The stress-strain state and potential crack trajectories in 2D elastic brittle materials from steady-state flow experiments

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A steady-state flow method is used to examine micromechanisms of brittle failure in 2D elastic cracked media submitted to uniaxial compressive stress. The steady-state flow experiments were conducted with an incompressible Newtonian fluid in a Hele Shaw cell. Thin linear rubber inclusions were inserted in the cell to model preexisting cracks and flow was visualised by a continuous injection of methylen blue dye. Several experiments with different configurations of inclusions were conducted: 1) one single inclusion inclined at different angles β to the flow direction, 2) left lateral shear of right or left-stepping en échelon inclusions with various overlapping and 3) several randomly-distributed inclusions. The flow lines around the inclusions show very strong similarity with the trajectories of growth of similarly arranged cracks in uniaxially compressed brittle plates. Although the similarity between Hele shaw flow and elastic deformation is not fully understood yet, the method may be used to visualise both crack-induced perturbation of the stress field and crack interaction and allows accurate predictions of the potential crack's trajectories for one or several pre-existing cracks in two dimensions.

Key words: **elasticity, stress field, brittle failure, crack-induced perturbations, elastic interactions, mode I crack growth, Hele Shaw cell, steady-state flow.**

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1 Introduction

The role of initial defects such as inclusions, pores and flaws in the mechanism of macroscopic failure of elastic-brittle materials has been known for a long time (e.g., Paterson, 1978). Defects may provide local sources of tensile stress concentration even when the applied stress is compressive and may thus promote nucleation, growth and interaction of opening cracks. It is generally accepted that these micromechanisms may control the macroscopic failure of brittle materials (e.g., Bombolakis, 1964; Horii and Nemat-Nasser, 1985; Hallam and Ashby, 1990). Propagation of individual cracks in a 2D elastic plate submitted to uniaxial compression and its relation with the perturbed stress field have been widely investigated from both the theoretical and experimental point of view (e.g., Hoek and Bienawski, 1965; Horii and Nemat-Nasser, 1982; Ashby and Hallam, 1986; Barquins et al., 1992; Germanovich et al., 1994). However, the rupture of real materials may involve a large number of cracks (Brace and Bombolakis, 1963; Hoek and Bienawski, 1965; Tapponnier and Brace, 1976; Horii and Nemat-Nasser, 1985 Hallam and Ashby, 1990). Complex interactions between cracks may occur (e.g., Kachanov and Montagut, 1986; Pijaudier-Cabot and Berthaud, 1991). Accurate prediction of the stress-strain state and potential crack trajectories may therefore become difficult, even for few cracks. Hence, the mechanisms by which an ensemble of microfractures produce a macroscopic failure still remain to be elucidated.
Although the mechanics of fluids seems to be completely different from the mechanics of crack growth, some analogy exists between the equations governing the steady-state flow and the equations of elasticity (e.g., Batchelor, 1974; Küntz et al., 1997). In particular, the resemblance between 2D Hele Shaw flow and elastic deformation in elastic plates submitted to uniaxial compression was already noted by Krylov (1954). This analogy may serve to analyse some basic problems of crack interaction, although the correspondence still remains to be completely elucidated (Küntz et al., 1997).

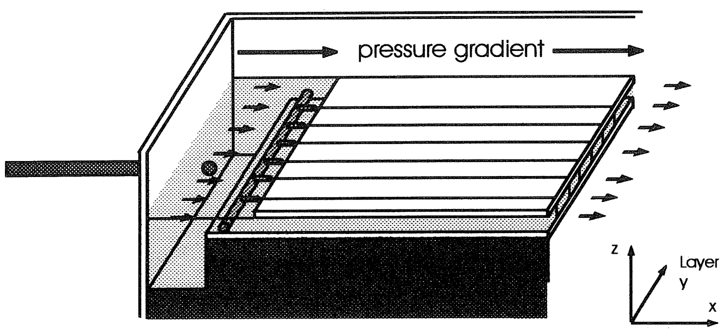

Fig. 1. Schematic view of the Hele-Shaw cell.

Preliminary results of a 2D steady-state flow method are presented in this paper in order to visualise the stress-strain state and potential crack trajectories in 2D elastic media. The steady-state flow experiments were performed in a Hele Shaw cell (Hele Shaw, 1898) (Figure 1). The experimental set-up is briefly sketched in section 2. The results of several steady-state flow experiments are presented in section 3 and compared with similar experiments on crack growth in 2D elastic-brittle plates in uniaxial compression. The nature and limits of the analogy are examined in the discussion. The technical advantages and main contributions of the steady-state flow method to fracture mechanics are also summarized.

