Analysis of the critical length of culms of bamboo in four-point bending tests

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Bamboo is a remarkable engineering material: it is a tapered tube with nodes, and the material is a composite with fibres in one direction. The bending behaviour has to be studied with a four point bending test. For pure bending a minimum length of the free span is essential. This minimum length is being discussed theoretically; the outcome is verified by tests.

Key words: **bamboo, bending test.**

1 Introduction

One of the major reasons for the social neglect of bamboo is the lack of acceptance by professionals in the construction field. Engineers and architects prefer to work with the determinancy of a well-known system or material, supported by a solid knowledge of its properties, backed by the existence of a minimum of code specifications, on which they can base their judgement and design decisions. An example of these code specifications is the ISO-standardisation used worldwide, which already exists for steel, concrete and wood. Bamboo does not have such a standardisation yet. One of the aspects that have to be researched on the way to such a standardisation is the behaviour of bamboo culms in bending. There are several ways to carry out bending experiments; a very useful method is the "four-point bending test.". A great advantage of the four-point bending test is the appearance of pure bending at the centre of the culm, thus without shear stresses. Research has pointed out that a culm with a span length that is relatively short shall fail on shear stresses and a relatively long culm fails on pure bending stresses. Therefore, there has to be a 'critical length' above which the culm normally fails due to bending stresses. If this critical length can be calculated then a framework can be created for the length of every specimen in four-point bending tests, namely a length above which the beam will fail due to bending stresses and not to shear stresses. Evidently a comparison between different experiments is only possible if the experiments are carried out in the same, proper way. On the way to an ISO-standardisation it is very important that experiments are carried out within such a framework. In this study a theoretical model for the calculation of the critical length has been developed as well as an experimental set-up. The study is based on the work by Arce (1993) and Gnanaharan (1994). A general introduction on bending in bamboo is given first.

HERON, Vol. 42, No. 2 (1997) ISSN 0046-7316

2 Bending in bamboo

Bending in bamboo is a phenomenon which deserves some explanation, because bamboo is a hollow tube, composed out of a anisotropic composite. Cellulose fibres are embedded in a matrix of lignin. The distribution can be observed in figure 1. This distribution shows that the percentage of fibres increases from the innerside of the cross-section towards the outer-skin. This is a clever natural detail: it causes an EI-value of the culm which is ten percent higher than in the case of an even distribution.

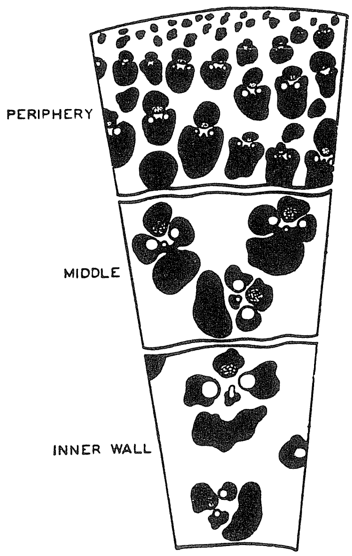

Fig. 1. Cellulose fibres in bamboo.

A bending moment causes compressive stresses in the top of the cross-section, parallel to the fibres which are strong enough to take these. But perpendicular to the fibres these compressive stresses cause lateral tensile strain, and the lignin is weak in this respect. The critical value for this lateral strain has been determined by Arce (1993) as 0.0011. With a Poisson's ratio of 0.3 (Janssen 1981) we can estimate the strain parallel to the fibres as 0.0011 / 0.3 = – 0.0037. Assuming for bamboo an E-modulus of 17 000 N / mm^2, we find an ultimate bending stress of 62 N / mm^2, a normal result in bending tests.
A bending moment causes also shear stresses in the neutral layer; in a hollow tube these are $\tau = 2F / A$, to be compared with the same in a massive rectangular beam: $\tau = 1.5(F / A)$. From tests it has been observed that this shear stress in bamboo is about 2.2 N / mm^2, which is the ultimate shear tress in bamboo. This means that failure occurs about simultaneously in the top and in the neutral layer. The structural design of bamboo really is very clever. With this introduction we can understand failure in bending in the case of bamboo.

