# Architectural pattern generation by discrete wavelet transform and utilisation in structural design

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Since computers were introduced in architectural design as a valuable tool, there was a growing need to develop tools that would support the designer from the initial phase of the design till the detailing. In a computer aided architectural design environment it is feasible to stimulate the spatial design ideas and create alternatives in an efficient way. *Pattern Grammar* approach is one of the design alternatives where patterns, based on complex spatial geometry, are used as an underlayer for a design. In this research, the *wavelets* techniques are used as pattern grammar and applied to spatial information processing for the generation and analysis of the architectural patterns as well as for supporting decision-makings in structural realisations.

*Key words*: pattern grammar, architecture, wavelets, multiresolution, space-frame

## 1    Introduction

Parallel to the advancements in the computer technology, there is a need for enhanced information processing virtually in all disciplines to cope with the incoming information. This is basically due to growing data and information processing, as a demand of modern technologies. In engineering disciplines, the growing need for information processing can be imagined due to the advancements in data acquisition and strong demands on real-time operations in dynamic engineering systems. In the same way, the large-scale systems and the associated complexity of modern age, put heavy demands on optimal decisions and/or solutions that are expected to be efficient and especially cost-effective. In this respect, in the last decade, information processing methods, methodologies, paradigms resulted in their associated emerging technology known as information processing technology. Multiresolutional decomposition is one of the emerging components of information processing technology that introduced new paradigm for enhanced information processing, at large. The decomposition is accomplished by means of some base functions known as wavelets so that the process is known as wavelet decomposition.
In this work we focus on the development of a new design tool for pattern generation especially for architectural design using multiresolution information processing. In general, design tools are essential in the initial phase of the architectural design. Although there are several alternatives of

pattern generation for implementation in architectural design, the multiresolution spatial decomposition provides accurate mathematical description of the design and decomposes the design information into multilevel information form. This is called multiresolution information. As it will be explained later in the work, here the implication of this approach is that, each level of multiresolutional information, that is function approximation, might be considered as a separate pattern as well as the difference information, so-called details, between the levels. With this interpretation, the decomposition provides immense variety in pattern generation simply by using the design information in parts and in a controlled way. At the same time, the mathematical details of the design are accurately maintained. Today wavelet decomposition is used virtually in all engineering fields as it provides a unique tool for simultaneous time and frequency analyses of signals. Therefore it gives insight into the signal analysis beyond the traditional Fourier analysis paradigm. Basically, the multiresolution form can be considered as a function approximation in Hilbert space [1]. One can think of Hilbert space, where functions are represented as vectors in a certain base, through the analogy with an Euclidean n-space denoted by $\mathbb{R}^n$. In the present context, the functions in Hilbert space subjected to systematic approximation yield the patterns with the associated grammar known as *pattern grammar*. Pattern grammar approach is one of the design alternatives where patterns, based on complex spatial geometry, are used as an underlayer for a design [2]. However, the application of wavelets for spatial information processing is rather limited and its utilisation in architectural design apparently provides a new tool for this very purpose. Hence, it is the goal of this research to describe the important role of wavelets in spatial information processing, taking the architectural pattern generation as a specific example area with application to structural design. The organisation of the paper is as follows. Section 2, introduces briefly wavelets and its role in multiresolution decomposition of spatial information. Section 3, describes geometric pattern generation by wavelets and utilisation of wavelets as a decision-making support for structural realisations that are followed by conclusions in Section 4.

## 2   Wavelets

Although computer aided design (CAD) facilities are quite comprehensive, automatic generation of patterns is always an important concern for both architectural and industrial design because of the variety of patterns needed together with their systematic description for easy reproduction. Although the formalism of the wavelet transform is slightly older than a decade, in the last decade as a new emerging technology, it has proved to be a powerful information processing tool in diverse disciplines like pure mathematics, physics, electrical engineering, image processing, computer graphics etc. In a CAD environment, the mathematical description of patterns using wavelets is an interesting co-ordination of these two modern technologies, yielding a productive outcome. The wavelet transform is a fairly simple yet very powerful mathematical tool that divides data or functions into localised components. Afterwards, it studies each with a resolution matched to its scale within this multiresolution scheme. There are many excellent text books on wavelets, e.g., [3–6]. However, a brief description of wavelets is in order here for the sake of completeness and the description of the work.

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The multiresolution theory can conveniently be described by the theory of function spaces. A function space is made of embedded subspaces $V_m$ where the limit of their union is $\mathbb{L}^2(\mathbb{R})$ as schematically shown in Fig. 1. In general,

$$\ldots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset \ldots$$

and

$$\bigcap_{m \in \mathbb{Z}} V_m = \{0\}, \qquad \bigcap_{m \in \mathbb{Z}} V_m = \mathbb{L}^2(\mathbb{R}) \tag{1}$$

where $\mathbb{R}$ is the set of all real numbers and $\mathbb{L}^2(\mathbb{R})$ indicates the integration with respect to Lebesgue measure for squared integrable functions; $\mathbb{Z}$ is the set of integers.

