Finite elements simulation of reflective cracking in asphaltic overlays

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Overlaying is one of the most popular and cost effective techniques of rehabilitation of cracked pavements. The placing of reinforcement between the overlay and the top layer of the cracked pavement is currently being utilised as a possible technique for delaying the development of cracks into the overlay. In order to assist the road designer with his attempt to determine the factors leading to the development of cracks into the overlay and in order to enable him to quantify the contribution of reinforcement in carrying tensile forces across cracks in overlayed pavements, CAPA, a user-friendly, PC based, finite elements system has been developed. A variety of options enable the simulation of discrete cracks in the pavement and their interaction with the surrounding materials. Starting from an initial crack length in the pavement, the system can automatically propagate the crack all the way to the surface computing at the same time the relevant fracture mechanics parameters at successive crack tip positions. A user friendly graphical screen input facility, together with a fully fledged mesh generator and extensive pre- and post-processing graphic facilities allow the designer to quickly evaluate the efficiency of various overlay techniques. By means of an example of an actual Dutch pavement profile it is shown that adequately bonded reinforcement can reduce the speed of crack propagation into the overlay and hence prolong the economic life of the structure.

Keywords: **pavement rehabilitation, reflective cracking, pavement reinforcement, discrete cracking, Barsoum elements, interface elements**

1 Introduction

Cracking in the various layers of pavements is one of the most commonly encountered causes of deterioration. Depending on the situation it can be due to traffic loads, subsidence, thermal gradients and combinations thereof. The rehabilitation of cracked pavements by overlaying is rarely a durable solution since after a while, the cracks propagate rapidly through the new layer. This phenomenon, commonly known as **reflective cracking**, occurs under various conditions and imposes heavy financial strains on national and local pavement maintenance authorities.

Cracks introduce physical discontinuities in the otherwise homogeneous – for engineering analysis purposes – body of the pavement, Fig. 1.1. Because their stiffness is usually low, if not zero, their presence plays a major role in determining the integrity and hence the load carrying capacity of the pavement. The discontinuous nature of cracks makes inappropriate the application of classical continuum based layered analysis methods. On the other hand, within the context of the FEM,

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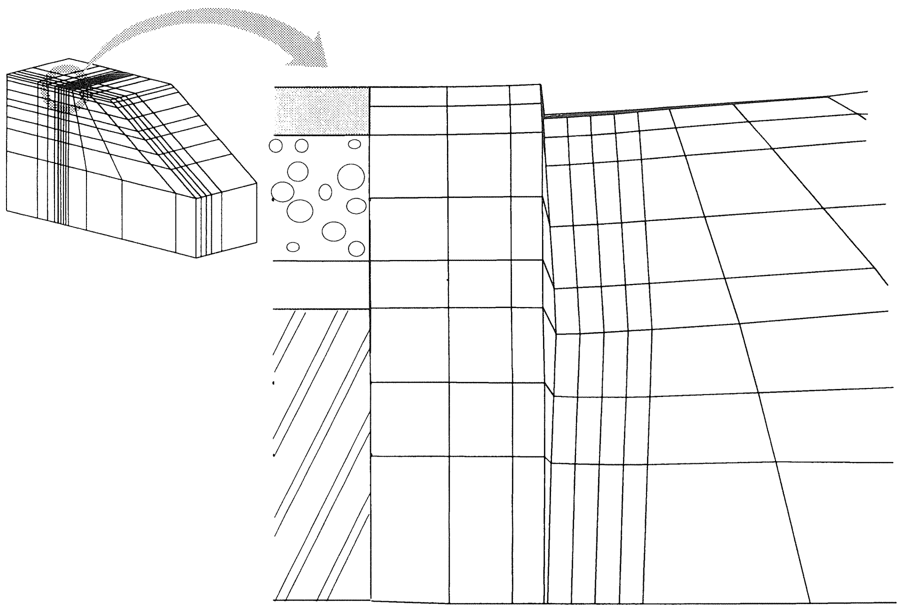


Fig. 1.1. Cracking introduces a plane of physical discontinuity in the pavement.

cracks in the body of the pavement can be easily simulated by disconnecting the nodes of elements on either side of the crack propagation path, Fig. 1.2.

The placing of reinforcement between the cracked pavement and the overlay is one of the currently available techniques for delaying the development of reflective cracking. Nevertheless, in the recent past, concerns have been raised as to the effectiveness of reinforcement in bituminous overlays. By enabling the detailed study of the local phenomena and the mechanisms that affect the interaction of reinforcement with the surrounding materials, the finite elements method (FEM) can assist the designer with his choice of materials and construction techniques.

In an effort to gain some insight on the mechanics of unreinforced / reinforced overlayed pavements, CAPA, a user-friendly, PC based, FEM system, has been developed at Delft University of Technology (TU-Delft) in the course of an STW sponsored cooperation project between the section of Structural Mechanics and the section of Road and Railway Engineering. By means of CAPA extensive studies have been undertaken of the response of overlayed pavements subjected to various load conditions, Bruijn et al. (1994), Pajunen (1995). In this contribution several features of the FEM simulation of pavement response by means of CAPA will be reviewed. An example of the utilization of the system in the design process of an actual pavement will be presented.

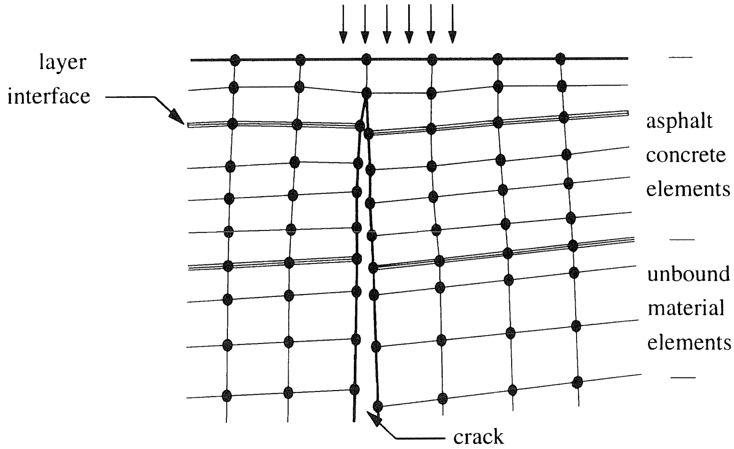


Fig. 1.2. Finite elements crack simulation.

2 Fracture mechanics aspects of FEM crack simulation

Within the context of the FEM, the singularity of strains in the vicinity of a crack tip can be modelled accurately and elegantly if the elements surrounding the tip node are substituted by specially developed elements. Several special elements have been developed over the years. Among the best performers are the elements developed by Barsoum (1976). He observed that by placing the mid-side nodes of two adjacent sides of a quadrilateral 8-noded isoparametric element on the quarter distance from the common corner node, Fig. 2.1(a), the desired singularity of strains is obtained within the element in the region surrounding the corner node.
He also showed that by collapsing a side of a quadrilateral 8-noded isoparametric element and by arranging the nodes as shown in Fig. 2.1(b), a triangular 6-noded singular element is obtained. This element can simulate both, an $1 / \sqrt{r}$ and an $1 / r$ singularity.

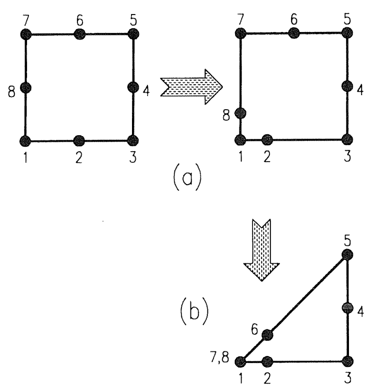


Fig. 2.1. Quadrilateral and triangular Barsoum finite elements.

