Preliminary design of high-rise outrigger braced shear wall structures on flexible foundations

J.C.D. Hoenderkamp, M.C.M. Bakker and H.H. Snijder
Eindhoven University of Technology, Eindhoven, The Netherlands

This paper presents a graphical method to optimise the position of outriggers on shear walls with flexible foundations. This location for the outriggers will cause a maximum reduction in lateral deflection at the top of the building. The method can be used for preliminary design of high-rise stuctures subjected to horizontal loading.
The method requires the calculation of six structural parameters: bending stiffnesses for the shear wall and outrigger structure, an overall bending stiffness contribution from the exterior columns, rotational stiffnesses for the shear wall and column foundations in addition to a newly suggested bending stiffness parameter representing the structural behaviour of the flexible foundation beam connecting the foundation of the shear wall to the exterior column foundations. These parameters allow the derivation of two compatibility equations for rotations at the intersections of the neutral lines of the shear wall with the outrigger and foundation structures. They yield expressions for the restraining moments at outrigger and foundation levels that act in the opposing direction to the bending moment from the horizontal loading on the structure. Maximising the influence of the restraining moments on the horizontal deflections leads to the optimum location of the outrigger structure. Combining all stiffness parameters into two non-dimensional characteristic structural parameters allows the optimisation procedure for this type of structure to be represented by a single graph that directly gives the optimum level of the outrigger.
It is concluded that all six stiffness parameters need to be included in the preliminary analysis of a proposed tall building structure as the optimum location of the outrigger as well as the reductions in horizontal deformations and internal forces in the structure can be significantly influenced by all the structural components.

Key words: Tall buildings, outrigger structures, shear walls, high-rise, design method

1 Introduction

The outrigger braced high-rise shear wall structure in Figure 1 comprises a centrally located wall with two equal length outrigger beams positioned at a distance x from the top of the structure. The cantilever outriggers are rigidly connected to the wall and pin-connected to the columns in the facade of the structure. The individual floor beams are taken to be hinge connected to the shear

HERON, Vol. 48, No. 2 (2003) ISSN 0046-7316

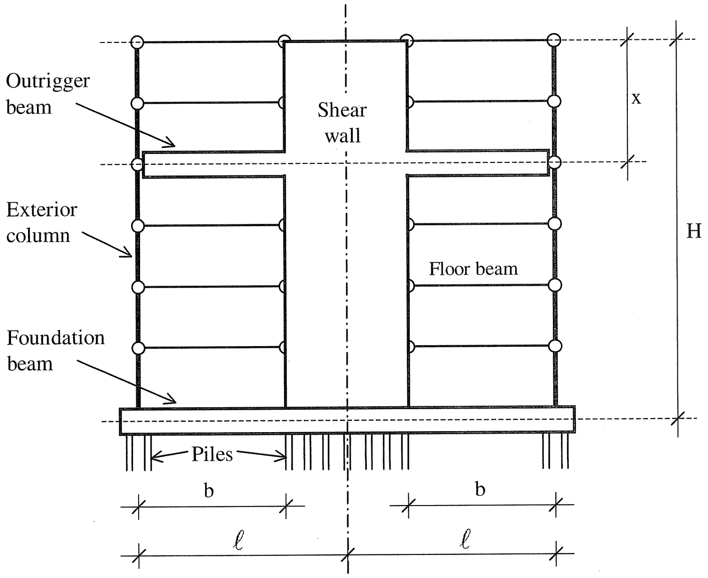

Fig. 1. Outrigger braced shear wall structure on flexible foundation

wall and columns, thereby not participating in the lateral resistance of the building. The column-to-column connections are also assumed to be non-moment resistant. The shear wall is rigidly connected to the foundation structure that is founded on piles which are located at the shear wall and exterior columns only. The foundation structure is a flexible system.

When horizontal load is applied to such a structure, the wall will interact with the exterior structure by introducing a tension force in the windward exterior columns and compression in the leeward columns. These forces form restraining moments at ourigger and foundation levels which act in the opposite direction to the bending moment from the horizontal load. Both effects will reduce the horizontal deflections of the structure. Combined with the laterally applied load, the restraining moments will force a triple-curvature in the shear wall up the height and double curvature in the outrigger and foundation structures between the exterior columns. Figure 2 displays the overall deflected configuration of the structure due to wind load and the restraining moments.

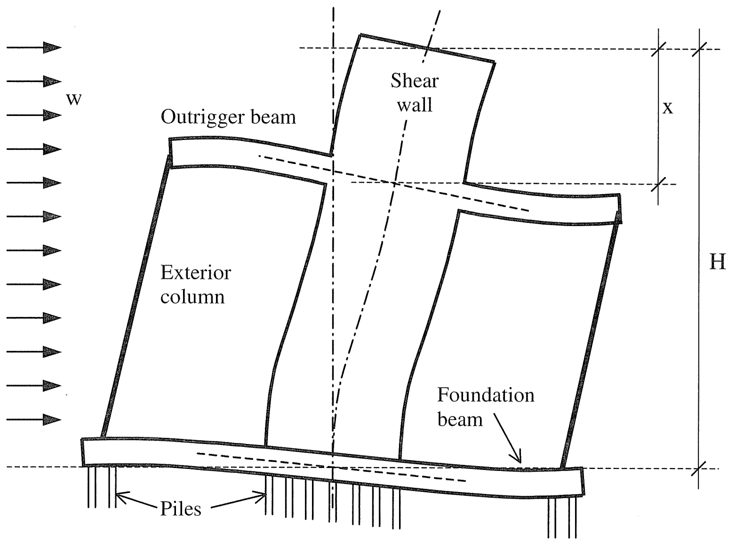

Fig. 2. Deflected shape of outrigger braced structures

The early development of simplified hand or graphical methods of analysis for outrigger braced structures (Taranath, 1974, 1975) considered belt trusses to have infinite bending stiffness and its location up the height of the structure to be the most significant factor influencing the reduction in horizontal drift. A solution was proposed for the optimum location for a structure with one outrigger. It was also shown (McNabb and Muvdi, 1975, 1977) that the structural properties of the shear wall and the columns are significant design parameters in reducing lateral deflections and a solution was suggested for a two-outrigger system. The influence of non-prismatic elements and non-uniformly distributed horizontal loading on outrigger braced structures was also included (Boggs and Casparini, 1983; Rutenberg and Tal, 1987). It was shown that the optimum location of the outrigger shifts towards the top when non-prismatic walls and columns are used.

