

Economical optimization of the maintenance of a river bed protection construction

H.K.T. KUIJPER

Grabowsky & Poort bv, Consulting engineers

J.K. VRIJLING

Delft University of Technology

Abstract

This example of optimizing the maintenance by minimizing the sum of the present values of the risk and the costs of repair in a certain plan period is a part of a study done by the author by order of the Delft University of Technology. In this study also a classification of ageing models and a definition of the usefulness of inspection is given. By means of this definition the inspection period can be optimized.

Keywords: river bed protection, economical optimization, ageing model, repair internal, maintenance, failure rate.

1 Introduction

This paper presents a fundamental approach for optimizing the maintenance of a river bed protection downstream of a weir by minimizing the sum of the present values of the costs of maintenance and the risk of failure of the bed protection in a certain plan period.

The function of the bed protection is to protect the original bed against scouring. The bed protection consists of a geotextile which is covered by a layer of riprap (see Figure 1).

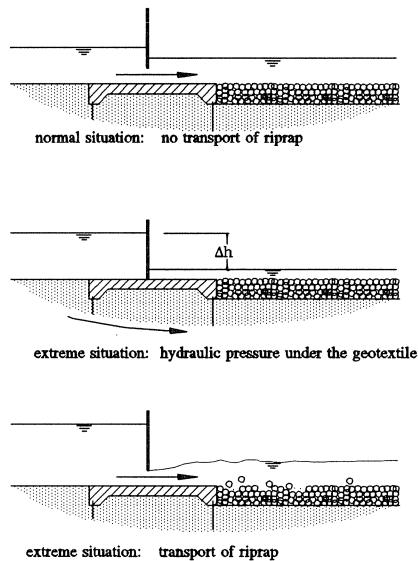


Fig. 1

The strength of the bed protection is defined by the mass of riprap per unit of area on the synthetic textile. Decrease of strength is caused by erosion of the riprap layer due to severe current. Therefore the flow velocity is a load which causes ageing of the bed protection.

In this example it is assumed that the flow velocity is low during eleven months of a year. However during one month of each year the discharge and with it the flow velocity will be extremely high and causes the transport of a certain amount of riprap. The flow velocity is assumed to be constant during this period but the extend is not exactly known on forehand (see Figure 2).

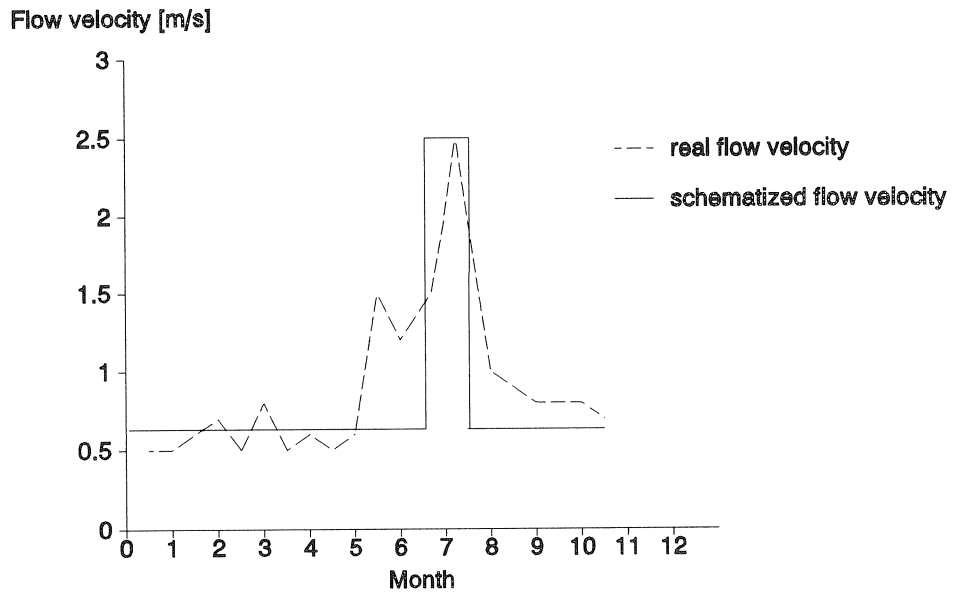


Fig. 2

The extreme hydraulic pressure under the geotextile will occur in periods in which the weir is closed. In these periods the difference between the up- and downstream water level causes a pressure gradient across the subsoil including the geotextile and riprap layer.

