# Numerical analysis of soil-structure interaction
## Evaluation and application to cofferdam construction

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**Abstract**

The aim of this contribution is to evaluate a number of geotechnical aspects associated to finite element analysis of soil-structure interaction problems. Some recently developed tools covering these aspects are demonstrated and evaluated by way of an example.

Before presenting the results of a finite element analysis of a typical soil-structure interaction problem the geotechnical aspects are highlighted: introducing the initial state of stress of the soil into the model, the execution of a stepwise (phased) analysis, drained and undrained soil behavior, modeling of soil-structure interaction and soil-pore fluid interaction.

All these aspects are incorporated in the numerical analysis of a cofferdam construction. A phased analysis is presented starting from an initial situation followed by a number of construction phases. Evaluation of the results shows that by using these tools it is possible to simulate the behavior of constructions of this type.

### 1 Introduction

All buildings have a foundation: the loading has to be transferred from the building to the underground subsoil. The foundation type is important with respect to the stress and strain trajectories in the structural members. In fact the construction and the surrounding soil can be defined as one mechanical system. So, it is preferable to analyze the behavior in one combined model. An example in which the importance of the interaction between soil and construction is clear is the underground infrastructure. At the moment there is growing interest in this way of building as it is considered to be a solution in crowded parts of the world. Evaluation of tools for finite element analysis of soil-structure interaction problems is therefore the major objective of this contribution.

As a demonstration project the numerical analysis of a cofferdam construction is presented. A cofferdam is one of the alternatives used in dike elevation projects. The main advantage of this type of construction is that no widening of the dike is necessary, only elevation. Sheet pile walls are placed at both sides of the dike, while the height between the top of these sheet piles and the old dike level is exactly the height of the elevation. This space is filled up with soil.

In this example various geotechnical problems are encountered. Before presenting the results of the cofferdam analysis some aspects of these problems will be highlighted.

## 2 Geotechnical aspects

### 2.1 *Initial state of stress*

The initial state of stress in the soil can be characterized by the volumetric weight of the soil ($\gamma$), the depth ($z$) and the lateral pressure ratio ($K_0$). This ratio is defined as the quotient of the horizontal (principal) effective stress $\sigma_1'$ and the vertical effective stress $\sigma_3'$:

$$K_0 = \frac{\sigma_1'}{\sigma_3'} = \frac{\sigma_1'}{\gamma_z} \tag{1}$$

In the case of a linear elastic plane strain model and a horizontal surface it can be demonstrated that $K_0$ is related to Poisson's ratio $\nu$ in the following way:

$$K_0 = \frac{\nu}{1-\nu} \tag{2}$$

After introducing the initial stresses into the model, all stress points in a diagram relating the isotropic stress to the deviatoric stress are located on the "$K_0$-line". An example is given in Fig. 1. Since the difference between the "$K_0$-line" and the failure envelope of the material is a global measure of the load bearing capacity, introducing the initial stresses into the model is necessary to give reliable results.

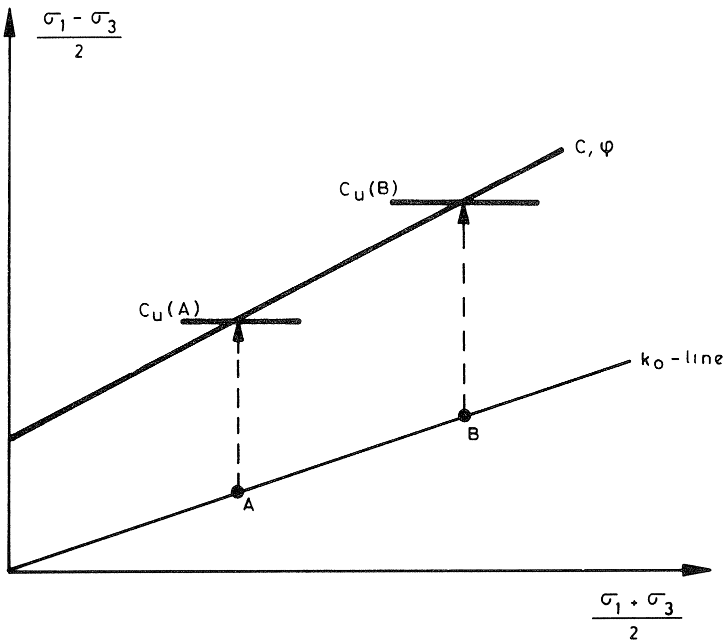


Fig. 1. Initial state of stress and relation between drained (Mohr-Coulomb) model and undrained (Tresca) model.