In the horizontal deflection analysis of outrigger braced shear walls on fixed foundations it has been shown (Stafford Smith and Coull, 1991; Stafford Smith and Salim, 1981) that the wall can be represented by a single flexural stiffness parameter. The outriggers were assumed to be prismatic members rigidly connected to the wall and hinge connected to the exterior columns. The resulting double curvature behaviour of these flexural members was represented by a single bending stiffness parameter. It was further taken that the columns are pin connected to a fixed foundation and could thus be represented by a parameter which represents the axial stiffnesses of the columns only. With three stiffness parameters representing the wall, outriggers and exterior columns it was possible to combine them in a single dimensionless parameter which allowed a graphical procedure to obtain the optimum location of the outriggers such that they would cause a maximum reduction in horizontal deflection at the top of the structure.

The basic concept described above has been taken further by placing outriggers in the end facades parallel to the shear wall(s) in the structure (Hoenderkamp and Snijder, 2000). In that method it is allowed to account for possible racking shear behaviour in the facade riggers due to strain in the diagonal members of the structure. In a first attempt to take foundation structures into account in the design of outrigger braced shear walls, a graphical method was suggested (Hoenderkamp, 2003; Hoenderkamp and Snijder, 2002) to obtain the optimum location of outriggers for buildings on separate wall and column foundations. The analysis used for that structural problem forms the basis for the suggested graphical method for preliminary design of outrigger braced structures on flexible foundations. Although most of the research so far has been concentrated on shear walls braced by outriggers or facade riggers, it has also been shown that the concrete shear wall can be replaced by a steel braced frame (Hoenderkamp and Snijder, 2003; Hoenderkamp and Bakker, 2003). In addition to the bending behaviour of this frame, it also requires its racking shear behaviour to be taken into account in the analysis.

To develop a rapid method of analysis for preliminary design of a complicated structure, it is necessary to introduce simplifications in order to take only the major modes of behaviour into account. Figure 3 shows a simplified model to be used for the analysis of the outrigger braced structure shown in Figure 2. In the structural analysis, the individual elements are only subjected to forces resulting from horizontal loading on the structure. The wide column behaviour of the shear wall is modeled by a flexural member on the neutral axis of the wall with rigidly connected beams of infinite bending stiffness at outrigger and foundation levels representing the width of the wall.

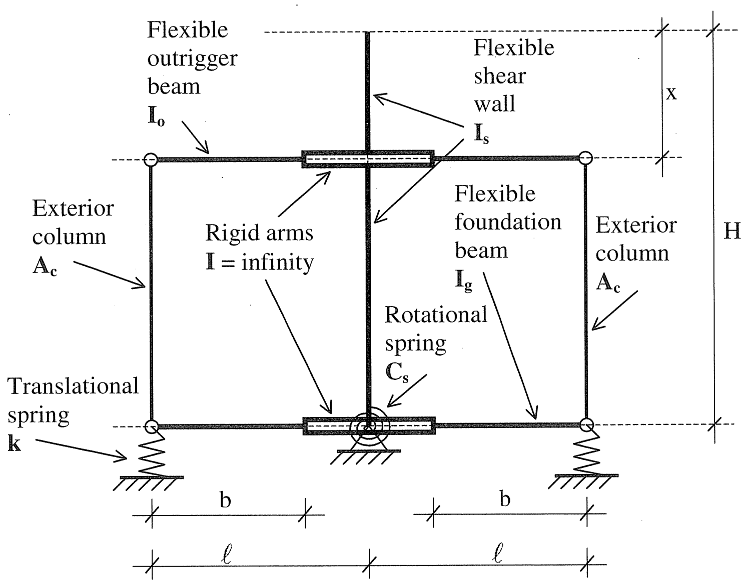

Fig. 3. Analytical model

The exterior columns are represented by pin-connected links only. Their flexural stiffness can be assumed to be negligible. The foundations under the pin connected façade columns are taken to be

pile foundations which act in a vertical direction only. They are modeled as linear springs. The shear wall foundation is only subjected to a bending moment. The net axial load on this foundation as a result of lateral loading is zero as it is positioned in the center of a symmetric structure. The foundation of the wall can thus be modeled by a rotational spring with a rotational spring constant. Further simplifying assumptions are: the structure behaves linear elastically; the sectional properties of the shear wall, exterior columns and outriggers are uniform througout their height or length; and the lateral loading is uniformly distributed along the height of the structure.
Compatibility equations are to be developed for the rotations in the wall, outrigger and foundation at the intersections of their neutral lines. This leads to two expressions for the restraining moment allowing the reduction in horizontal deflection at the top to be determined. Maximising this reduction will yield the optimum location of the outrigger.

2 Rotations at outrigger level

The rotations in the shear wall at outrigger level are the result of a uniformly distributed horizontal load w in addition to restraining moments M_r and M_f which are the result of reverse action by the outrigger and foundation beams. The rotations in the outrigger structure are caused by the restraining moments only.