3 The nature variations in the properties of bamboo

Each culm of bamboo has different properties. Depending on these properties, like wall thickness, external radius and Young's modulus, the critical length will vary. Evidently, the critical length is a function of these specific properties. Underlying these specific properties there are some factors which will not be taken into account, despite their undoubted influence on the critical length. There are two reasons for this:

1. For some factors, like the age of the culm, the influence on the properties from a theoretical point of view is not known (yet). Indeed, sometimes there have been established relations in experiments, for example between age and bending stress capacity, but these relations do not have a theoretical foundation.
2. For certain factors, such as nodes in the bamboo, it might be possible to establish the theoretical influence on the critical length but it is not relevant in this stage of the study on four-point bending tests.

The major reason is that taking such factors into account greatly complicates the analysis, while the actual result of the study only can have a global character.
By excluding a number of factors the critical length in practice will become a critical range above which the length of the culm has to be in the case of tests on the bending capacity and under which the length has to be in the case of tests on shear capacity. If the accuracy of the experiments increases the critical range becomes smaller. Clearly this emphasizes the importance of the accuracy of the experiments. Therefore, the experiment is carried out in steel first, in order to check this accuracy. (The outcome was satisfying; the report is not included in this publication).
For developing a criterion a number of assumptions have to be made.

- *Each culm has a cross-section that is perfectly circular (with a constant external radius).*
  In fact, bamboo almost never has a circular cross-section; the diameter in one direction differs some mm from the diameter in a perpendicular direction. Furthermore, the external radius varies significantly over the length of a culm; the culm is tapered. In the criterion a fictive culm is assumed with a constant external radius that equals the average value of the real culm.
- *Each culm has a constant wall thickness.*
  Actually, bamboo does not have a constant wall thickness all over the cross-section, nor over the length of the culm. In the criterion a fictive culm is postulated with a constant wall thickness that equals the average wall thickness of the real culm.
- *Plane cross-sections remain plane after loading.*
  This hypothesis of Bernouilli is assumed to be valid for bamboo just as it is valid for other materials like steel *(Janssen, 1981).*
- *Young's modulus has a constant value over the length of a culm.*
  The value of Young's modulus at a certain point on the outer side of the cross-section varies substantially over the culm. This value increases if the external radius of the cross-section decreases, and between the nodes Young's modulus might be twice as large as at the nodes *(Arce, 1993).* In the derivation, a fictive culm is assumed with a constant value of Young's modulus over the length of the culm that equals the average value of the real culm.

- *Young's modulus varies linearily over the cross-section.*
  Earlier research *(Janssen, 1981 & Duff, 1940)* has pointed out that the value of Young's modulus at the outer side of the cross-section is much larger than at the inner side due to the differences in cellulose content. These earlier studies showed that there is a three-fold difference between these two values. There is no reason to assume that Young's modulus does not vary linearly between the outer and inner side of this cross-section.
- *The material's behaviour is linearily elastic.*
  It is assumed that bamboo behaves linearily elastic (according to Hooke's law) until failure. Experiments have shown that there is only a slight plastic deformation which need not to be taken into account.
- *The strain varies linearily over the height of the cross-section and linearily over the cross-section itself.*
  This follows from Bernouilli's hypothesis.

4 Development of the theoretical model

As pointed out in the assumptions the strain ε varies linearly in two directions. This is illustrated in figure 2. Consider a certain element e, somewhere in the cross-section of a culm of bamboo in the case of pure bending. It is assumed that ε(e) is a linear function of the height of e in the cross-section (in this case $r.\sin(\varphi)$) and of the place of e in the wall, with a maximum at the outer skin.

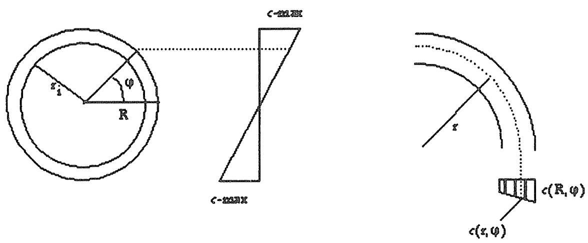

Fig. 2. Bamboo element e.

