(d - h_x) \cdot n \cdot \alpha_{t,s}}$$

In this case:

$$\alpha_{t,s} = \frac{\sigma_{s,cr-2}}{\varepsilon_{max} \cdot E_s}$$

Notes: 1. If $\alpha_{t,s} > 1$, the mean tensile stress in the reinforcement in a crack is $\alpha_{t,s} \cdot \sigma_{s,mean}$. This steel stress $\sigma_{s,mean}$ is the tensile stress in the reinforcement of the tension element.

2. Especially in the case of structural concrete with relatively large prestress the value of the tensile stress $\sigma_{s,cr-2}$ in a crack may be small and therefore also l_{pt} and l_{cr}.
As already mentioned in 4.2.1.2, in the case where the crack spacing is less than $8\varnothing_k$ the model is of no value. It can be shown that in that case the minimum crack spacing to be taken into account will be $8\,\varnothing_k$. The value of $8\,\varnothing_k$ takes account of twice the end distortion on both sides of a crack ($2 \times 2\,\varnothing_k$) and twice the length ($2\,\varnothing_k$) of the zone over which the tensile force of a bar is transmitted to the concrete. However, the crack width can be calculated from the formulas of Appendix 1.

7 Example structural concrete T-beam

T-beam (see Fig. A2-2).
Concrete strength B 35; $f_{cc}=35$ N/mm²; $f_{ct}=2.4$ N/mm².
Normal reinforcement 8 $\varnothing_k$ 12; $f_{sy}=400$ N/mm².
Prestressing steel 518 mm²; $f_{pk}=1860$ N/mm²; $\sigma_{p,\infty}=1150$ N/mm².
Section modulus $W_{c2}=48.8\times10^6$ N/mm²

$$k_1 = 119 \text{ mm}$$
$$A_c = 410\times10^3 \text{ mm}^2$$
$$y_2 = 520 \text{ mm}$$

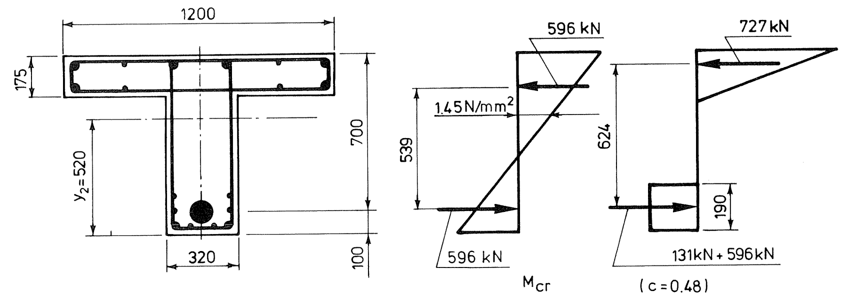

Fig. A2-2. Cross-section of T-beam.

Calculation of $\sigma_{cr.fl}$ if σ_{cr-1} is 2.2 N/mm²:

$$P_\infty = 596 \text{ kN}$$
$$\sigma_{cm} = -1.45 \text{ N/mm}^2$$

After some iterations it is found:

$$a = 340 \text{ mm}$$
$$\sigma_{cr,fl} = (0.8+0.4\times(0.68)^{-0.6})\times2.2 = 2.9 \text{ N/mm}^2$$

Calculation of M_{cr} and z_{cr}:

$$M_{cr} = 2.9\times48.8\times10^6 + 596\times10^3(520-100+119)$$
$$M_{cr} = 462.8 \text{ kNm}$$

$$z_{cr} = \frac{462.8}{596} = 0.78 \text{ m (see Fig. A2-3)}$$

According to Appendix 1:

$$c = \frac{1}{\sqrt{7\times7}}\times\frac{12}{3.6} = 0.48$$

To show the influence of the bond of the tendons (factor c) on the stresses, three values of c are chosen in the calculation (0, 0.48 and 1).

$c=$	0.0	0.48	1.0	
h_x	182	189	193	mm
z_{cr}	639	637	636	mm

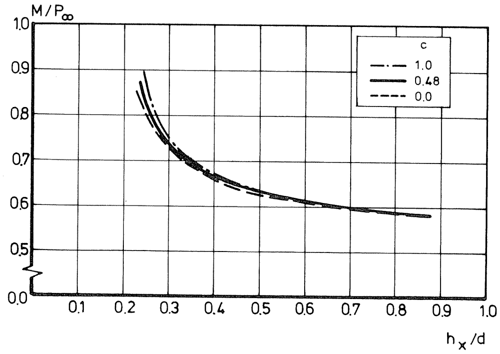

Fig. A2-3. $M/P - h_x$ relationship ($\alpha_{t,s} = 1$).

Calculation of crack width if $\varepsilon = \varepsilon_{max}$:
Because:

$$N_{cr-1} = \frac{M_{cr}}{z_{cr}} - P_\infty$$

$$\sigma_{s,cr-1} = \frac{N_{cr-1}}{A_{sp}}$$

$$\sigma_{s,cr-2} = 1.2 \times \sigma_{s,cr-1}$$

$c=$	0.0	0.48	1.0	
N_{cr-1}	128.3	130.5	131.7	kN
$\sigma_{s,cr-1}$	141.7	113.2	92.6	N/mm^2
$\sigma_{s,cr-2}$	170.1	135.9	111.1	N/mm^2
w_{cr}	0.06	0.04	0.03	mm
Δl_{gem}	130	109	94*	mm

* The crack spacing is equal to the minimum distance 8 $\varnothing_k$

Calculation of the crack width if $M = M_{max}$:

$$M_{max} = 508 \text{ kNm}$$

$$\frac{M}{P_\infty} = 0.85 \text{ m}$$

$c=$	0.0	0.48	1.0	
h_x	161	170	175	mm
z_{max}	646	643	642	mm
N_c	190	194	195	kN
σ_s	210	168	137	N/mm^2
$w(\alpha_{t,s}=1)$	0.13	0.09	0.06	mm
$w(\alpha_{t,s}\neq 1)$	0.10	0.07	0.05	mm