2.1 Shear wall

The rotations in the concrete shear wall at a distance x from the top of the structure are the result of bending in the wall and rotation of the wall foundation as shown in Figure 4.

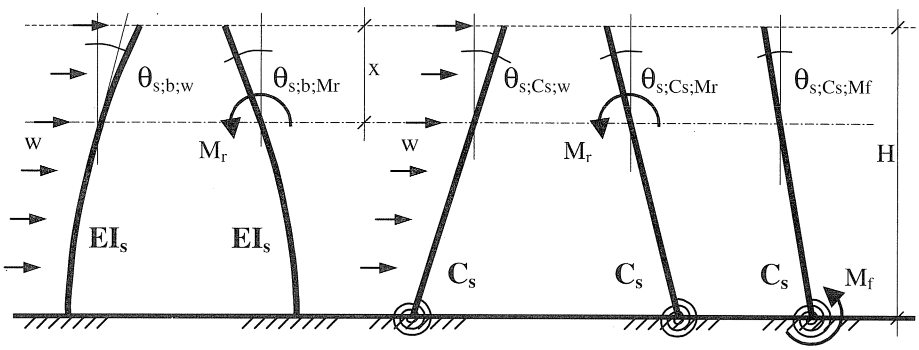

Fig. 4. Rotations of shear wall at outrigger level

2.1.1 Rotations due to lateral loading w

The rotations in the shear wall caused by the horizontally applied uniformly distributed load w as shown in Figure 4, are in the clockwise or positive direction. For bending this will be

$$\theta_{s;b;w} = \frac{w\left(H^3 - x^3\right)}{6EI_s} \tag{1}$$

where w is the uniformly applied lateral load on the structure, H represents the total height of the building, x is the distance measured from the top and EI_s is the bending stiffness of the shear wall. The rotation of the wall foundation can be expressed as follows

$$\theta_{s;C_s;w} = \frac{wH^2}{2C_s} \tag{2}$$

in which C_s represents the rotational stiffness of the shear wall foundation.

2.1.2 *Rotations due to restraining moment* M_r

The restraining moment M_r will cause bending deformations in the shear wall and and rotation in the wall foundation. For bending in the shear wall, the rotation at outrigger level is

$$\theta_{s;b;M_r} = -\frac{M_r(H-x)}{EI_s} \tag{3}$$

The rotation in the wall due to rotation of the wall foundation can be written as

$$\theta_{s;C_s;M_r} = -\frac{M_r}{C_s} \tag{4}$$

The rotations resulting from the outrigger restraining moment are in a counter clockwise or negative direction.

2.1.3 *Rotations due to restraining moment* M_f

The restraining moment M_f will cause rotation of the wall foundation. The rotation in the wall at outrigger level due to rotation of the wall foundation can be written as

$$\theta_{s;C_s;M_f} = -\frac{M_f}{C_s} \tag{5}$$

The rotation resulting from the foundation restraining moment is also in a counter clockwise or negative direction.

2.2 *Outrigger structure*

The rotations in the outrigger structure at a distance x from the top of the building are the result of bending in the flexible outriggers, axial strain in the exterior columns and deformations in the column foundations as shown in Figure 5.

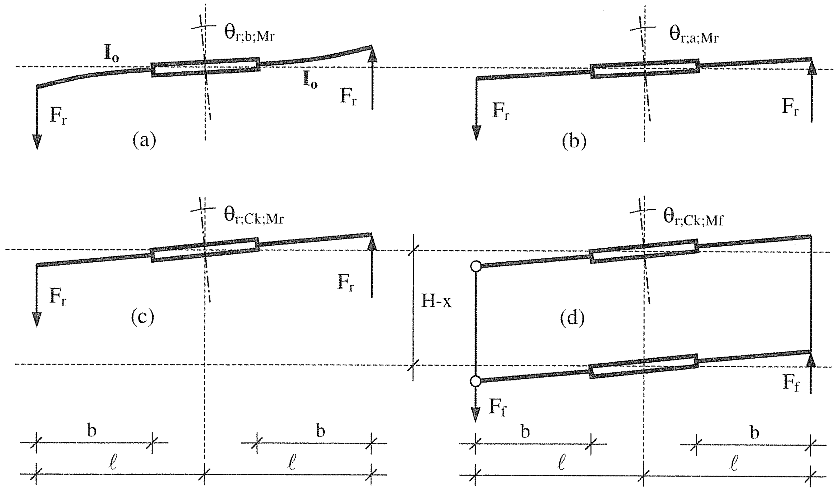

Fig. 5. Rotations of outrigger structure

2.2.1 Rotations in outriggers due to restraining force M_r

The rotations in the outrigger structure caused by the restraining forces F_r in the exterior columns are in the anti-clockwise or negative direction. The rotation due to bending in the outrigger structure is the result of the response of the two outriggers as shown in Figure 5a and can be expressed as

$$\theta_{r;b;M_r} = -\frac{2F_r \ell}{\Sigma\left\{\frac{3EI_o}{\ell}\left(\frac{\ell}{b}\right)^3\right\}} = -\frac{M_r \ell}{6EI_r} \tag{6}$$

where EI_o is the bending stiffness of the flexible part of the outrigger which is between the face of the shear wall and the exterior column, this distance is indicated by b, ℓ is the distance between the neutral line of the shear wall and an exterior column; EI_r is the equivalent flexural stiffness of a prismatic outrigger structure between the neutral lines of the shear wall and an exterior column. The bending stiffness of this member, taking the so called wide column action of the shear wall into account, can be written as follows

$$EI_r = EI_o\left(\frac{\ell}{b}\right)^3 \tag{7}$$

The axial shortening and lengthening of exterior columns will cause a rotation of the outrigger structure as shown in Figure 5b

$$\theta_{r;a;M_r} = -\frac{F_r(H-x)}{\ell\, EA_c} = -\frac{M_r(H-x)}{EI_c} \tag{8}$$

where EI_c is a global bending stiffness parameter pertaining to the axial stiffnesses of the exterior columns only and can be expressed as follows

$$EI_c = 2\ell^2 EA_c \tag{9}$$

in which A_c is the cross sectional area of the exterior column.