If we denote by $P_m$ the orthogonal projection operator onto $V_m$, then from (1) we can conclude that $\lim_{m \to -\infty} P_m f = f$ for all $f \in \mathbb{L}^2(\mathbb{R})$. However, central to the multiresolution aspect is that

$$f(x) \in V_m \Leftrightarrow f(2x) \in V_{m-1}. \tag{2}$$

which means, all the spaces are scaled versions of the fundamental space $V_0$. Alternatively, $V_m$ spaces have the property that for each function $f(x) \in V_m$, a contracted version of it is contained in the subspace $V_{m-1}$. The function spaces $V_m$ are schematically shown in Fig. 1a. A function $f$ and its projections onto $V_{m-1}$ and $V_m$ spaces is shown in Fig. 2.
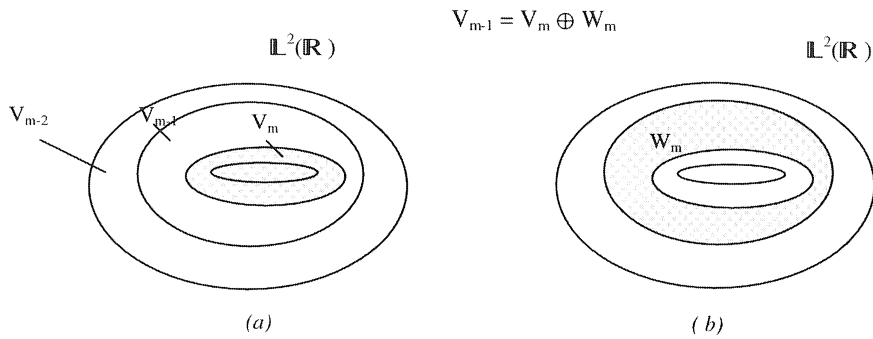


$$V_{m-1} = V_m \oplus W_m$$

Fig. 1.    *Multiresolution representation of $\mathbb{L}^2(\mathbb{R})$. (a) $V_m$ spaces where the shaded ellipse represents the function space $V_m$. (b) $W_m$ spaces where the shaded ring represents the difference space between $V_m$ and $V_{m-1}$ so that $V_{m-1} = V_m \oplus W_m$. The symbol $\oplus$ indicates the summation of the orthogonal spaces.*
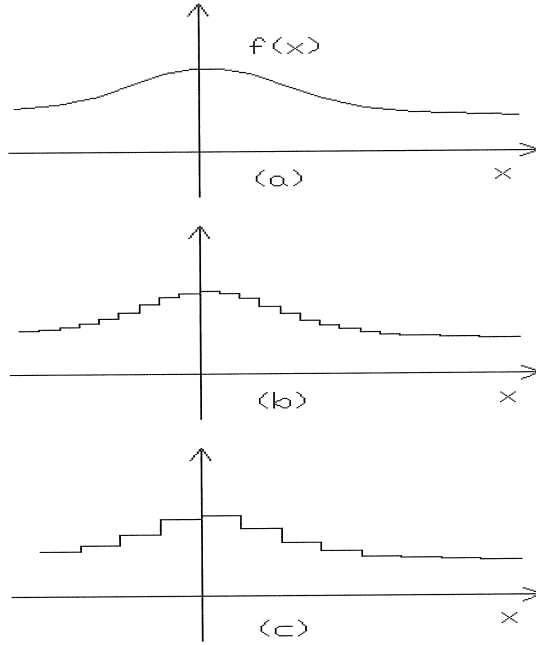
*Fig. 2. (a) A function f(x). (b) Projection of f(x) onto $V_{m-1}$. (c) Projection of f(x) onto $V_m$, ($V_m \subset V_{m-1}$).*

In $\mathbb{L}^2(\mathbb{R})$, the functions

$$\phi_{m,n}(x) = 2^{-m/2}\phi(2^{-m}x - n) \tag{3}$$

form an orthonormal basis for $V_m$. These are called scaling functions and for $m = 0$, we basically write

$$\phi_{0,n}(x) = \phi(x - n) \tag{4}$$

The function $f(x)$ in each subspace can be expressed by these orthogonal base functions as approximation in such a way that $f_m(x) \in V_m$ and

$$f(x) = \lim_{m \to -\infty} f_m(x) \tag{5}$$

All functions in $V_m$ can be represented using linear combinations of the scaling functions. In other words, $f_m(x)$ is an orthogonal projection of $f(x)$ onto $V_m$,

$$f_m(x) = \sum_n <\phi_{m,n}(x), f(x)> \phi_{m,n}(x) = \sum_n c_{m,n}\phi_{m,n}(x) \tag{6}$$

where $< \phi_{m,n}(x), f(x) >$ is the inner product defined by

$$< \phi_{m,n}(x), f(x) > \;=\; \int\limits_{-\infty}^{\infty} \phi_{m,n}(x) f(x) dx \qquad (7)$$

Note that, the number of points representing the function increase twice in the $V_{m-1}$ space relative to $V_m$ space.

*Here some illustrations are in order. Below, wavelet decomposition of a zero-mean function f(x) made available with 512 points is presented. In the multiresolution decomposition sense, this is $f_{-9}(x)$. In figure 3a, $f_{-8}(x)$ (lowermost with 256-points), is the first approximation to the function $f_{-9}(x)$ given with 512 points. Total four consecutive approximations are indicated upon each other. The vertical axis is with arbitrary units and used for the indication of relative approximation function amplitudes. Figure 3b indicates the data points of figure 3a. The numbers of the data points are halved at each consecutive coarse approximation towards to the top. These are the function approximations in $V_m$ spaces.*

In Fig. 1, the difference spaces can be represented by $W_m$ which are defined as the orthogonal complement of the spaces $V_m$ with respect to $V_{m-1}$, so that,

$$V_{m-1} = V_m \oplus W_m \qquad (8)$$

Where $\oplus$ is the summation operator for orthogonal spaces. The difference spaces $W_m$ are schematically shown in Fig. 1b.

