Size effects in plastic hinges of reinforced concrete members

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Reasons for size dependence of rotation capacity of plastic hinges are discussed. The increase of ductility with decreasing member size is interpreted from the viewpoint of fracture mechanics of concrete. The results of the introductory test series on simply supported slender beams loaded in three-point bending are discussed. A new model for the calculation of the rotation capacity is described, which takes into consideration the strain localisation in the damage zones in the hinge region. In particular, the way of implementing the Fictitious Crack Model, the Compressive Damage Zone Model and a new fracture mechanics based bond model that takes into account the steel stress level (e.g. the yielding of reinforcement) is explained. The results of parameter studies on size effects in plastic hinges are described. Special attention is given to the influence of reinforcement arrangement and of the flexural crack pattern. In conclusions the importance of the size effect in practical design situations is examined and the need for altering the existing design rules in the light of the possible member size dependence of the rotation capacity of plastic hinges is evaluated.

Key words: **Reinforced Concrete, Plastic Hinge, Rotation Capacity, Size Effect, Strain Localisation, Damage Zone, Bond, Yielding**

1 Problem statement

Design of concrete structures can be based on various principles, like linear or non-linear elasticity or plasticity. Especially in the case of the last two methods, the capacity of reinforced concrete members to redistribute moments is essential. With regard to safety, the ductility of structures may

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be as important as their strength. Therefore in the past considerable research has already been carried out in the field of rotation capacity of structures. The introduction of new types of steel, and high performance concrete's again raised the interest into this subject. Furthermore it was acknowledged, that none of the models that have been developed to determine the rotation capacity is totally satisfactory. It has for instance been suggested by some researchers [HIL 88], [HIL 89], [MAR 93] that the rotation capacity of plastic hinges is member size dependent, approximately inversely proportional to the beam height. Some experimental work in this field has been done in the past, where such a tendency has been noticed [MAT 65], [COR 66], [CED 90], [BOS 92]. However, so far the phenomenon of size dependence of rotation capacity of plastic hinges remains non-explained nor fully understood. This phenomenon may be quite important since structures in daily practice are generally much larger than laboratory specimens. Therefore verification of calculation models with laboratory tests could, in spite of apparent agreement between tests and theory, lead to unconservative design recommendations.

For a long time, the size effect has been explained statistically as a consequence of the randomness of the material strength, particularly by the fact that in a larger structure it is more likely to encounter a material area of smaller strength, [MIH 81], [MIH 83]. Later, however, it has been recognized that structural size effects are mostly a consequence of the way cracks propagate. It has been postulated that the size effect can properly be explained by the energy release due to fracture growth, producing damage localisation instabilities and that the randomness of material fracture properties plays only a negligible role in setting the size effect, [BAZ 84], [BAZ 91]. Hence, one of the possible explanations for the increase of ductility with decreasing member size originated from studies on failure localisation in concrete in compression. It has been found in concentric compression tests on concrete prisms that the ductility of the specimen apparently increases when its height decreases i.e. the compressive failure tends to be less brittle with decreasing size of the specimen [MIE 84], [MIE 91], [VON 92]. Such a behaviour can be explained by strain localisation, in a way similar as for softening in tension: just like cracking in tension, the deformation in compression in the post-peak regime is localised in certain zones, whereas the material outside these zones unloads when the specimen is subjected to imposed deformation.

The principle of failure localisation in compression and the consequent size dependence of the stress-strain behaviour of compressed concrete may be adopted for the analyses of flexural members as well: the size-related increase of ductility can be associated with the effect of the fracture processing within a damage zone of limited length and with the size dependent value of the ultimate strain of the concrete, approximately inversely proportional to the beam height [HIL 89], [MAR 93]. The complete stress-strain curve of concrete in compression, determined on the basis of this approach, has thus a size dependent softening branch and is not a pure material property but rather a structural characteristic. Still, it must be remembered that compressive failure of concrete is a three-dimensional process, highly sensitive to the boundary restraints. With respect to the fracture process in the compression zone of the flexural member this means that both the degree of confinement provided by the structure and the crack formation at the tensile side of the beam influence the strain development in the compressive damage zone. Accordingly, the strain localisation in the tensile zone of the member has to be considered as well when evaluating the size dependent phenomenon of plastic hinge formation in RC members.

This links the problem directly to the mechanism of tension stiffening, or - more specific - to the correlated mechanisms of deformation of stressed reinforcement and bond between steel and surrounding concrete. While the characteristics of the reinforcing steel can easily be established, a lot of uncertainties remain concerning the (size dependent) bond behaviour in a hinge region. To be capable of modelling deformations in a plastic hinge, a bond model must represent the bond properties in a wide range of steel strains, including the post-yield regime. Most currently available bond models have been based on pull-out tests with short embedment lengths where yielding of the steel was not reached. Consequently, they reflect only the softening of the concrete surrounding the bar, but not the effects associated with 'softening' of the bar itself. Additionally, since in most of the tests the bar diameter has not been varied systematically, experimental bond stress - slip relationships generally cannot represent the effect of the bar size. Yet, it has been experimentally observed [SHI 87], [ENG 92] that the local bond stress-slip relationship is considerably effected by yielding of the reinforcing steel and that the plastic deformations are strongly underestimated when calculations are based on an unsuitable bond stress-slip relationship, e.g. [ELI 83], [CEB 93]. Furthermore, internal cone-shaped cracking, which is likely to cause a significant part of the local slip, is bar diameter dependent.

An investigation described in detail in [BIG 99], was directed to achieve a better understanding of the size dependent behaviour of reinforced concrete on the basis of experimental work and extensive parameter studies, developing a new rational model for analysing the behaviour of plastic hinges in RC members. The analysis of flexural beams and slabs under monotonically increasing loads was considered. Only bending failure modes were considered: it was assumed that all other failure modes can be excluded because of sufficient resistance against shear, torsion etc. In particular slender members without confining reinforcement, representative for most slabs in practice, were studied.

2 Introductory test series

2.1. Test set-up

In order to generate the basis of a consistent model for the calculation of the rotation capacity of plastic hinges in RC members a systematic experimental study has been carried out. The study has been limited to the case of bending without axial load, utilising simply supported slender beams loaded at mid-span. The scope of the test series covered the investigation of three major parameters: member size, reinforcement ratio and concrete type. Specimens of three different sizes were tested, each of them having the same ratio of width b, effective height h and span l. Moreover, for the whole test series the ratio of the loading plate length to its width was kept constant. Table 1 gives the characteristics of all tested beams. Figure 1 shows a comparison of specimen dimensions for a set of beams with equal reinforcement ratio.