Both restraining moments will cause differential settlement in the column foundations resulting in a rotation of the outrigger structure as shown in Figure 5c. The rotation due to M_r is

$$\theta_{r;C_k;M_r} = -\frac{F_r}{\ell k} - \frac{M_r}{C_k} \tag{10}$$

where C_k is an overall rotational stiffness parameter representing the vertical spring stiffness of the piles of the column foundations

$$C_k = 2\ell^2 k \tag{11}$$

in which k represents the translational stiffness of the piles.

2.2.2 *Rotations in outrigger due to restraining moment M_f*

The differential settlement in the two exterior column foundations caused by the restraining force F_f will result in a rotation of the outrigger structure as shown in Figure 5d and can be expressed as follows

$$\theta_{r;C_k;M_f} = -\frac{F_f}{\ell k} = -\frac{M_f}{C_k} \tag{12}$$

The rotation in the outrigger structure due to the restraining moment at foundation level is also in the anti-clockwise direction.

2.3 *Compatibility equation at outrigger level*

At outrigger level, a distance x from the top of the structure, the sum of the rotations in the shear wall and the rotations in the outrigger structure at the intersection of their neutral lines is zero, thus

$$\theta_s + \theta_r = 0 \tag{13}$$

Substituting for the five rotations in the shear wall and four rotations in the outrigger structure yields the following simplified compatibility equation

$$\frac{w(H^3 - x^3)}{6EI_s} + \frac{wH^2}{2C_s} = M_r \left\{ \frac{(H-x)}{EI_s} + \frac{H-x}{EI_c} + \frac{\ell}{6EI_r} + \frac{1}{C_s} + \frac{1}{C_k} \right\} + M_f \left\{ \frac{1}{C_s} + \frac{1}{C_k} \right\} \tag{14}$$

in which the restraining moments M_r and M_f are two unknowns.

3 Rotations at foundation level

The rotations in the shear wall at foundation level are caused by the uniformly distributed horizontal load w in addition to restraining moments M_r and M_f. The rotations in the foundation structure are caused by the restaining moments only.

3.1 *Shear wall*

The rotations in the shear wall at the base of the structure are all due to deformations in the shear wall foundation as shown in Figure 6.

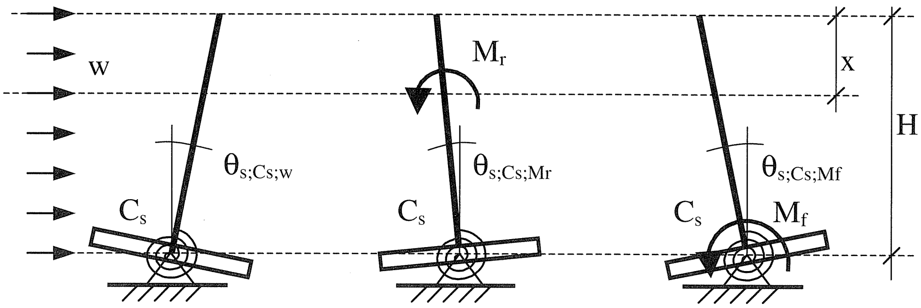

Fig. 6. Rotations of shear wall at foundation level

3.1.1 Rotations due to lateral loading w

The rotation of the wall foundation caused by the laterally applied load w as shown in Figure 6 is in a clockwise or positive direction and can be expressed as

$$\theta_{s;C_s;w} = \frac{wH^2}{2C_s} \tag{15}$$

3.1.2 Rotations due to restraining moment M_r

The restraining moment at outrigger level will rotate the wall foundation in a counter clockwise or negative direction. The foundation rotation then is

$$\theta_{s;C_s;M_r} = -\frac{M_r}{C_s} \tag{16}$$

3.1.3 Rotations due to restraining moment M_f

The rotation of the wall foundation due to the restraining moment at the base of the structure becomes

$$\theta_{s;C_s;M_f} = -\frac{M_f}{C_s} \tag{17}$$

which is also in the counter clockwise direction.

3.2 *Foundation structure*

The rotations in the foundation structure at a distance H from the top of the building are the result of bending in the flexible ground beams and deformations in the column foundations as shown in Figure 7.

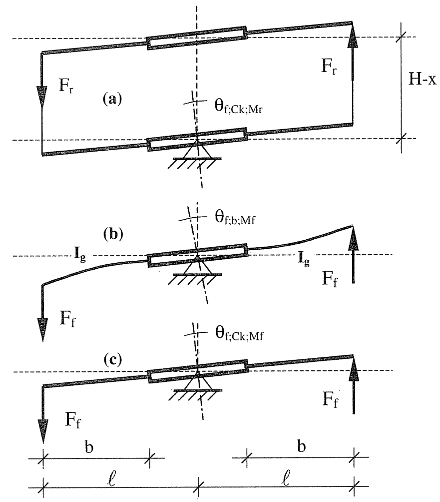

Fig. 7. Rotations in foundation structure

3.2.1 Rotations due to restraining moment M_r

The restraining moment at outrigger level will cause a differential settlement in the exterior column foundations through the axial forces in the exterior columns. Figure 7a shows the resulting rotation in the foundation structure in the counter clockwise or negative direction, thus