## 2.2 *Phased analysis*

The execution of geotechnical works generally involves a number of phases. Starting from the initial situation (with dead weight load and pore pressure load) a sequence of the following phases may occur.

- Addition of piles, sheet piles, anchors, geotextile.
- Excavation.
- Construction of foundation.
- Elevation.
- Removal of piles, sheet piles or anchors.

The execution history influences the final deformations and stress situation. If the finite element model changes from one phase to another, phased analysis is needed for an accurate modeling of the execution history [1].

Phased analysis enables addition or removal of elements and constraints between different stages in a finite element analysis, taking into account the previously calculated stresses.

Phased analysis is an incremental method allowing model changes. The incremental displacements ${}^{\mathrm{i}}\Delta \boldsymbol{u}$ of phase $i$ are calculated from incremental effective loads ${}^{\mathrm{i}}\Delta \boldsymbol{f}$ with the linear or nonlinear stiffness ${}^{\mathrm{i}}\boldsymbol{K}$.

$$ {}^{\mathrm{i}}\boldsymbol{K}\,{}^{\mathrm{i}}\Delta \boldsymbol{u} = {}^{\mathrm{i}}\Delta \boldsymbol{f} \tag{3} $$

If a number of load steps $ns$ is involved in each phase the equation changes to:

$$ \sum_{\mathrm{s}=1}^{\mathrm{ns}} {}^{\mathrm{s}}\boldsymbol{K}\,{}^{\mathrm{s}}\Delta \boldsymbol{u} = {}^{\mathrm{i}}\Delta \boldsymbol{f} \tag{4} $$

## 2.3 *Drained and undrained behavior*

Depending on the loading rate the behavior of soft soil layers like clay and peat can be characterized as drained or as undrained.

This implies that during some phases of the analysis these layers may react drained, while during other phases undrained.

In this contribution the constitutive behavior of the soil is restricted to "perfect elastic – perfect plastic" models. No hardening effects will be taken into account and a non-associated flow-rule will be used [2]. In this part only the relation will be given between the drained and undrained stiffness and strength.

*Stiffness*

Undrained behavior basically implies that the material behaves like an incompressible medium. For the elastic part of the model this means that the undrained Poisson's ratio, $\nu_{\mathrm{u}}$, will be equal to 0.5 and the undrained bulk modulus, $K_{\mathrm{u}}$, will be (nearly) infinite. The undrained shear modulus, $G_{\mathrm{u}}$, will be approximately equal to the drained modulus, $G_{\mathrm{d}}$. So, one has to conclude that the undrained Young's modulus, $E_{\mathrm{u}}$, is not equal to the drained one. $E_{\mathrm{u}}$ is related to $E_{\mathrm{d}}$ in the following way:

$$E_u = \frac{3E_d}{2(1+\nu_d)} \tag{5}$$

*Strength*

The undrained (incompressible) plastic behavior of soft soils can be modeled by the Tresca material model. This implies that the difference between the maximum and minimum principle stresses is limited to two times the undrained cohesion $c_u$:

$$\max\left[\,|\sigma_1' - \sigma_2'|, \quad |\sigma_2' - \sigma_3'|, \quad |\sigma_3' - \sigma_1'|\,\right] \leq 2c_u \tag{6}$$

Drained behavior of the same material can be modeled by the Mohr-Coulomb model:

$$\sigma_1'(1+\sin\phi) - \sigma_3'(1-\sin\phi) \leq 2c_d \cos\phi \tag{7}$$

in which:

$c_d$ = drained cohesion of the soil
$\phi$ = angle of internal friction

Depending on the initial state of stress $(\sigma_1', \sigma_3')$ the relation between the drained $(c_d, \phi)$ model and undrained $(c_u)$ model is given in equation 8. This is shown in graph form in Fig. 1.

$$c_u = c_d \cos\phi + \frac{\sigma_1' + \sigma_3'}{2} \sin\phi \tag{8}$$

The undrained cohesion $c_u$ is automatically determined in the finite element program after the calculation of the initial state of stress.

## 2.4 *Soil-structure interaction*

When large localized strains are involved between construction and soil, special elements are needed to describe the interface behavior correctly. Two mechanisms can be distinguished at the interface: the contact-gapping mechanism and the frictional shearing mechanism.