Now, let $\psi(x) = \psi_{o,o}(x)$ be a basis function of $W_o$. Note that $\psi_{o,o}(x) \in W_o \subset V_{-1}$ and therefore can be expressed in terms of basis functions $\phi_{-1,n}(x)$ and consequently, we can also define functions $\psi_{m,n}(x)$ that are shifted and dilated versions of one prototype function $\psi(x)$ of the form

$$\psi_{m,n}(x) \;=\; 2^{-m/2}(2^{-m}x - n) \qquad (9)$$

The functions $\psi_{m,n}(x)$ are similar to the wavelets described in (3). Explicitly, $\phi(x)$ is defined for the vector space $V_m$ and $\psi(x)$ is defined for the vector space $W_m$. Therefore, there are strong relations between $\phi(x)$ and $\psi(x)$. The introduction of the wavelet functions enables us to write any function $f(x)$ in $\mathbb{L}^2(\mathbb{R})$ as a sum of projections on $W_j$, $j \in \mathbb{R}$, of the form

$$f(x) \;=\; \sum_{j=-\infty}^{\infty} w_j(x) \qquad (10)$$

where

$$w_j(x) \;=\; \sum < \psi_{j,k}(x), f(x) > \psi_{j,k}(x) \qquad (11)$$

Considering a certain scale $m$, the function $f(x)$ can be written as the sum of a low resolution part $f_m(x) \in V_m$ and the detail part which is constituted by the wavelets $w_j(x) \in W_j$ so that
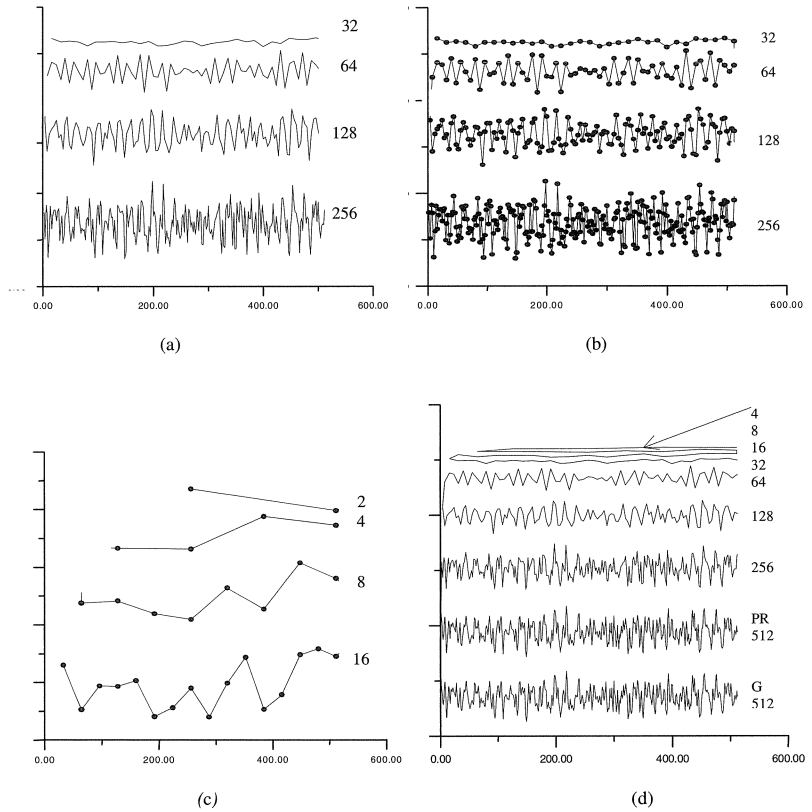
Fig. 3.    (a) Four consecutive function approximations in $V_{-8} - V_{-5}$ spaces to a function $f_{-9}$, given with 512-points are indicated upon each other. The numbers next to the approximations are the number of points representing the function. The linear vertical axis is with arbitrary units and used for the indication of relative approximation function amplitudes. (b) Explicitly indicated data points of figure 3a. The number of data points of each representation is halved at each consecutive coarse approximation. These are the function approximations in $V_m$ spaces from $V_{-8}$ to $V_{-5}$. The coarse approximation at the top has 32 points. (c) Further function approximations in $V_{-4} - V_{-1}$ from 16 points down to 2 points to be added to those in figure 3a. (d) Reconstruction of the signal $f_{-9}(x)$ starting from 4-point representation (top, in $V_{-2}$). The given function (indicated by G) subjected to decomposition is the lowermost and its perfectly reconstructed counterpart (indicated by PR) is next to it above (second from the bottom). Note that in all figures (3a–d) all the variations are zero-mean functions and here their variation and relative amplitudes are meant for representation. Therefore, the units of the linear vertical axes are arbitrary. The data points representing functions in space/time, span the horizontal axis.

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$$f(x) = f_m(x) + \sum_{j=-\infty}^{m} w_j(x) \tag{12}$$

$$= \sum_n <\phi_{m,n}(x), f(x)> \phi_{m,n}(x) + \sum_{j=-\infty}^{m} \sum_k <\psi_{j,k}(x), f(x)> \psi_{j,k}(x) \tag{13}$$

which can be expressed as

$$f(x) = \sum_n c_{m,n} \phi_{m,n}(x) + \sum_{j=-\infty}^{m} \sum_k d_{j,k} \psi_{j,k}(x) \tag{14}$$

The coefficients $d_{j,k}$ in the equation above, are known as the wavelet coefficients. From (11), the multiresolution decomposition is represented by an approximation i.e., the first term with $\phi_{m,n}(x)$ functions, and the detail part i.e., the second term with the $\psi_{j,k}(x)$ functions. The variable $m$ indicates the scale and is called *scale factor* or *scale level*. If the scale level $m$ is high, it indicates that the function in $V_m$ is a coarse approximation of $f(x)$, so the details are neglected. On the contrary, if the scale level is low, a detailed approximation of $f(x)$ is achieved.

With respect to above-described multiresolution scheme, the orthogonal and compact supported Daubechies wavelets [7,8] with 12 points are used in this study. The important multiresolution properties expressed by (14) are the virtually exact reconstruction of the decomposed functions as well as the multiresolution decomposition of functions. To see this, consider the detail parts of the decomposition in the $W_m$ spaces and the approximations in the $V_m$ spaces in Fig. 1 . Approximations $f_m$ and $f_{m-1}$, are related by the relationship

$$f_{m-1} = f_m + \sum_k d_{m,k} \psi_{m,k}(x) \tag{15}$$

that represents the one-step reconstruction.