$$\theta_{f;C_k;M_r} = -\frac{F_r}{\ell\, k} = -\frac{M_r}{C_k} \tag{18}$$

3.2.2 Rotations due to restraining moment M_f

Similar to the rotations in the outrigger structure, the rotations in the foundation structure caused by the restraining forces F_f are in the anti-clockwise or negative direction as shown in Figure 7b. For bending this must also be summed for the two foundation beams

$$\theta_{f;b;M_f} = -\frac{2F_f\,\ell}{\Sigma\left\{\left[\frac{3EI_g}{\ell}\left(\frac{\ell}{b}\right)^3\right]\right\}} = -\frac{M_f\,\ell}{6EI_f} \tag{19}$$

where EI_g represents the bending stiffness of the flexible foundation beam which is between the face of the shear wall and the exterior column, EI_f is the equivalent flexural stiffness of a prismatic foundation structure between the neutral lines of the shear wall and the exterior column. The bending stiffness of this member, taking the so called wide column action of the shear wall into account, can be written as follows

$$EI_f = EI_g \left(\frac{\ell}{b}\right)^3 \tag{20}$$

The restraining moment at foundation level shown in Figure 7c will also cause a differential settlement in the exterior column foundations resulting in a counter clockwise rotation of the foundation structure

$$\theta_{f;C_k;M_f} = -\frac{F_f}{\ell k} = -\frac{M_f}{C_k} \tag{21}$$

3.3 *Compatibility equation at foundation level*

At the base of the structure, the sum of the rotations in the shear wall and the rotations in the foundation structure at the intersection of their neutral lines is zero, thus

$$\theta_s + \theta_f = 0 \tag{22}$$

Substituting for the three rotations in the shear wall and three rotations in the foundation structure leads after substitution to the following simplified compatibility equation

$$\frac{wH^2}{2C_s} = M_r\left\{\frac{1}{C_s} + \frac{1}{C_k}\right\} + M_f\left\{\frac{\ell}{6EI_f} + \frac{1}{C_s} + \frac{1}{C_k}\right\} \tag{23}$$

Similar to the compatability equations for rotation at outrigger level, at foundation level the restraining moments M_r and M_f are the only two unknowns.

4 Lateral deflection at the top

The two simultaneous compatibility equations given in Eqs. (14) and (23) must now be solved for the unknown restraining moments M_r and M_f. The complexity of the final expressions for these moments can be simplified by reducing the six characteristic stiffnesses of the total structure to two characteristic flexiblity parameters (S_v and S_h) and a non-dimensional constant (K). Setting for the two vertical structural elements

$$S_v = \frac{H}{EI_s} + \frac{H}{EI_c} \tag{24}$$

and for the outrigger and foundation structures combined

$$S_h = \frac{\ell}{6EI_r} + \frac{K}{C_s} + \frac{K}{C_k} \tag{25}$$

where

$$K = \frac{\ell / 6EI_f}{\ell / 6EI_f + 1/C_s + 1/C_k} \tag{26}$$

leads to an expression for the restraining moment at outrigger level

$$M_r = \left\{ \frac{w(H^3 - x^3)}{6EI_s} + \frac{wH^2}{2C_s} K \right\} \left\{ \frac{H}{(H-x)S_v + HS_h} \right\} \tag{27}$$

which causes a bending moment reduction on the wall foundation as well as in the shear wall between outrigger level and the foundation. It will result in reduced horizontal deflections along the height of the structure. The restraining moment at the base of the structure then becomes

$$M_f = \frac{wH^2}{2C_s} \left\{ \frac{K}{\ell / 6EI_f} \right\} + M_r(K-1) \tag{28}$$

This moment will also cause a bending moment reduction on the wall foundation and reduce horizontal deflections along the height of the structure.

The deflected shape of the shear wall in the structure due to the horizontally applied load w is shown in Figure 8. The maximum deflection at the top can be obtained from a further simplified model where only the shear wall with its foundation are subjected to the lateral load in addition to the two restraining moments. The expression for the horizontal deflection at the top then simply can be written as follows

$$y_{top} = \frac{wH^4}{8EI_s} + \frac{wH^3}{2C_s} - \frac{M_r(H^2 - x^2)}{2EI_s} - \frac{M_r H}{C_s} - \frac{M_f H}{C_s} \tag{29}$$

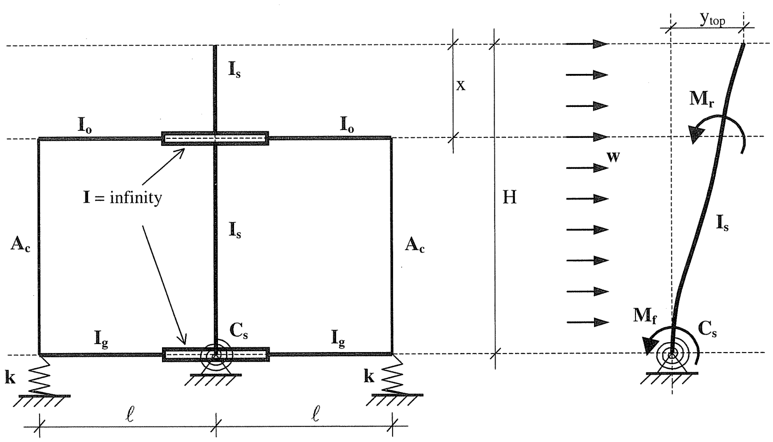

Fig. 8. Deflected shape of outrigger structure

The last term in Eq. (29) represents the reduction in deflection of the wall due to restraining moment M_f which after substitution can be written as

$$y_{s;C_s;M_f} = \frac{wH^3}{2C_s^2}\left\{\frac{K}{\ell/6EI_f}\right\} + \frac{M_rH}{C_s}(K-1) \tag{30}$$