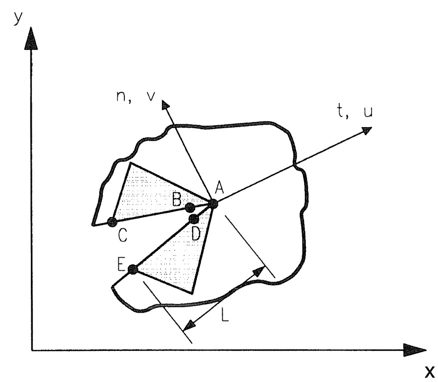


Fig. 2.2. Crack tip nodal lettering for Eq. 2.1.

As it is known from the theory of fracture mechanics, a crack in a solid can be stressed in three different modes and their combinations. Modes I and II refer to in-plane purely flexural and purely shear-sliding displacements respectively while mode III refers to out-of-plane displacements.
The energy available at the crack tip for crack propagation in each mode can be measured in terms of the corresponding stress intensity factors (SIF) K_I, K_{II} and K_{III}.
By correlating the displacements of the crack tip nodes computed via the FEM with the theoretical expressions for the displacements, Ingraffea et al. (1978) have derived closed form formulae for computation of the SIF on the basis of the displacements of the nodes along the faces of the crack. For the case of a crack tip simulated by means of collapsed Barsoum singular elements, Fig. 2.2, in the crack local coordinate system (t, n):

$$K_{II} = \sqrt{\frac{2\pi}{L}}\frac{G}{\kappa + 1}\left[4(v_B - v_D) + v_E - v_C\right] \tag{2.1a}$$

$$K_{II} = \sqrt{\frac{2\pi}{L}}\frac{G}{\kappa + 1}\left[4(u_B - u_D) + u_E - u_C\right] \tag{2.1b}$$

with: $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress

$\kappa = 3 - 4\nu$ for plane strain

G is the shear modulus and ν is Poisson's ratio.

3 Simulation of crack and layer interface regions

Depending on the overall pavement geometric and material characteristics but also on the nature of the applied load, cracks which may have initially developed under prevailing Mode I conditions may later encounter a compression field along their propagation path. The presence of a compression field in the vicinity of a crack tip is indicated by non-positive K_I values. As a result, Mode I crack opening displacements are restrained and subsequent crack propagation can only take place under Mode II shear sliding conditions.

Within the context of the finite elements method, non-positive K_I values signify "numerical" overlap of the crack faces, Fig. 3.1. This may occur not only if an attempt is made to extend a crack into a compression field but also if, as a result of a change in the position and the nature of the applied loads, a compression field develops perpendicular to the faces of an existing crack, Fig. 3.2(a).

Numerical overlap of the crack faces can be prevented by the introduction into the appropriate crack locations, in the finite elements mesh, of specially developed "interface elements". Their role is to simulate the physical conditions at the contact region between opposite crack faces.

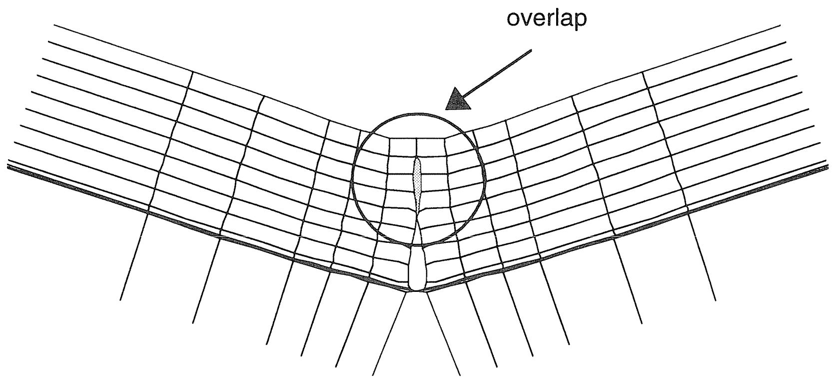


Fig. 3.1. Crack face overlap due to surrounding compression field.

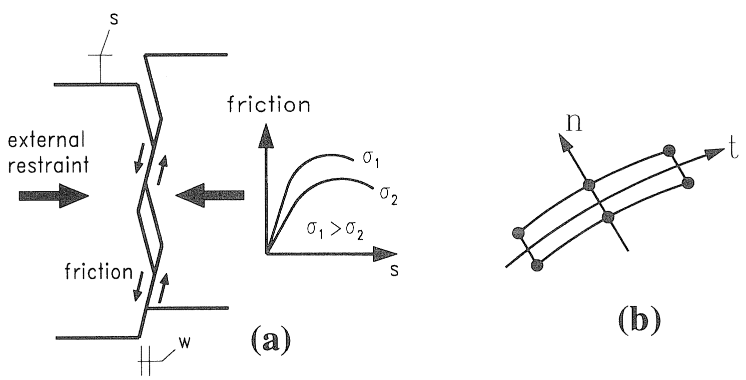


Fig. 3.2. Contact region between opposite crack faces.

A typical interface element is shown in Fig. 3.2(b). It consists of two pairs of nodes, one pair on each side of the element axis. The thickness of the element in its undeformed configuration can be very small or even zero. One normal n and one transverse t axes of material anisotropy can be associated with the principal local axes of the element. The constitutive relation associating local stresses to relative local displacements is:

$$\begin{Bmatrix} \tau \\ \sigma \end{Bmatrix} = \begin{bmatrix} D_{tt} & D_{tn} \\ D_{nt} & D_{nn} \end{bmatrix} \cdot \begin{Bmatrix} s \\ w \end{Bmatrix} \tag{3.1}$$

The stiffness coefficients depend on both the state of stress and the deformation, Scarpas (1994). Determination of their values has been the topic of an extensive program of experimental investigation carried out at Delft University of Technology, de Bondt et al. (1993), (1994a) and de Bondt (1996).

Precracked rectangular asphalt concrete specimens were subjected to 4-point shear tests, Fig. 3.3. The experimental set-up is shown in Fig. 3.4. A horizontally acting jack enabled the application of normal stresses to the crack interface plane.

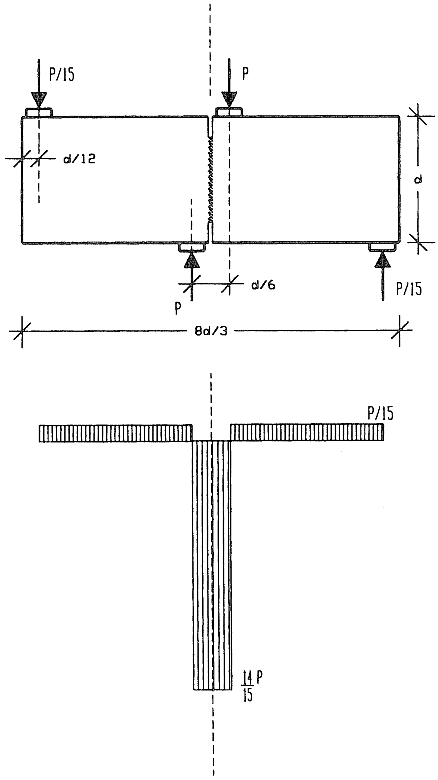


Fig. 3.3. 4-point shear test.

Some typical experimental results are shown in Fig. 3.5. As the magnitude of the applied normal stress increases both, the shear stiffness and the shear carrying capacity of the crack interface increase as well.

Interface elements are also needed for simulation of the contact regions between the successive material layers in a pavement. These are seldomly, if ever, rigidly bonded to each other. Depending on the nature of the materials and the construction techniques involved, some magnitude of interlayer slip can occur. The degree of interlayer bonding can influence significantly the overall structural response.

