

# Ponding on flat roofs: A different perspective

Johan Blaauwendraad

Faculty of Civil Engineering and Geosciences, Delft University of Technology, the  
Netherlands

Rainwater ponding in flat roofs is studied with aid of two simplified models. Distinction is made to two categories of rainwater ponding on flat light-weight roofs. In the first category strength is governing failure of roofs and in the other one stability is governing.

For the first category the nonlinear strength problem is discussed in a way which closely leans towards the  $\frac{1}{8}ql^2$ -method for a homogeneously distributed load  $q$ , familiar to structural engineers. The approach applies for horizontal roofs, sloped roofs, roofs with initial deflection due to permanent load and camber and for roofs which are composed of primary and secondary members and profiled steel sheeting. The deflection of all these components is accounted for. Application examples demonstrate that the model is an easy-to-use design tool. The design method clarifies the effect of the profiled steel sheeting on the safety. Also other practical hints are included in the paper.

For the second category of ponding, in which stability governs the problem, a separate check on the ultimate capacity is available. The paper is a designer-oriented version of a research paper published in Engineering Structures [1]. Here, designer-oriented features are summarized and the discussion is extended to aspects of interest to structural engineers and code developers.

*Key words: Flat roof collapse, rainwater ponding, structural safety, method of analysis, design method*

## 1 Introduction

Factory halls, super stores, swimming pools and other public buildings are often covered with spacious flat or nearly flat roofs. High steel grades and thought-out design methods lead to very light roof structures, which are extremely sensitive to pond forming in case of incidental, short-lasting, extremely heavy rain showers. If water cannot run-off, it will cause additional deflection, which results in more water on the roof, causing new

additional deflection, and so on. Water is accumulated. National design codes present different rules to avoid the danger of failure due to water accumulation. Some just request a minimum slope where others may require an iterative analysis. Yet, in practice, roofs may fail under extreme weather conditions. It appears that emergency drainage systems, supposed to take over when the regular drainage pipes are blocked, are not always accurately designed or positioned. Despite much attention paid to the subject in the past, structural engineers still struggle with water accumulation. Fortunately, commercial software packages increasingly include options for checking the sensitivity of roofs to ponding. These programs are based on an iterative procedure and are typically useful for checking purposes. They tell the structural engineer whether the displacement due to water accumulation will converge or diverge, so whether a stable state is achieved or not. If the structure is stable, information about the bending moments will be obtained, but no information how to adapt the design when necessary. For this purpose simple to use design models are welcome. In the past such models have indeed been proposed; for a survey of which it is referred to [1]. The majority of all papers is restricted to simply supported members and based on the assumption of a half sine wave deflection. As a rule, a fourth-order differential equation is derived and solved. In rare cases clamped end conditions and two-way roof systems are considered. The recent publication of Dutch colleagues in Eindhoven University of Technology is worth mentioning because it focuses on sensitivity of structures to imperfections [2]. Interesting work has also been done by the Department of Structural Engineering in the Technical University of Milan, [3] - [6]. In the context of the present paper the publication of Urbano [5] is appreciable, who applies a discrete model with a finite number of degrees of freedom to a horizontal flat roof in presence of resting camber after application of the permanent load. His charming method does not need differential equations and easily covers two-way roof systems. Unfortunately, he does not meet the tricky problem of pitched roofs, and the pity is that he neglects the influence of the profiled steel sheeting. In the present paper we summarize new thoughts about the theme as published in [1] and extend the discussion to aspects of interest to structural designers and code developers. The promoted method covers one-way and two-way roof systems, camber, slopes and the influence of the profiled steel sheeting. In connection with the Dutch code supplementary [7] it is presupposed that water accumulation is an extraordinary load, for which limitations to the deflections need not be considered. As a consequence, differently from practice for permanent and variable loads like wind and snow, just a unity-check on strength is performed and no check on

deformation. In water accumulation studies it is handy to define a measure to judge the stiffness, for which a stiffness parameter  $n$  is introduced. It appears that two different categories of structures must be distinguished, one consisting of relative stiff roofs which can be completely inundated by water ( $n > 1$ ) and one of extremely flexible roofs on which only partial inundation is possible ( $n \leq 1$ ). Stiff roofs fail due to shortage of strength and flexible roofs due to loss of stability.