$\varepsilon_{r,\varphi}$ can be written as a function of the maximum strain at the outer side of the cross-section as in equation (1).

$$\varepsilon_{r,\varphi} = \varepsilon_{\max} \sin(\varphi)\frac{r}{R} \tag{1}$$

In this element e, Young's modulus only relates to the value of r (not to φ) and hence can be written as:

$$E_r - \frac{r - r_i}{R - r_i}(E_R - E_i) + E_i \tag{2}$$

As the specimen shows linear elastic behaviour, Hooke's law can be applied and after several substitutions the corresponding internal bending moment is given by

$$M = \frac{\pi\varepsilon_{\max}}{20R}\left[\frac{E_R - E_i}{R - r_i}(4R^5 - 5R^4 r_i + r_i^5) + 5E_i(R^4 - r_i^4)\right] \tag{3}$$

As stated in the assumptions there is a three-fold difference between the values of Young's modulus at the outer and inner skin. Knowing this, E_i can be written as $0.33\, E_R$. Introducing a new parameter x, with

$$x = \frac{t}{R}$$

and using the moment of inertia of a fictive perfectly round tube, σ (= MR/I) can be written as

$$\sigma = \frac{\varepsilon_{max} E_R}{15(x-2)^3}[14x^3 - 60x^2 + 100x - 80] \qquad (4)$$

The shear stress in this cross-section is assumed to be a sine function of φ with a maximum at the neutral axis (this assumption falls outside the scope of this article).The maximum stress is given by

$$\tau_{max} = \frac{2V}{\pi t(2R - t)} \qquad (5)$$

Experimental results *(Arce, 1993)* indicate that bamboo fails at a particular strain rather than at a particular stress, in the longitudinal as well as in the tangential direction. This is an interesting result since the calculation of maximum strains could be used to check the ultimate capacity. These maximum longitudinal and tangential strains can be directly derived from the above model and they correspond to a maximum bending and shear stress respectively.
In a homogenous and isotropic material, it is possible to relate these two maximum stresses to a general formula for the maximum stress in any element *e* in the cross-section. When a specimen fails due to a combination of both stresses, there is a need for a combined stress or strain criterion. Experimental results *(Janssen, 1981)* indicate that the cracking in the case of bamboo initially starts either at the neutral axis, or at the top or bottom part of the cross-section. The specimen will fail due to pure shear stresses or pure bending stresses respectively. In a four-point bending test with a load F and a span l, the maximum moment (M) equals $Fl/3$ and the maximum transversal force (V) equals F. The force that causes failure due to bending stresses in the cross-section, $F_{max,b}$, is a function of the maximum bending stress. The force that causes failure due to shear stresses, $F_{max,s}$, is a function of the maximum tangential stress.
A specimen with a length that equals the critical length shall theoretically fail on shear stress, as well as on bending stress at the same time. Therefore $F_{max,b}$ equals $F_{max,s}$.
Using equation (3), (4) and (5) the critical length is given by

$$l_c = \frac{R\varepsilon_{max} E_R}{\tau_{max}}\left[\frac{14x^3 - 60x^2 + 100x - 80}{20(x-2)}\right] \qquad (6)$$

In order to establish the critical length, according to equation (6), experiments have to be carried out with which the values of *Young's modulus are measured first*. They were carried out on a relatively short specimen (heigth: around 100 mm).
In carrying out these experiments several problems arise which have to be dealt with:
A first problem is that the wall thickness is not constant. Therefore, a certain amount of stress concentration (and strains) may occur during testing. To minimize this effect on the results, the strains

were measured at 3 places on the outer side of the specimen, at equal distances from each other. The average value of these three strains was used in the calculation of the critical length, except when one of them significantly differed from the other two. Such a strain gauge was not taken into account. The strains were measured with strain gauges.