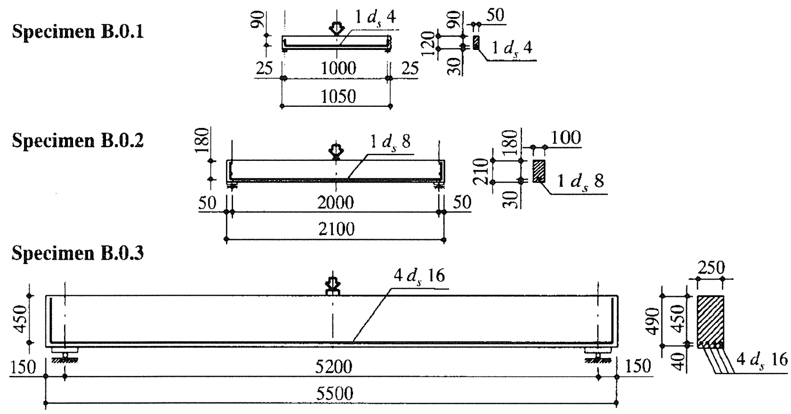

Figure 1. Geometry of test specimens in a series with reinforcement ratio ρ_s = 0.28%

The specimens were provided with longitudinal bars at the tensile side of the beam only; no stirrup reinforcement was used. Two different reinforcement ratios ρ_s were chosen, namely 0.28% and 1.12%. All specimens were reinforced with ribbed bars. Use was made of hot rolled ribbed reinforcing steel FeB 500 HWL in all but one test specimen: B.0.1.4. was reinforced with cold worked ribbed steel FeB 500 HKN. The actual values of the yield stress f_y, the tensile strength f_t and the elongation of the steel at maximum load from ε_u for the whole test series are given in Table 1 along with the concrete characteristics: the cube compressive strength f_{cc} and the splitting tensile strength f_{cts}. Two different concrete mixes were used with the maximum aggregate size $d_{a\,max}$ varying from 4 to 16 mm (as denoted by the last digits of the specimens identification codes). In total ten beams were tested, each one with a unique combination of test variables.

Table 1. Characteristics of test specimens

	Size			Concrete			Steel		
	d	B	L	f_{cc}	f_{cts}	ρ_s	f_t	f_y	ε_u
Code	[mm]	[mm]	[mm]	[MPa]	[MPa]	[%]	[MPa]	[MPa]	[%]
B.0.1.4	90	50	1000	31.71	2.47	0.280	590	678	3.60
B.0.2.4	180	100	2000	34.40	2.37	0.279	562	641	9.17
B.0.2.16	180	100	2000	40.57	3.16	0.279	562	641	9.17
B.0.3.4	450	250	5200	33.52	2.31	0.279	568	641	9.36
B.0.3.16	450	250	5200	37.25	2.77	0.279	568	641	9.36
B.1.1.4	90	50	1000	33.12	2.51	1.118	562	641	9.17
B.1.2.4	180	100	2000	35.27	2.33	1.117	573	661	9.31
B.1.2.16	180	100	2000	39.76	2.91	1.117	573	661	9.31
B.1.3.4	450	250	5200	32.26	2.26	1.116	550	650	9.27
B.1.3.16	450	250	5000	35.43	2.73	1.116	550	650	9.27

The test series has been intended to determine the size effect on the rotation capacity of plastic hinges and to investigate the localisation process in the hinge region, in particular in the compression zone of the member. To measure longitudinal deformations high accuracy clip gauges were applied, whereas to register deformations in the transverse (lateral) direction linear variable displacement transducers were used. The steel strains were determined on the basis of the deformations registered with clip gauges at the side faces of the beams in the locations corresponding to the position of the reinforcing bars. All tests were carried out load-controlled until the specimen reached about 85% of its estimated maximum capacity and displacement-controlled afterwards.

2.2 Discussion of test results

The relation between rotation capacity and effective height of the beam has been analysed. Conform the definition given in [CEB 93], the rotation capacity has been defined as the difference between the total rotation of a member at peak load and its total rotation at the onset of yielding of the reinforcement neglecting the post-peak deformation capacity. The rotation has been calculated by integration of the curvature along the longitudinal axis of the member, while the curvature was obtained directly at each location from measurements of the strain in the compression zone and at the position of the reinforcing bars.

A clear relation between the member size and its rotation capacity was found: members with smaller dimensions showed a much more ductile response to the load than members with the same characteristics but larger dimensions. This relation has been observed for both reinforcement ratios investigated, i.e. both for members which fail due to exceeding the deformation capacity of the steel bars and for members in which crushing of the concrete prevails after yielding of the steel. For a reinforcement ratio of $\rho_s = 0.28\%$ a twofold increase of the rotation capacity as the effective height of the beam decreased from 450 to 180 mm, while for a reinforcement ratio of $\rho_s = 1.12\%$ a fourfold increase of the rotation capacity was observed with the effective height of the beam decreasing from 450 to 90 mm, see Figure 2. The low value of the rotation capacity obtained for specimen B.0.1.4 shows that the use of cold worked steel can reduce the member ductility significantly, compared to cases where hot rolled (or heat treated) steel is used.

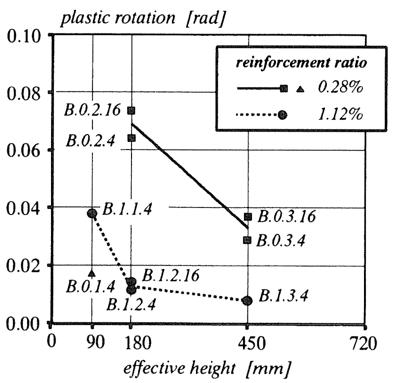

Figure 2. Rotation capacity at peak load versus effective height of the member

The strong sensitivity of the rotation capacity of plastic hinges to the member size is likely to be explained by strain localisation in the hinge region, e.g by the size dependence of the concrete in compression arising from the member size independent dimensions of the localisation zone. This believe corresponds with earlier observations concerning the size effect on beams that failed due to concrete crushing, given in [MAT 65] and [COR 66]. In the tests discussed here, in the case of members which failed due to concrete crushing no direct link is found between the member size and the extreme concrete compressive strain values measured in the hinge region, although it should be stressed that the strain values were far in excess of the conventionally assumed value of the ultimate concrete compressive strain of 0.35% and amounted even up to around 1%, see Table 2. On the contrary, as far as the average compressive strain values are concerned (both calculated over one and two times the effective member height) clearly larger strains were found for the smallest specimens. Note that the values interpreted as the concrete compression strain were assessed as the displacement measured between the measuring points at the outer face of the member (surface shortening) divided by the gauge length. Besides the clearly visible effect of the measuring length, also the position of the measuring device with respect to the failure localisation zone influences the measurements and shall be involved in the through interpretation of the measurements. In the case of members that fail due to steel rupture a large variation in maximum measured compressive strain values is found, which is not surprising pondering the previously mentioned effect of the measuring length and of the measuring device location. Besides, the deformation of the reinforcement and, hence, the actual (ultimate) stress-strain characteristics of the steel effect significantly the equilibrium of stresses in the hinge region. As far as the average compressive strain values are concerned, a tendency for decreasing strain values with increasing member size is found also in case of failure due to rupture of reinforcement. Obviously this last conclusion holds solely for members reinforced with steel characterised by similar ductility: the results for specimen B.0.1.4. are not involved in this analysis, pondering the significantly lower ductility of the reinforcement used in this case.