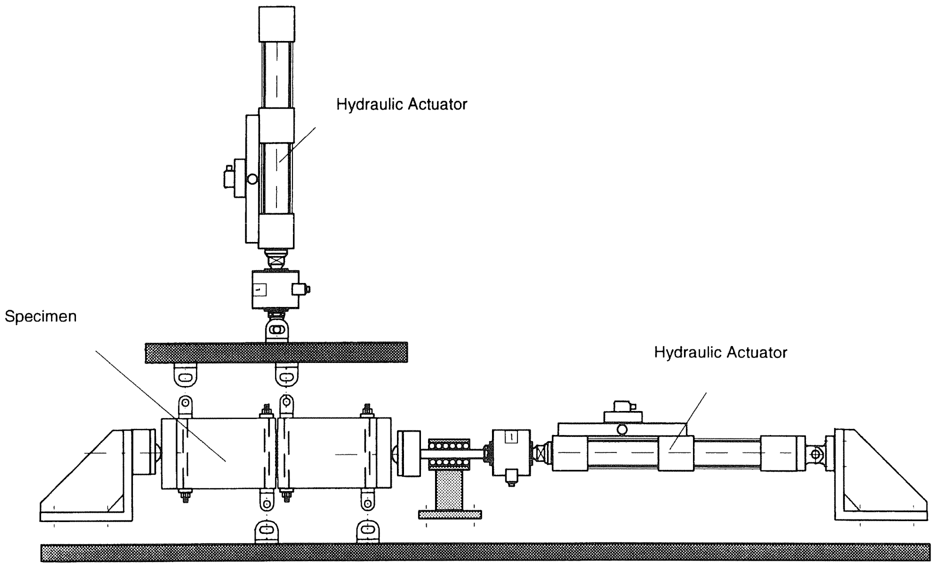


Fig. 3.4. TU-Delft experimental test set-up.

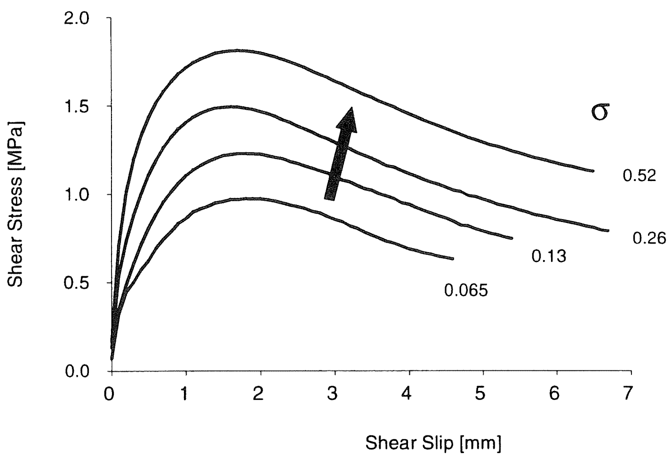


Fig. 3.5. Experimental shear stress – shear slip response of asphalt concrete cracked interfaces.

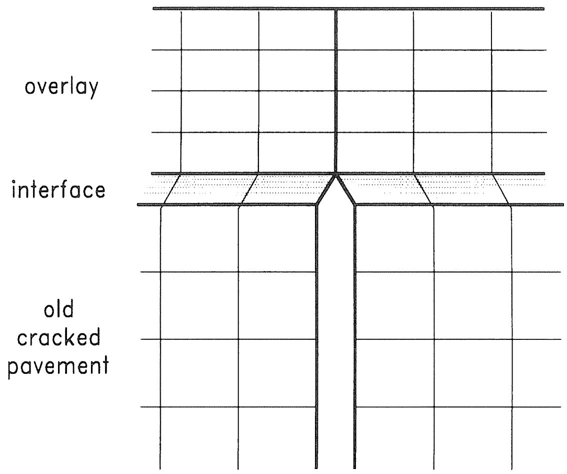


Fig. 3.6. Schematic of FEM simulation of a cracked overlayed pavement.

A schematic representation of a typical finite elements simulation of a cracked overlayed pavement is shown in Fig. 3.6. Quadrilateral finite elements can be used for modelling the geometry of the pavement layers and interface elements for simulation of the bond region between the layers and, if necessary, of any crack regions in compression.

4 FEM aspects of crack propagation simulation in pavements

Since in most rehabilitation cases cracking has already reached the top of the old pavement, the original crack tip is normally placed at the bottom of the overlay. Depending on the situation, the existing crack may span several layers into the underlying materials.
Within the context of the FEM, crack extension requires a series of successive analyses, in each of which the mesh is progressively modified manually by disconnecting the nodes of elements on either side of the crack propagation path. This is a laborious and error prone task.
In the CAPA-2D finite elements system, an incorporated remeshing technique completely automates the above task. Starting from the initial cracked pavement configuration as input by the user, the following steps are performed by the system:

1. the ordinary finite elements surrounding the crack tip are substituted by singular elements (see Fig. 2.1).
2. an analysis is performed for the specified load combination and material properties. The SIF at the crack tip is computed on the basis of the crack face nodal displacements by means of Eq. 1.1.
3. the singular elements surrounding the current crack tip are automatically replaced by ordinary elements and the new crack tip node is identified.
4. the elements on either side of the incremental crack extension path *MN*, Fig. 4.1, are disconnected and their nodal loads redistributed to the pavement.

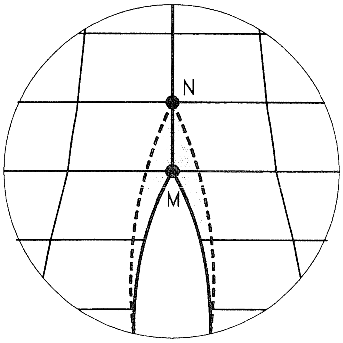


Fig. 4.1. Incremental crack propagation in CAPA*-2D.*

As illustrated in Fig. 4.2, the above procedure is repeated until the crack reaches the top of the overlay. Once a set of pairs of SIF values vs. crack length has been obtained, the system estimates the number of load repetitions required, after repair, for reflective cracking to appear at the top of the overlay by numerical integration of Paris' law, Molenaar (1983). No user intervention is necessary at any stage of the calculation.

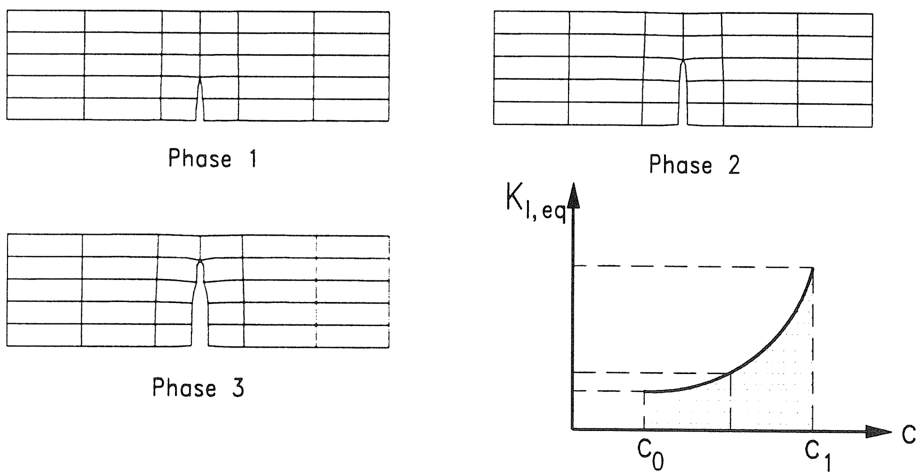


Fig. 4.2. Discrete crack propagation via CAPA*-2D.*

5 Aspects of FEM simulation of reinforced overlayed pavements

Adequately anchored reinforcement can delay the speed of crack propagation within the overlay by allowing the development of a tensile force across the crack faces. This plying action of the reinforcement not only restrains crack dilatancy (and hence reduces stress concentration at the crack tip) but also enables the development of the aggregate interlock mechanism, Fig. 5.1, Scarpas (1994).