Basically two kinds of interface constitutive equations are used to model these mechanisms in numerical simulations [3].

1. The soil-structure interface is considered as a thin continuum layer.
2. The continuum equations are degenerated in such a way that the interface zone is replaced by a bi-dimensional constitutive relation [4].

The second approach is adopted in this contribution. The interface behavior is described in terms of a relation between the normal and shear tractions, $t_n$ and $t_s$, and the normal and shear relative displacements across the interface, $\Delta u_n$ and $\Delta u_s$. In geomechanics, when the stresses are within the elastic region, the normal and shear relations are generally assumed to be independent:

$$\begin{Bmatrix} t_n \\ t_s \end{Bmatrix} = \begin{bmatrix} D_{11} & 0 \\ 0 & D_{22} \end{bmatrix} \begin{Bmatrix} \Delta n_u \\ \Delta u_s \end{Bmatrix} \tag{9}$$

The component $D_{11}$ stands for the relation between normal traction and normal relative displacement: the contact-gapping mechanism. $D_{11}$ resembles an elastic spring stiffness, a very stiff spring to simulate contact up to a specified maximum normal traction. If the normal traction exceeds this maximum value a discrete "gap" arises between the construction and the soil and the normal traction reduces to zero: instantaneous, linear or nonlinear.
Generally the friction mechanism, $D_{22}$, is described as follows: up to a specified shear stress level, depending on the normal stress at the interface, "elastic" shearing-behavior is assumed. If the shear stress exceeds this threshold, plastic slip deformation occurs at the interface. In that case the maximum shear stress $\tau_{\max}$ can be defined by the Coulomb friction model:

$$\tau_{\max} = a + \sigma_{\mathrm{n}} \tan \delta \tag{10}$$

in which:

$a$ = the adhesion between the construction and the soil
$\sigma_{\mathrm{n}}$ = the normal stress at the interface
$\delta$ = the interface friction angle

## 2.5 *Soil-pore fluid interaction*

The material behavior of soil is based on the effective stress $\sigma'$, which is affected by the pore fluid pressure $p$.

$$\sigma'_{\mathrm{ij}} = \sigma_{\mathrm{ij}} - \delta_{\mathrm{ij}}\, p \tag{11}$$

The total linearized set of equations for the coupled problem of soil-pore fluid interaction, following from spatial discretization, is given by [5]:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} - \boldsymbol{Q}\boldsymbol{p} = \boldsymbol{F}_{\mathrm{u}} \tag{12a}$$

$$\boldsymbol{Q}^{\mathrm{T}}\dot{\boldsymbol{u}} + \boldsymbol{S}\boldsymbol{p} + \boldsymbol{H}\boldsymbol{p} = \boldsymbol{F}_{\mathrm{p}} \tag{12b}$$

with $\boldsymbol{u}$ being the displacement, $\boldsymbol{p}$ the pressure, $\boldsymbol{C}$ the damping matrix, $\boldsymbol{M}$ the mass matrix, $\boldsymbol{K}$ the stiffness matrix, $\boldsymbol{H}$ the permeability matrix, $\boldsymbol{S}$ the hydraulic capacity matrix, and $\boldsymbol{Q}$ the coupling matrix.
The definition of the matrices follows below:

$$\boldsymbol{M} = \int_{\mathrm{V}} \boldsymbol{N}_{\mathrm{u}}^{\mathrm{T}} \varrho \boldsymbol{N}_{\mathrm{u}}\, \mathrm{d}V, \quad \boldsymbol{K} = \int_{\mathrm{V}} \boldsymbol{B}_{\mathrm{u}}^{\mathrm{T}} \boldsymbol{D} \boldsymbol{B}_{\mathrm{u}}\, \mathrm{d}V, \quad \boldsymbol{S} = \int_{\mathrm{V}} \boldsymbol{N}_{\mathrm{p}}^{\mathrm{T}} s \boldsymbol{N}_{\mathrm{p}}\, \mathrm{d}V$$