A second problem is that the steel plates of the apparatus act as a lateral reinforcement *(Arce, 1993)*, resulting in possible overestimations of the mechanic properties. Friction keeps the specimen trapped between the plates so that it can sustain some extra force. A special support was used by Arce in the Musschenbroek laboratory at the Eindhoven University of Technology. The support is made of a steel plate with grease, teflon and radial sliding elements of galvanized steel (1 mm) on it. These radial elements can displace radially with the lateral expansion of the specimen, as a result of remaining friction. The layer of grease and teflon separates the radial elements from the actual support. In the experiments done in this study, this type of support was also used.

A third problem is that due to the irregular shape of the cross-section, it is very difficult to avoid certain excentricities of the load. Most of the time the irregular shape leads to specimens with surfaces that are not completely parallel in their contact with the support, which causes excentricities. An increase in the height of the specimen implies increments on these excentricities of load. This results in a concentration of stresses. A solution for this problem is mentioned above (under "a first problem").

5 Discussion of test parameters

The *compression tests to determine E* were carried out with hinged supports using balls that could rotate almost frictionlessly (see fig. 3). Lateral friction at these supports was avoided this way.

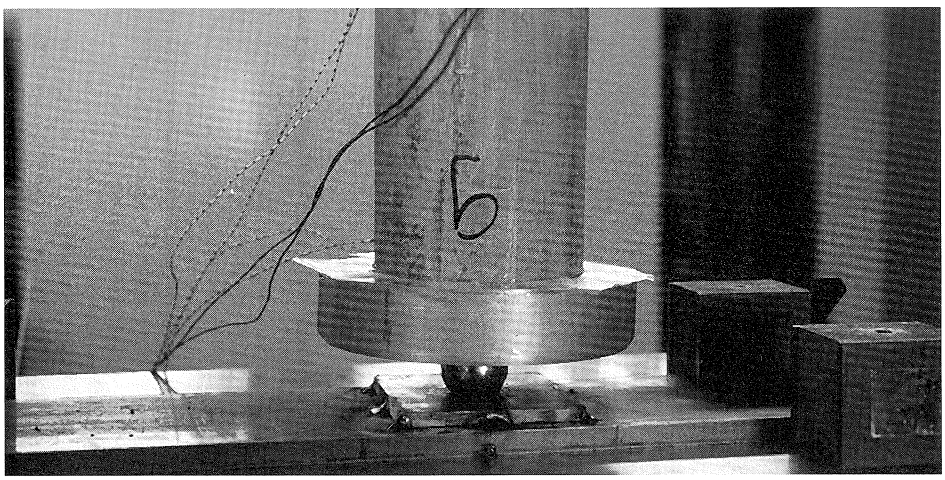

Fig. 3. Compression test.

The results of each specimen can be plotted in a σ-ε-diagram, an example of which is given in fig. 4. Such a diagram reflects the whole mechanism of the test, including the settlements at the beginning till all the pieces fit with each other, and the plastic deformations of the specimen at the end.
The E-modulus can be measured in the part of the elastic area where the machine interaction is minimal. On the basis of earlier studies *(Arce, 1993)*, it was decided that the E-modulus could be calculated as the average slope in the area between the 20 % and 80 % values of the strains.

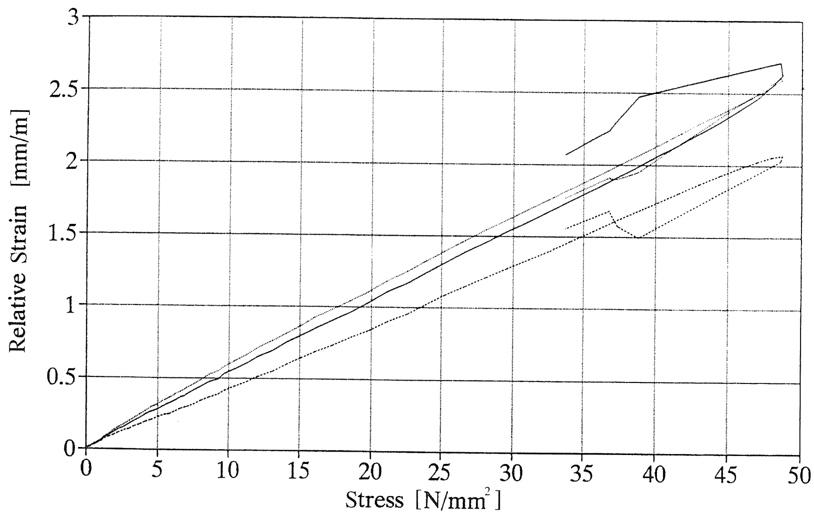

Fig. 4. Stress-strain diagram compression test; data from three strain-gauges on one specimen.