Table 2. Measured concrete compressive strains at maximum load

Effective heigth h [mm]	Average ε_c^{max} ($l_{gauge} = 2h$)	Average ε_c^{max} ($l_{gauge} = h$)	Largest ε_c^{max} ($l_m = l_{gauge}$)	Test ; l_{gauge} [mm]	Failure type
90	0.00425	0.00652	0.01354	B.1.1.4 ; l_{gauge} = 28 mm	concrete crushing
180	0.00282	0.00326	0.00546	B.1.2.4 ; l_{gauge} = 28 mm	
	0.00233	0.00268	0.00510	B.1.2.16 ; l_{gauge} = 28 mm	
450		0.00398	0.01283	B.1.3.4 ; l_{gauge} = 28 mm	
90	0.00095	0.00108	0.00167	B.0.1.4 ; l_{gauge} = 28 mm	steel rupture
180	0.00109	0.00144	0.00454	B.0.2.4 ; l_{gauge} = 28 mm	
	0.00096	0.00124	0.00750	B.0.2.16 ; l_{gauge} = 28 mm	
450		0.00054	0.00090	B.0.3.4 ; l_{gauge} = 28 mm	
	0.00088	0.00090	0.00194	B.0.3.16 ; l_{gauge} = 28 mm	

The analyses of the crack width development reveal in all cases a strong concentration of deformations at the mid-span cracks. Moreover, after the peak load had been reached, the deformation in the main mid-span cracks increased while some other crack widths decreased. It appeared that Bernoulli's principle of plain sections remaining plain loses its validity at a certain stage of loading. A comparison of the measured crack width values with these estimated on the basis of the [CEB 93] showed a strong discrepancy, in particular in the range beyond steel yielding. A variation of the concrete mix composition by increasing the maximum aggregate size in the range investigated does not seem to have a significant influence on the available plastic rotation of the members. For a detailed discussion of the test results reference is made to [BIG 99].

3 Calculation model for rotation capacity

3.1. Model formulation

In this study the mechanism of plastic hinging has been analysed taking into account the strain localisation in the damage zones in the hinge region, both in compression and in tension, and modelling bond between steel and concrete by virtue of a fracture-mechanics-based approach. This complex strategy permits a fundamental description of various aspects of the size dependence of plastic hinge behaviour.

3.3.1. Modelling of plastic hinge behaviour

From a number of experimental and theoretical studies ([DIL 66], [BAC 67], [BAC 70], [SVE 91], [SIG 94], [SIG 95]) it is concluded that, depending on the magnitude of the shear force in the critical region of the RC member, two significantly different types of plastic hinges can develop. So-called flexural crack hinges (FC-hinges) occur in the member zone in which the bending moment is predominant, while shear crack hinges (SC-hinges) develop in the member zone where in addition to a bending moment a considerable shear force is acting. In the case of FC-hinges, plastic deformations concentrate in a single or a very few cracks, so that the hinge rotation capacity remains relatively low. On the contrary, SC-hinges exhibit a significantly increased rotation capacity due to the inclined flexural-shear cracks which enlarge the length of the plastic hinge, provided that the member possesses a sufficient shear capacity. Since basing the analysis on a FC-type of hinge yields lower limit values for the ultimate rotation and pondering that this type of hinge prevails in slender (slab) members, which are going to be analysed in the scope of this study, the proposed model deals solely with the FC-type of plastic hinge.

The rotation of a plastic hinge is directly related to the length of the plastic hinge. In this respect, not only the crack spacing and bond of steel to concrete between the cracks but also the typography of the developed crack pattern is of influence. There is nearly an equal probability of developing a symmetrical crack pattern with and without crack at the mid-span. Depending on the position of the cracks in the presence of a moment gradient, the length of the bars where plastic deformations localise significantly differs and a huge supplementary rotation capacity is foreseen if a favourable crack pattern occurs. Therefore in the proposed model a distinction is made between two particular failure modes: ductile failure with dispersed cracking (which provides a lower bound for the rotation capacity) and non-ductile failure with a single major crack (which provides an upper bound for the rotation capacity). Note that it is indispensable to keep this phenomenon in mind when interpreting test results and that it is not conservative to consider anything but the lower bound values when evaluating the available rotation capacity of flexural members in a general case.

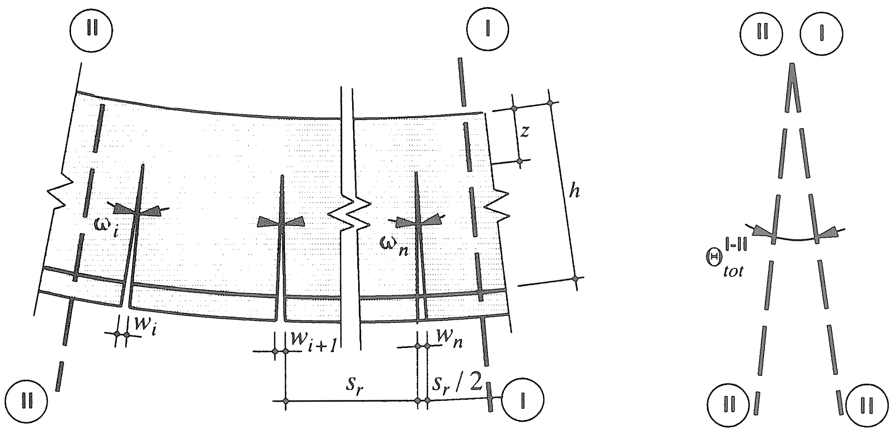

Figure 3. Modelling approach - discretisation of FC-hinge

To predict the deformation capacity of a member with given dimensions, reinforcement layout and material characteristics, firstly the crack spacing s_r must be determined. Secondly, the distribution of bending moments along the member axis has to be calculated at the loading level considered. Next, a RC member with a crack spacing s_r is subdivided into cracked elements (FC-elements) of length s_r, considering a feasible pattern of flexural cracks, as shown in Fig. 3. It is assumed that each of the discrete flexural crack elements (FC-element) is subjected to a bending moment only.
The sectional forces at each crack are determined by sectional analysis, assuming the validity of Bernoulli's principle of plane sections remaining plane. Since the constitutive law of concrete in compression, described in section 3.3.2, takes into account failure localisation in the damage zone an iterative procedure is required: value of the compressive strain of the mostly stressed fibre is corrected until the depth of the compression zone and the size of the localization zone are found which satisfy the equilibrium of forces and compatibility of deformations. Knowing the stresses and strains at every crack, the elongation of the tensile chord of strains each FC-element can be derived in an iterative calculation procedure using a bond model suitable to describe the bond behaviour in the hinge region, see section 3.3.3. Strain localisation in compression is accounted for when calculating the total deformation of the mostly stressed compressive fibre by assuming a constant strain value over the length corresponding to the computed extension of the localisation zone, for the case that damage localisation in compression is found to occur, and a linear strain distribution is assumed in the range outside the damage zone. The elementary rotations of the FC-elements