$$\boldsymbol{H} = \int_{\mathrm{V}} \boldsymbol{B}_{\mathrm{p}}^{\mathrm{T}} \boldsymbol{k} \boldsymbol{B}_{\mathrm{p}}\, \mathrm{d}V, \quad \boldsymbol{Q} = \int_{\mathrm{V}} \boldsymbol{B}_{\mathrm{u}}^{\mathrm{T}} \boldsymbol{m} \boldsymbol{N}_{\mathrm{p}}\, \mathrm{d}V, \quad \boldsymbol{m}^{\mathrm{T}} = \{1, 1, 1, 0, 0, 0\}$$

with $\boldsymbol{N}_{\mathrm{u}}$ being the displacement interpolation matrix, $\varrho$ the density of the porous medium, $\boldsymbol{B}_{\mathrm{u}}$ the strain interpolation matrix, $\boldsymbol{D}$ the stiffness tensor, $\boldsymbol{N}_{\mathrm{p}}$ the pressure interpolation matrix, $s$ the hydraulic capacity, $\boldsymbol{B}_{\mathrm{p}} = \nabla \boldsymbol{N}_{\mathrm{p}}$ and $\boldsymbol{k}$ the permeability.

Full dynamic analysis necessitates a total or staggered coupled solution, whereas in static analysis a staggered solution is more convenient. The pressure $\boldsymbol{p}$ is solved first and displacement $\boldsymbol{u}$ is solved next.

$$p = H^{-1}F_{\mathrm{p}} \tag{13b}$$

$$u = K^{-1}(Q^{\mathrm{T}}p + F_{\mathrm{u}}) \tag{13a}$$

In the example treated in the next paragraph, a separate static analysis for pressure and displacement is executed, based on the same structural mesh. Quadratic $C_0$ interpolation of displacement $\boldsymbol{u}$ is combined with linear $C_0$ interpolation of pressure $\boldsymbol{p}$, to ensure compatibility between pressure and stress. The results from the pressure calculation are automatically input for the displacement calculation. This input includes both pore pressure load and additional weight load.

## 3 Cofferdam construction

### 3.1 *Introduction*

For various reasons the analysis of soil-structure interaction problems is very complicated. Mostly two or even three dimensions have to be taken into account. A number of construction phases have to be distinguished. In addition, the soil behavior is highly nonlinear and rate dependent. So, up to 10 or 20 years ago, one was obliged to use empirical or semi-analytical models. This approach, as applied to a cofferdam construction, is available from the literature [6, 7]. A cofferdam is one of the alternatives in dike elevation projects. The main advantage of this type of construction is that no widening of the dike is necessary, only elevation. Especially in areas in which houses are situated at the slopes of the dike, this way of constructing is a feasible alternative. In this paragraph the numerical analysis of a cofferdam construction is presented. The geotechnical aspects worked out in paragraph 2 are incorporated in the analysis. A phased analysis is presented starting from an initial situation (without a cofferdam) followed by a number of construction phases.

A cofferdam is constructed as follows. At both sides of the dike to be elevated sheet pile walls are placed. The height between the top of these sheet piles is exactly the height of elevation. This space is filled up with soil.

The geometry of the specific situation analyzed is presented in Fig. 2. This situation is representative for the neighborhood of a little village named Sliedrecht. A large number of buildings are situated at the sides of the dike, which is located between the Merwede and the Alblasserwaard. The soil-profile is schematized into four different layers as summarized in Table 1.

The initial groundwater level on the river side is 0.0 m -NAP and on the polder side 2.0 m -NAP. The following stages can be distinguished:

Construction analysis

- The initial situation (the stress state in the soil mass caused by the dead weight and pore pressure loading).

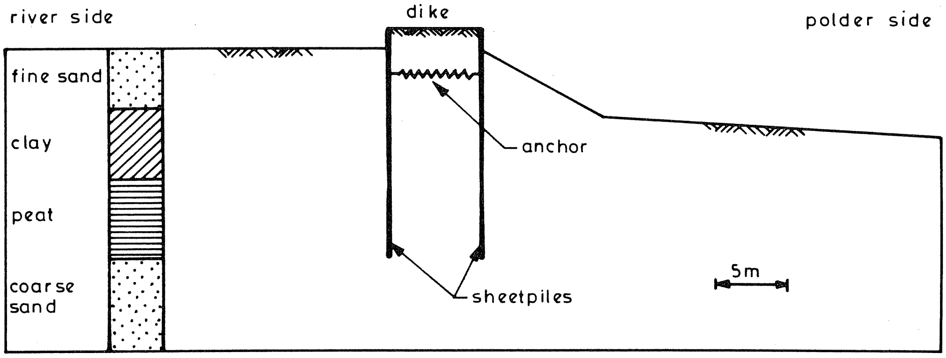


Fig. 2. Cross section of cofferdam construction.