2 Decrease of the strength in a year

The maximum flow velocity in one year is not known on forehand and therefore it will be schematized by a stochastic variable. For this example we will assume that it can be described by a normal distribution with a mean value (μ) of 2 m/s and a standard deviation (σ) of 0.4 m/s.

According to this distribution a very large and even negative flow velocity is possible. However the probability of occurrence of a very large or negative value of the flow velocity is very small and the schematization is therefore acceptable for our purpose.

The resistance of the bed protection against erosion is defined by the flow conditions and by the weight and the shape of the individual stones. In this example a fictive relation between the properties of the riprap, the flow velocity and the decrease of strength will be adopted. It is given by:

$$\text{if } (U - U_{cr}) > 0 \text{ then } \Delta R(1) = A \cdot (U - U_{cr})^n + \varepsilon \text{ else } \Delta R(1) = 0 \quad (1)$$

In which:

- $\Delta R(1)$ = decrease of the strength in a year [kg/m² year]
- A, n = parameters depending on the geometry of the construction and the resistance of the riprap against transport
- U = flow velocity
- U_{cr} = critical flow velocity; erosion will occur only when the critical flow velocity is exceeded
- ε = schematization error

All the above mentioned parameters are stochastic variables. For the sake of simplicity we will assume that A, n and U_{cr} are exactly known and we will neglect ε .

Without further explanation the values of A, n and U_{cr} will be given:

$$A = 1260 \text{ kg s}^2/\text{m}^4\text{year}, \quad n = 2 \quad \text{and} \quad U_{cr} = 2 \text{ m/s}$$

The resulting equation for the decrease of the strength of the bed protection in a year as a function of the flow velocity is now given by:

$$\begin{aligned} U \geq U_{cr} &\Rightarrow \Delta R(1) = 1260 \cdot (U - U_{cr})^2 \\ U < U_{cr} &\Rightarrow \Delta R(1) = 0 \end{aligned} \quad (2)$$

The stochastic variables in this equation are U and $\Delta R(1)$ (as a function of U). In Figure 3 this equation as well as the probability density function of the flow velocity is shown in a graph.

The relation between the probability density function of the decrease of the strength in a year and the probability density function of the flow velocity is given by:

$$f_{\Delta R(1)}(\xi) \cdot d\xi = f_U(\zeta) \cdot d\zeta \Rightarrow f_{\Delta R(1)}(\xi) = f_U(\zeta) \cdot \frac{d\zeta}{d\xi} = f_U(\zeta) \cdot \frac{dU}{d(\Delta R(1))}(\xi) \quad (3)$$

This results in a probability density function for the decrease of strength in a year, viz.:

$$\xi > 0 \Rightarrow f_{\Delta R(1)}(\xi) = \frac{1}{\sqrt{5040 \cdot \xi}} \cdot \exp\left(-\frac{\xi}{403,2}\right) \quad (4)$$