Θ_{el} are then obtained integrating the calculated strains in the reinforcement and in the upper fibre of the member compression zone, over each element length s_r . The summation of the elementary Θ_{el} rotations gives the total rotation in the hinge Θ_{tot}. In this step each FC-element, in which plastic steel deformations occur at the ultimate loading state, must be taken into account. Accordingly, the rotation capacity of the hinge $\Theta^{(p)}$ may be computed as the difference between the total rotations in the hinge at ultimate load $\Theta^{(u)}$ and at the onset of yielding of the reinforcement $\Theta^{(y)}$.

The adequacy of the model was evaluated in a step-by-step verification procedure. Firstly, the elementary models included in the calculation model for the rotation capacity (such as the CDZ model for concrete in compression and the new bond model) were separately verified using the results of the initial tests developed especially to study these phenomena, see [MAR 93], [BIG 95]. Note that the extremely high values of the concrete compressive strain measured in the tests discussed above provided important information for the validation of the CDZ model applied to describe the concrete behaviour in the compression zone of a flexural member. Secondly, the proper link between the elementary models was verified and its ability to model a selected aspect of the structural member behaviour was checked, e.g. flexural crack spacing and tension stiffening effect, see [BIG 96]. Finally, the prediction of the deformation capacity for a broad scope of variables was examined, both with respect to the influence of structural materials and member geometry, using the test results reported in [BIG 92] and [BÜH 91], see Section 3.2 and [BIG 99] for more detail. It is believed that with this verification the validity of the model has been confirmed so that it can be used for further parameter studies.

3.1.2. Modelling of concrete and steel

The constitutive models used for concrete in tension and in compression are discussed below. The essential issue in studying size effects in structural concrete is taking the failure localisation into account. To fulfil this primary requirement, the concrete was modelled by virtue of a fracture mechanics approach. To that end the Fictitious Crack Model and the Compressive Damage Zone Model were adopted to describe the behaviour of concrete in tension and in compression, respectively. Such a routine offers a possibility to study explicitly the influence of concrete fracture toughness on the member behaviour.

The Fictitious Crack Model (FCM) is a well known general tensile-softening model characterising the fracture of concrete in tension, the fundamental principles of which are given in [HIL 83]. In the proposed calculation model the FCM was used with the following simplifications. Linear elastic response in tension was assumed prior to the peak, modelled as a straight line with a slope equal to the modulus of elasticity of the concrete E_c. A bi-linear softening relation, as proposed by [ROE 80], was chosen. For normal density concrete the localised deformation at failure w_o was assumed to be concrete strength independent, while the values of the model constants defining the slope of the softening branches were linked to the strength of the concrete, see [BIG 99]. In order to obtain a complete stress-strain relationship, which could be used in sectional analyses, it was assumed that the distance between the successive damage zones equals the crack spacing s_r, and consequently the length of one localisation zone becomes s_r.

To model failure of concrete under compression the Compressive Damage Zone (CDZ) model was applied [MAR 93]. This model describes the response of concrete under compression using the same principal ideas as used in the FCM: prior to the peak load the longitudinal strain in the concrete is regarded as uniformly distributed along the whole length of the specimen, whereas after the peak load longitudinal tensile cracks and lateral deformations, which lead to axial splitting and the formation of a localised shear-band in the damage zone, are assumed to occur within a limited part of the specimen (damage zone). In modelling cases where a strain gradient is present, e.g. the bending case, it is considered that the difference in deformation between different fibres influences both the strength and the ductility. In the proposed calculation model the CDZ model was used with the following simplifications. A bi-linear curve represented the pre-peak response, with the slope of the unloading curve assumed to be equal to the modulus of elasticity of the concrete E_c. The softening curves, both describing the energy absorption in the longitudinal cracks and in the localised shear band, were approximated with straight lines. To formulate the complete stress-strain relationship in the case of bending it was assumed that the damage zones are close together and that the distance between the successive damage zones, i.e. the length containing one damage zone, is equal to the damage zone length.

It is indispensable to provide a proper description of the reinforcing steel behaviour in order to secure a satisfactory representation of the member performance. Information concerning steel properties for different steel ductility classes are available e.g. from [RUS 93]. Note that in test simulations the actual steel properties should be used, if available. For the purpose of application in the proposed calculation model the stress-strain relation of the reinforcing steel was simplified by a polygon with 6 points, which permitted a sufficiently accurate representation of the characteristic shape of this curve. Pondering the uncertainties in defining the post-peak part of the steel stress-strain curve and considering that steel necking under constant load leads to brittle failure, only the ascending part of the stress-strain curve was taken into account.

3.1.3. Modelling of bond

As far as the bond model for reinforcing units is concerned, an essential lack of information has been found, especially with regard to the bond behaviour of yielding steel. With this concern a study reported in [BIG 95] was performed, aiming at developing the basis for a general bond model for ribbed bars, which accounts for the effect of steel strain on bond in normal strength and high strength concrete. The new bond model presented in [UIJ 96] takes into account the concrete quality, the bar contraction (significant after steel yielding), the degree of confinement and the corresponding mode of bond failure.

For the formulation of this new general bond model the concrete confining capacity was used as a starting point. It was found to have a decisive influence on the ultimate bond resistance and on the mode of bond failure. Bond splitting prevails when the confinement is not sufficient to prevent the radial cracks from penetrating the whole concrete cover. With enough confinement the concrete teeth in front of the ribs are sheared off resulting in a new cylindrical sliding plane and pull-out failure takes place. In the proposed model it is taken into account that the confinement delivered by the concrete not only depends on the smallest concrete cover, but a greater part of the concrete cross-section may contribute to it and the actual member geometry must be involved in determining the degree of confinement.