The total deflection reduction of the wall due to both restraining moments is represented by the last three terms on the right hand side of Eq. (29). Substituting Eq. (30) for the last term leads to the following equation for the deflection reduction

$$y_{red} = \frac{M_r\left(H^2-x^2\right)}{2EI_s} + \frac{M_rH}{C_s} + \frac{M_rH}{C_s}(K-1) + \frac{wH^3}{2C_s^2}\left\{\frac{K}{\ell/6EI_f}\right\} \tag{31}$$

which can be simplified to

$$y_{red} = \frac{M_r}{2EI_s}\left\{H^2 - x^2 + \frac{2H^2}{\gamma H}\right\} + \frac{wH^3}{2C_s^2}\left\{\frac{K}{\ell/6EI_f}\right\} \tag{32}$$

where a characteristic non-dimensional parameter has been defined as

$$\gamma H = \frac{C_sH}{KEI_s} \tag{33}$$

The maximum deflection at the top of the structure can now be written as

$$y_{top} = \frac{wH^4}{8EI_s} + \frac{wH^3}{2C_s}\left\{1 - \frac{H/EI_s}{(\gamma H)(\ell/6EI_f)}\right\} - \frac{M_r}{2EI_s}\left\{H^2 - x^2 + \frac{2H^2}{(\gamma H)}\right\} \tag{34}$$

Substituting Eq. (27) for M_r into Eq. (32) and simplifying leads to the following equation for the reduction in deflection

$$y_{red} = \frac{wH^5}{12(EI_s)^2}\left[\left\{1-\bar{x}^2-\bar{x}^3+\bar{x}^5+\frac{5-3\bar{x}^2-2\bar{x}^3}{\gamma H}+\frac{6}{(\gamma H)^2}\right\}\left\{\frac{1}{1-\bar{x}+\omega}\right\}\right] \tag{35}$$

in which a characteristic dimensionless structural parameter is defined as follows

$$\omega = \frac{S_h}{S_v} \tag{36}$$

and a location parameter as

$$\bar{x} = \frac{x}{H} \tag{37}$$

The deflection reduction can be maximized by differentiating Eq. (35) with respect to $\bar{x}$, setting it equal to zero and solving for $\bar{x}$. It can quite easily be shown that the outcome is a function of the two non- dimensional parameters γH and x/H only. This allows the definition of a simple diagram as shown in Figure 9 for determining the optimum location of the outriggerstructure. For this location, the reduction in horizontal deflection is maximised.

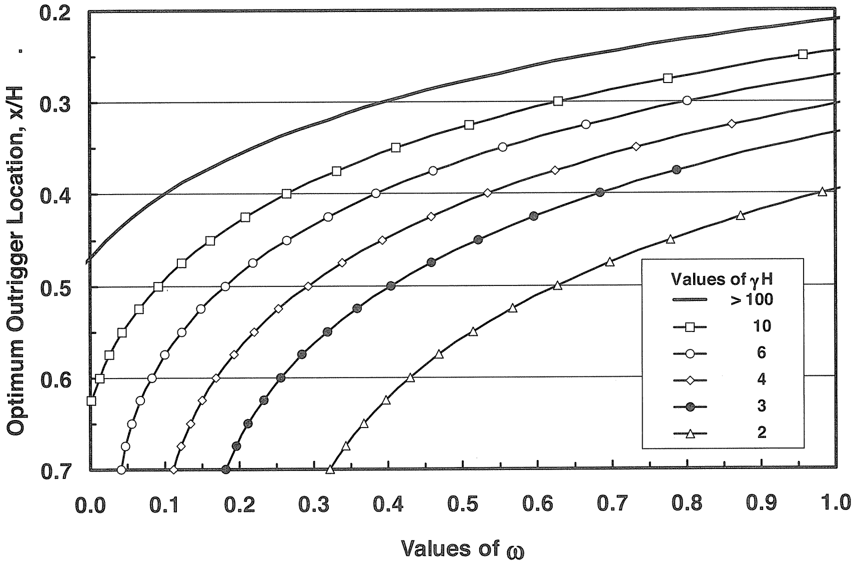

Fig. 9. Optimum location of outrigger

The assumption of infinitely stiff foundation beams will reduce the constant K in Eq. (26) to a zero value and thus give the non-dimensional parameter γH an infinite value. From the diagram in Figure 9 it can be seen that the outrigger will then be located above its optimum location in the structure. Giving the foundation beam a zero stiffness will result in a maximum value for K, i.e. 1.0, which will reduce the restraining moment at the base of the structure to zero. The parameter γH now has a minimum value and the outrigger will be placed below its optimum position. Solutions for this situation have been given earlier (Hoenderkamp, 2003; Hoenderkamp and Snijder, 2002). Increasing the bending stiffness of the outrigger structure will reduce the value of the flexibility parameter S_h thereby also reducing the non-dimensional parameter ω and thus locating the outrigger too far down the structure.

5 Worked example

The structural floor plan of a 29 storey, 87 m high building in Figure 10 shows the arrangement of four identical shear walls with outriggers on both sides. Each rigger and ground beam is 9 m long and has a bending stiffness $EI_o = 2.25 \times 10^7$ kNm^2 and $EI_g = 1.44 \times 10^8$ kNm^2 resp. The flexural stiffness of the reinforced concrete shear wall EI_s = 1.5 x 10^9 kNm^2. For the exterior columns in steel A_c = 3.12 x 10^{-2} m^2. The elastic modulus of steel is E = 2.1 x 10^8 kN/m^2. The rotational stiffness of the shear wall foundation C_s = 1.0 x 10^8 kNm and the translational stiffness of the column foundations k = 4.0 x 10^5 kN/m. The building is subjected to a uniformly distributed lateral load of 1.6 kN/m^2 leading to a line load of 18 kN/m for each of the four outrigger-shear wall systems.