Table 1. Schematized soil profile

| layer | soil type | depth (in m -NAP) |
|---|---|---|
| 1 | fine-grained sand | surface to 0.0 m -NAP |
| 2 | clay | from 0.0 to 3.0 m -NAP |
| 3 | peat | from 3.0 to 8.5 m -NAP |
| 4 | deep, coarse-grained sand | below 8.5 m -NAP |

- The sheet pile walls are placed and connected by anchors.
- The soil elevation between the sheet piles is added.

Failure analysis

- The (ground)water level rises on the river side up to the top of the dam.
- Due to a storm the water level rises above the top of the dike and water curls over the dike and the soil on the inner (passive) side of the dam is washed away.

The main question to be answered is whether the dike is stable or not after the storm simulated in the final stage.

### 3.2 *Numerical model*

The problem definition gives rise to 4 calculation phases, with corresponding FEM models.

The soil layers are modeled with plane-strain elements, the sheet piling is modeled with beam and interface elements and the anchor is modeled with one truss element. All elements are quadratic interpolated and isoparametric, except the truss element. The material properties of the soil layers for both drained and undrained conditions are specified in Table 2. Plasticity of drained materials is modeled with the Mohr-Coulomb yield criterion. Deformations are assumed to be small. The nonlinear calculation of each phase is executed with an incremental iterative method (maximum 40 iterations), using the initial linear stiffness and an energy based accuracy condition ($\varepsilon = 10^{-4}$).

Table 2. Soil properties

| property | values (N, m, sec, kg, deg) fine sand | clay | peat | coarse sand |
|---|---|---|---|---|
| drained Young mod. $E_d$ | 15.E6 | 3.5E6 | 1.5E6 | 40.E6 |
| undrained Young mod. $E_u$ | – | 4.E6 | 1.62E6 | – |
| drained Poisson's ratio $\nu_d$ | 0.31 | 0.33 | 0.4 | 0.29 |
| undrained Poisson's ratio $\nu_u$ | – | 0.49 | 0.49 | – |
| dry density $\varrho$ | 1700. | 1100. | 800. | 1700. |
| drained cohesion $c_d$ | 3.9E3 | 5.4E3 | 10.E3 | 0. |
| drained friction angle $\phi_d$ | 27 | 21.5 | 25 | 35 |
| dilatation angle $\psi$ | 0 | 0 | 0 | 5 |
| $K_0$-ratio | 0.58 | 0.5 | 0.666 | 0.41 |
| permeability $k$ | 1.E-4 | 1.E-6 | 1.E-6 | 1.E-3 |
| porosity $n$ | 0.3 | 0.5 | 0.5 | 0.3 |

A description of the calculation phases follows below. The finite element models corresponding to these phases are presented in Fig. 3.

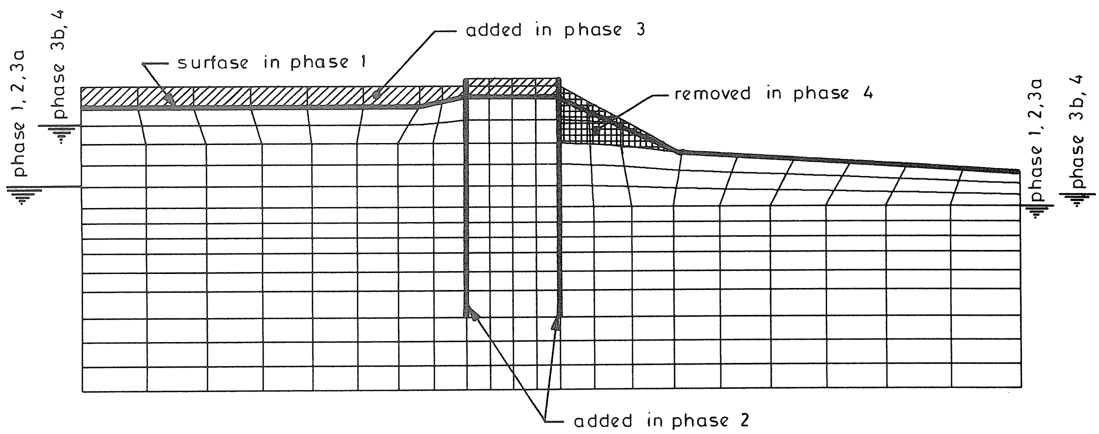


Fig. 3. Finite element model for phased analysis.