## 2 Roofs failing by shortage of strength

In this paper we consider roofs of a regular rectangular plan. The roof structure consists of primary members, secondary members and of profiled steel sheeting, see the floor plan in Figure 1. The primary members have spans  $l_p$ , the secondary members spans  $l_s$  and the steel sheeting spans  $l_{sh}$ . It is convenient to introduce a separate symbol  $a_s$  for the in-between distance of the secondary members, which - of course - is equal to  $l_{sh}$ . Until further notice it is supposed for the roof plan in Figure 1 that secondary members are simply supported by the primary members and the steel sheeting by the secondary members. Each of these three components will normally deflect because of finite stiffness, but we start considering a roof of infinitely stiff primary members and steel sheeting. So for the time being we will restrict ourselves to a secondary member, a simply supported beam on rigid supports. We suppose that the beam is straight after application of permanent loading and in perfect horizontal position. Later, we will generalize the discussion to roofs with initial deflections, initial camber and/or slopes. Until further notice we will skip the subscript  $s$ .

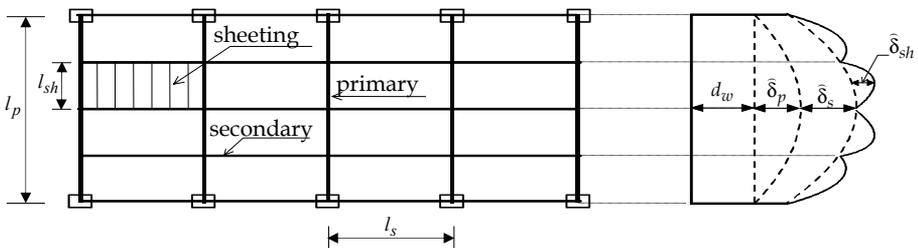


Figure 1: Roof of primary members, secondary members and steel sheeting

The roof secondary members have length  $l$  and in-between distance  $a$ . The regular discharge pipes are blocked and the emergency discharge is in stationary operation. In this

paper the emergency discharge is always thought to be installed at the edge of the roof. It is assumed that a stable equilibrium state exists with a water level  $d$  above the support as shown in the left part of Figure 2. This water depth is the initial depth before the member starts to bend.

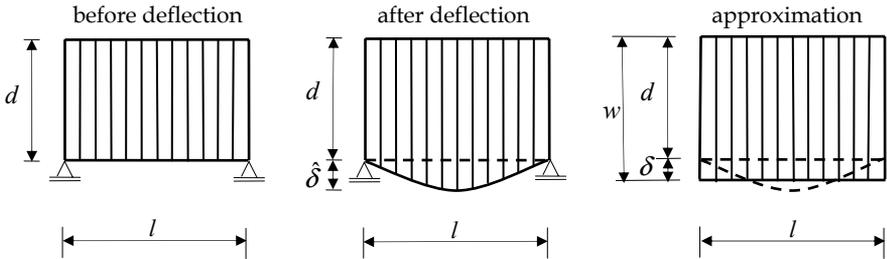


Figure 2: Initial (left), real (mid) and idealized water loading (right)

The beam will deflect over a distance  $\hat{\delta}$  and the deflected shape is filled with water as is shown in the middle part of the figure. The water loading on the beam now consists of two parts, one homogeneously distributed over the span (initial water depth  $d$ ) and one varying over the span (additional water with maximum depth  $\hat{\delta}$ ). From here on we work with homogeneously distributed loads only and substitute the varying part by a statically equivalent constant part with water depth  $\delta$ . Statically equivalent means that the same bending moment occurs in the mid of the span. Assuming a sine shape of the deflection and introducing the specific water weight  $\gamma$  results in a half-wave sine shaped bending diagram with maximum value  $M = (\gamma a \hat{\delta})l^2/\pi^2$ . The equivalent homogeneously distributed load causes the bending moment  $M = (\gamma a \delta)l^2/8$ . Equating these moments yields  $\delta = 8/\pi^2 \approx 0.8\hat{\delta}$ . The factor 0.8 will play a key role in the theory and appears not to hold true for simply supported members only, but also applies for other support conditions. The initial water depth and the additional water depth having the same shape, we introduce the piston-spring model of Figure 3. The weightless piston represents the part of the roof surface above the considered support structure and the spring the support structure itself. The left part of the figure shows the empty state before the rain storm starts and the right-hand part the stationary state during the rain storm. The distance  $d$  includes the additional height above the emergency outlet needed to have the water flowing. The displacement  $\delta$  is the average displacement discussed above. In fact, the support structure