and

$$f_{\Delta R(1)}(0) \cdot d\xi = \int_0^2 f_U(\zeta) d\zeta = 0,5$$

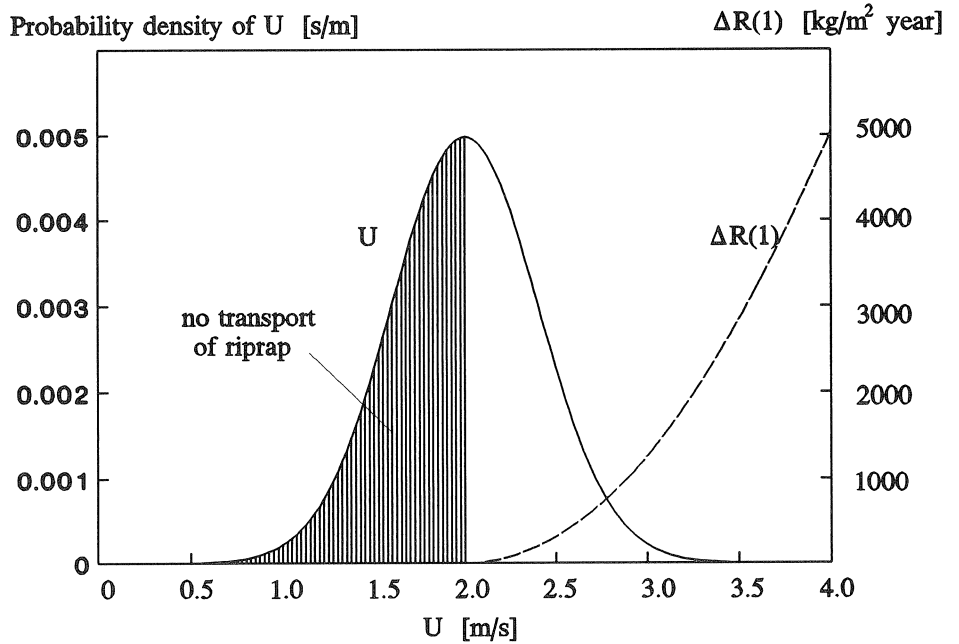


Fig. 3

This probability density function is shown in Figure 4. It shows a great similarity with a Gamma-Distribution. The mean value of the decrease of the strength in a year is $\mu_{\Delta R(1)} = 101 \text{ kg/m}$ and the standard deviation amounts $\sigma_{\Delta R(1)} = 226 \text{ kg/m}^2$. These values are obtained by numerical integration.

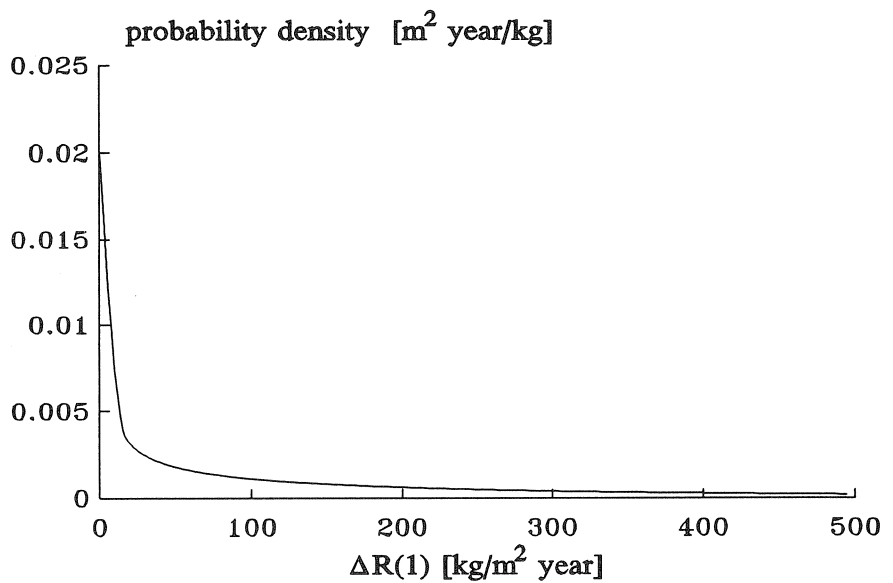


Fig. 4

3 Ageing model

The ageing model describes the decrease of strength as a function of a period without maintenance. It is an addition of the decrease of strength in the succeeding years of the period. In formula:

$$\Delta R(t) = \sum_{i=1}^t \Delta R(1)_i \quad (5)$$

in which: $\Delta R(t)$ = decrease of strength after t years
 $\Delta R(1)_i$ = decrease of strength in the i^{th} year

The here presented ageing model is a so called Random Walk Model. This means that the decrease of strength for the succeeding years are assumed to be stochastic independent. For this reason the mean value and the standard deviation of the decrease of strength after t years can be computed with:

$$\mu_{\Delta R(t)} = t \cdot \mu_{\Delta R(1)} \quad \text{and} \quad \sigma_{\Delta R(t)} = \sqrt{t} \cdot \sigma_{\Delta R(1)} \quad (6)$$

in which: $\mu_{\Delta R(1)}$ = mean value of the decrease of the strength in one year
 $\sigma_{\Delta R(1)}$ = standard deviation of the decrease of the strength in one year

The probability density function of the decrease of strength after t years is not easy to be obtained. It can be estimated by using the Monte Carlo simulation technique or numerical integration. For this example no further explanation will be given.