The concrete confining capacity was analytically estimated using the model of a thick-walled cylinder-based on the partially-cracked-elastic-ring concept, established in [TEP 79], and further developed in [VEE 90]. The fracture mechanics based description of the behaviour of concrete loaded in tension incorporated the FCM. For the transition from the relationship between the radial stress and the radial displacement, which represents the concrete confining capacity, to a bond stress-slip relationship the rebar was conceived as a smooth conical bar transferring the bond stress through dry friction to the concrete. The effect of the transverse deformations of the pulled bar and of the smoothening of the sliding (interface) surface on the release of the radial strain in the confining concrete was included.

By this means, the degree of confinement delivered by the outer surrounding concrete was related not only to the relative displacement at the bar-to-concrete interface but to the state of stress in the rebar itself as well. As long as rib bearing is the force transferring mechanism, the state of stress in the rebar has a negligible influence on the bond resistance of ribbed bars. Hence, in the case of splitting bond failure and in the absence of additional confinement the bond stress-slip relationship is directly related to the clamping action of the concrete surrounding the bar, which is proportional to the depth of the effective concrete cover. When for pull-out failure the force transfer mechanism changes from rib bearing to friction the local transverse deformation can not be disregarded and in this case an increasing steel stress results in a release of the radial strain due to the Poisson effect and, thus, in a reduction of the bond stress, which becomes pronounced when the steel starts to yield. Therefore in the case of pull-out failure the bond stress is primarily a function of the slip and the steel strain. Note that this implies a significant influence of the stress-strain characteristics of the reinforcing steel on the bond of yielding of steel in case of pull-out failure.

Hence, for given construction materials characteristics and well defined specimen geometry there is an unique bond stress - steel strain - slip relationship, however the bond stress-slip relationship is not unique and dependent on the actual loading and boundary conditions. Figure 4 shows examples of bond stress-slip relationships obtained for two types of bond failure for a bar diameter d_s = 20 mm and a concrete strength f_{cc} = 35 MPa. Note that the proposed bond model has been tuned for bars with a medium relative rib area.

(a)

(b)

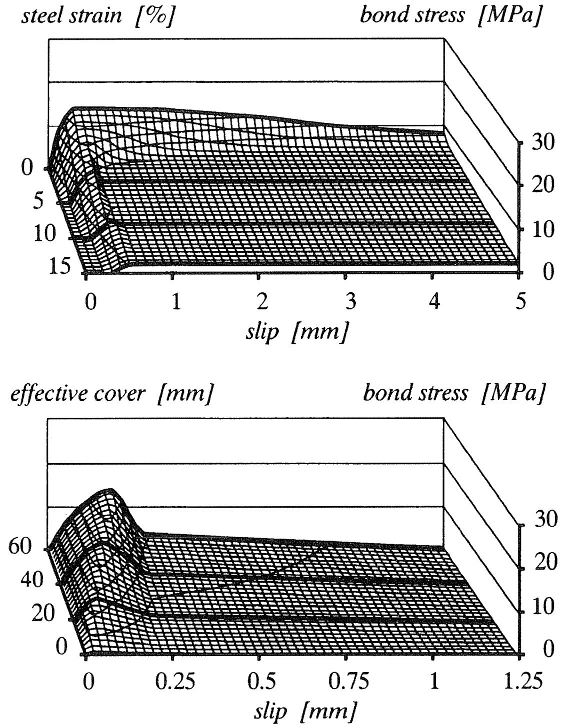

Figure 4. Bond stress – slip relationships for bar diameter d_s = 20 mm and concrete strength f_{cc} = 35 MPa: (a) pull-out bond failure for; (b) splitting bond failure

3.2. Model verification

An extensive verification of the proposed model for calculation of the rotation capacity has been reported in [BIG 99]. Here emphasis is put only on its capacity to predict the member size dependence of the rotation capacity and to capture the effect of steel ductility on the deformation capacity of the members. In Figure 5 the results of the simulation of the introductory tests series described in Section 2 are compared with the actual measurements of this test series. In the diagrams the crack pattern dependent lower and upper simulation bounds are indicated, between which the actual value is to be expected (the solid mark represent the simulation with the crack pattern closest to that actually observed in the test). In general, a satisfactory correlation between the measured and the calculated values is found. A very strong effect of the type of crack pattern (with or without crack at the mid-span) on the available rotation capacity is clearly visible. Especially in cases with low reinforcement ratio this effect becomes very pronounced, for the combination of parameters used in the simulations. For the members analysed here the difference in the available rotation capacity may amount in some cases to more than 50%, which is of similar magnitude as the variation attributed to the member size influence in the range investigated. This should be kept in mind, considering that in practice the location of the first crack, and thus the type of crack pattern, depends on the stochastic distribution of the material strength or is enforced by the presence of transverse reinforcing bars. In both cases this is hard to predict (contrary to laboratory tests, in practice the location of transverse bars is never known) and it is therefore not conservative to assume anything but the lower bound condition when evaluating the available rotation capacity of flexural members in a general case.

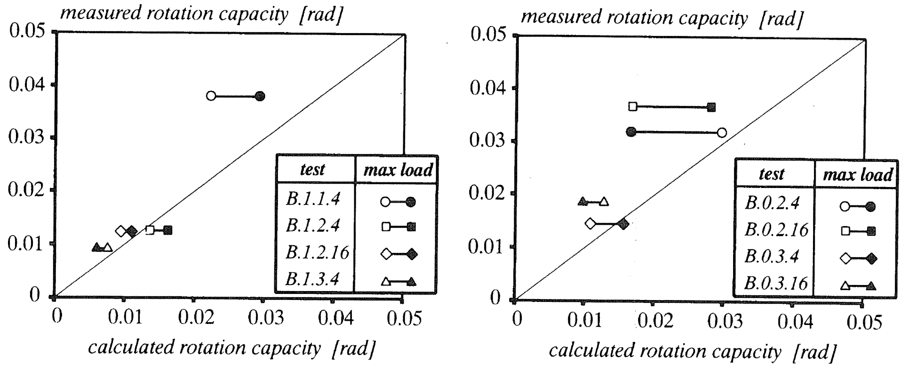

Figure 5. Comparison of rotation capacity measured in the introductory test series and calculated with the proposed model for two reinforcement ratios: 1.12% (left) and 0.27% (right).