2 Experimental set-up

The Hele Shaw cell essentially consists of two parallel and horizontal 60 cm long and 30 cm wide glass plates. The two plates are 2 mm apart and are sealed along the two long sides (Figure 1). Injection of water at a low and constant velocity (v ~ 3 cm s^{-1}) along one of the short sides through a controlled water head induces a small pressure gradient through the cell. A low Reynold's number steady-state flow is established, which may be described by Darcy's law. Thin inclusions are inserted between the two glass plates to simulate pre-existing defects. Methylen blue dye is continuously injected into the cell trough a set of capillary tubes to visualise the streamlines of flow (Figure 1). The streamlines, which are the lines tangent to the local velocity vectors, materialise the velocity field and thus any inclusion-induced perturbation of the stationnary flow.

3 Experimental data

3.1 *Single crack growth*

The perturbations of the steady-state velocity field induced by a single obstacle inclined at an angle β of about 40° to the applied pressure gradient is visualised on Figure 2a. Close to the tips of the inclusion, two adjacent streamlines steeply diverge and become parallel to the obstacle. These lines delimit two narrow triangular zones of high pressure and low velocity, which act as indenters into the flow. Then the streamlines progressively curve and become parallel to the main flow direction far from the inclusion. The perturbation may affect the flow over more than half the length of the preexisting crack in the direction of the applied pressure gradient. Note also that the location of the perturbed zone along the inclusion depends on the orientation β of the crack with the principal flow direction and migrates toward the tips of the inclusion for decreasing values of the angle β (Figure 2b). The mode I crack propagation of an inclined preexisting crack in an elastic-brittle plate submitted to uniaxial compression is represented in Figure 3 for comparison. Two symmetric opening cracks nucleate near the tips of the initial planar defect where the tensile stress is maximum (Bombolakis, 1964; Horii and Nemat-Nasser, 1982; Ashby and Hallam, 1986; Barquins et al., 1992). They propagate first at sharp angles from the preexisiting crack to become progressively parallel to the direction of the compressive far-field uniform applied stress (Figure 3), until they attain a stable configuration and then cease to grow. It must be pointed out that the similarity between the two types of experiments is almost complete. The two branch cracks (wings) nucleate on the pre-existing slot at locations corresponding to the zones of flow divergence and exhibit the same curvature as the streamlines. It is worth noting that the potential crack trajectory may be visualised over large distances from the pre-existing slot.

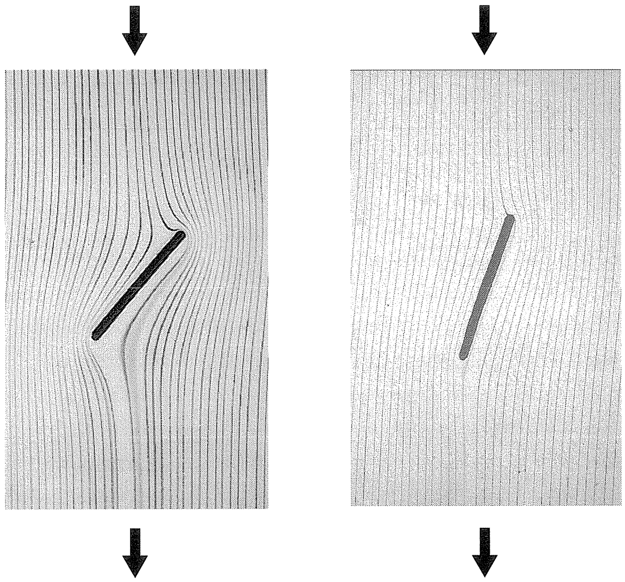

Fig. 2. Steady-state flow around a single inclusion oblique on the far-field applied pressure gradient. a.) β ~.40°. β.) β ~.10°.

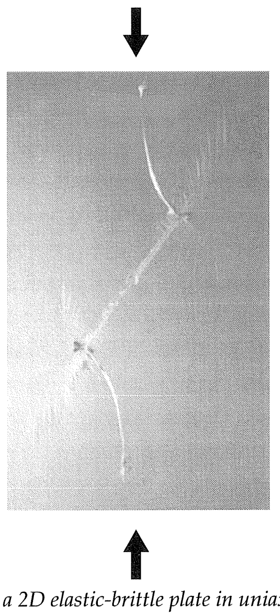

Fig. 3. Crack- induced wing flaws in a 2D elastic-brittle plate in uniaxial compression (from Barquins and Petit, 1992; reprinted with permission).