is an elastic-plastic spring. When the force in the spring reaches a specific level, the friction element starts to slip keeping the supporting force constant. When this starts to occur the strength limit is reached.

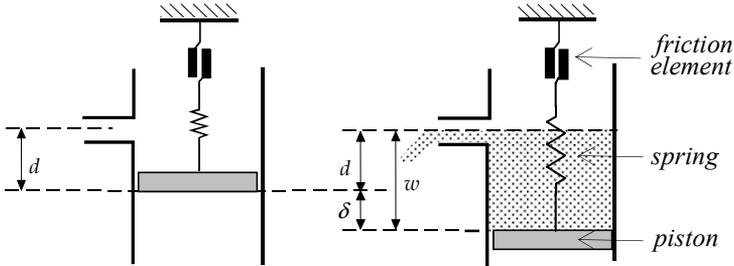


Figure 3: Piston-spring model of a roof member. Left without, right with water

It appears handy to introduce symbols  $W$  and  $D$ , of which  $W$  is used for the weight of one meter water on the piston and is expressed in kN/m and  $D$  is the spring stiffness, also expressed in kN/m. Note that the symbols  $W$  and  $D$  have the same units. It means that we can write for the load of the water and the resistance of the roof:

$$\begin{aligned} F_{water} &= W w \\ F_{roof} &= D \delta \end{aligned} \tag{1}$$

where  $F_{water}$  is the total distributed water load over the span and  $F_{roof}$  the total distributed load that can be resisted.

### 2.1 Determination of $W$ and $D$

The calculation of  $W$  is straightforward. The area of the roof part which is assigned to the roof member is  $al$ , so the weight of one meter water on this roof part is:

$$W = \gamma a l . \tag{2}$$

The stiffness  $D$  defines the relationship between the load  $F_{roof}$  on the considered roof part and the equivalent displacement  $\delta$ . It depends on the support conditions. For the time being, we restrict ourselves here to a simply supported member. The relation between the

resistance  $F_{roof}$  and the equivalent displacement  $\delta$  is derived in the following way. For a homogeneously distributed load  $q$  on the roof it holds  $\hat{\delta} = (5/384)(ql^4/EI)$ . Multiplying both members of this relationship by 0.8 yields  $\delta = (1/96)(ql^4/EI)$ . Next, we replace the complete load  $ql$  by  $F_{roof}$  and rearrange the relation, ending up with:  $F_{roof} = 96(EI/l^3)\delta$ . Hence:

$$D = 96 \frac{EI}{l^3} \tag{3}$$

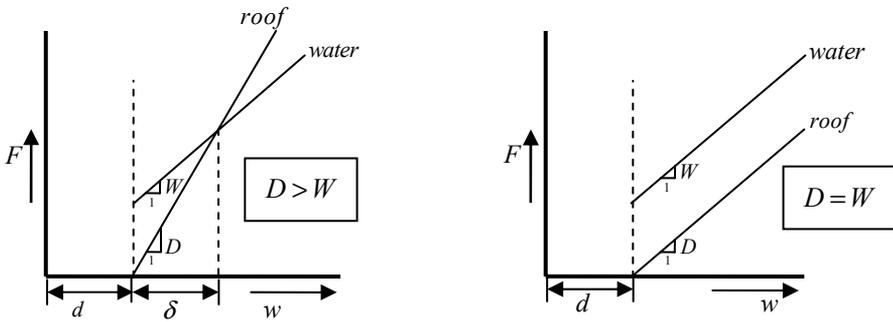


Figure 4: Graphical representation of the relationship between  $F_{water}$  and  $F_{roof}$  and  $w$