The results of the simulation for the lower reinforcement ratio (failure due to rupture of steel) shown in Figure 5 tend to fall slightly bellow the measured values – which is likely to be the result of assuming in the simulations the lower bound ultimate strain value of 9% (compare with the average values in Table 1). This points directly at the strong influence of the material properties on the available rotation capacity. Numerous tests have been carried out to study the correlation between the deformation capacity of the flexural members and the ductility of the reinforcing steel. The ability of the proposed model to cope with this problem is analysed for a wide range of steel ductility classes used in tests on single span slabs, reported in [BÜH 91] (H x B x L = 160 mm x 800 mm x 2000 mm, f_{cc} = 33 MPa, reinforcement ratio of ρ_s = 0.24%). In this test series numerous samples were taken from the steel bars to determine the main steel properties in series of independent tests performed in three laboratories. Considering the large scatter in the reported values in the simulations discussed here in some cases, besides the average for the whole set of standard tests also the average for the single series (one laboratory) was used to perform calculation of rotation capacity. In Table 3 the results of the simulations are compared with the reported experimental values of rotation capacity $\Theta^{(p)}$. Considering the imposed crack pattern (sheet-metal strip inserted at the mid-span) only the lower bound simulation results are listed. The agreement between the measurements and the simulation results is in general very good, thus showing the capability of the model to take into account the influence of the steel characteristics on the available rotation capacity. Differences obtained for only slightly altered steel characteristics indicate the sensitivity of the structural response to changes in the specific material characteristics of the reinforcement, in particular in cases where failure is governed by the rupture of reinforcement.

Table 3. Summary of test results from [BÜH 91] and corresponding simulation results

	Test results	Steel characteristics				Simulation results	
Code	$\Theta^{(p)}$ [rad]	average from	f_y [MPa]	f_t [MPa]	ε_u [%]	$\Theta^{(p)}_{calc}$ [rad]	$\Theta^{(p)} / \Theta^{(p)}_{calc}$ [-]
RPL1	0.0118	whole set	703	732	2.00	0.0059	2
		selected series	672	716	2.75	0.0079	1.49
RPL2	0.0292	whole set	502	594	10.4	0.0393	0.74
		selected series	521	607	10.4	0.0337	0.87
RPL4	0.0110	whole set	590	629	4.40	0.0096	1.12
		selected series	585	626	4.50	0.0098	1.12
RPL5	0.0161	whole set	590	629	4.40	0.0096	1.68
		selected series	590	636	5.20	0.0129	1.25
RPL6	0.0165	whole set	532	612	4.30	0.0168	0.98
		selected series	-	-	-	-	-

In this respect it should be stressed that for the assessment of the rotation capacity from the experiments the accuracy of the estimation of the maximum load is significant. Considering the shape of the load-deflection diagram, which is almost horizontal at the level close to the maximum load, a possible inaccuracy in the interpretation of data may take place. This is especially noticeable in the case of members with low reinforcement ratio where steel rupture is critical, pondering the very low hardening ratio in the range of the highest steel strain. This should be kept in mind while conducting and interpreting this kind of experimental investigation.

4 Parameter studies

4.1. Parameters choice

Interpreting the size effect in terms of system toughness is a necessary step that must be taken when reinforced concrete members are analysed. If the toughness of a reinforced structure is expressed by the quotient of the energy needed to fracture the structure and the stored elastic energy [ELF 89], the fracture characteristics of both materials, the structural dimensions and the stiffness, related to bond characteristics and reinforcement ratio, will prove to be influential. Such a large number of independent influential parameters causes a difficulty in formulating a simple member size dependence rule for reinforced concrete members. The purpose of the parameter study is thus only to evaluate the importance of the size effect in practical design situations.

When analysing the member size dependence of RC members not only the overall member dimensions must be regarded but a number of other geometry dependent factors as well. Here the consequences of scaling the members, as encountered in practice with regard to the detailing of the reinforcement, are minded. Note that the bar size and effective concrete cover thickness dependence are implied in setting the bond between steel and concrete. Furthermore, the tension stiffening of the steel, which is related to bond strength, bonded area and tensile cord reinforcement ratio, effects the overall member deformations. Finally, the proportion between the reinforcement ratio of the tensile cord and the reinforcement ratio of the member must be considered, in order to account for the effect of a non-uniformly distributed reinforcement on the bending stiffness of members with different sizes, amount and arrangement of reinforcement.

Although it is acknowledged that a number of parameters is of importance for the magnitude of the member size dependence of the rotation capacity, in this study some limitations were applied. Only single reinforced slender members without shear and confining reinforcement were analysed. The concrete strength was not included as a parameter in the analysis and the following concrete characteristics were used in the simulations: f_{cc} = 35 Mpa, f_{ct} = 2.80 MPa, E_c = 29055 MPa, G_f = 109.2 N/m. Since the characteristics of reinforcing steel are likely to have a minor influence on the magnitude of the size effect [BIG 99], the simulations were performed with one type of hot rolled steel, characterised by f_y = 550 MPa, f_t = 594 MPa (f_t / f_y = 1.08), ε_u = 5.0%.

The major parameters in this analysis are related to the geometry of the members. The effect of a proportional increase of the member dimensions (effective height h, width b, span l) while keeping the reinforcement ratio ρ_s constant was studied. The simulations were performed for one slenderness ratio l/h = 12, for five h values (varying from 100 to 1200 mm), and for three reinforcement ratios ρ_s (0.25%, 0.50% and 1.00%). With respect to the latter, the difficulties in scaling the members, as encountered in practice with regard to the detailing of the reinforcement, were minded. Numerous possibilities to achieve the same reinforcement ratio with a different number of bars - and thus different bonded area - asked for analysing the effect of reinforcement layout on the deformation capacity of the member. It is well known that in general increasing the number of bars while decreasing the single bar diameter causes a more dense crack spacing and smaller crack widths at the serviceability limit state. It is however important to know to what extend the ultimate deformation capacity will be influenced and what impact it will have on the magnitude of the member size dependence. Different reinforcement layouts were studied in combination with different member sizes in order to find the answer to this last question. The following cases have been chosen:

- **I** constant bar diameter d_s = 16 mm, bar spacing adjusted to the actual member size and to the required reinforcement ratio
- **II** constant bar spacing s_s = 100 mm, bar diameter adjusted to the actual member size and to the required reinforcement ratio
- **III** constant bar spacing s_s = 50 mm, bar diameter adjusted to the actual member size and to the required reinforcement ratio
- **IV** constant bar diameter d_s = 8 mm, bar spacing adjusted to the actual member size and to the required reinforcement ratio

In order to limit the number of geometry-dependent variables, the bottom cover on the bars was in all cases kept equal to 2 d_s.

4.2. Simulation results

In Figure 6 the effect of member size on the rotation capacity is illustrated. In all cases both the lower and the upper bound values are shown, that follow from assuming different critical crack patterns in the simulations (discussed in Section 3.3.1). The actual layout of the reinforcement is indicated.