The strength after a period of t years is defined by:

$$R(t) = R(0) - \Delta R(t) \quad (7)$$

in which: $R(0)$ = strength directly after construction or repair
 $R(t)$ = strength after a period of t years

So $R(t)$ is a stochastic variable defined by the initial strength and the ageing model. In this example it is assumed that $R(0)$ is exactly known, viz.:

$$R(0) = 4000 \text{ kg/m}^2$$

In practice $R(0)$ will also be a stochastic variable. This effects the probability density function of the time-dependent strength.

4 The costs of repair

A period between two succeeding times of repair is called the repair interval. The amount of repair after a repair interval depends on the decrease of strength during this interval. So the expected amount of repair in the plan period depends on the length of the repair interval, the length of the plan period and the ageing model.

For this example we will distinguish initial costs which are the same for every time of repair and variable costs which depend on the amount of repair. The variable costs are given to be directly proportional with the amount of repair.

The present value of the future costs of repair is:

$$PV_{\text{repair}} = (C_i + c_v \cdot \mu_{\Delta R(1)} \cdot TR) \cdot \sum_{j=1}^n (1 + r')^{-j \cdot TR} \quad (8)$$

in which: C_i = initial costs
 c_v = variable costs
 TR = repair interval (fixed)
 n = number of repair intervals in the plan period
 r' = effective interest

5 Probability of failure in a year (failure rate)

There are two mechanisms that can cause failure of the bed protection.

The bed protection can fail in case there is not any ballast left on the geotextile. The textile will be carried along by the current which will result in a severe erosion of the bed. This mechanism will be called “*total erosion*”.

The bed protection can also fail in a period in which the weir is closed when the hydraulic pressure under the geotextile exceeds the weight of the remaining mass of riprap on the textile. The geotextile will be pushed up and will collapse. This mechanism will be called “*uplift*”.

The two mechanisms are fully related to each other by the strength (mass) of the bed protection. Therefore the probability of failure of the bed protection in a year is equal to the maximum probability of one of the mechanisms. In this case the mechanism “*uplift*” is always determinant.

The pressure gradient will be at a maximum when the upstream water level is maximal (level of the crest of the weir) and the downstream water level is minimal (bottom level). It will be at a minimum when the water levels at both sides of the weir are the same. This means that the extreme load is limited with a maximum and a minimum.

The extreme hydraulic pressure under the geotextile can be converted into an equivalent mass of riprap per unit of area that is necessary to keep the geotextile in its place by dividing the effective pressure by g (gravitational acceleration). This mass will be

schematized by a truncated Gumbel distribution. The probability of exceeding an equivalent mass by the effective pressure divided by g in a year is:

$$P\{S > \xi\} = 1 - F_S(\xi) = 1 - \exp\left(-\exp\left(-\frac{\xi - a}{b}\right)\right) \quad (9)$$

provided that: $\xi < M$

in which: $P\{S > \xi\}$ = probability that S exceeds ξ

S = load: mass per unit of area that is necessary to keep the geotextile in its place

a, b = parameters of the Gumbel distribution; without further explanation the values for a and b are given, viz.:

$a = 155 \text{ kg/m}^2$ and $b = 78 \text{ kg/m}^2$

M = maximum value of the equivalent mass; in this example $M = 600 \text{ kg/m}^2$

The reliability function or limit state function in a certain year τ can be written as:

$$Z(\tau) = R(\tau) - S(\tau) \quad (10)$$

in which: $R(\tau)$ = the strength in year τ

$S(\tau)$ = the maximum load in year τ

The failure rate gives the probability of failure in a certain year. It is described by:

$$f_L(t) dt = P\{Z(\tau) < 0 | \tau \in (t, t + dt) \wedge Z(\tau) > 0 | \tau \in (0, t)\} \quad (11)$$

In this example an upper limit for the failure rate will be used for further calculations, viz.:

$$f_L(t) dt \leq \int_{-\infty}^c (1 - F_S(\zeta)) \cdot f_{R(t)}(\zeta) d\zeta \quad (12)$$

in which: $f_{R(t)}$ = probability density function for the strength of the protection after t years without maintenance

Figure 5 shows the results of (12). The failure rate increases more or less exponential. In some cases a limitation of the probability of failure is prescribed. In these cases an optimization of the maintenance by minimizing the total costs is often not possible, e.g. because the consequences of failure cannot be capitalized. Then the length of a repair interval is almost fully related to the failure rate.