### 2.2 Determination of $n$

The two equations in (1) can be displayed in an  $F - w$  space as is done in Figure 4. It is clear that the two straight lines do only meet if  $D > W$ . For  $D = W$  the two lines are parallel and meet at an infinite value of  $w$  and for  $D < W$  no intersection can occur at all. So it is convenient to introduce the dimensionless stiffness parameter  $n$ :

$$n = \frac{D}{W} \tag{4}$$

If  $n > 1$  equilibrium exists, if  $n = 1$  equilibrium only exists for an infinite value of  $w$  (so of  $F_{water}$ ) and if  $n < 1$  no equilibrium state is found anymore. The model does not apply in that case. The equilibrium state is marked by the intersection of the two lines in the left-hand part of Figure 4. Elementary mathematics results into:

$$w = \frac{n}{n-1}d; \quad \delta = \frac{1}{n-1}d \quad (5)$$

The factor  $n/(n-1)$  is an amplification factor, applied to the initial water load  $d$  before accumulation. In practice, structural engineers will first design the roof taking permanent loading into account, as well as wind and snow loading and check afterwards if the roof will be safe for the extreme situation of water accumulation. So the structural engineer starts the water accumulation check on basis of a given span  $l$ , in-between distance  $a$ , member flexural rigidity  $EI$  and water depth  $d$ . This implies that he will work as follows. He determines  $n$ , computes the amplification factor  $n/(n-1)$  and achieves at a water column  $w$  by application of equation (5) to  $d$ . From this the water load  $F_{water}$  is computed, which in turn, facilitates computing the bending moment  $M$ :

$$F_{water} = Ww \quad (6)$$

The homogeneously distributed water load is  $F_{water}/l$ , hence:

$$M = (F_{water}/l)l^2/8 = Fl/8 \quad (7)$$

This moment  $M$  is added to the moment due to the permanent loading and a unity check for the total moment can be done as required by the governing design code.

At this place a comment has to be made about the calculation of  $n$ . If we substitute (2) and (3) in (4) we find:  $n = 96 EI / (\gamma a l^4)$ , which alternatively can be written as:

$$n = \frac{EI}{EI_{cr}}, \quad \text{where } EI_{cr} = \frac{\gamma a l^4}{96} \quad (8)$$

The critical flexural rigidity  $EI_{cr}$  depends on geometry and specific fluid weight only. In water accumulation literature the ratio  $n$  of  $EI$  and  $EI_{cr}$  is a familiar quantity, be it that the critical rigidity is always derived on basis of an assumed sine shape for the deflection. In that case  $\pi^4$  is found instead of 96, which is very close (difference 1.5%). Both values  $\pi^4$  and 96 follow from an assumption about the deflected shape, however, the assumption which

leads to 96 is more general. The sine shape assumption is only valid for simply supported members, whilst the approach with the average water depth can be applied to all kinds of member end conditions.

### 2.3 Initial deflection and camber

We can easily extend the discussion to roofs with an initial deflection due to permanent loading (or construction imperfections). Again the total roof area is covered with water. The mid-span value of the initial deflection is  $\hat{\delta}_i$ . This is replaced by the constant value  $\delta_i = 0.8\hat{\delta}_i$  as shown in Figure 5a. The water depth at the edge of the roof is denoted by  $d_w$ . In the computation of the bending moment we simply work with the effective water depth  $d = d_w + \delta_i$ .

We can also consider the case of camber. If a camber remains (maximum value  $\hat{\delta}_c$ ) after application of the permanent load we must introduce the value  $\delta_c = 0.8\hat{\delta}_c$  in the computation and use the effective water depth  $d = d_w - \delta_c$ . This is shown in Figure 5b. In fact a camber is just an initial deflection with negative sign. De correctness of this approach has been confirmed by computer analyses with commercial software.