*Here again some illustrative examples are in order. The implication of (15) is illustrated in Figure 4. Figure 4a is the progressive enhancement of the spatial representation of the approximation with 2-points shown in figure 3a, for better visualisation. Note that, this is not the increase of the resolution for function represen-tation. From 4-point (uppermost) to 512-point (lowermost) representation is obtained by taking the wavelet coefficients zero i.e., $W_m$ spaces are zero so that $V_m$ ($m = -1$) space remains the same. Figure 4b is the similar progressive enhancement of the spatial representation of the difference of two approximations, i.e. $W_{-1}$, from $V_{-2} = V_{-1} \oplus W_{-1}$. Note that, this is not the increase of the resolution spatial enhancement of the difference representation. From 4-point ($W_{-1}$ uppermost) to 512-point (lowermost) representation is obtained by taking the wavelet coefficients zero i.e., $W_m$ spaces ($m < -1$) are zero so that $W_m$ ($m = -1$) space remains the same. In accordance with the relationship $V_{-2} = V_{-1} \oplus W_{-1}$, the summation of two functions in $V_{-1}$ and $W_{-1}$ yields the function approximation ($f_{-2}$) in $V_{-2}$ space with enhanced representation by 512 points and this is illustrated in Figure 4c. The low-resolution function approximation $f_{-2}$ without spatial enhancement is already given before (second from the top in figure 3c, with 4 points).*
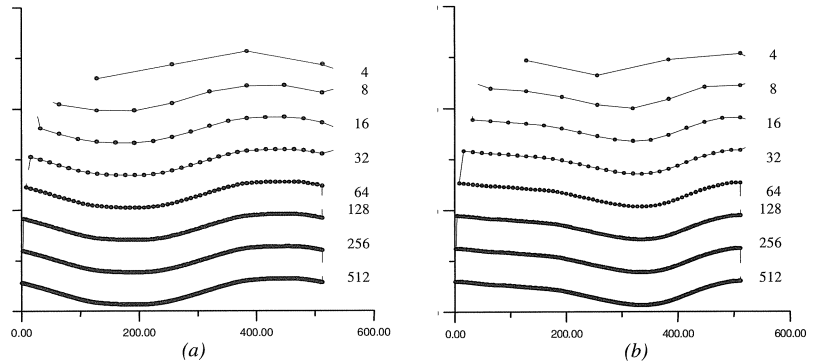
*Fig. 4.*     *(a) The enhancement of the spatial representation of the approximation for better visualisation.*
*Note that, this is not the increase of the resolution for function representation. This is spatial*
*enhancement of the function given in figure 3a with 2 points. Hence, from 4-point (uppermost) to*
*512-point (lowermost) representation is obtained by taking the wavelet coefficients zero i.e., $W_m$*
*spaces are zero in (15). (b) Illustration of the same enhancement as in 4a but for $W_{-1}$. Here $W_m$*
*spaces for $m < -1$ are zero so that the difference space $W_{-1}$ remains the same. Consequently, the*
*variation shown is the difference between two function-approximations $f_{-2}$ and $f_{-1}$ in $W_{-1}$ space.*
*Note that all the variations are zero-mean functions and here their variation and relative*
*amplitudes are meant for representation. Therefore, the units of the linear vertical axes are*
*arbitrary. The data points representing functions in space/time, span the horizontal axis.*
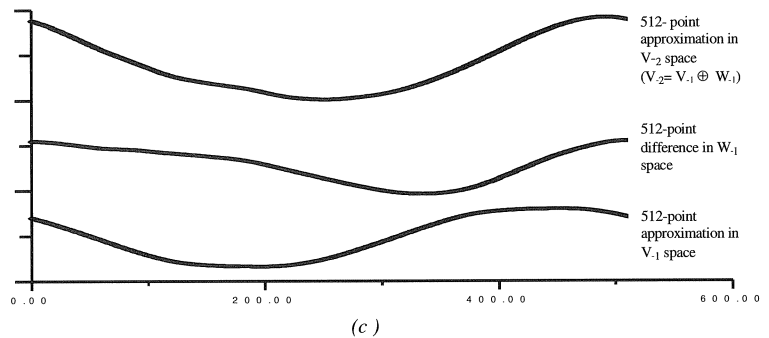


*Fig. 4 (cont'd): (c) The summation of two zero-mean approximations in $V_{-1}$ (lower) and $W_{-1}$ (middle) spaces*
*where the functions are given by enhanced spatial representation with 512 points. Their*
*summation yields the function approximation in $V_{-2}$ space with 512 points (top) and this is the*
*same approximation given in figure 3c with only 4 points (second from the top) without spatial*
*enhancement; the enhancement of the spatial representation of the 4-point approximation for*
*better visualisation would yield the same approximation in $V_{-2}$, shown above. Note that all the*
*variations are zero-mean functions and here their variation and relative amplitudes are meant for*
*representation. Therefore, the units of the linear vertical axes are arbitrary. The data points*
*representing approximation and detail functions in space/time, span the horizontal axis.*

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# 3. Wavelets in Architecture

## 3.1 Pattern generation

Multiresolution decomposition as expressed in (14) and (15), has several implications in architectural and structural design, as described below.

