Durability aspects of probabilistic ultimate limit state design

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An example calculation is given to demonstrate the various steps in a probabilistic approach of the durability aspects of structural reinforced concrete design. The results are translated into a verification procedure based on code type limit state formulations, characteristic values and partial factors. This way the durability aspect is a natural extension of the classical resistance verification where deterioration effects normally are neglected. The paper draws attention to a recent BRITE EURAM project on this topic.

Keywords: **reliability, durability, concrete, chloride ingress**

1 Introduction

Codes of Practice often formulate requirements related to durability using implicit statements while requirements for serviceability and load bearing capacity are formulated in a time independent manner. This way the treatment of load carrying capacity and durability has been separated, not only in the codes but also in the profession of structural engineering. It would be much better to integrate the durability aspects in the standard requirements for structural design. This paper reports on a recent BRITE EURAM project which describes the design framework for such an approach related to concrete structures (for a list of partners on this project, see Annex A).
The first step in the approach is the definition of the required performance of the structure. In practice this means that one formulates the adverse states of the structure that should be avoided. Examples of adverse states are collapse, spalling of concrete, inesthetic appearance, large deflections and so on. As a second step one has to specify the periods of time and levels of reliability for which these adverse states should be avoided. The target reliability levels may be different for each adverse state: for collapse normally a higher reliability is required as for large deflections or visible inesthetic.
The next formal step is to make an inventory of the various mechanical and environmental actions on the one side and the resulting processes and mechanisms on the other. In order to quantify the reliability level of reaching the adverse states, the various actions, processes and mechanisms need some degree of modelling. In general models can be split up in two parts: a deterministic description of the physical phenomena and a statistical description of the various uncertainties.

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Given the models and target reliability levels for the listed adverse states, a design format should be chosen by which a designer may verify whether a given design meets the performance requirements or not. In the BRITE EURAM project mentioned before the concepts of the Eurocode system have been chosen as the basis for design. The Eurocodes are based on the so called limit state approach in combination with a system of characteristic values and partial factors. The basic idea is to use this framework and to extend the various limit state formulations by adding the deterioration processes. In some cases also some new limit state formulations, not yet encountered in the present codes, may have to be added.
The present paper is a follow up of (Siemes 1999) where, besides general aspects, the focus is on a serviceability limit state example.

2 Limit state formulation

The definition of a limit state can in general be stated as: *A Limit State is the border that separates desired states from the adverse states.* Two types of limit states are usually distinguished, the ultimate limit state and the serviceability limit state (Eurocode 1.1, 1994, Background Documentation for Eurocode, 1996, ISO-DIS 3294,1996).
The ultimate limit states refer to collapse, fracture, overturning, lifting or sliding and other events where the safety of the structure is of importance. The serviceability limit states refer to comfort for the user, the functionality (fitness for purpose) and aesthetic or cosmetic aspects. For serviceability it is often useful to distinguish between irreversible cases and reversible cases.
Most ultimate limit states are of the step kind: a structural failure usually leads to large losses. Many serviceability limit states on the other hand are of the gradual kind: there are no sharp borders between large and small deflections or between acceptable and unacceptable appearances. In this project however, following present day practice, only sharp type limit states will be formulated, also for the serviceability limit state.
In most cases durability concerns the serviceability of the structure. The most simple category for instance are problems of aesthetics. But also if limits for necessary repair or maintenance actions are exceeded, this is usually considered as a serviceability problem.
However in cases where deterioration might go on unobserved the durability problem can be directly associated with an ultimate limit state. As an example consider prestressed concrete members which are permanently underground. Cracking followed by corrosion might easily go undetected and collapse of the structure may be the eventual result. So, from a formal point of view, durability is always aspect of either an ultimate or a serviceability limit state.
The description of a limit state may require one or more *limit state functions*. The general notation of the adverse state of a structure is mathematically given by:

$$g(X) < 0 \tag{1}$$

where $g(X)$ is the limit state function. The vector X in the limit state function may represent variables like:

F = loads (either mechanical or environmental)
f = material properties (mechanical or physical / chemical)
a = geometrical properties
θ = model uncertainties