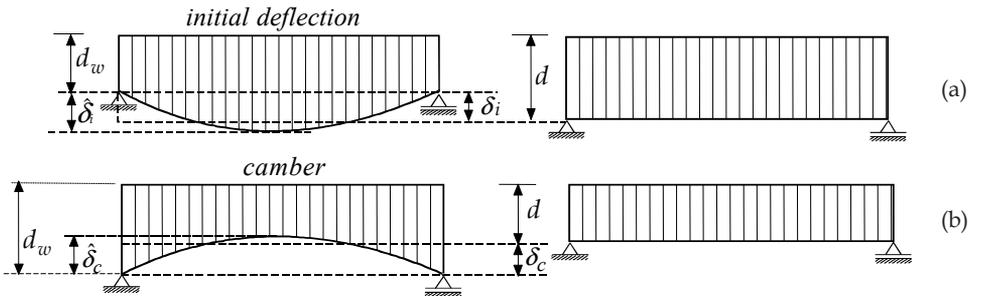


Figure 5: An initial deflection  $\delta_i$  increases the water column  $d_w$  and a camber  $\delta_c$  decreases it

#### 2.3.1 Application

We consider a horizontal roof part with span  $l = 15$  m, simply supported secondary IPE 450 members, in-between distance  $a = 5$  m and  $EI = 7.09 \cdot 10^4$  kNm<sup>2</sup>. The profiled steel sheeting is supposed to be infinitely stiff. The initial deflection  $\hat{\delta}_i$  due to permanent loading is 0.016

m. The initial water depth is  $d_w = 0.10$  m. Here we restrict the application to the calculation of the bending moment due to water accumulation. The water depth is:

$$d = 0.8 \hat{\delta}_i + d_w = 0.8 \cdot 0.016 + 0.100 = 0.113 \text{ m. The specific weight of water is } \gamma = 10 \text{ kN/m}^3.$$

The stiffness  $D$  is:

$$D = \frac{96EI}{l^3} = \frac{96 \cdot 7.09 \cdot 10^4}{15^3} = 2017 \text{ kN/m}$$

The specific water weight is:

$$W = \gamma a l = 10 \cdot 5 \cdot 15 = 750 \text{ kN/m.}$$

Hence:

$$n = \frac{D}{W} = \frac{2017}{750} = 2.69$$

The water column  $w$  is:

$$w = \frac{n}{n-1} d = \frac{2.69}{1.69} \cdot 0.113 = 0.180 \text{ m.}$$

The total water load is:

$$F_{water} = Ww = 750 \cdot 0.180 = 135 \text{ kN.}$$

The bending moment in the member becomes:

$$M = \frac{1}{8} F_{water} l = \frac{135 \cdot 15}{8} = 253 \text{ kNm.}$$

For comparison: the commercial software package ESA.PT with functionality for water accumulation yields the same solution 253 kNm.

### 2.3.2 Remark

An alternative way to determine  $n$  is application of (8). This results, taking into account

$$E_{steel} = 2.1 \cdot 10^8 \text{ kN/m}^2, \text{ into:}$$

$$EI_{cr} = E \cdot \frac{\gamma a l^4}{96} = 2.1 \cdot 10^8 \cdot \frac{10.0 \cdot 5.0 \cdot (15.0)^4}{96} = 2.64 \cdot 10^4 \text{ kNm}^2.$$

$$EI = 7.09 \cdot 10^4 \text{ kNm}^2.$$

$$n = \frac{EI}{EI_{cr}} = \frac{7.09}{2.64} = 2.69.$$

The same result is obtained as before.

2.4 Various member end conditions

So far it has been assumed that all roof members are simply supported. However, this is only one special case out of more possibilities, (case e in Figure 6). Secondary members may continue over primary members and, as a consequence, the secondary member behaves as clamped-in at both ends (case a). Profiled steel sheeting often has the length of two in-between distances of secondary members and will behave as two-span members continuing over a mid support. Both spans may be loaded by water (case c) or just one span (case d). Steel sheeting elements may also be placed jumping, in pattern (case b), such that they do not all end at the same secondary member but alternating to the one secondary member and the other. Figure 6 presents the deflection diagram for all these cases. If the stiffness of the simply supported member is set to 1 all other cases have a higher stiffness, called the *stiffness ratio*, given in Figure 6. Stiffnesses  $D$  are still calculated by equation (2), but must be multiplied by the stiffness ratio of the present case. The effect is that  $n$  increases for a given  $EI$ . Those who prefer the formula  $n = EI/EI_{cr}$  can reach the same effect if they divide  $EI_{cr}$  by the corresponding stiffness ratio. This way was chosen in the Dutch code supplementary [7].