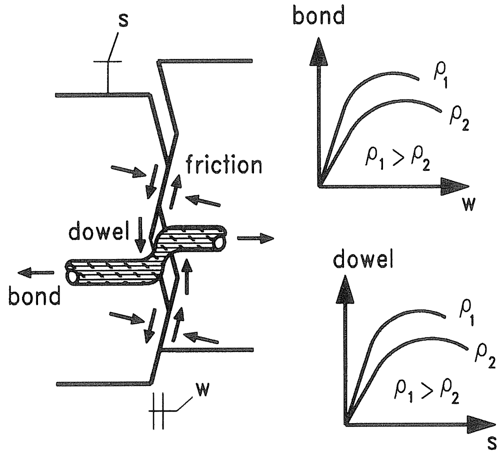


Fig. 5.1. Force transfer at a reinforced crack (rho: reinforcement percentage).

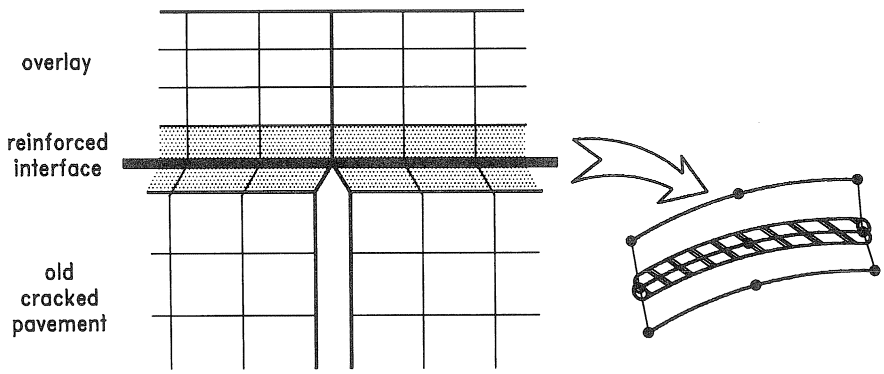


Fig. 5.2. FEM simulation of a reinforced overlay.

A schematic representation of a typical finite elements simulation of a cracked reinforced overlayed pavement is shown in Fig. 5.2. Ordinary quadrilateral finite elements can be used for modelling the geometry of the pavement layers and bar type elements for modelling the reinforcement (see inset in Fig. 5.2). For the simulation of bond between the reinforcement and the surrounding pavement materials interface elements can be utilized. By varying the transverse

stiffness D_{tt} of the interfaces various types of bond can be specified.
Because of construction difficulties during paving, the bond between the reinforcement and the top of the old cracked pavement may have completely different characteristics from that between the reinforcement and the overlay. By means of the arrangement of Fig. 5.2, different bond characteristics can be assigned to the interfaces on either side of the reinforcement.
The pullout characteristics of reinforcement embedded between asphalt concrete layers were studied by means of the 4-point shear test set-up of Fig. 3.4. Since the contribution of the friction mechanism had been already determined from the unreinforced tests of Section 3, the contribution of the bond mechanism could be determined once the externally applied forces were known.
A typical example of such an inverse determination of the contribution of reinforcement in the experimental response of the specimen is shown in Fig. 5.3.

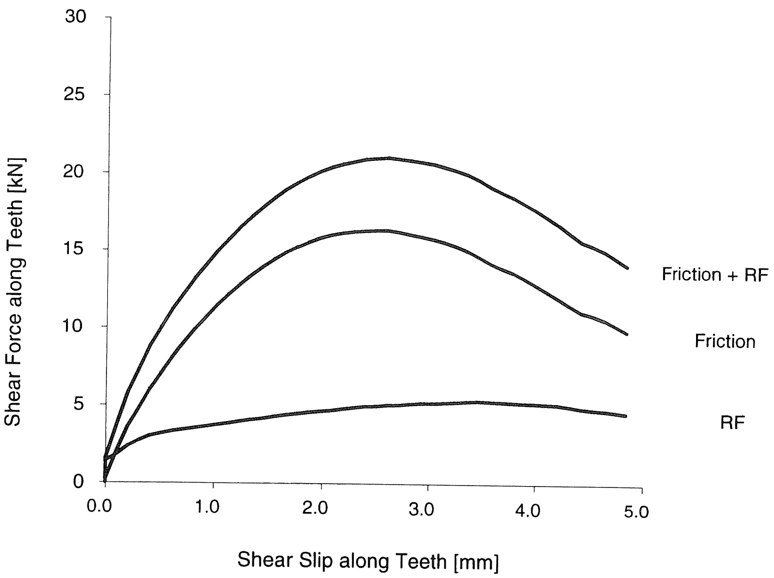


Fig. 5.3. Contribution of force transfer mechanisms at a reinforced crack interface.

In case of grids pullout resistance is mainly generated by bearing resistance in the apertures, Fig. 5.4, whereas in case of wovens resistance to pullout is developed via adhesion along the longitudinal strands, Fig. 5.5. Only for grids, straining in one direction engages the material in the cross direction as well.

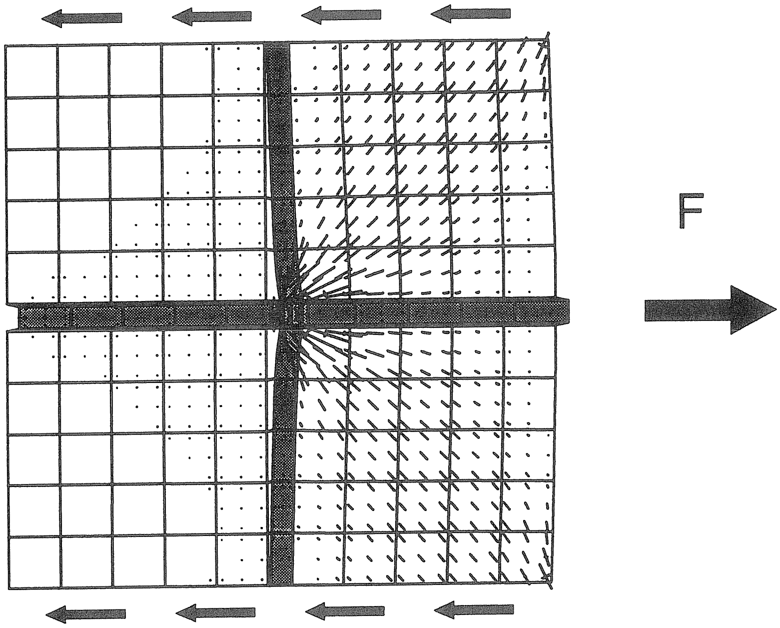


Fig 5.4. Pullout simulation of a grid.

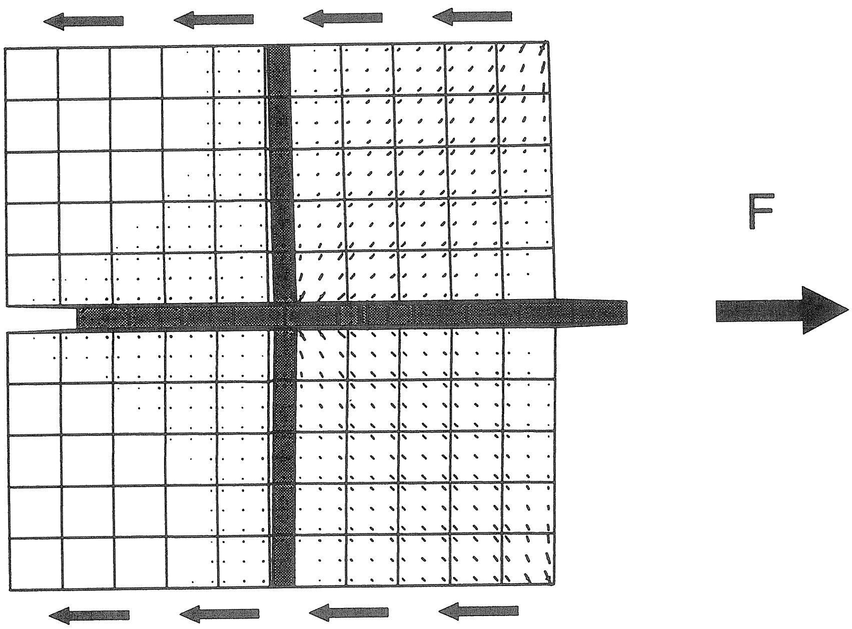


Fig 5.5. Pullout simulation of a woven.