Model uncertainties express the accuracy of the models used. They should cover the scatter observed in tests that cannot be attributed to any of the parameters in the model (e.g. due to simplifications) and possible discrepancy between laboratory type of tests and reality.
All variables $\boldsymbol{X} = (X_1, X_2, ...X_n)$ may be random or deterministic. The randomness may depend on the degree of information. For some variables it is possible to measure the values in course of time, reducing their randomness. In some cases such a system of inspection is the only way to achieve designs that meet the reliability in an economical way.
In structural engineering it is a custom to write limit state functions like (1) in two parts, representing respectively the capacity and the loading:

$$g(\boldsymbol{X}) = R(\boldsymbol{X}) - S(\boldsymbol{X}) \tag{2}$$

R = the model that describes the capacity of the structure
S = the model that describes the load effect on the structure

In the case of durability problems, time enters the limit state function explicitly. This leads to the general formulation that failure occurs if:

$$g(\boldsymbol{X},t) < 0 \tag{3}$$

for some point in time t in the time interval $(0,T)$ where T is reference period under consideration. The resulting failure probability is either the probability that the structure fails in the period $(0,T)$ or that it fails in year T. This depends on the definition of the limit state and the random variables. We will readdress this topic later.

3 Example structure

As an example, consider a concrete reinforced beam (see Figure 1 and Table 1), subject to a combination of mechanical loading and chloride penetration. The total failure process can be split up as follows:

1. the chloride concentration raises in the concrete parts and reaches its critical value at the reinforcement (initiation period);
2. corrosion of the reinforcement starts (propagation period);
3. the load bearing capacity is lower than the load.

We may model this problem starting from the following expression for the limit state function:

$$g(X,t) = M_{res}(t) - M_{load}(t)$$

In here $M_{res}(t)$ is the bending moment capacity and $M_{load}(t)$ is the midspan bending moment due to self weight and live load. In this case we will only consider the moment due to the live load P. The resistance $M_{res}(t)$ is a constant function of time during the first step of the total process and a monotonically decreasing function of time during the second step. Failure occurs if $g(X,t) < 0$, which may be either in the first, or the second step.

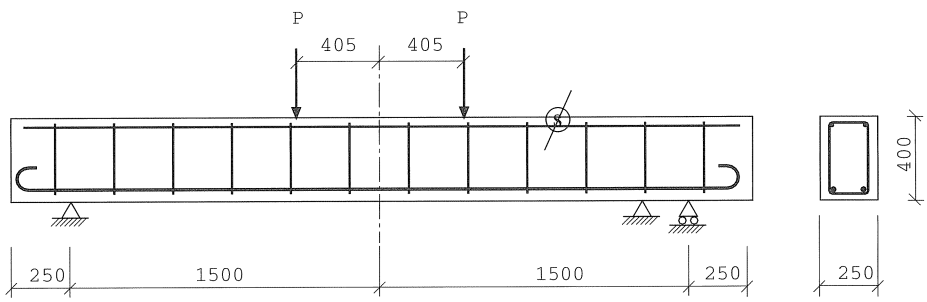

Fig. 1. Concrete reinforced beam and loading condition, measures in mm.

Table 1. Characteristic properties of the concrete and reinforcement.

L	Length of beam	3000 mm
A	Distance from support to load	1095 mm
H_{eff}	Effective height	351.5 mm
B	Width of beam	250 mm
A_{s0}	Initial cross sectional area of steel	1185 mm^2
f_y	Main bars yield stress	550 N/mm^2
f_c	Compressive strength	35 N/mm^2
d_c	Concrete cover	30 mm
w/b	Water/binder ratio	0.50

4 Physical modelling

Let us again consider the beam example of section 3 and define models for the three steps that have been defined. It should be kept in mind that the exact way of modelling is not the essential issue in this paper. Each model may be replaced by a better one if that is available.