1. *Initial conditions.* After a potential flow calculation, the finite element model with soil layers is loaded with dead weight and pore pressure. The initial stresses at the start of the nonlinear structural analysis are derived from the calculated linear elastic vertical stresses and the specified $K_0$ ratios. The nonlinear calculation is executed with 4 load steps.
2. *Addition of cofferdam.* The beam elements, interface elements (sheet piles) and truss element (anchor) are added to the model in a separate phase to prevent false deformations in these elements from the initial load in phase 1. The nonlinear calculation is executed with 2 load steps.

3a. *Elevation.* New elements are added at the top of the dike, introducing additional stiffness and dead weight load. As stated previously, the layers with cohesive material show undrained (incompressible) behavior under this load. Therefore the elastic and plastic material properties of these layers are adapted in accordance with Table 2. The nonlinear calculation is executed with 5 load steps.

3b. *Phreatic level rise.* After the addition of elements an additional pore pressure load and weight load is applied. This load follows from a calculated rise in the phreatic level and is applied to check the stability under these new conditions. The nonlinear calculation is executed with 5 load steps.

4. *Erosion of polder side.* To study the effect of possible erosion of the polder side, a number of elements are removed from the actual model, again assuming undrained properties for the cohesive materials. The nonlinear calculation is executed with 20 load steps.

### 3.3 *Results*

Fig. 4 shows the plots of the calculated plastified area after each phase. Fig. 5 shows a plot of the total deformation due to phases 3a and 3b. Fig. 6 shows a plot of displacement increments in phase 4. Fig. 7 shows a graph of the calculated horizontal displacement of the top of the left and right sheet piles as a function of the load. The calculated displacements fall within the range of expected values. Fig. 8 shows the calculated bending moments in both sheet piles after phase 3b.

## 4 Evaluation

- Phased analysis.
  This example clearly demonstrates that phased analysis is needed for a proper treatment of model changes during the calculation.
  - The use of the $K_0$ ratio combined with the addition of the construction in a separate phase is an excellent way of establishing correct initial stress conditions. Control over superposition of incremental displacements is useful to achieve zero initial displacements.
    Without phased analysis, tricks are needed to reduce the initial displacement and false deformations in the construction part.
  - Excavation or elevation is modeled both elegantly and correctly. The changed contributions to dead weight load, stiffness, strength and drained/undrained properties are automatically incorporated.
    Without phased analysis, changes in dead weight load have to be applied manually. The stiffness and strength changes cannot be modeled exactly.
- Groundwater load.
  The load from a calculated pressure field is automatically generated and has important effects on the effective stresses and therefore also on their resulting lateral component acting on the sheet piling.