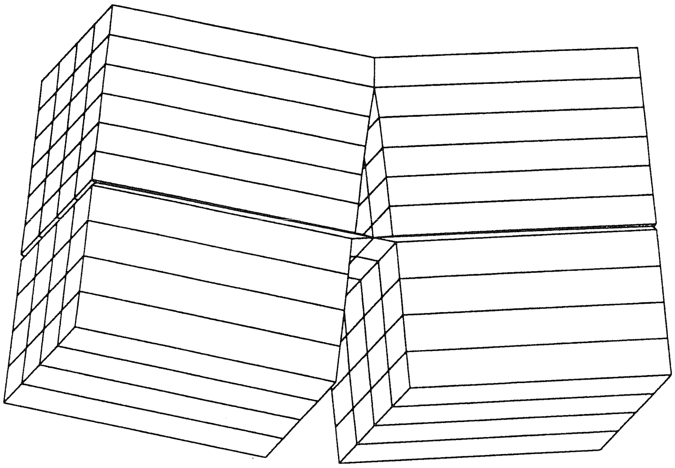


Fig. 5.6. CAPA simulation of reinforcement pullout.

In the finite elements analysis of actual pavements, simulation of every reinforcing strand consisting the reinforcing medium would be impractical. Instead average bond stiffness values are determined by means of laboratory pull-out tests. These "smeared" values can then be utilised in Eq. 3.1 (c.f. also Fig. 5.2) for simulation of bond between the reinforcement and the surrounding materials, Fig. 5.6.

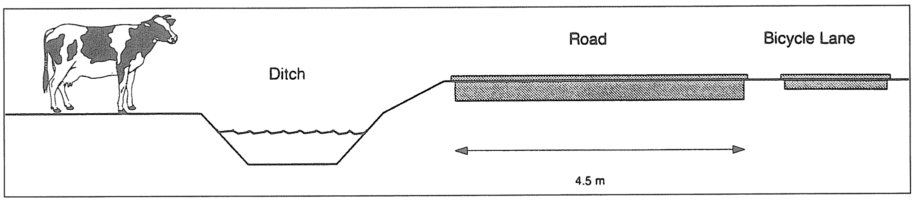
6 Finite elements study of reflective cracking in an agricultural road

Agricultural roads in areas with soft soils often show severe cracking, Fig. 6.1. The wide cracks enable the penetration of water into the structure which causes rapid deterioration of the pavement. After a while the serviceability has dropped to such a level that traffic has to reduce its speed to guarantee a safe passage. It is clear that at that moment maintenance should be carried out. Fig. 6.1 gives an impression of the type of cracking which is often observed at (narrow) agricultural roads.

Characteristic for these type of roads is that:
a) verges only generate a negligible amount of support,
b) a proper base is missing and
c) geotechnical actions occur (subsidence).

These factors cause that soon after overlaying the new surface shows the same crack pattern as the old one before overlaying; reflective cracking is the predominant mechanism. In this section the behaviour of the Slagterslaan in Berkhout will be presented, de Bondt et al. (1992). This narrow agricultural road, is close to a ditch; the subgrade consists of peat. In fact the pavement is 'floating' in the mud. During an average working day 35 trucks pass in each direction.

Fig. 6.1. Typical damage in an agricultural road.



In the past the Slagterslaan has turned from a cart-track via several maintenance measures (surface dressings and overlays) into a modern road. In 1991 an overlay was constructed, because the pavement surface showed severe uneveness and cracking. It is clear that only by means of the finite element method the behaviour of this overlay can be examined.
Figure 6.2 shows the mesh which was utilized and the specified boundary conditions. The pavement structure consists of an overlay (thickness 0.05 m) placed on top of an 0.08 m thick old bituminous concrete layer and a 0.24 m thick unbound granular material layer. In this way it is taken into account that some of the very old overlays from the past have degraded into granular materials. The width of the new overlay is slightly larger than the existing pavement (4.7 m versus 4.5 m) because in practice agricultural roads are, although not intended, often widened during the construction of a new wearing course.
The effect of a wide longitudinal crack at a distance of 1.1 m from the edge of the old surface at the side of the ditch was analyzed. At its bottom this crack was assumed to extend into the peat in order to prevent the transfer of forces which do not occur in reality. Due to the soft nature of peat, a plane stress analysis was thought to be the best simulation. Tables 6.1 and 6.2 list the material properties which were used.

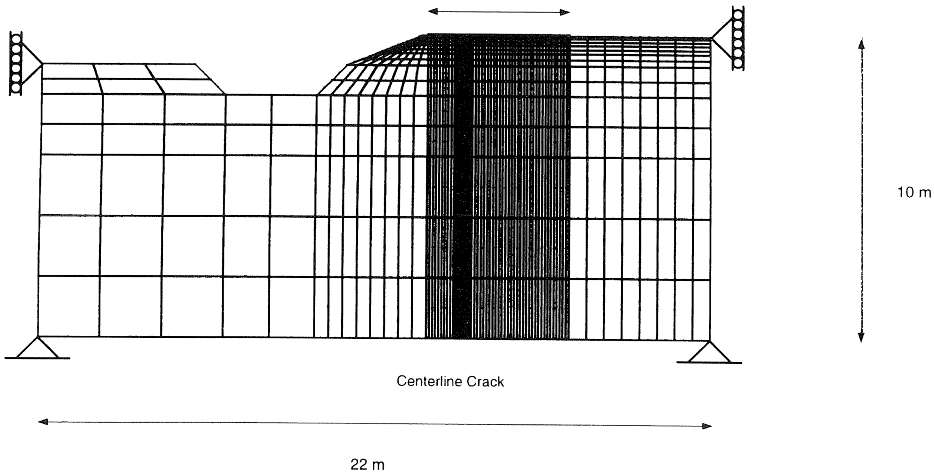


Fig. 6.2. Finite elements simulation of Slagterslaan.

Table 6.1. Layer material properties.

Layer	E modulus [MPa]	Poisson's ratio	Density [kg/m³]
Overlay	5500	0.35	2300
Old bituminous concrete	3500	0.35	2300
Granular material	200	0.35	2000
Soil	20/40(1)	0.35	1100

(1) increasing with depth

Table 6.2. Reinforcing system properties.

Case	D_{tt} (bottom interface) $[(N/mm^2)/mm^2]$	EA $[N]$	D_{tt} (top interface) $[(N/mm^2)/mm^2]$
no reinforcement	0.5	0.25	0.5
reinforcement type A	0.5	250	913
reinforcement type B	0.5	2500	71.6
reinforcement type C	0.5	25000	5.61

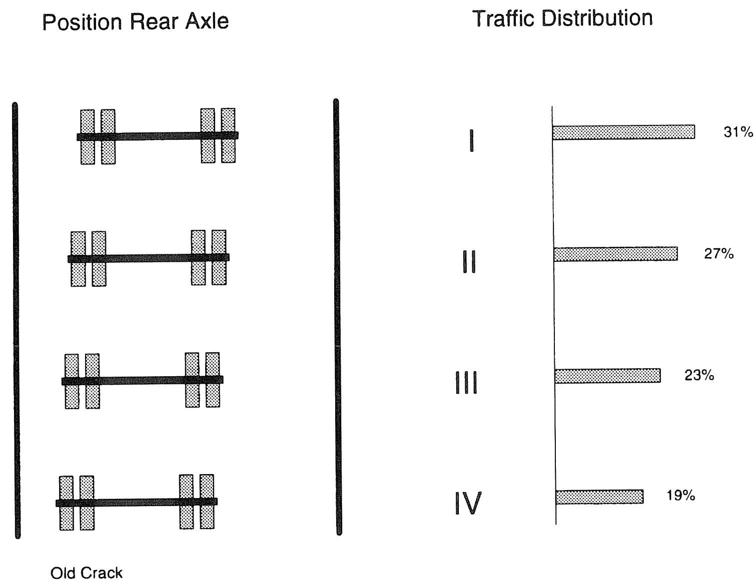


Fig. 6.3. Top view of rear axle lateral positions w.r.t. the crack axis.