3.2 *Crack interaction*

Pertubations induced by the left shear of left or right-stepping crack arrays with different step-over were modeled to investigate the effects of crack interaction (Figure 4). For two left-stepping cracks with a negative step-over (Figure 5a), the interaction is almost completely confined in a narrow zone between the two pre-existing defects. This zone is delimited by two potential branch cracks, which may eventually connect the tips of the two neighbouring slots, at least for this initial geometry. For positive step-over (Figure 5b), the two pre-existing defaults interact over a large area and delimit a large screened zone. It can be seen that the potential cracks curve when they arrive near the neighbouring pre-existing flaws, thus preventing any connection between the two slots.

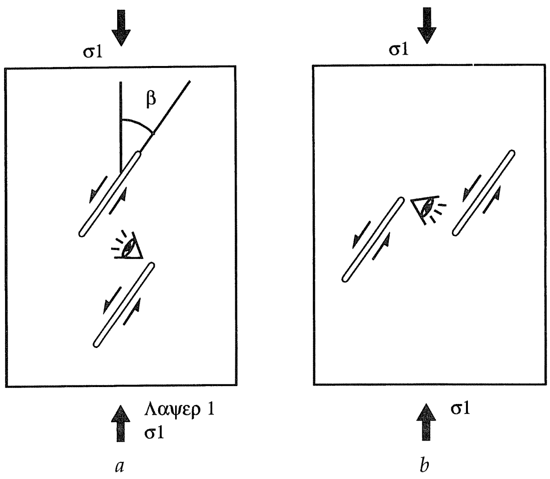

Fig. 4. Schematic view of a letft shear and a) left-stepping, or b) right-stepping en échelon crack array (from Segall and Pollard, 1980).

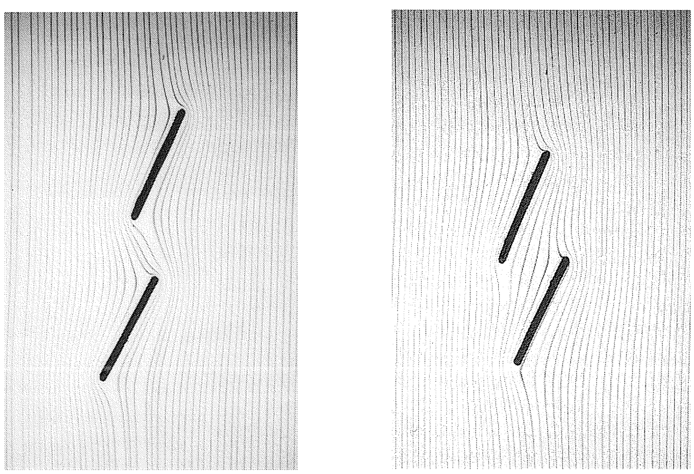

Fig. 5. Interaction of a two left-stepping cracks array a.) with a negative step-over, b.) with a large positive step-over.

These experiments compare well with the crack growth experiments performed for similar crack configurations in 2D PMMA plates by Hallam and Ashby (1990). It is shown that coalescence may only occur for left-stepping with a negative step-over (Figure 6a). In this case, the branch cracks which initiate close to the tips of the pre-existing slots progressively curve until they link the tips of two neighbouring flaws. It is worth noting that the simultaneous branching of the two wing cracks produces a small isolated piece of PMMA, which may be the analogue of the zone delimited by the potential crack trajectories on Figure 5a. For large positive step-over (Figure 6b), the tensile cracks may propagate close to but do not connect the pre-existing defaults (see also Brace and Bombolakis, 1963; Bombolakis, 1964). These data essentially confirm the predictions made from the flow experiments. Crack coalescence is likely to be associated to left-stepping arrays with a negative step-over. It is evident from these experiments that the interaction of crack within the array and the mechanism of crack propagation are very sensitive to their spatial distribution. This is best illustrated on Figure 7 where slight variations of the relative position of two neighbouring pre-existing defects may or may not favor their coalescence (also compare with Figure 6a). On the contrary, linkage is not related to right stepping crack arrays, whatever the relative position and step-over of the en échelon cracks (Figure 8).