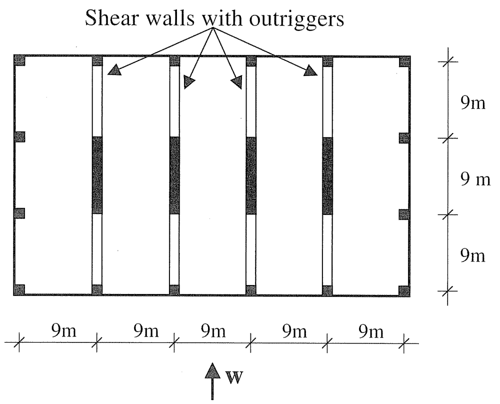

Fig. 10. Structural floor plan

The detailed calculations are shown here for a single shear wall with outriggers and ground beams. The flexural stiffness of one 13.5 m long outrigger structure is given by Eq. 7:

$$EI_r = EI_0\left(\frac{\ell}{b}\right)^3 = 2.25\times10^7\left(\frac{9.0+4.5}{9.0}\right)^3 = 7.594\times10^7\ kNm^2$$

The global second moment of area of the exterior columns can be obtained from Eq. 9:

$$EI_c = 2EA_c\ell^2 = 2\times2.1\times10^8\times3.12\times10^{-2}\times(13.5)^2 = 2.388\times10^9\ kNm^2$$

An overall rotational stiffness parameter for the foundations under the exterior columns is given by Eq. 11:

$$C_k = 2\ell^2k = 2\times13.5^2\times4\times10^5 = 1.458\times10^8\ kNm$$

The flexural stiffness of one 13.5 m long foundation beam is obtained from Eq. (20):

$$EI_f = EI_g\left(\frac{\ell}{b}\right)^3 = 1.44\times10^8\left(\frac{9.0+4.5}{9.0}\right)^3 = 4.860\times10^8\ kNm^2$$

The constant K is given by Eq. (26)

$$K = \frac{\ell / 6EI_f}{\ell / 6EI_f + 1/C_s + 1/C_k} = \frac{13.5/(6\times4.860\times10^8)}{13.5/(6\times4.860\times10^8)+1/(1.0\times10^8)+1/(1.458\times10^8)} = 0.2154$$

The characteristic parameters S_v and S_h are given by Eqs. (24) and (25):

$$S_v = \frac{H}{EI_s} + \frac{H}{EI_c} = \frac{87}{1.5\times10^9} + \frac{87}{2.388\times10^9} = 9.443\times10^{-8}\,(kNm)^{-1}$$

$$S_h = \frac{\ell}{6EI_r} + \frac{K}{C_s} + \frac{K}{C_k} = \frac{13.5}{6\times7.594\times10^7} + \frac{0.2154}{1.0\times10^8} + \frac{0.2154}{1.458\times10^8} = 3.326\times10^{-8}\,(kNm)^{-1}$$

The characteristic non-dimensional parameters for the structure can now be obtained from Eqs. (33) and (36):

$$\gamma H = \frac{C_s H}{KEI_s} = \frac{1.0\times10^8\times87}{0.2154\times1.5\times10^9} = 26.93$$

$$\omega = \frac{S_h}{S_v} = \frac{3.326\times10^{-8}}{9.443\times10^{-8}} = 0.3522$$

From the diagram in Figure 9 it can now quite easily be determined that the optimum location of the outrigger will be at $x/H = 0.33$. Locating the outrigger at a mid-story level nearest to the theoretical optimum location, i.e. 28.5 m from the top, and using Eq. (27) yields the restraining moment at this level,

$$\begin{aligned} M_r &= \left\{\frac{w(H^3 - x^3)}{6EI_s} + \frac{wH^2K}{2C_s}\right\}\left\{\frac{H}{(H-x)S_v + HS_h}\right\} \\ &= \left\{\frac{18(87^3 - 28.5^3)}{6\times1.5\times10^9} + \frac{18\times87^2\times0.2154}{2\times1.0\times10^8}\right\}\left\{\frac{87}{(87-28.5)\times9.443\times10^{-8} + 87\times3.326\times10^{-8}}\right\} \\ &= 1.4650\times10^4\,kNm \end{aligned}$$

This is a 21.5% reduction in the bending moment at the base of the shear wall. It is not the maximum possible moment reduction for the frame; this will occur with the outrigger at a lower position. The bending moment reduction occurs between outrigger level and the base of the structure. The horizontal deflections and reductions at the top of the structure can be obtained by rewriting Eq. 34 as follows:

$$\begin{aligned} y_{top} &= \frac{wH^4}{8EI_s} + \frac{wH^3}{2C_s} - \frac{wH^3}{2C_s^2}\left\{\frac{K}{\ell/6EI_f}\right\} - \frac{M_r}{2EI_s}\left\{H^2 - x^2 + \frac{2H^2}{(\gamma H)}\right\} \\ &= \frac{18\times87^4}{8\times1.5\times10^9} + \frac{18\times87^3}{2\times1.0\times10^8} - \frac{18\times87^3}{2(1.0\times10^8)^2}\left\{\frac{0.2154}{13.5/(6\times4.860\times10^8)}\right\} - \\ &\quad \frac{1.4650\times10^4}{2\times1.5\times10^9}\left\{87^2 - 28.5^2 + \frac{2\times87^2}{(26.93)}\right\} \\ &= 0.08593 + 0.05927 - 0.02757 - 0.03574 = 0.08189\,m \end{aligned}$$

This indicates that the "free" lateral deformation at the top of the structure would be 85.9 + 59.3 = 145.2 mm if the outriggers were absent. The reduction in deflection due to the restraining moments is 27.6 + 35.7 = 63.3 mm. The total reduction in horizontal deflection is thus 43.6%. The values in Table 1 give both absolute and relative reductions for lateral deflections at the top of the structure and for bending moments in the shear wall for the optimum outrigger location. This location, x_{opt}, is defined by a distance from the top for which the deflection reduction has a maximum value. The bottom row gives the horizontal deflections at the top of the structure. In column (b) marked "Flexible" all stiffness parameters have been given finite values. In column (c) the rotational stiffness of the shear wall foundation has been given an infinite value, the other stiffnesses remaining unchanged. Similar reasoning has been applied to columns (d), (e), (f) and (g).