The first implication, from the architectural viewpoint, is the pattern generation by wavelets, or in other words, wavelets can play the role of pattern grammar. Patterns subject to creation can be in multidimensional space. The examples presented here will be in 2D space for simplicity. To create a pattern, at the beginning a 2D function is expressed in parametric form as illustrated in Fig. 5a. Since the 2D function is expressed with the 1D functions in parametric form, the function is decomposed to some detail and approximation levels, generally, treating the 1D functions separately and combining the results for 2D representation again. As result, the detailed parts are shown in Fig. 5b-d, for a given function with 32 points shown in figure 5a. The approximations starting from the given function onwards are shown in Fig. 6b-d. Any function belonging to the detailed or approximation parts or to the function itself can be subjected to low level sampling, simply by regular skipping some points in the function. This means, the function is low-pass filtered. However, because of the higher frequency components in the function that are more than the twice of the sampling frequency, the form information is altered due to the effect known as *aliasing*. The spatial enhancement of the new shape can be increased for a better visualisation that means without adding any detail (shape) information. The Hilbert-space representation of this concept is illustrated in figure 2 and (14), (15) give its mathematical form. The implication of (14) and (15) is that the increase of the number of points i.e. enhancement of the spatial representation of a closed shape is known as *node insertion*. Such implementations are important for tilings in computer graphics, for instance, that is reconstructing a surface from a set of planar contours [9].

The result of the process described above is given in Fig. 7. In that process, initially, Fig. 5d, i.e., spatially enhanced approximation function, from 4 points to 32 points, in the $V_{-1}$ function-space is selected as a pattern subject to the development of a new pattern according to the wavelet-based grammar. The initial pattern that is a function with 32 points is reduced to 8 points by low sampling rate, that is, by taking one of each consecutive four nodes. The spatial resolution of this new function is increased by node insertion up to 32 points, using wavelets. In the terminology of wavelets, this corresponds to function representation remaining in the same space $V_{m}$ where the function is represented with more number of wavelets, as this is explained before. This process is repeated while the shape of the pattern is modified in each case due to the aliasing effect mentioned above. However, since in each such consecutive process, the aliasing effect reduces and eventually it diminishes, such a process converges to a certain pattern as Fig. 8a illustrates. The final shape is further increased from 32 to 128 points for a better visualisation using the spatial enhancement technique by wavelets, described above.
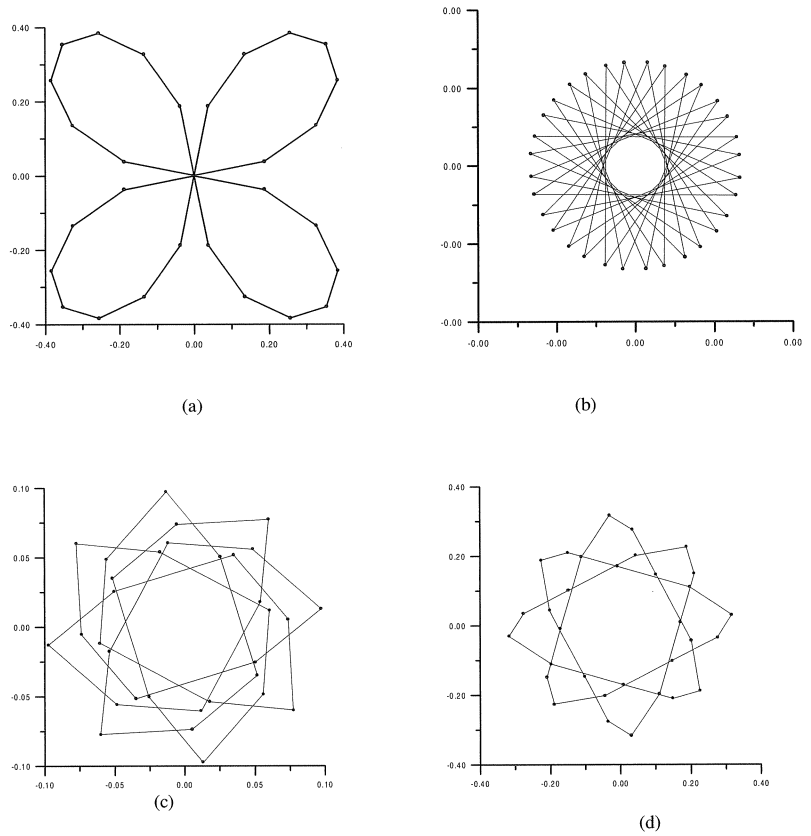
(a)

(b)

(c)

(d)

Fig. 5. *Detail components in the wavelet spaces of a decomposed function given in parametric form in 2D as $x = sin(\omega t) \, cos^2(\omega t)$, $y = cos(\omega t) \, sin^2(\omega t)$. Note that all details are expressed in enhanced spatial representation with 32 points for better visualisation. (a) the function ($f_{-5}$) given with 32 points; (b) the first difference in the wavelet space $W_{-4}$, after wavelet decomposition. The fraction of the divisions is represented with two digits. However, since the difference between $V_{-5}$ and $V_{-4}$ is small they are all indicated as null; (c) the second difference in the wavelet space $W_{-3}$; (d) third difference in the wavelet space $W_{-2}$.*
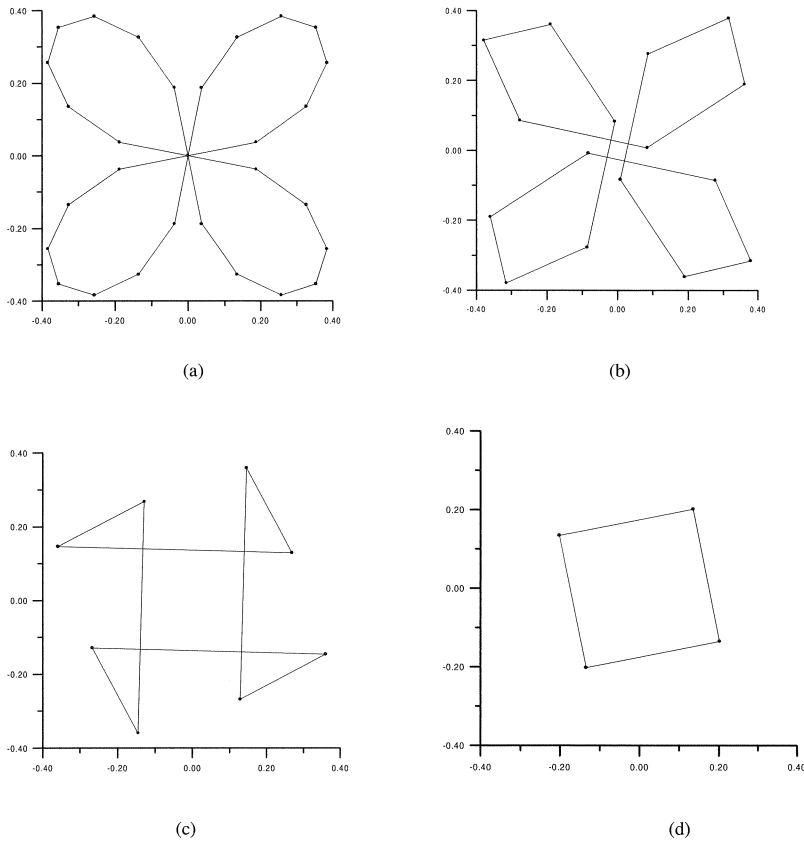
(a)