The calibration of the mesh, that is the determination of the thickness of the soil layer, was carried out by matching measured and computed surface deflections under the loading centre in an uncracked area of the pavement, de Bondt et al. (1994b).
Typical for narrow agricultural roads is that in general traffic travels near the pavement centre. Only in case of passing cars and trucks, edge loads occur. In this analysis four lateral positions of the 100 kN rear axle were simulated, Fig. 6.3. The tire pressure was set to 0.7 MPa.
An unreinforced case and three reinforced cases were analyzed. Figure 6.4 shows the overall deformation of the mesh for a specific case (load position I).

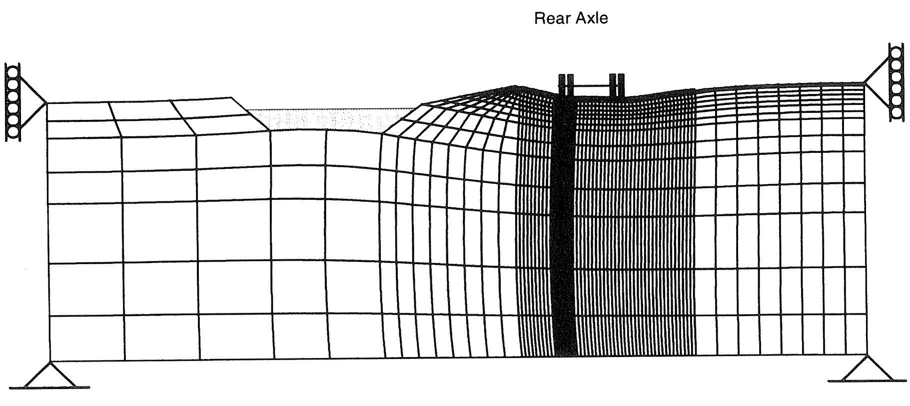


Fig. 6.4. Overall mesh deformation (Load position I).

Figures 6.5 and 6.6 present the computed stress intensity factors (SIF) during the propagation of the crack through the overlay for load position I. The length of the initial crack in the overlay was set to 10 mm.

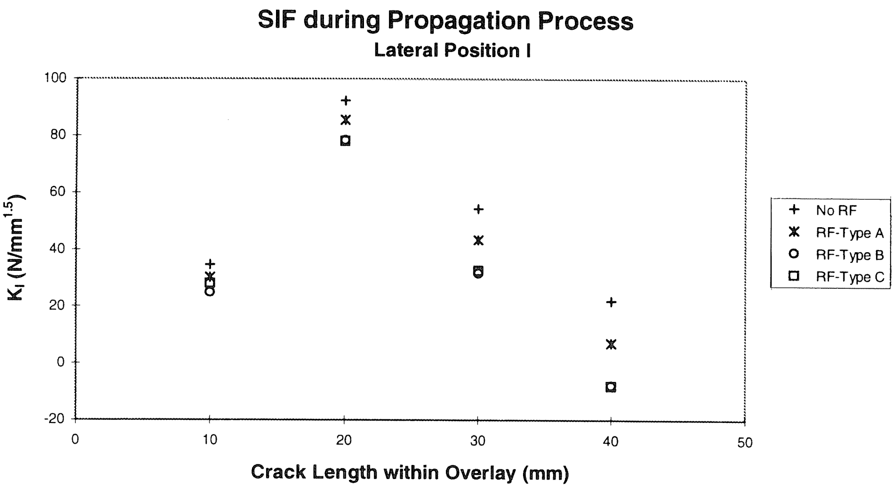


Fig. 6.5. Mode I SIF for load position I.

It can be seen that in general the mode I stress intensity factors are much higher than the mode II stress intensity factors; bending is thus the predominant mechanism. Furthermore it is clear that the reinforcement reduces the mode I stress intensity factor; even negative values are generated indicating that the specified amount of reinforcement was capable of restraining Mode I induced crack propagation.

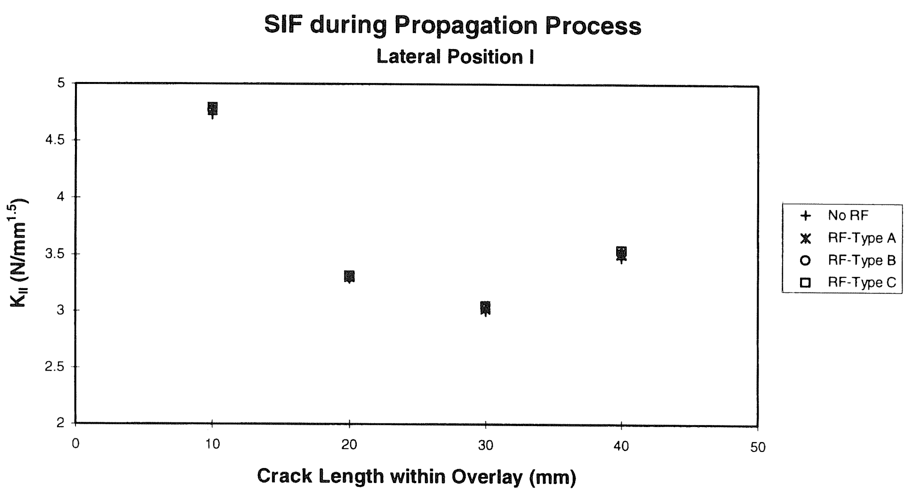


Fig. 6.6. Mode II SIF for load position I.

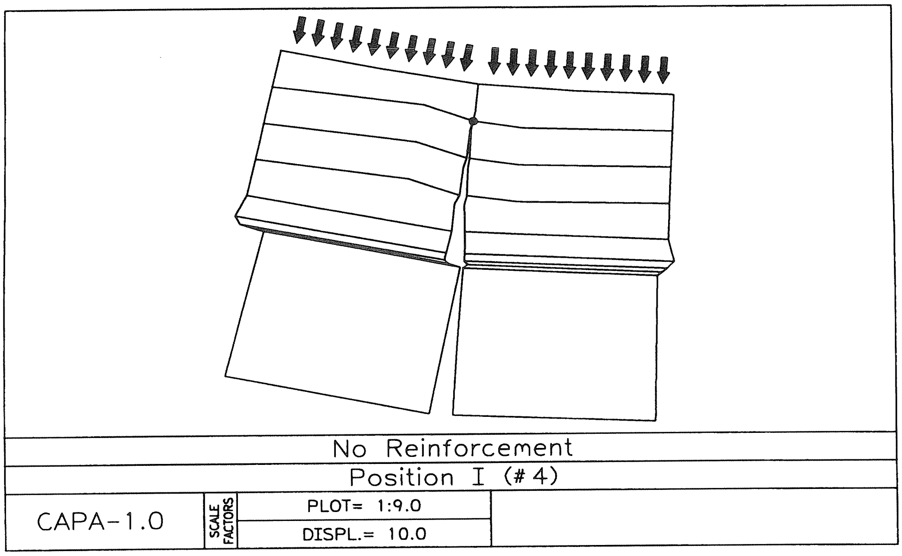


Fig. 6.7. Crack tip deformations for unreinforced mesh.