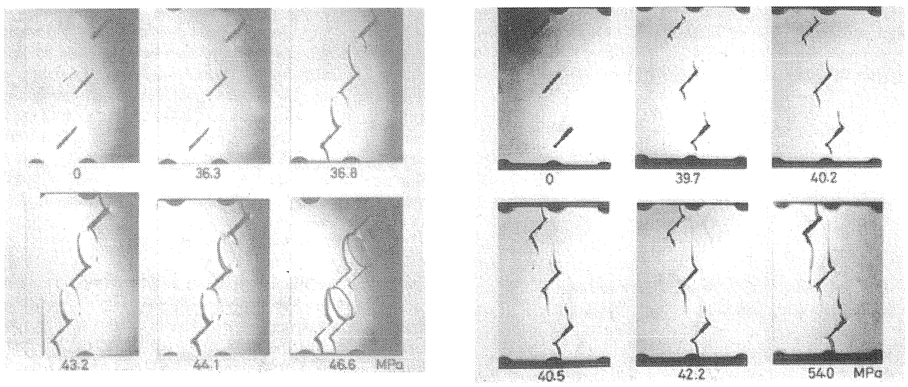

Fig. 6. *Crack growth of a left-stepping en échelon crack array in a 2D elastic plate sumitted to uniaial compression. a.) with a negative step-over, and b.) with a positive step-over (after Hallam and Ashby, 1990; reprinted with permission).*

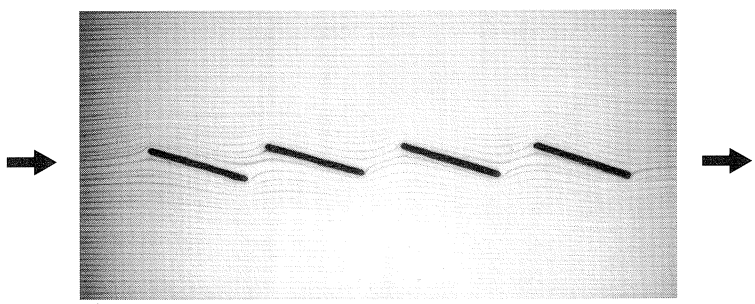

Fig. 7 *Four left-stepping cracks arrays. This pictures illustrates the sensitivity of crack's interaction to the slight changes of the geometry of en échelon cracks (compare also with figure 4a).*

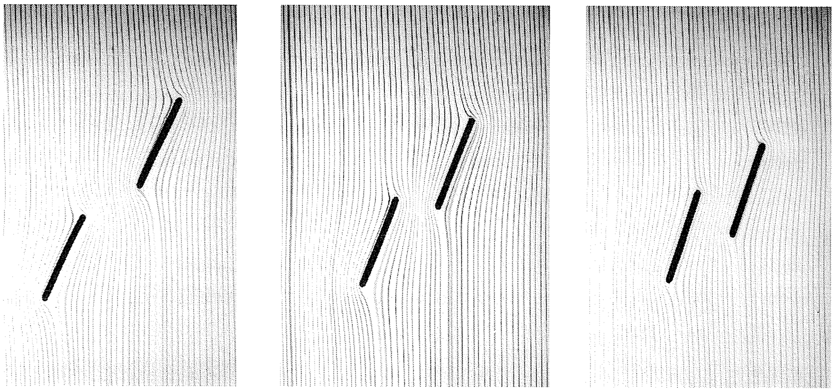

Fig. 8. Different configurations of right-stepping en échelon cracks arrays.

An illustration of the potential crack paths in a muli-cracked elastic medium is given on Figure 9. The pre-existing flaws were more or less randomly distributed in the model to approximate a real material. It must be pointed out that, even for few defects, large surface of the experiment is affected by the crack-induced pertubations. Mechanisms of screening, or enhancing effects induce complex interactions which may involve more than two cracks, as seen in the centre of the experiment. It is worth noting that for this particular distribution of cracks there is no clear evidence of coalescence between the pre-existing slots which could indicate the development of a macroscopic fracture.

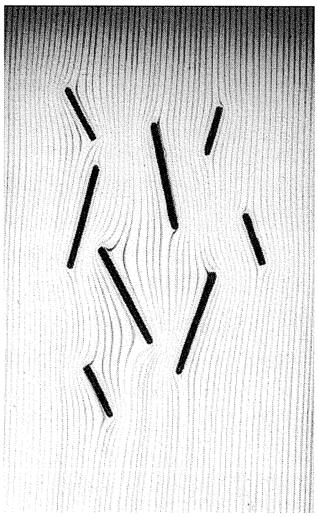

Fig. 9. Potential crack trajectories in a multi-cracked 2D elastic plate.