(b)

(c)

(d)

Fig. 6.     Approximation components in the function spaces of a decomposed function given in parametric form in 2D as $x = sin(\omega t)\ cos^2(\omega t)$, $y = cos(\omega t)\ sin^2(\omega t)$. (a) The function $(f_{-5})$ given with 32 points; (b) The first approximation in the $V_{-4}$ function space; (c) The second approximation in the $V_{-3}$ space; (d) The third approximation in the $V_{-2}$ space. In each case, the number of nodes reduces by a factor of two. In contrast with the detail parts shown in the preceding figure, no spatial enhancement is made here to illustrate the sharp (edged) geometric shapes formed. The close relationships between figures 5 and 6 can easily be identified. For instance, in accordance with the relationship $V_{m-1} = V_m \oplus W_m$ given in (8), the summation of two functions in $W_{-4}$ (figure 5b) and in $V_{-4}$ (figure 6b) yield the given function $(f_{-5})$ in $V_{-5}$ space with 32 points being illustrated in Figure 6a. Proceeding in the same way, summation of the figures 5c and 6c results in figure 6b. To carry out the summation procedures, figure 6b–d is subjected to spatial enhancement as is done in figure 5b-d.
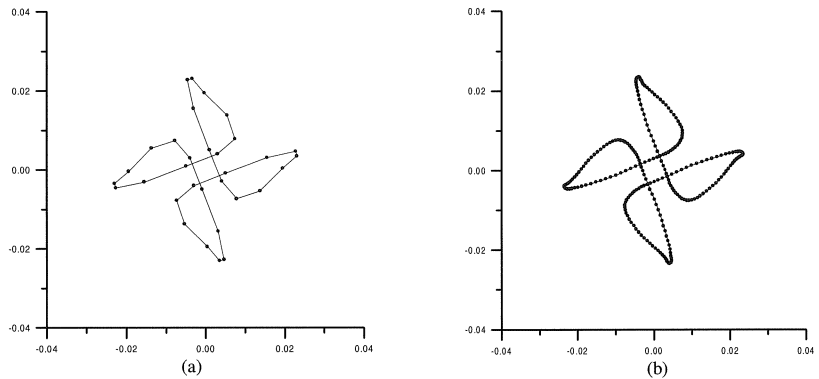
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Fig. 7.    (a) A new pattern by wavelet grammar using figure 5c; (b) Representation of figure 7a with spatial enhancement from 32 points to 128 points.

Various patterns can be obtained in the way as described above. Here wavelets play the role of pattern grammar. Additionally, by the translation, rotation and scaling still further patterns in variety can be formed.

### 3.2    Structural design application

The second implication of (14) and (15) concerns the space-frame and tensile structured architectural building designs. Namely, since the low-resolution function representations are the best representations in the mean-squared error sense [3], the nodes representing the functions also indicate the optimal locations for the joints in a space-frame structure subject to the design conditions.

A trivial example of this as a simple design is explained below.

For this purpose we consider a full sphere as a basic geometrical shape. The wavelet decomposition down to 8-points from 32-point circles obtained from equally spaced cross sections perpendicular to z axes (i.e. by planes parallel to **x-y** plane) is shown in Fig. 8a. In other words, figure 8a is obtained by multiresolution wavelet transform from 32-point discretized representation of concentric circles. Such a trivial decomposition can show the important implications of wavelet transform in engineering designs as a counterpart of architectural designs. The wavelet decomposition from 32 points to 8 points of the semi-circle in **x-z** plane is shown in Fig. 8b.
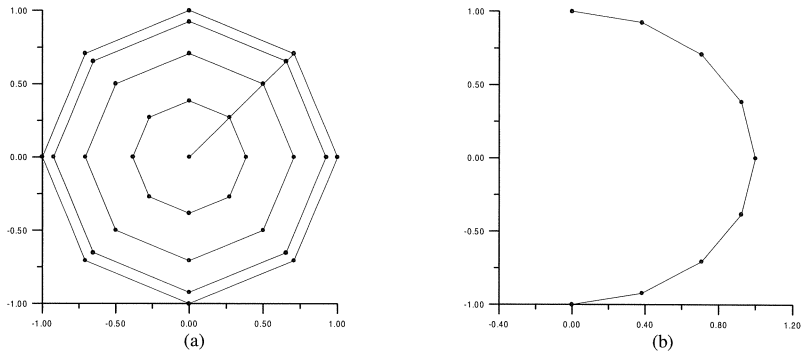
Fig. 8.    *Representation of the circles with eight nodes in eight planes parallel to **x-y** plane, in the **z** direction where (a) Representation of the sphere with eight points on the eight planes parallel to **x-y** plane (b) Representation of the cross-section of sphere with **x-z** plane. Here the half of the cross-section (i.e. semicircle) is represented with eight points. Each point belongs to a different plane parallel to the **x-y** plane, in figure 8a .*