Four-point bending tests were done on nine specimens. Five (long) specimens have been cut from five culms, and four (short) specimens from two other culms, in total seven different culms, so 14 tests were carried out to measure the E-moduli of both sides of each culm. Considering that the value of the E-modulus varies linearly over the culm (see equation (2)), it is possible to estimate the E-modulus in every place of the culm. The prediction of l_c is based on the average value of the E-modulus over the specimen. In the same way the average value of t, R and A can be calculated. The value l_c can be calculated with eq. (6).
The parameters mentioned above are given in table 1.

Table 1. The wall thickness, radius, area and E-modulus of each specimen.

specimen	t [mm]	R [mm]	E [N/mm^2]	A [mm^2]	l [mm]	l_c [mm]
1	6,4	39,9	20350	1401	3220	1778
2	9,1	46,4	18250	2628	3220	1808
3	6,5	41,2	22500	1439	955	2033
4	8,2	36,5	22450	1587	2680	1714
5	18,0	53,9	16250	3882	2475	1707
6	9,7	42,0	17350	2246	1570	1519
7	5,9	36,6	20450	1295	1255	1841
8	5,7	36,5	21250	1179	2790	1703
9	6,9	41,3	22850	1203	1985	2056

Having found the E-modulus and a theoretical formula for the critical length in a four-point bending-test, it is possible to carry out *the bending experiments themselves* to test this theoretical formula. Research *(Janssen, 1981)* has indicated that the initial cracking might be located at two places. This means that just by watching the pattern of failure one can see whether the chosen length of the specimen is beneath (in the case of failure by shear stresses and inital cracking at the neutral axis) or above the critical length (in the case of failure by bending stresses and inital cracking at the top or bottom of the cross-section). The setup of the experiment needs to be chosen carefully, in order to approximate the mechanical model optimally. Therefore, four-point bending tests were done on steel first. Steel is a homogeneous material and the response to a certain load is easy to predict. By comparing the theoretical predictions with the experimental values the chosen experimental set-up can be qualified. There appears to be only a slight difference (less than 1 percent) between the experimental results and the theoretical values.
The critical value of l_c should be determined doing four-point bending-tests on specimens with a length far above the l_c. The value of the maximum shear stress, τ, can be found doing tests on the shear capacity. Arce (1993) has given indications of these values for different kinds of bamboo.
In this study the tests have only been carried out on *Bambusa Bluemana.* For each specimen the critical length has been calculated with the values for the maximum ε and τ as in the introduction.

6 Discussion of test results

There are two ways to find out how the specimen fails. First, it can be seen during the experiment. The location of the initial cracking indicates the kind of failure. Second, it can be seen from the strains measured by the strain gauges. If the strain at the top and bottom of the cross-section are still in the linear elastic area while the specimen cracks, then it is obvious that the specimen fails on shear capacity, otherwise it fails on bending capacity. As indicated it is also possible that the specimen will fail on a combination of bending and shear stresses. This can be observed by the researcher because the cracking does not start at the top, the bottom or at the neutral axis. (see figure 5 for each test).