4 Summary and conclusion

The experiments presented in the previous section demonstrate that the flow method may be used to predict the potential crack path of one or several pre-existing cracks in 2D elastic plates submitted to uniaxial compressive loading. They further indicate that the streamlines may be equivalent to the stress trajectories or isostatics of σ_1. The flow experiments may then provide not only the potential crack trajectories but also accurate pictures of the stress-strain state in elastic cracked media that otherwise may be difficult to investigate. In fact, the method may be applied for any crack shape and any crack distribution, has the advantage of being cheap and is much more easily implemented than any other whole field method. The flow method could therefore be an useful and complementary approach to fracture mechanics.

The resemblance between 2D Hele-Shaw flow and elastic deformation has still to be completely elucidated. The formal equivalence between the steady-state velocity field of an incompressible Newtonian fluid and the displacement field in an incompressible solid in equilibrium may not apply in this case because the displacement field can not be derived from a scalar potential in the elastic domain (Küntz et al., 1997). The analogy between Hook's and Darcy's laws was proposed as an explanation by Küntz et al. (accepted). However, it was pointed out that for the given assumptions, the analogy was only valid for hydrostatic pressure and then incompatible with uniaxial compression. In fact, the equivalence is only rigorously established between the steady-state flow of a incompressible Newtonian fluid and the rigid translation of an elastic body around fixed inclusions. Further work is required to determine the exact nature and limit of the analogy. The modeling of cracks by rigid inclusions may also constitute an other limit to the method. The usual no-slip condition imposes that the velocity of the fluid is null at the fluid-solid interface. Although the boundary layer thickness may be negligible, the flow perturbations induced by the inclusion may locally differ from that induced by a real crack in an elastic solid. However, Nemat-Nasser (1983) reports that a similar crack nucleation and growth regime results if a rigid inclusion instead of the initial crack is embedded in an elastic plate, suggesting that the approximation may be neglected at first glance.

Even without an exact equivalence, the steady-state flow method provides new insights into the mechanism of crack propagation. The experiments confirm that the mode I branch cracks initiate on the slot in the zones of tensile stress concentration. The streamlines indicate that the direction of the maximum tensile stress is parallel to the pre-existing flaw, suggesting the open crack may only initiate at an angle of $\theta = 90°$ to the pre-existing slot (Barquins et al., 1992) instead of the theoretical value $\theta = 70.6°$ (Nemat-Nasser and Horii, 1982; Horii and Nemat-Nasser, 1986). The position of the tensile zone along the slot is a function of the angle β between the crack and the direction of the far field applied compressive stress. The branch cracks (wings) start to nucleate toward the centre with increasing β (Barquins et al., 1992). The steady state flow experiments further suggest that the potential fracture trajectory may be completely determined even at the early stage of loading. This may explain the previous and apparently paradoxical observations that growth of a new branch (wing) crack may not perturb significantely the stress distribution around the initial pre-existing flaw (Brace and Bombolakis, 1963, and Bombolakis, 1964, with reference to the *"simulated branch fractures"* experiments; Barquins et al., 1992). Furthermore, by assuming that the streamlines actually correspond to the σ_1 stress trajectories, the mode I crack path may follow an isostatic of σ_1

(see also Barquins et al., 1992). The progressive decrease of the angle the growing crack makes with the direction of compression far from the pre-existing slot is then naturally explained.
The advantage of the method is more pronounced for several cracks, for which analytical solutions or numerical approximates may be hard to obtain and experiments on real elastic materials become too difficult to implement. The flow experiments confirm that nucleation and growth of tensile branch cracks may be the dominant mechanism of rupture at the microscopic scale in 2D (for 3D mechanisms of crack growth see Dyskin et al., 1994). They further indicate that crack coalescence may only be observed for a few favorable configurations of the pre-existing cracks (Hoek and Bienawski, 1965; Horii and Nemat-Nasser, 1985; Hallam and Ashby, 1990). This may question the generally well accepted hypothesis of a macroscopic fracture mechanism resulting from the interaction of multiple growing tensile wing microcracks. Other mechanisms may thus take place to account for the macroscopic failure of brittle materials. The formation of en échelon mode I microcracks induced by high shear-stress concentration (Pollard et al., 1982; Petit and Barquins, 1988; Barquins et al., 1992) may be one of them.

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References

M.F. Ashby and S.D. Hallam (1986): The failure of brittle solids containing small cracks under compressive stress states. Acta metall. 34, 497-510.