Here, each of the multiresolution representations is the best approximation to the actual closed function (that it is the shape of the cross section and it is a circle in the present case) in the least mean square sense. For engineering considerations such representations might be especially of interest. Consider determining the location of the structural forces in space-frame structures of a cupola. The wavelet spatial resolution enhancement from 8 points to 32 points for concentric circles is shown in Fig. 9 for 32 evenly separated different $z$ values determined by analytical computations.
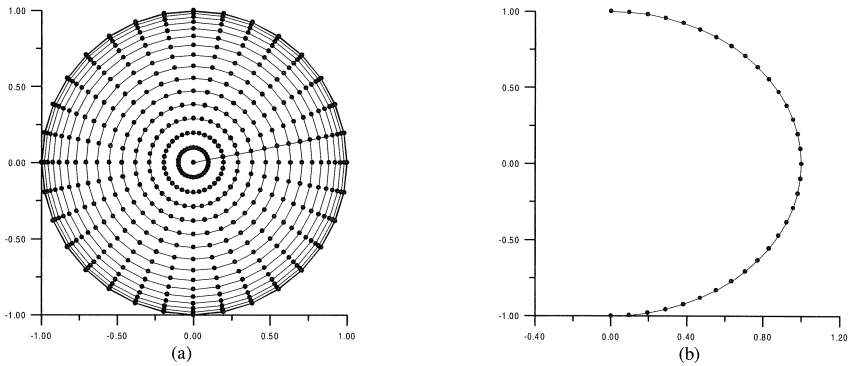


Fig. 9.    *(a) Spatial enhancement of the circles in figure 8 by wavelets from 8 up to 32 points in the planes parallel to **x-y** plane for 32 evenly divided different **z** values determined by analytical computations (b) Representation of the cross-section of sphere with **x-z** plane. Here the half of the cross-section (i.e. semicircle) is represented with 32 evenly separated points. Each point belongs to a different plane parallel to the **x-y** plane, in figure 9a.*

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The wavelet multiresolution decomposition of concentric circles in Fig. 9 is shown in Fig. 10 where decomposition yields four-point representation of these circles so that the closed polygons become squares. Fig. 10b is obtained by wavelets in contrast with Fig. 9b that is from analytical computations.
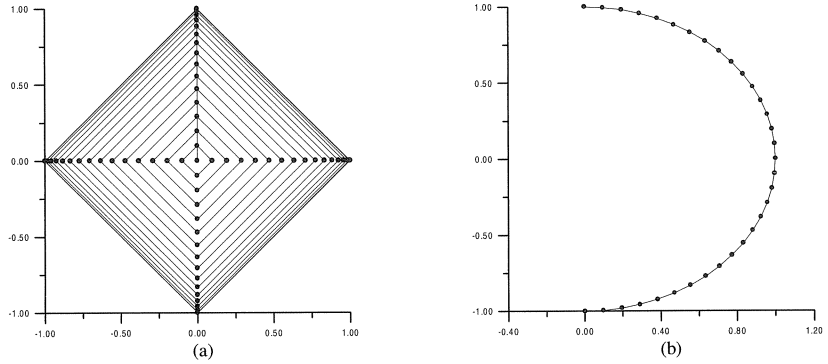


Fig. 10.    (a) For 32 different z values, wavelet decomposition of the circles in figure 9a, from eight down to four points in the planes parallel to x-y plane (b) Spatial enhancement of the semicircle in figure 9b, from 8 up to 32 by addition of more wavelets.

Since the geometric shape that we consider is a sphere, the results shown in Fig. 9 and Fig. 10 are trivial and quite obvious. However, the results are quite general since no assumption during the spatial enhancement in Fig. 9 or multiresolution decomposition in Fig. 10 is made. Also, it is worthy mentioning that, although the above analyses dealing with wavelet decomposition and spatial resolution enhancement can readily be carried out by analytical computations, the latter can only be done for analytically known shapes. In other words, such analytical computations cannot be done for analytically unknown shapes. However, wavelet approach decomposes the spatial information into lower resolution levels or reconstructs the information in higher resolution levels with or without adding new spatial information but simply by processing the information represented by given points defining the shape. This practically means any shape and the spatial resolution enhancement can be seen as a visualisation process for given representation. This process should not be confused with the conventional smoothing approaches like spline methods or others. Such approaches may be good enough for applications where mere smoothing or continuity is needed. However, grammar based pattern generations cannot tolerate arbitrary information addition or deletion for persistent architectural tilings or designs and may also be important in certain computer graphics applications. This is because, arbitrary information involvement in such analyses can impair the symmetry of the pattern subject to exploitation, for instance. In this context, the reconstruction of the shape information in higher spatial resolution levels by wavelets can be obtained for visualisation without adding new information. This practically implies enhanced spatial resolution without disturbing the symmetry inherently present in the shape at hand, i.e. spherical shape in the present example, subjected to the analyses. Such designs can easily be

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represented as well as processed from different architectural design viewpoints by means of advanced graphical software.