- Specimen 1: (see figure 6): At the time of the initial cracking the deformation of specimen 1 becomes inelastic. This occurs at the centre of the specimen, which indicates that it is not caused by the introduction of the force. Since this is plastic deformation of the top of the cross-section, the specimen appears to fail on bending capacity. During the test the cracks developed from the top to the neutral axis of the cross-section.
- Specimen 2: The cracking starts at the top and bottom of the cross-section. From there on further cracking is very similar to the development of the cracking of specimen 1, but this time it includes the lower part of the cross-section. This specimen clearly failed on bending capacity too.
- Specimen 3: The cracking starts at the neutral axis (as expected: it is a short specimen, which should fail in shear). When the cracking at the neutral axis begins, the longitudinal strain (which normally should fluctuate around zero) increases. During research on the specimen after the test, it became clear that the specimen actually had failed on the bending and shear stresses in the partition, at the node where F was introduced. This partition contained several vertical and diagonal cracks.

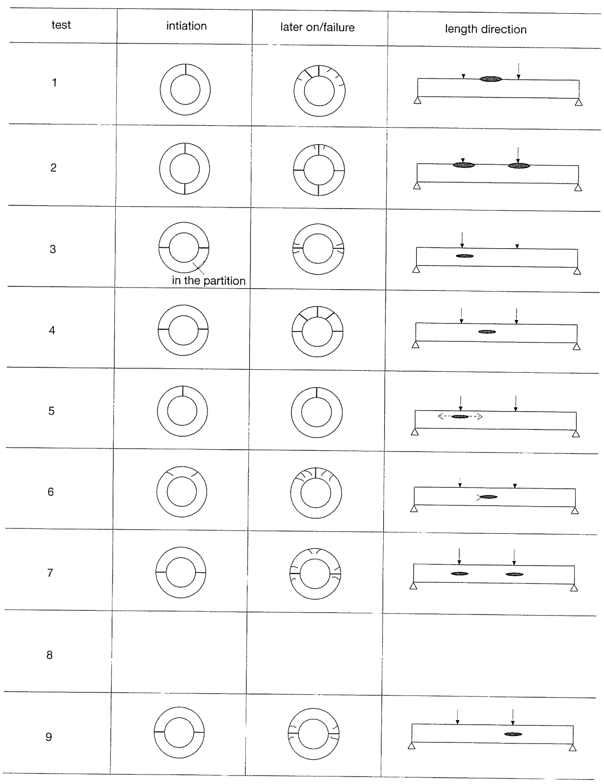

Fig. 5. Cracking patterns for the various tests.

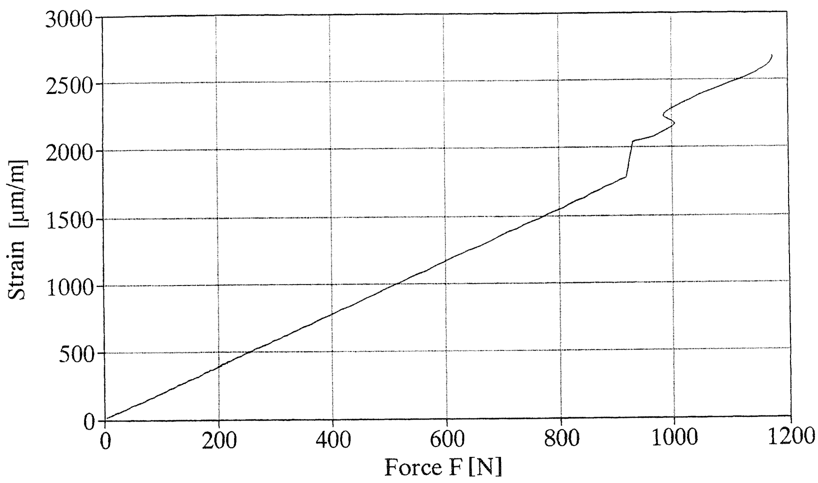

Fig. 6. Stress-strain diagram bending test.