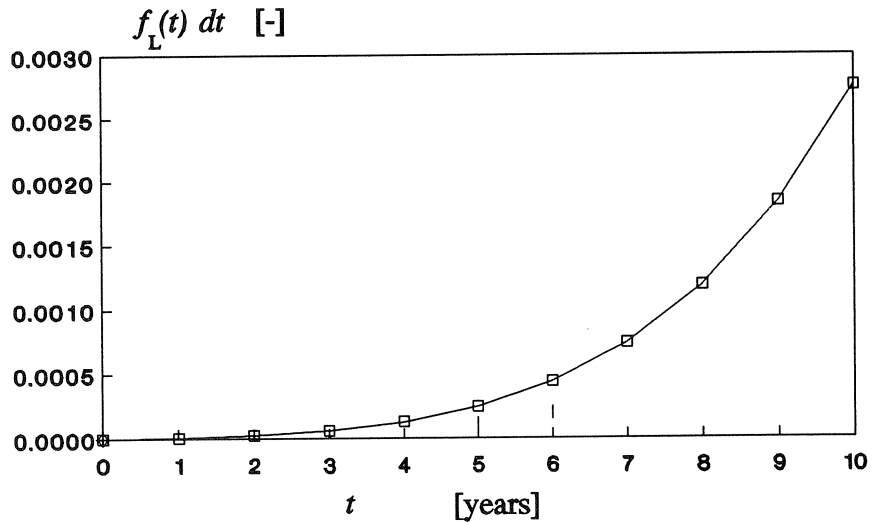


Fig. 5

6 Present value of the risk

Risk is defined as the probability of failure multiplied with the consequence of failure. For the economical optimization of the maintenance the consequence of failure has to be expressed by an amount of money. To compare the costs of the risk with the costs of repair it is necessary to compute the present value of the risk. This is done with the formula:

$$PV_{\text{Risk}} = \int_0^{t_p} \frac{D f_L(t)}{(1+r')^t} dt \quad (13)$$

in which: t_p = end of the plan period
 D = consequence of failure
 r' = effective interest

7 Optimizing the repair interval by minimizing the total costs

The present value of the total costs is defined by the sum of the present value of the repair costs and the present value of the risk. The present value of the total costs can be described as a function of the repair interval.

In Table 1 and Figure 6 the results are shown of a computation of the costs for a plan period of 500 years. For this computation the next values are adopted:

$$\begin{aligned}
D &= 40,000,000.- && [\text{Dfl}] \\
C_i &= 10,000.- && [\text{Dfl}] \\
c_v &= 207.- && [\text{Dfl m}^2/\text{kg}] \\
r^i &= 0.04 && [-]
\end{aligned}$$

The graph of Figure 6 shows that there is a minimum of the present value of the total costs for a repair period of about 4 years.

The probability of failure in a repair period of 4 years is:

$$Pr \{L < 4\} \leq \int_0^4 f_L(t) dt \approx 1,6 \cdot 10^{-4} \quad (14)$$

This probability of failure is considered to be acceptable.

Table 1. Costs as function of the size of the repair interval

T_{repair}	PV_{risk}	PV_{repair}	$PV_{\text{total costs}}$
1	3523	772500	776023
2	9619	634804	644422
3	20417	582233	602650
4	38301	551047	589348
5	66435	528495	594930
6	108898	510329	619227
7	170802	494727	665529
8	258425	480778	739203
9	379218	467977	847195
10	541931	456018	997949