The simulations prove the empirical observation that the member size influences the deformation capacity. Both rotation at maximum load and rotation capacity decrease with increasing member size, while the rotation at the onset of yielding of reinforcement is almost size independent. The decrease of the rotation at maximum load and of the rotation capacity as a function of the increasing effective member height is fast for small members and tends to be slower as the dimensions of the members increase, which agrees with experimental evidence. The reinforcement ratio has an obvious effect on reached rotation values. Note that for the combination of parameters studied in these simulations, failure is governed by concrete crushing for ρ_s = 0.50% and 1.00%, and by steel rupture for ρ_s = 0.25%. For the investigated sizes it is found that, unless the fracture properties of the concrete are kept constant, members which posses higher rotation capacity and, hence, exhibit more stable crack growth are more sensitive to the change of size, both in the case of steel and concrete failure. On the contrary, members which posses a lower rotation capacity due to the less stable crack growth are less sensitive to a size effect.

There is no meaningful distinction between the intensity of the size effect observed for the reinforcement ratios 0.25% and 0.50%, although the effect is slightly less pronounced for the lower of the two ratios. This slight difference it is not very surprising, ponder the fact that for a lower reinforcement ratio, in the case of steel rupture, the concrete in compression will be less exhausted, so that its contribution to the member size dependence will be effectively reduced. This corresponds well with the theories that explain member size dependence of the rotation capacity through size dependence of the concrete response in compression.

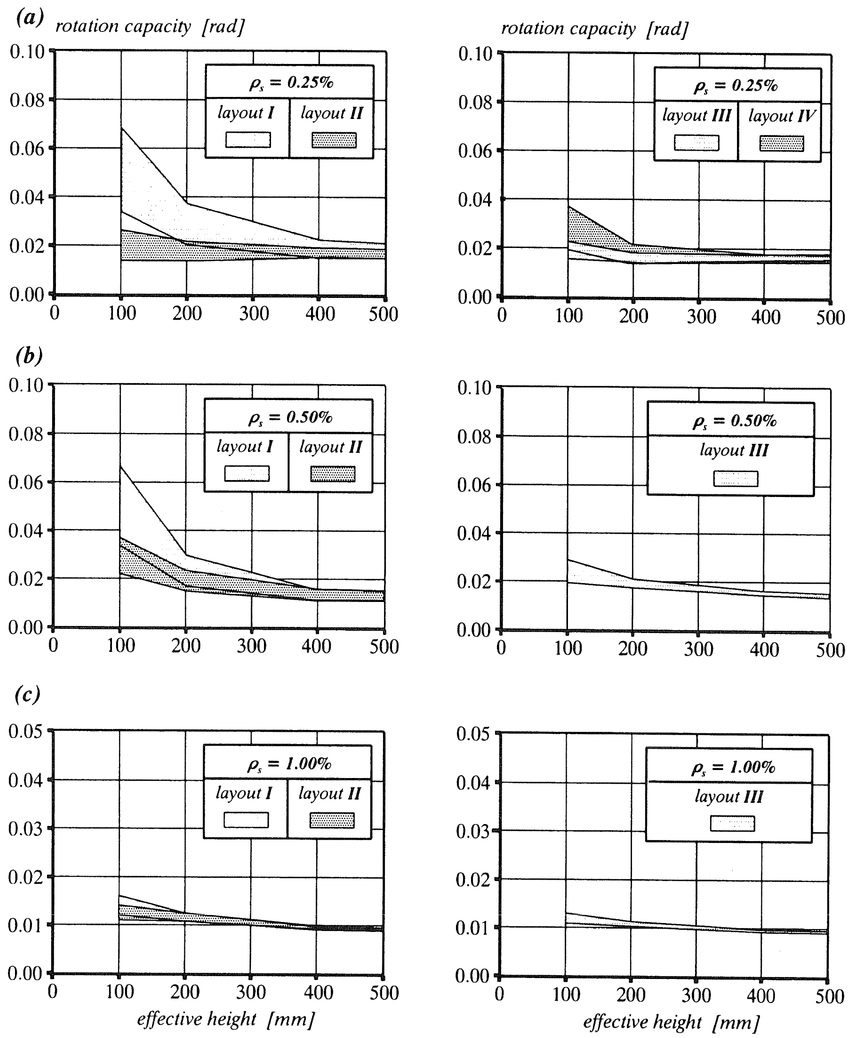

*Figure 6. Rotation capacity as a function of the effective member height – simulation results: (a) for ρ_s = 0.25% and reinforcement layouts **I-IV**; (b) for ρ_s = 0.50% and reinforcement layouts **I-III**; (c) for ρ_s = 1.00% and reinforcement layouts **I-III***

For the investigated sizes, the size effect is clearly stronger for ρ_s = 0.50%, and is much less pronounced, yet still visible for ρ_s = 1.00%. This points at the effect that the deformation of the tensile cord of the member can have on the size dependence of the overall member response. Because of requirements of compatibility and equilibrium, the depth of the compression zone at failure and the corresponding concrete deformations are related to the steel elongations at ultimate load. Considering the effect of strain localisation in the concrete in compression, it can be understood that a decrease of the depth of the compression zone must result in an active increase of the member size dependence. In this sense, the deformations of the tensile cord enhance the effect of strain localisation in the concrete compression zone on the overall size dependence of the plastic hinge behaviour. It should be remembered that general tensile cord deformations are, on their own, sensitive to a number of size and geometry dependent factors, e.g. there is bar size and effective concrete cover thickness dependence involved in setting the bond between steel and concrete. The importance of the bond dependent size effect can be evaluated comparing the results of the simulations obtained for different reinforcement layouts. In the case of $\rho_s = 0.25\%$, the influence of the reinforcement detailing is found to be very pronounced. A specific choice of the reinforcement layout can nearly eliminate the size dependence of the rotation capacity, if understood as a relation to the effective member depth. Here it must be stressed that this does not mean, that the size effects are eliminated from the plastic hinge performance, but they rather are superimposed in such a way that the overall member response is finally affected by a mixed type of size effect. This can be clearly seen when for $\rho_s = 0.25\%$ the relations corresponding to the layouts **I** and **IV** or **II** and **III** are compared. In the first case the reduction of bar diameter and doubling of bar number results in a decrease of the rotation capacity and a reduction of the degree of size dependence. A similar effect is observed in the second case, where again the bar number is doubled and the bar diameter reduced, but with a different initial set of values. Therefore, the major conclusion is that when speaking about member size dependence of reinforced concrete members not only the overall member dimensions must be regarded but the reinforcement configuration as well.

Finally, the results of the simulations clearly show that there is a very strong effect of the crack pattern type (with or without crack at mid-span) on the available rotation capacity. Especially in cases with low reinforcement ratio this effect becomes very pronounced, for the combination of parameters used in this simulation. For the members analysed here the difference in the available rotation capacity is of similar magnitude as the variation attributed to the member size influence in the range investigated. This should be kept in mind, considering that in practice the location of the first crack, and thus the type of crack pattern, depends on the stochastic distribution of the material strength or is enforced by the presence of transverse reinforcing bars, both being hard to predict in a general case.