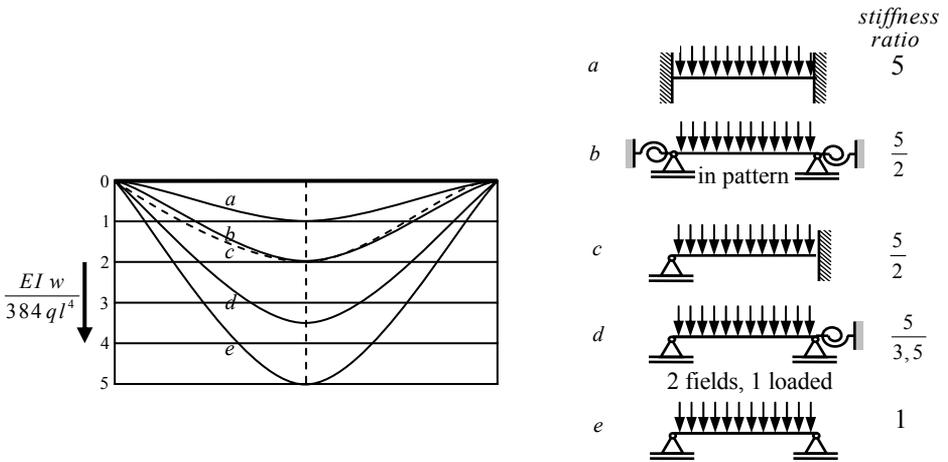


Figure 6: Stiffness ratio of various member end conditions (in case the preceding and/or the next member have a similar span)

The calculation of the bending moments needs extra attention. The bending moment diagram will differ from a simply supported member and the largest value of the moment can vary in place and magnitude. The bending moment diagrams corresponding with the different cases in Figure 6 are presented in Figure 7. One can still use equation (7) to

calculate the bending moment, however multiplied by the *moment ratio* listed in the figure. A minus sign indicates that the moment occurs at a support. For steel sheeting in pattern (case b of Figure 7) it must be kept in mind that the computed moment is the average value along a support line. In reality the moment is in turn zero or maximum. Therefore, to calculate the real bending moment at the mid support of a two-span sheeting element in pattern the factor  $-1/2$  must be multiplied by a factor 2, resulting into a moment ratio  $-1$ . Attention is drawn to the support reactions in case c of Figure 6 for steel sheeting not in pattern. They are not equally divided over the supports, and the increase of the mid support reaction is 25%. One can account for this by increasing  $a_s$  with a factor 1.25 when calculating  $W_s$ .

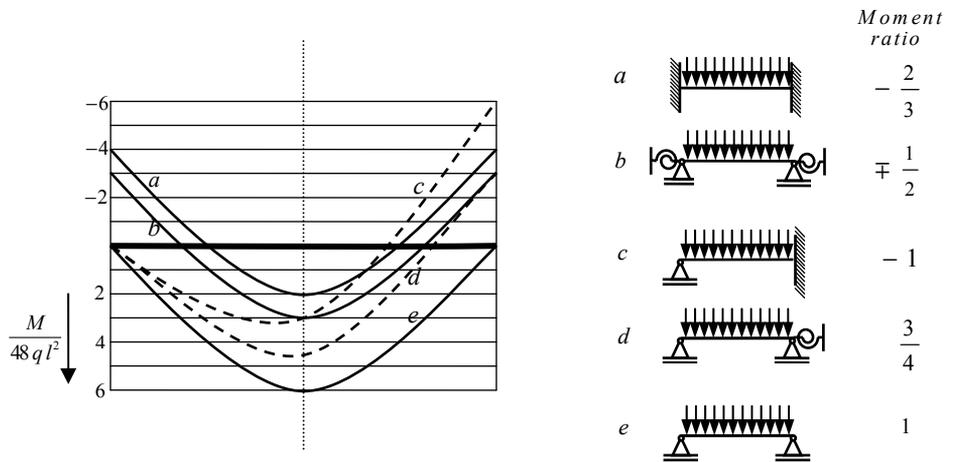


Figure 7: Moment ratio for various member end conditions  
(in case the preceding and/or next member have a similar span)