Model 1. Chlorides penetration

Chlorides can penetrate into the concrete from the environment or they can be mixed in the fresh concrete. External sources are sea water, de-icing salt solutions and smoke from fires wherein chloride containing substances are burnt. Cement is capable to bind chloride for a few tenths of a percent by its mass. This chloride is fixed in the solid phase and does not cause depassivation. The binding results in a certain tolerance for low levels of chloride. Above a certain chloride concentration in the pore water, or a certain chloride content of the concrete, corrosion is initiated. The boundary value is called the critical chloride content, c_{cr}, which depends quite strongly on the concrete composition and on the environment (Bamforth & Chapman Andrews, 1994, Schießl & Breit 1996). So, corrosion is initiated at time t_o when the chloride concentration at the depth of the concrete cover d_c reaches the critical value:

$$c(d_c, t_o) = c_{cr}$$

where $c(x,t)$ is the chloride concentration at a depth x after a period t, which may be found from the following diffusion model:

$$c(x,t) = c_s\,[1 - erf\,(Y)] \quad \text{and} \quad Y = x/\,2\sqrt{\,(\theta\, D\,(t_r/\,t)^n\, t)}$$

In this model *cs* represents the chloride concentration at the concrete surface, D the coefficient of diffusion, t_r a reference period, n an exponent taking care of the time dependency of the diffusion progress and θ a model factor.; *erf* stands for the standard error function. In the present example the initial content of chloride is assumed to be zero, but other values could easily have been chosen without changing the principles.

Model 2: Corrosion of steel

A simple model describing the deterioration of the reinforcing steel is given by:

$$A_S(t) = A_{S_0}(1 - \alpha(t))$$

$$\alpha(t) = \frac{t - t_0}{t_{corr}}\left(2 - \frac{t - t_0}{t_{corr}}\right) \qquad \text{for} \quad t_0 < t < t_{corr}$$

where $A_S(t)$ is the area of steel at time t and A_{S_0} is the initial area of steel, t=0. The parameter t_{corr} represent the period from initiation to total deterioration. Of course $\alpha(t)$ and 0 if $t < t_0$ and $\alpha(t) = 1$ if $t \geq t_0 + t_{corr}$.

Model 3. Structural collapse

The ultimate bending capacity may be calculated as follows:

$$M_{\mathrm{res}}(t) = b(h_{eff})^2 f_c \Theta \left(1 - \frac{\Theta}{2}\right)$$

$$\Theta = \frac{A_s(t) f_y}{b\, h_{\mathrm{eff}} f_c}$$

where b is the width of the concrete beam, h_{eff} the effective height, f_c the compression strength of the concrete, $A_s(t)$ the cross sectional area of the reinforcing steel at time t and f_y the yield stress of the reinforcing steel.

The Midspan moment M_{load} is simply calculated as

$$M_{\mathrm{load}} = \alpha P$$

This completes the physical model.

5 Modelling of random variables

Random variables are formally described by a probability distribution (see Figure 2). This is normally done by specifying the distribution type (e.g. the normal distribution or an exponential distribution) and some parameters, unusually the mean μ and the standard deviation σ (Madsen, Krenk & Lind, 1986). The mean is the quick characterisation for the order of magnitude of a quantity. The standard deviation is the quick characterisation for the scatter. As an alternative one often uses the coefficient of variation $V = \sigma / \mu$. Of course, this measure is meaningful only if μ is a positive quantity. In many cases a probabilistic model needs also take into account the random variations in time and space. We will, however, not go in such details here.

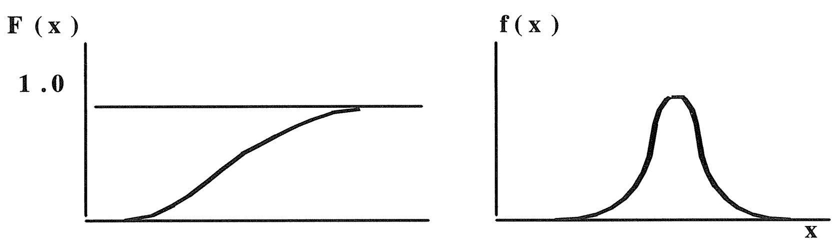

Fig. 2. Probability distribution function F(x) and probability density function f(x).

The statistical properties of the random variables for the example in this paper is presented in Table 2. We will not discuss here the arguments and backgrounds for these numbers. Reference is made to the final reports of the BRITE EURAM project which will be published in the course of 1999. In these reports even other choices may be present. The purpose of this paper is only to demonstrate the procedure.