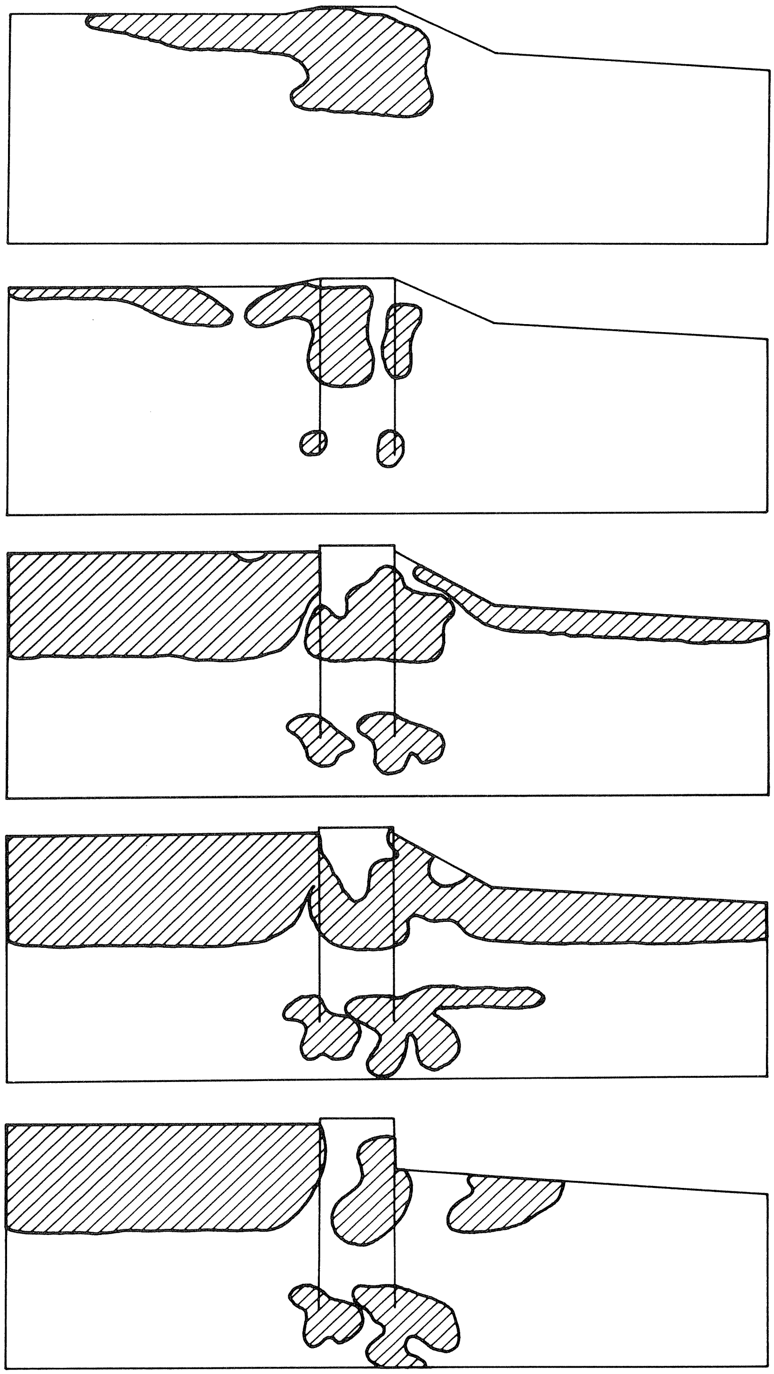

Fig. 4. Spread of plastified area; (a) phase 1; (b) phase 2; (c) phase 3a; (d) phase 3b and (e) phase 4.

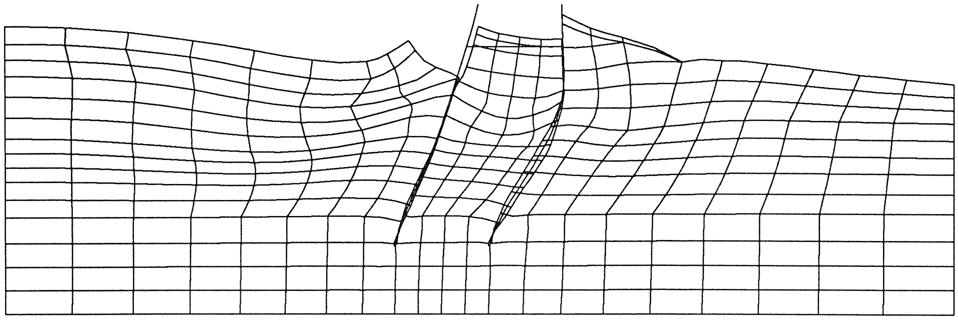

Fig. 5. Deformation due to phase 3a and 3b.

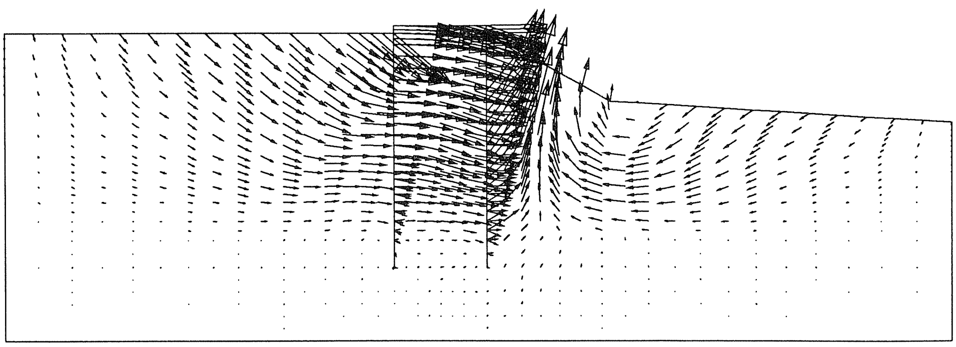

Fig. 6. Displacement increments in phase 4.