M. Barquins, J.P. Petit, D. Maugis and K. Ghalayini (1992): Path and kinetics of branching from defects under uniaxial and biaxial compressive loading. Int. J. Fracture, 54, 139-163.

M. Barquins and J.P. Petit (1992): Kinetic instabilities during propagation of branch crack: effect of loading conditions and internal pressure. J. Struct. Geol., 14, 893-903.

G.K. Batchelor (1974): Transport properties of two phase materials with random structure. In "Ann. Rev. Fluid Mech.", 6, M. van Dyke, W.G. Vincenti and J.V. Wehausen eds., pp. 227-255. Annual Reviews, Palo Alto, CA.

E.G. Bombolakis (1964): Photoelastic investigation of brittle crack growth within a field of uniaxial compression. Tectonophysics, 1, 343-351.

W.F. Brace and E.G. Bombolakis (1963): A note on brittle crack growth in compression. J. Geophys. Res., 68, 3709-3713.

A.S. Dyskin, R.J. Jewell, H. Joer, E. Sahouryeh and K.B. Ustinov (1994): Experiments on 3-D crack growth in uniaxial compression. Int. J. Fract., 65, R77-R83.

L.N. GERMANOVICH, R.L. SALGANIK, A.V. DYSKIN and K.K. LEE (1994): Mechanisms of brittle fracture of rock with multiple pre-existing cracks in compression. *Pure and Applied Geophysics (PAGEOPH)*, Vol. 143 1/2/3, p. 117-149.

S.D. HALLAM and M.F. ASHBY (1990): Compressive brittle fracture and the construction of multi-axial failure maps. In "Deformation processes in minerals, ceramics and rocks", D.J. Barber and P.G. Meredith ed., Unwin Hyman, 84-108.

H.S. HELE SHAW (1898): The flow of water. Nature, 58, 34-36.

E. HOEK and Z.T. BIENAWSKI (1965): Brittle fracture propagation in rock under compression. Int. J. Frat. Mech., 1, 137-155.

H. HORII and S. NEMAT-NASSER (1985): Compression-induced microcrack growth in brittle solids: axial splitting and shear failure. J. Geophys. Res., 90, 3105-3125.

H. HORII and S. NEMAT-NASSER (1986): Brittle failure in compression: splitting, faulting and brittle-ductile transition. Phil. Trans. R. Soc. London, A 319, 337-374.

M. KACHANOV and E. MONTAGUT (1986): Interaction of a crack with certain microcraks arrays. Eng. Fract. Mech., 25, 625-636.

A. KRYLOV (1954): Preface of the first edition of the book "Some Basic Problems of the Mathematical Theory of Elasticity" by N.I. Muskhelishvili, p. , Noordhoff, Leiden, The Netherlands.

M. KÜNTZ, J.C. MARESCHAL and P. LAVALLÉE (1997): Determination of elastic properties of very heterogeneous media with cellular automata. J. Geophys. Res., 102, 7647-7658.

M. KÜNTZ, P. LAVALLÉE and J.C. MARESCHAL: Steady state flow experiments to visualise the displacement field and potential cracks trajectories in 2D elastic-brittle cracked media in uniaxial compression. Accepted for publication in International Journal of Fracture.

S. NEMAT-NASSER (1983): Non-coplanar crack growth. In proceedings of the ICF International Symposium on Fracture Mechanics, Science Press, pp, 185-197, Beijing, People's Republic of China.

S. NEMAT-NASSER and H. HORII (1982): Compression-induced non planar crack extension with application to splitting, exfoliation and rockburst. J. Geophys. Res., 87, 6805-6821.

M.S. PATERSON (1978): Experimental rock deformation: the brittle field. Springer Verlag.

J.P. PETIT and M. BARQUINS (1988): Can natural faults propagate under mode II conditions? Tectonics, 7, 1243-1256.

PIJAUDIER-CABOT and Y. BERTHAUD (1990): Effets des interactions dans l'endommagement d'un milieu fragile: formulation non locale. C.R. Acad. Sci. Paris, 310, 1577-1582.

D.D. POLLARD, P. SEGALL and P.T. DELANEY (1982): Formation and interpretation of dilatant en échelon cracks. Geol. Soc. Am. Bull., 93, 1291-1303.

P. SEGALL and D.D. POLLARD (1980): Mechanics of discontinuous faults, J. Geophys. Res., 85, 4337-4350.

P. TAPPONNIER and W.F. BRACE (1976): Development of stress-induced microcracks in Westerly Granite. Int. J. Rock mech. Min. Sci. & Geomech. Abstr., 13, 103-112.