In general, however, it should be noted that most numbers are based on a mix of observations (in nature as well as in the laboratory) and expert judgement. The necessity to include expert opinions for estimating scatters and uncertainties is often considered as a weak point of the approach. However, one should keep in mind that other methods also rely on these expert opinions and often even in a more obscure and untraceable way.

Table 2. Statistical properties for the basic variables.

Variable	Designation	unit	type	mean μ	$V = \sigma/\mu$
c_s	Chloride surface content	-	lognormal	0.0095	0.20
c_{cr}	Critical chloride content	-	normal	0.0016	0.10
D	Diffusion Coefficient	mm^2/yr	lognormal	30	0.35
Θ	Model uncertainty	-	lognormal	1.0	0.20
t_r	Diffusion reference period	year	deterministic	0.10	-
n	Exponent	-	lognormal	0.10	0.25
d_c	Concrete cover	mm	lognormal	30	0.15
P	Load (maximum in one year)	kN	Gumbel	80	0.10
t_{corr}	Time of total deterioration	year	lognormal	500	0.35
f_c	Compression strength concrete	N/mm^2	lognormal	43	0.12
f_y	Yield stress steel	N/mm^2	lognormal	573	0.08

6 Reliability analysis

In the previous sections we have defined the deterministic models for the physical processes and statistical models for the uncertainties in the variables. We may now start the actual reliability analysis. Using available software we may calculate the failure probability as a function of time (Madsen, Krenk & Lind, 1986). Again we will not go into details. The result can be found in Figure 3. Presented is the so called reliability index BETA. As can be seen: the reliability index is first almost constant and then drops down as a result of the deterioration process in the reinforcement.

The reliability index is directly related to the failure probability P_f. Table 3 gives an overview of this relationship. Although there is a one to one relationship between the reliability index and the probability P_f most experts prefer the index BETA above the failure probability for a number of reasons.

Table 3. Failure probability versus reliability index.

β	0.0	1.3	2.3	3.1	3.8	4.3	4.8	5.2	5.6
P_f	0.5	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}

Looking into Figure 3 we find that, for instance, after an exposure time of 50 years, the BETA is equal to 4.7. From the Table 3 we may derive that the corresponding failure probability is equal to $1.5*10^{-6}$. We will discuss shortly the interpretation of this result in the time domain and in the space domain.

In the time domain the resulting failure probability represents an annual failure probability, so to speak the probability that the structure fails between the age of 50 years and 51 years. This follows from the fact that the load P has been defined by its maximum in one year. To be exact, the probability is an upper bound for the annual failure probability as we have not explicitly excluded the effect of failure in the period up to 50 years.

In the space domain the result represents the probability that an arbitrary beam fails. Normally, however, a structure consists out of a large number of elements. From the above analysis it cannot be concluded whether all members in the structure will fail simultaneously or whether one member may fail locally while a neighbouring member is still far from the limit state. In order to derive conclusions in that respect a more extended probabilistic modelling is necessary. In the present stage, however, this is not considered to be essential.

The next question is whether the resulting reliability index is high enough. The explicitly or implicitly chosen reliability level should reflect (Sorensen, Kroon & Faber, 1994):

- the consequences of failure in terms of risk to life, injury, potential economic losses and the level of social inconvenience;
- the expense and effort required to reduce the probability of failure.

However, in many cases the target reliability level is chosen in such a way that the design according to the probabilistic method is, on the average, the same as the design according to current practice. The present Eurocode 1 uses $\beta = 3.8$ as a life time or 50 years target reliability or equivalently $\beta = 4.7$ for a one year period.

If a structure does not meet the safety target one may take various measures to improve the situation. The most simple measure is to increase structural dimensions. Other possibilities are to take protective measures or to plan a system of inspection and maintenance. It goes too far for this paper to indicate the impact of the latter two measures on the calculation of the failure probabilities.