- Specimen 4: This specimen already had two large cracks at the neutral axis before the experiment. The specimen initially cracked at the neutral axis (due to these premature cracks) and from there the cracking continued to the top. The results of this test made clear that although a specimen already has a crack at the beginning of loading, it will still behave in a linear elastic way. This indicates that a culm of bamboo can be loaded time after time, whereas the behaviour of the culm at relatively low forces remains the same.
- Specimen 5: The initial cracking starts (as expected: it is a long specimen, which should fail in bending) at the top of the cross-section. However, the individual cracks do not appear in the longitudinal direction. This type of crack is caused by reaching the shear capacity, which is due to the introduction of F. Further examination of this specimen after testing proved that the cracking mainly appeared near the node, right under the place F is introduced. This illustrates the fact that the fibres fail on shear stresses. The node itself is able to carry the shear stress, thanks to the partition, where the shear stress actually becomes an axial stress. Near the nodes, however, the current shear stresses are carried by the fibres themselves. It is in these fibres that the capacity is reached first.
- Specimen 6: The length of this specimen is very near to the critical length. It initially failed on very small cracks right between the neutral axis and the top of the cross-section (under an angle of 45 degrees with the neutral axis). Failing on a combination of shear and bending stresses theoretically can only take place at the critical length. This experiment indicates that there is a small area in which specimens fail on such a combination. Further research is needed to confirm this indication.
- Specimen 7: The stresses in the top and bottom of the cross-section both remained in the elastic area, even when the specimen actually failed. This clearly indicates failing on shear capacity.

During the experiment it could also be observed that the initial cracking took place at the neutral axis and the actual failing occured due to these cracks.
- Specimen 8: No results due to failing of the apparatus.
- Specimen 9: This specimen failed, like specimen 5, on shear in the fibres with the introduction of F. The stresses in the top of the cross-section are still in the elastic area (at the centre of the specimen), where the specimen actually has failed already. Due to these two specimens the question arises whether the tests cannot be carried out in a better way to avoid this premature kind of failing.

It may be concluded that the seven experiments as carried out in this study indicated the existence of a critical length near the theoretical length; six out of seven specimens showed a behaviour that was expected.

7 Concluding remarks

This paper describes the development of a theoretical model for a critical length, above which a bamboo culm should fail in bending, and under which a culm should fail in shear. This model has been verified with tests on seven specimens. For a full validation of the model, more tests should be carried out. Further research should be done on the way the load F is applied, in order to avoid failure modes as observed in specimens 5 and 9. Theoretical ideas that might be a solution to this problem have been developed in our laboratory.

List of symbols

τ_{crit} the critical tangential stress [N/mm^2]
σ_{crit} the critical longitudinal stress for bamboo [N/mm^2]
$\varepsilon_{r,\varphi}$ the strain of an element x with a distance r from the centre of the cross-section and under an angle φ with the neutral axis
ε_{max} the maximum strain in the cross-section; $\varepsilon_{R,\pi}$ in case of pure bending
A the area of the cross-section [mm^2]
e element of bamboo (figure 1)
E_r Young's modulus of element e with a distance r from the centre of the cross-section [N/mm^2]
E_R Young's modulus on the outer skin [N/mm^2]
E_i Young's modulus on the inner side of the cross-section. [N/mm^2]
l free span in bending tests [mm]
t the wall thickness [mm]
R the external radius [mm]
r the distance between the centre of the cross-section and an element e [mm]
r_i the internal radius [mm].

References

ARCE-VILLALOBOS, O.A. 1993. **Fundamentals of the design of bamboo structures**. Ph.D thesis , University of Technology, Eindhoven, the Netherlands.

DUFF, C.H. 1941. Bamboo and its structural use. **The Engineering society of China.** session 1940–41, paper # ICE 1. Institution of Civil Engineers of Shanghai, pp. 1–27.

GNANAHARAN, R.; 1994; **Bending strength of bamboo: Comparison of testing procedures with a view to standarisation** (IDRC- report).

GONZALEZ, G.; 1992, Técnicas de preservacion de culmos y esterillas de bambu. **Seminario Centroamericano usos del Bambu,** Pocora, Costa Rica.

JANSSEN, J.J.A.; 1981, **Bamboo in building structures,** Ph.D. Thesis, University of Technology of Eindhoven, The Netherlands.

XIU-XIN, L.; LIU-KEQING, W.; 1985, A study on the physico mechanical properties of culm wood of *Phylostachys glauca* of Shandong. **Journal of bamboo Research,** Vol 4, #2, July.