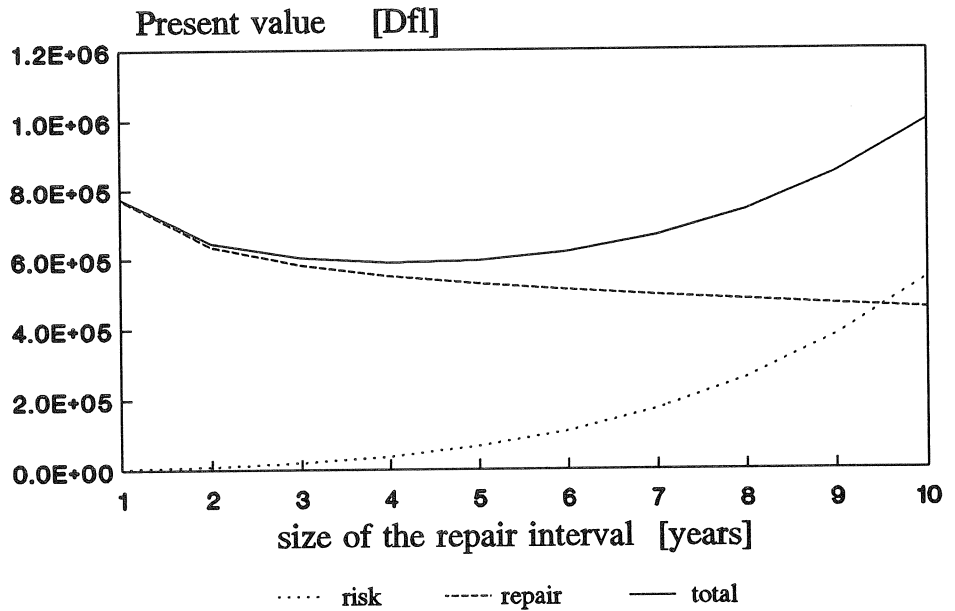


Fig. 6

Notation

- A, n = parameters depending on the geometry of the construction and the resistance of the riprap against transport
 a, b = parameters of the Gumbel distribution
 C_i = initial costs
 c_v = variable costs
 D = consequence of failure
 $f_{R(t)}$ = probability density function for the strength of the protection after t years without maintenance
 M = maximum value of the equivalent mass
 n = number of repair intervals in the plan period
 $P\{S > \xi\}$ = probability that S exceeds ξ
 $R(0)$ = strength directly after construction or repair
 $R(t)$ = strength after a period of t years
 $R(\tau)$ = the strength in year τ
 r' = effective interest
 $\Delta R(1)$ = decrease of the strength in a year [kg/m² year]
 $\Delta R(t)$ = decrease of strength after t years
 $\Delta R(1)_i$ = decrease of strength in the i^{th} year

- S = load: mass per unit of area that is necessary to keep the geotextile in its place
 $S(\tau)$ = the maximum load in year τ
 t_p = end of the plan period
 TR = repair interval (fixed)
 U = flow velocity
 U_{cr} = critical flow velocity; erosion will occur only when the critical flow velocity is exceeded.
 ε = schematization error
 $\mu_{\Delta R(1)}$ = mean value of the decrease of the strength in one year.
 $\sigma_{\Delta R(1)}$ = standard deviation of the decrease of the strength in one year.

References

- BENJAMIN, JACK R. and ALLIN CORNELL, C. 1970, Probability, Statistics and Decision for Civil Engineers, McGraw-Hill Book Company, New York.
 GIJSBERS, F.B.J. 1987, Optimalisatie van onderhoud, TNO-IBBC rapport B-86-549.
 KOK, M. januari 1988, Stormvloedkering Oosterschelde; optimaliseren van inspectie en onderhoud natte werken, Waterloopkundig Laboratorium rapport Q 606.
 KOK, M. 1990, Onderhoud; methoden voor rationeel onderhoud van civiele constructies, Waterloopkundig Laboratorium rapport Q 606.
 KUIJPER, H.K.T. 1992, Maintenance of hydraulic structures (Dutch), Delft University of Technology.
 MEERMANS, W. 1990, Handleiding computerprogramma AFDA.PAS, Technische Universiteit Delft.
 OORSCHOT, H.J.L VAN 1989, Operationele Analyse en Risico Analyse, Technische Universiteit Delft.
 VROUWENVELDER, A.C.W.M. en LENOS, S. 1987, Optimalisatie van onderhoud - deel II, TNO- IBBC rapport B-87-346.
 VROUWENVELDER, A.C.W.M. en LENOS, S. 1987, Optimalisatie van onderhoud, Berekening van kostenverwachting, TNO-IBBC rapport B-87-717.
 VROUWENVELDER, A.C.W.M. en VRIJLING, J.K. Probabilistisch ontwerpen, Technische Universiteit Delft, april 1986