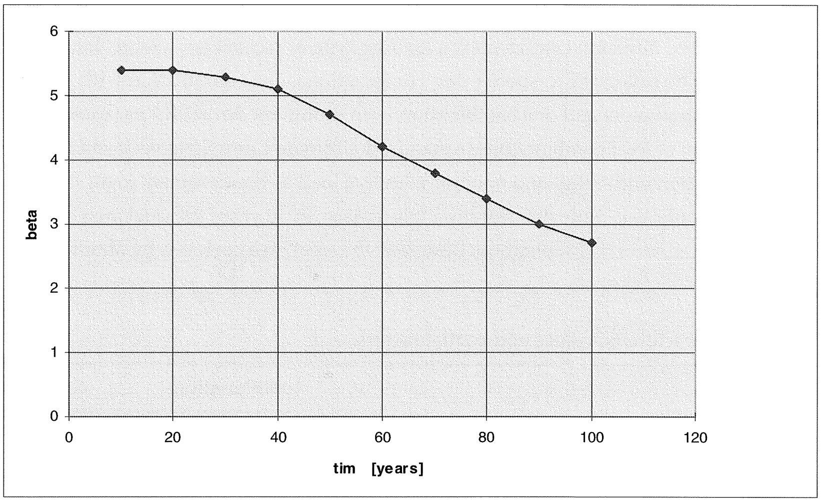

Fig. 3. Reliability index β versus time.

7 Practical design procedures in codes

Designers in practice normally do not want to perform a probabilistic analysis. For this reason the above full probabilistic verification procedure should be translated into the well known semi probabilistic design formats. This procedure can be summarised as follows:

1. define characteristic values X_k
2. define partial factors γ
3. calculate design values γX_k (for loads) or X_k / γ (for resistance)
4. check whether the limit state function is positive for the design values

For resistance properties the characteristic values are usually defined through a lower fractile of the corresponding probability distribution function. In the present code a characteristic value corresponding to the 5 percentile is chosen. For load variables the characteristic values are defined though their return period. The loads considered in the present context are characterised by a value corresponding to a return period of 50 years.

Whereas the characteristic values are maintained for all design situations the partial factors may be differentiated in accordance with the required reliability levels. The partial factors depend formally on:

- the target reliability level β for the limit state
- the statistical variability V of the action or resistance parameter
- the sensitivity of the structure to the action or resistance parameter

It is generally the task of a code writer to provide such values for partial factors that the reliability targets are met. For the example structure under consideration a set of characteristic values and partial factors meeting the target reliability is presented in Table 4. In the partial factor columns it is indicated whether one should multiply (load parameter) or divide (resistance parameter). Note for instance that the diffusion coefficient is a loading type of parameter: a large value is unfavourable. It turns out to be necessary to specify two combinations, one combination with dominant mechanical parameters and one combination with dominant durability parameters. This is similar to the situation of load combinations in standard Ultimate Limit State verifications: one may have a load combination with wind and another with live load as the dominant load. In the first combination only very little deterioration takes place: in 50 years the steel area reduces only by 3 percent. In the second combination, however, the steel area corrodes by about 40 percent.

Table 4. Characteristic values and partial factors.

				combination 1		combination 2	
variable	designation	unit	X_k	γ	X_d	γ	X_d
c_s	Cl surface content	-	0.0095	* 1.00	0.0095	* 1.00	0.0095
c_{cr}	Critical Cl content	-	0.0016	/ 1.00	0.0016	/ 1.00	0.0016
n	Exponent	-	0.10	/ 1.00	0.10	/ 1.00	0.10
d_c	Concrete cover		30	/ 1.00	30	/ 1.25	24
D	Diffusion Coefficient	mm^2/yr	30	* 1.00	30	* 2.00	60
P	Load *)	kN	100	* 1.50	150	* 1.00	100
t_{corr}	Total deterioration time	year	500	/ 1.00	500	/ 3.00	170
f_c	Concrete strength	N/mm^2	35	/ 1.20	28	/ 1.00	35
f_y	Yield stress steel	N/mm^2	550	/ 1.15	490	/ 1.00	550

*) maximum in one year

8. Closure

This paper demonstrates an alternative approach to the durability aspect of structural design. It is proposed to treat deterioration mechanisms in a way that is similar to mechanical failure mechanisms. In short this comprises the definition of limit states, reliability targets, physical models and statistical models. For practical purposes this system has to be translated into a design format on the basis of characteristic values and partial factors. The simple example in this paper shows that such an approach is feasible in principle. In a recent BRITE EURAM project (final reports to be expected this year) a more elaborate treatment of the subject is presented and direct recommendations are given.

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