Table 1: Structural performance

(a)	(b)	(c)	(d)	(e)	(f)	(g)
	Flexible	$C_s = \infty$	$C_k = \infty$	$EI_f = \infty$	$EI_f = 0.0$	$C_s = C_k = EI_f = \infty$
ω	0.352	0.343	0.347	0.314	0.492	0.314
γH	26.9	∞	18.3	∞	5.8	∞
$x_{opt.}$ in m	**28.5**	**25.5**	**28.5**	**28.5**	**31.5**	**28.5**
% M_{red}	21.5 *(21.5)*	19.0 *(19.0)*	22.7 *(22.7)*	20.0 *(20.0)*	26.6 *(26.6)*	20.0 *(20.0)*
M_{red} in kNm	14650 *(14655)*	12949 *(12955)*	15436 *(15441)*	13645 *(13651)*	18137 *(18140)*	13645 *(13651)*
% y_{red}	**43.6** *(43.6)*	**34.8** *(34.8)*	**54.8** *(54.8)*	**45.4** *(45.4)*	**38.3** *(38.3)*	**35.8** *(35.8)*
y_{top} in mm	81.9 *(81.9)*	56.1 *(56.1)*	65.7 *(65.7)*	79.3 *(79.3)*	89.7 *(89.7)*	55.2 *(55.2)*

The results show that increasing the translational stiffness of the exterior column foundations C_k and bending stiffness of the ground beam EI_f in column (d) and (e) in Table 1 respectively, yield larger reductions for the horizontal deflection in the shear wall. Increasing the rotational stiffness of the shear wall foundation C_s as indicated in column (c), will reduce the horizontal deflection at the top of the structure but will be less effective in terms of relative reductions. If the presence of a foundation beam is ignored, the lateral deformations in the structure will be conservatively estimated but the bending moment reduction in the shear wall may show large errors on the unsafe side as shown in column (f) of Table 1. Column (g) assumes infinite stiffnesses for the foundations. This can lead to seriously underestimating the deflections for real cases.

The results given in brackets indicate values obtained from finite element computer analyses (ESA-Prima Win) and clearly show that the suggested method gives very accurate results.

6 Conclusions

The suggested method of analysis for high-rise outrigger braced shear wall structures on flexible foundations allows the design engineer to:

- determine the optimum location for an outrigger structure where it will cause a maximum reduction in horizontal deflection at the top of the structure;
- obtain the maximum horizontal displacement in addition to the bending moments in the structure resulting from horizontal loading;
- investigate the influence of the various structural parameters on the lateral stiffness behaviour of the structure;
- rapidly assess the overall behaviour of the structure in the very early stages of the design of a proposed tall building structure and
- check the reasonableness of finite element computer analyses.

References

Boggs, P.C. and Casparini, D.A. (1983). Lateral stiffness of core outrigger systems, *Eng. J., AISC,* **20**(4): 172-180.

Hoenderkamp, J.C.D. Shear wall with outrigger trusses on wall and column foundations, Struct. *Design Tall and Special Build. (in press)*

Hoenderkamp, J.C.D. and Bakker, M.C.M. (2003). Analysis of high-rise braced frames with outriggers, Struct. Design Tall Spec. Build. 12, 335-350

Hoenderkamp, J.C.D. and Snijder, H.H. (2000). Simplified analysis of facade rigger braced high-rise structures, *Struct. Design Tall Build.* **9**, 309-319

Hoenderkamp, J.C.D. and Snijder, H.H. (2002). Outrigger structures on non-fixed foundations, *Proceedings of the Second Structural Engineers World Congress,* Yokohama, Japan, pp. T1-2-c-5

Hoenderkamp, J.C.D. and Snijder, H.H. (2003). Preliminary analysis of high-rise braced frames with facade riggers, *Journal of Structural Engineering,* ASCE, **129**(5), pp. 640-647

McNabb, J.W. and Muvdi, B.B. (1975). Drift reduction factors for belted high-rise structures, *Eng. J., AISC,* **12**(3): 88-91.

McNabb, J.W. and Muvdi, B.B. (1977). Discussion: Drift reduc-tion factors for belted high-rise structures, *Eng. J., AISC,* **14**(1): 44-47.

Rutenberg, A. and Tal, D. (1987). Lateral load response of belted tall building structures, *Engineering Structures,* **9**(1): 53-67.

Stafford Smith, B. and Coull, A. (1991). *Tall Building Structures,* John Wiley & Sons, New York

Stafford Smith, B. and Salim, I. (1981). Parameter study of outrigger-braced tall building structures, *Journal of the Structural Division,* ASCE, **107**, No. ST10, pp. 2001-2013.

Taranath, B.S. (1974). Optimum belt truss location for high-rise structures, *Eng. J., AISC,* **11**(1): 18-21.

Taranath, B.S. (1975). Optimum belt truss location for high-rise structures, *Structural Engineer,* **53**(8): 345-347.