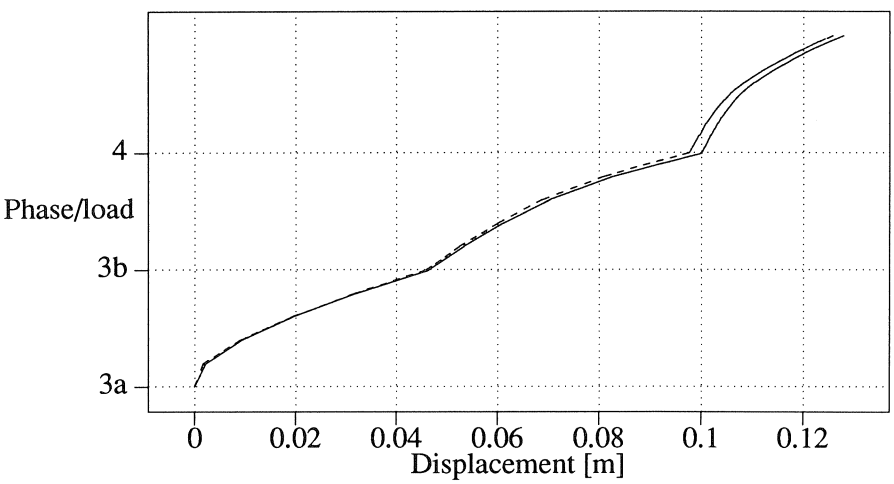


Fig. 7. Horizontal displacement of top sheet pile walls.

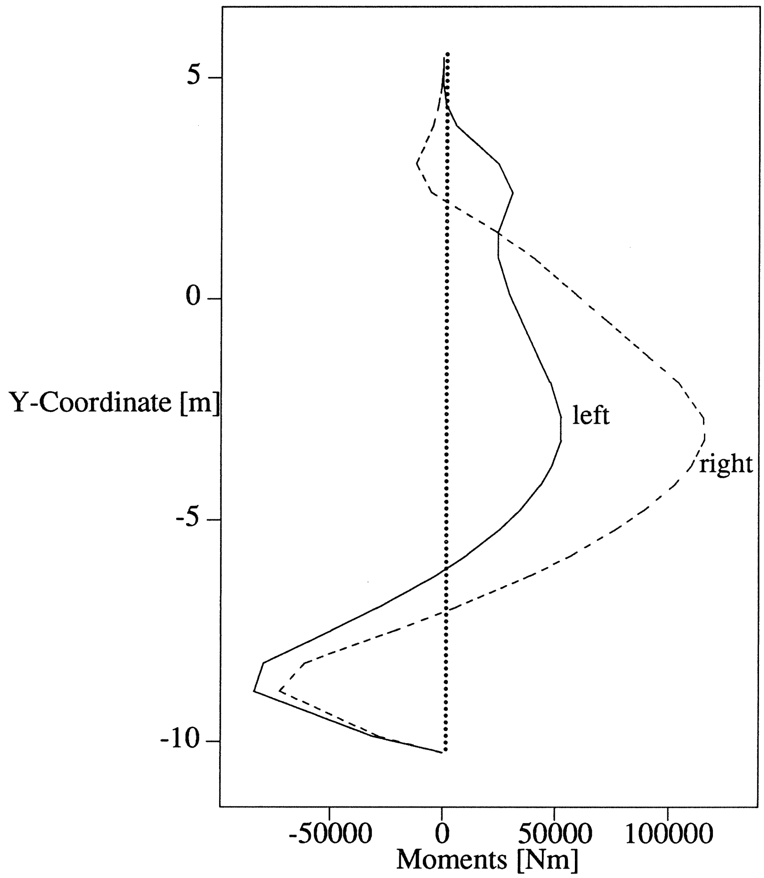


Fig. 8. Bending moments in sheet pile walls after phase 3b.

- Interface elements.
  The use of interface elements is essential for modeling the frictional behavior in the contact zone between soil and structure. Gapping in interface elements is, for obvious reasons, not easily combined with a traditional linear stiffness iteration method. Therefore, if gapping occurs and if a linear stiffness iteration method has to be used, modifications to the stiffness contribution of the interface elements are desirable.

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