Fig. 6.7 and Fig. 6.8 show the deformations at the tip of the crack when the crack length within the overlay is 40 mm.
It can be seen that the presence of reinforcement implies less opening of the 'new' crack in the overlay.

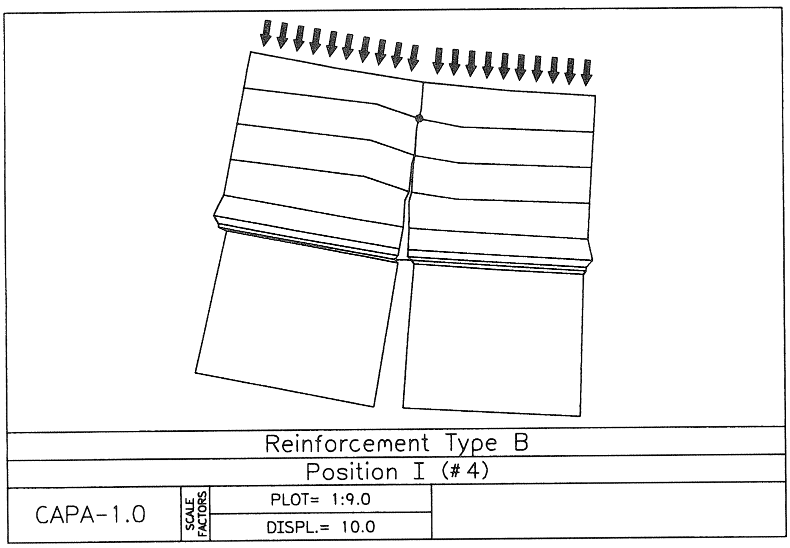


Fig. 6.8. Crack tip deformations for reinforced mesh.

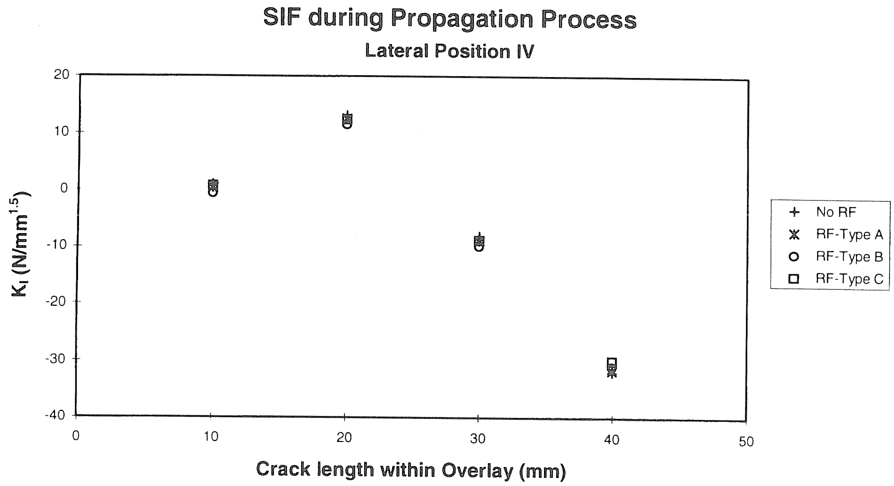


Fig. 6.9. Mode I SIF for load position IV.

Figures 6.9 and 6.10 present the computed stress intensity factors during the propagation of the crack through the overlay for load position IV.

It can be seen that the mode I stress intensity factor is smaller and the mode II stress intensity factor larger than in case of the previous load position (c.f. Figures 6.6 and 6.7). For this load position, the presence of reinforcement hardly influences these values.

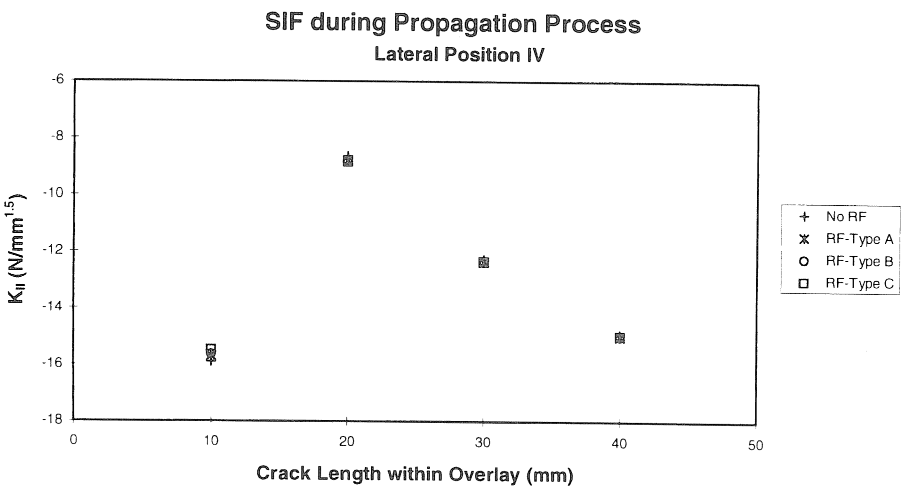


Fig. 6.10. Mode II SIF for load position IV.

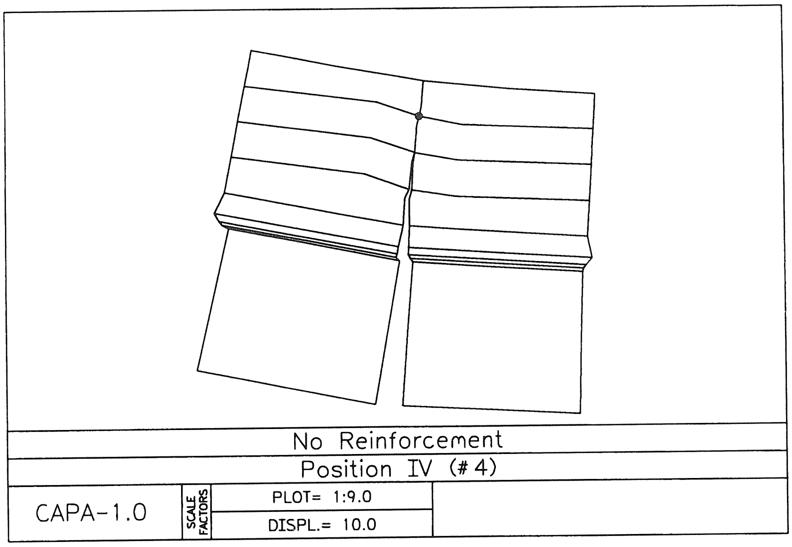


Fig. 6.11. Crack tip deformations for unreinforced mesh (Crack increment 4)

Fig. 6.11 and Fig. 6.12 show the deformations at the tip of the crack when the crack length within the overlay is 40 mm.

Although the stress intensity factors hardly differ for these cases (see previous figures), the deformations are certainly not similar and overlap occurs for the reinforced case.