From a design point of view it is important to know to what extent member size dependence may manifest in usual engineering practice. Here an attempt is made to answer this question using a simple statistical evaluation of all results obtained in the simulations discussed above.

The definition of the reference value $\Theta^{(p)}_{ref}$ is based on the following consideration. When estimating the rotation capacity in engineering practice only single influential factors are taken into account. These are basically the steel ductility, the reinforcement ratio and (to some extend) the concrete strength. The influence of factors such as such as the member size or the detailing of the reinforcement is, however fully neglected. The justification for this approach can only be given with the hypothesis that the currently used semi-empirical design rules are tuned on the basis of set of experimental results, which in safe way deals with the influence of the abovementioned geometrical factors. In other words, before the basis for validation of design rules is accepted it is necessary to know what can be the consequence of limiting it to certain range of member sizes and allowing for variety of reinforcement layouts. Imagine the situation where for a fixed reinforcement ratio results for one – reference – member size are used as the basis for estimating the available rotation capacity for a general case. As shown in the preceding simulations, even for one fixed member size a range of values of rotation capacity will be obtained if the detailing of the reinforcement is varied. Now, in order to quantify the variation in rotation capacity when changing the geometrical characteristics of the member, the reference value is defined as $\Theta^{(p)}_{ref}$, which is the average of all results obtained for the reference size member range when varying the layout of the reinforcement. In the next step for each reinforcement ratio a degree of size dependence $\Theta^{(p)}$ / $\Theta^{(p)}_{ref}$ is analysed, which is the ratio between the rotation capacity of a member with any size and the reinforcement layout $\Theta^{(p)}$ and the reference value $\Theta^{(p)}_{ref}$ defined above (the rotation capacity of a member with reference size and "average" reinforcement layout).

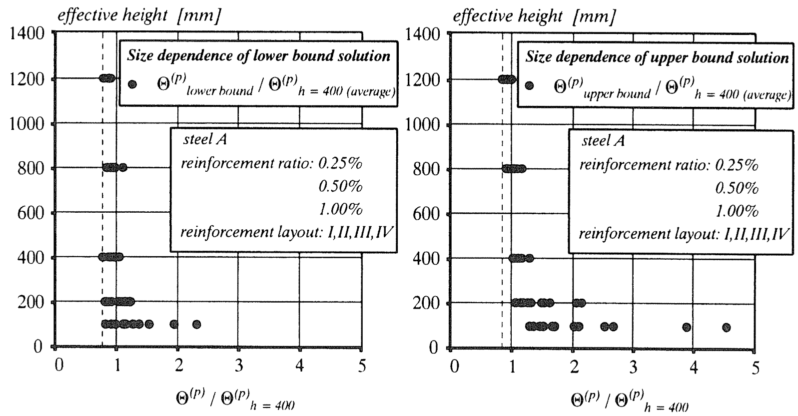

Figure 7. Degree of size dependence for a reference member size h = 400 mm as a function of the effective member height (upper and lower bounds shown)

In Figure 7 the degree of size dependence defined above is shown for the reference member height h = 400 mm. It shows that when the estimate of available rotation capacity is based on the evaluation of the behaviour of a single reference member (in this case a member with reference size and "average" reinforcement layout), an overestimation of the actual member rotation is very likely. The possible error increases significantly with decreasing member size: similar evaluation performed for reference member height h = 200 mm shows that overestimation of rotation capacity can amount up to almost 50% if h = 200 mm while only up to about 20% overestimation of rotation capacity may occur if h = 400 mm is taken as the reference member size. Here it must be stressed that this conclusion follows from the evaluation of a limited number of cases, and does not cover all the extremes e.g. with respect to the reinforcement layout. In any case, however, it is clear that taking a larger member size as a reference for further parameter studies is more conservative, i.e. a more safe approach. Therefore fixing the reference member size at h = 400 mm seems a justified choice for general experimental and analytical studies, considering the type of members studied (slabs with no shear reinforcement and slender beams with light confinement) and the dimension limits following from the range of their application.

5 Final remarks

This investigation was intended to achieve a better understanding of the behaviour of plastic hinges and their structural dependence on the basis of experimental work and extensive parameter studies, using a rational model developed to analyse the phenomenon of plastic hinging in RC members. It was found essential to consider strain localisation in the damage zones in the hinge region when analysing the deformations in the plastic hinge. To that end a fracture mechanics approach to modelling the behaviour of concrete and bond between the concrete and the reinforcement was taken. The phenomenon of size dependence as well as construction materials dependence of rotation capacity could reliably be studied by virtue of this approach.

In the course of this experimental and analytical study it has been proven that there is a clear relation between member size and rotation capacity. While the rotation at the onset of yielding of the reinforcement is almost size independent, both the rotation at maximum load and the rotation capacity increase with decreasing member size in the case of members which fail due to rupture of steel as well as in the case when crushing of concrete prevails after yielding of the steel. Yet, due to a large number of independent parameters that are involved no general size dependence rule for RC members can be formulated. Besides structural dimensions, the occurrence and the growth of the system of cracks in RC member strongly depends on the fracture properties of the construction materials and on the structural stiffness, related to the bond characteristics, the area of reinforcement and its distribution.

The importance of the geometrical effect in practical design situations has been evaluated, showing that an overestimation of the actual member rotation is very likely if the available rotation capacity is based on the evaluation of the behaviour of the reference members within a limited size range. It can amount up to about 20% if h = 400 mm or up to almost 50% if h = 200 mm is taken as the reference value. The bigger the size of the member used as a reference, the larger the choice of adequate reinforcement arrangement and the wider the size range of proportionally scaled members, which safety is not negatively influenced by consequences of the ductility reduction due to the size dependence. In engineering practice one should be aware of the fact that a poor design in combination with unavoidable size dependence of the deformation capacity of RC members can endanger the safety margin of the codes provisions.

In order to have a ductile RC structure the arrangement of the reinforcement should be related to the member size. A choice of the reinforcement arrangement can nearly eliminate the size dependence of the rotation capacity, if understood as a relation to the effective member depth. This does not mean, that size effects are eliminated from the plastic hinge performance, but that they are rather superimposed. An optimisation of the distribution of reinforcement from the point of view of the potential member size dependence is possible.

To increase the knowledge of size effects on rotation capacity further experimental and analytical research is needed. In particular attention should be given to the influence of concrete grade and brittleness, covering besides high strength also fibre reinforced concrete. Furthermore, the influence of detailing of reinforcement as well as the effect of confinement must be carefully analysed when estimating the deformation capacity of the members, see also the recent study reported in [MAY 02].

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