The tensile and space-frame structures realisations according to above described wavelet multi-resolution decomposition results are presented in Figs. 11–15, respectively.
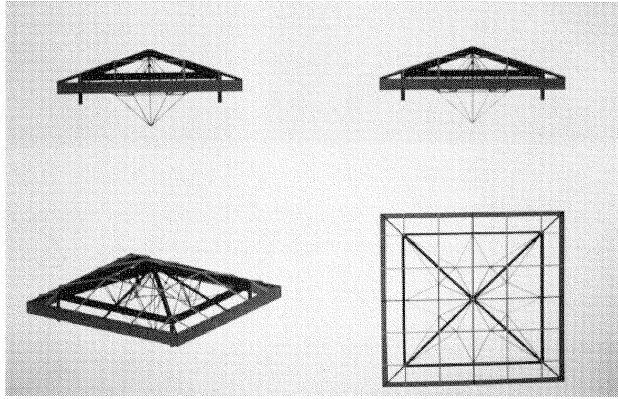


Fig. 11. Computer generated model of tensile cupola for "Provincichuis Limburg" in Hasselt, Belgium [11].
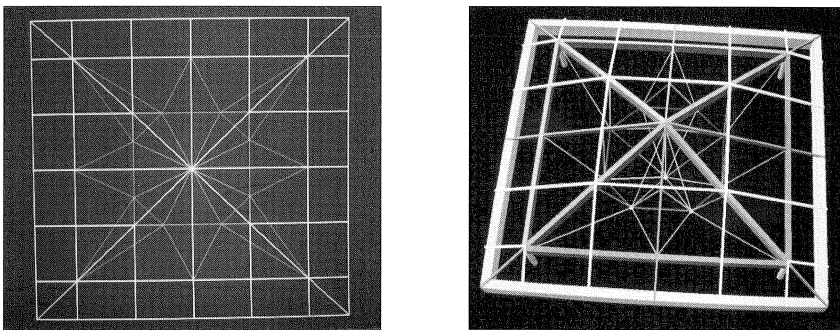


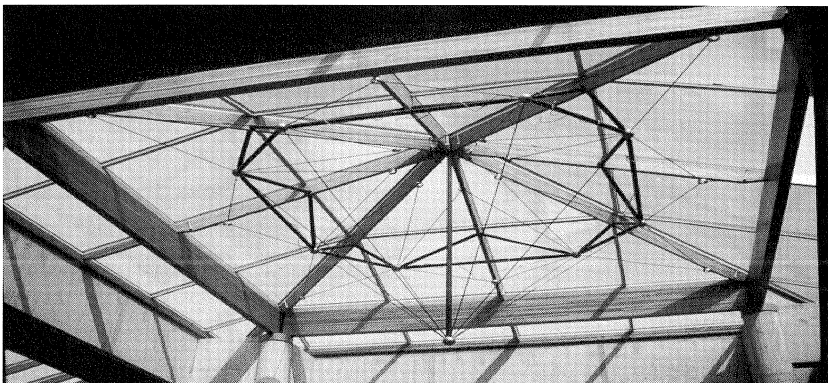Fig. 12. Top and perspective views of the same cupola shown in figure 8.



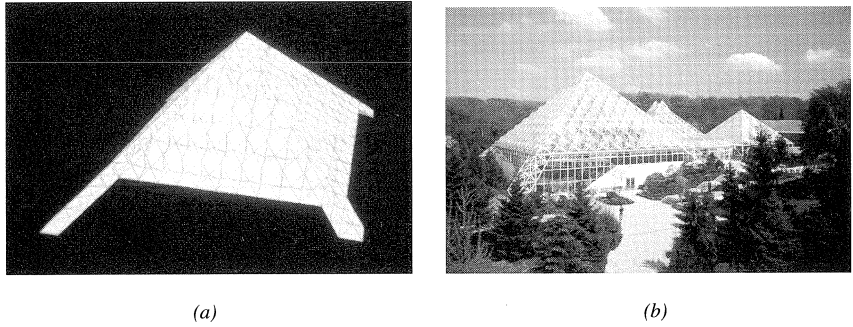Fig. 13. Realisation of "Provinciehuis Limburg" in Hasselt, Belgium.

Fig. 14. (a) Computer generated model (b) realisation of a space-frame pyramidal structure by Mero Space Structures Company, Wurzburg. Photos by dr. Jaime Sanchez.
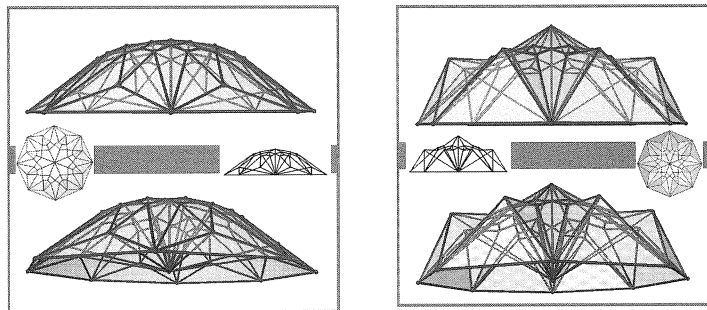


Fig. 15. Two examples of tensile structures using the same pattern generation yielding different profiles (cross sections) [10].

## 4 Conclusions

We have considered wavelets for geometric pattern generation in architectural tiling and possible application for space-frame and tensile structure generation. However, such applications have to be considered and verified from the engineering point of view. The pattern may be obtained from multiresolution decomposition of a geometric shape or it may be the given geometric shape itself subject to wavelet decomposition. After repeated process of spatial enhancement by wavelets followed by low sampling can yield very diverse patterns with complex configurations or compositions where wavelets play the role of pattern-grammar. Note that, this systematic generation is carried out by wavelet analysis without recourse to any analytical formulation of the initial patterns. The examples here presented are obtained from a circular and spherical geometry as these can offer a number of varieties. Symmetry plays an important role in grammar-based pattern generation. However, in principle, the method can be extended to other types of geometric closed shapes without symmetry.

With respect to structural realisation, an essential point in orthogonal wavelet decomposition is that, in low resolution the number of points approximating the higher resolutions is optimal in representing a given pattern. In other words, they are the unique representations with minimum error in least-mean square sense in their approximations. In this respect, the approximations and node positions relative to each other are important and the implication of this can be essentially used in structural realisations as support for decision-makings.

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