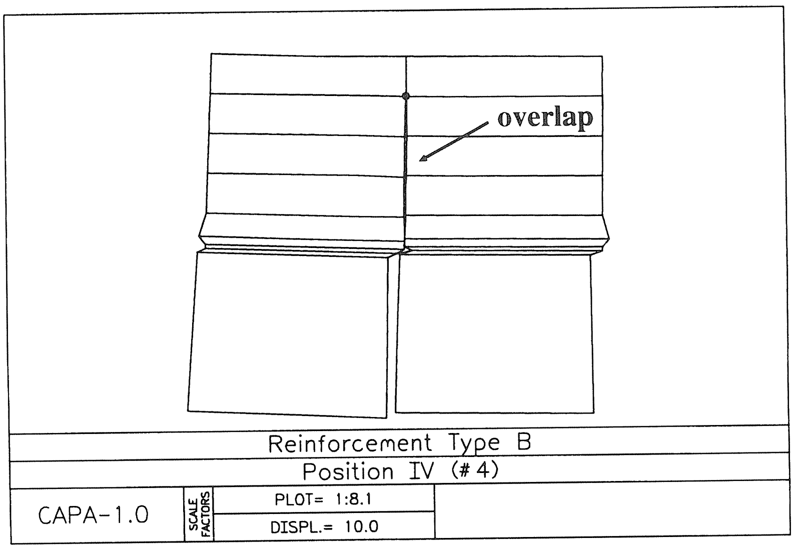


Fig. 6.12. Crack tip deformations for reinforced mesh (Crack increment 4)

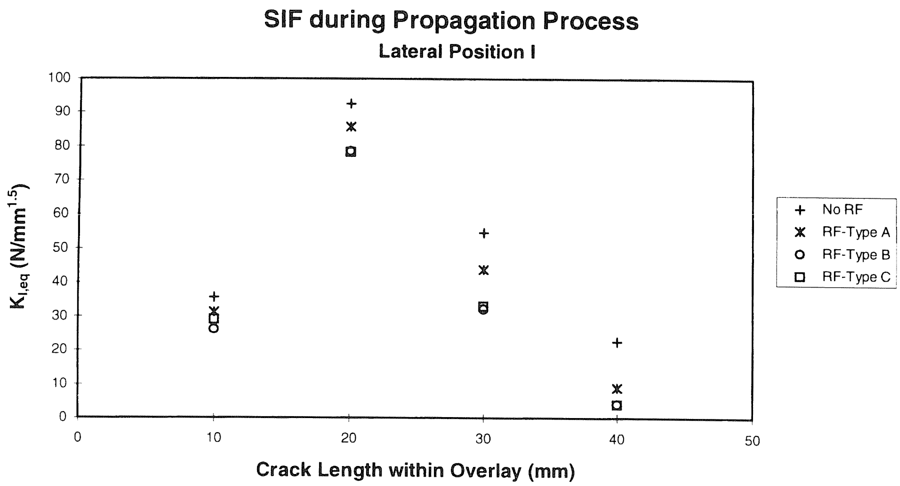


Fig. 6.13. Equivalent Mode I SIF for load position I.

The individual Mode I and Mode II stress intensity factors can be utilized to obtain the equivalent Mode I stress intensity factors, Fig. 6.13 and Fig. 6.14.
Although it seems as if lateral position I is predominant for all cases, this is only for the unreinforced situation. In case of the presence of reinforcement, the values for load position I are reduced. As a result, the values for load position IV (i.e. trucks travelling at the edge of the road) become dominant and have therefore the most detrimental effect.

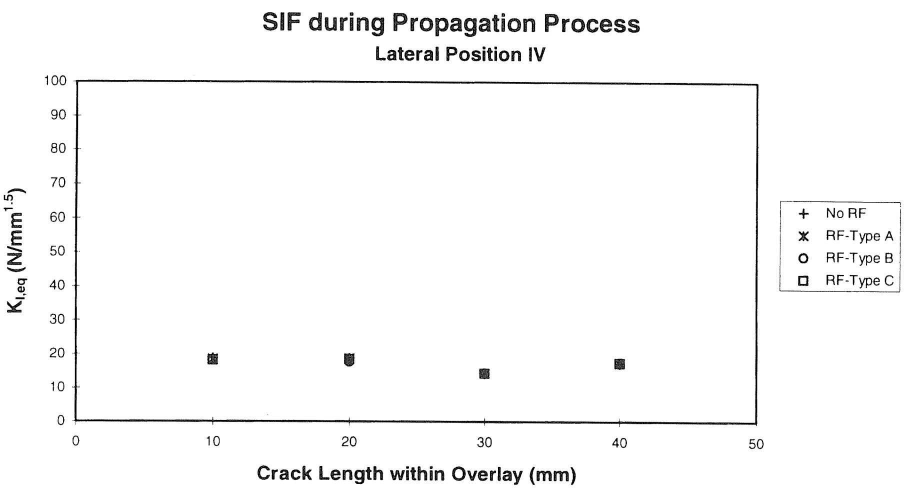


Fig. 6.14. Equivalent Mode I SIF for load position IV.

By integration of the well-known Paris' law the number of allowable load repetitions for each lateral position can be computed. For the load positions distribution of Fig. 6.3, the design life N of the overlay can be computed as:

$$\frac{1}{N_{ov}} = \frac{0.31}{N_1} = \frac{0.27}{N_{II}} + \frac{0.23}{N_{III}} + \frac{0.19}{N_{IV}} \qquad (6.1)$$

For the following values of the parameters of Paris' law for the asphaltic overlay material:

$A = 1.\, e^{-7}$ and $n = 4$

it can be computed that reinforcement type A implies an improvement in overlay life by a factor 4.2. The corresponding values for reinforcement types B and C are 6.3, and 6.0 respectively.

It is clear that given these improvement factors, although initially more expensive, utilisation of reinforcement becomes an attractive option from the economical point of view. Until recently, manufacturers did not optimise their glass, polyester, polypropylene or steel products because it was unknown if the product itself or the bond characteristics should be improved. Fig. 6.15 indicates that by increasing the bond stiffness even higher improvement factors can be obtained, especially for stiff reinforcing products.

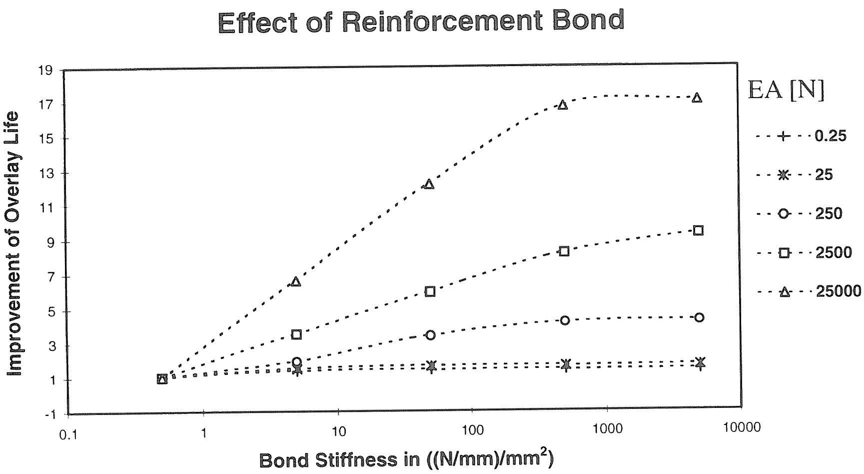


Fig. 6.15. Improvement in overlay life as a function of increasing bond stiffness.

7 Conclusions

The finite elements method is a powerful computational tool capable of assisting pavement designers in their evaluation of alternative rehabilitation strategies. A few years ago, utilization of finite elements for ordinary pavement design calculations was impractical.

The success of the method depends on the availability of computers powerful enough to solve the resulting large systems of equations. For this reason, until recently, it was considered the sole domain of researchers and academics.
Nowadays developments in micro-computer engineering enable, at present, implementation of the method to very modest sized personal computers commonly used by engineers for routine design calculations. Since the early stages of its development CAPA was targeted for this platform range. In addition and in order to enable the designer to concentrate on the engineering aspects of his project rather than on data input, handling and reduction, extensive graphically controlled facilities have been developed. These include, a fully fledged mesh generator and extensive pre- and post-processing graphic facilities.
From the pavement case studies investigated so far, it appears that utilisation of reinforcement in asphaltic overlays is an appealing anti-reflective cracking measure. It must be realized that the beneficial effect of reinforcement can be only materialised if the reinforcing product remains firmly attached onto the old surface during the overlay construction phases. No wrinkling should be allowed to occur.
By means of the methodology developed in the context of the STW project as presented in the foregoing, it becomes possible to optimize a specific reinforcing product and/or a reinforcing system and delineate its range of application and utilization in practice.

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