### 2.5 Composed roof systems

The approach is easily generalized to roof systems composed of *primary members*, *secondary members* and *profiled steel sheeting*. For the definition of the lengths  $l_p$ ,  $l_s$  and  $l_{sh}$  of the primary members, secondary members and steel sheeting respectively it is referred to Figure 1. Similarly, for convenience, the in-between distances  $a_p$ ,  $a_s$  and  $a_{sh}$  are introduced, of which  $a_p = l_s$  and  $a_s = l_{sh}$ . We consider a roof part which corresponds with one primary member, so a roof area of length  $l_p$  and width  $a_p$ . This primary field is divided in  $N_s$  secondary fields ( $N_s = 4$  in Figure 1). It is presupposed that the number of secondary

members is sufficiently big to assume that the loading on the primary members is homogeneously distributed. The same applies for the loading due to the steel sheeting on the secondary members. Product information of profiled steel sheeting for roofing specifies flexural rigidity per meter width of the sheeting. Therefore,  $a_{sh} = 1$  m and  $N_{sh}$  steel sheeting members occur, each one meter wide. The same water load  $F_{water} (= W w)$  must be carried by the primary member, the  $N_s$  secondary members and the  $N_{sh}$  steel sheeting elements, because  $W$  is common to all. Therefore, to determine  $N_{sh}$  a summation must be done over all secondary fields in the roof part under consideration (area  $a_p l_p$ ). We start assuming that all members are simply supported. The total displacement is a summation of the separate deflection of the primary members, secondary members and steel sheeting. Each of the composing elements can be modelled by its own spring and the three springs act as a series chain, see Figure 8. Each of the springs will be loaded by the same water load  $Ww$ . We apply equation (2), taking  $l_p$  for  $l$  and  $a_p$  for  $a$ :

$$W = \gamma a_p l_p \tag{9}$$

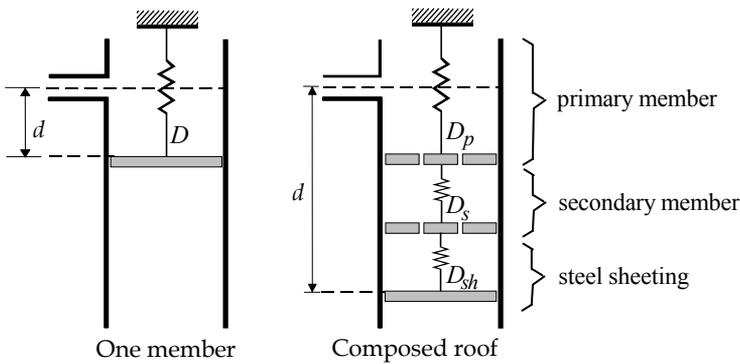


Figure 8: Series chain of springs for a composed roof system

As before, we define the stiffness  $D_p$  of the primary member. Because  $W$  is related to the roof part with area  $a_p l_p$ , all stiffnesses must be computed for this area. The joint stiffness of  $N_s$  secondary members is  $D_s$  and the joint stiffness of  $N_{sh}$  steel sheeting members  $D_{sh}$ .

For a series chain of springs we must add the reciprocal values to find the stiffness  $D$  of the total system:

$$\frac{1}{D} = \frac{1}{D_p} + \frac{1}{D_s} + \frac{1}{D_{sh}} \quad (10)$$

Formulae (4), (5) and (6) still apply to calculate  $n$ ,  $w$  and  $F_{water}$ . Because the water load  $F_{water}$  holds for the area  $a_p l_p$ , the bending moments in each primary member, each secondary member and per meter width of steel sheeting become respectively:

$$M_p = \frac{1}{8} F_{water} l_p; \quad M_s = \frac{1}{8} \frac{F_{water} l_s}{N_s}; \quad M_{sh} = \frac{1}{8} \frac{F_{water} l_{sh}}{N_{sh}} \quad (11)$$

Formula (10) can be presented in another way if we multiply both the left-hand and right-hand member of the equation by  $W$ . Then it reads:

$$\frac{1}{n} = \frac{1}{n_p} + \frac{1}{n_s} + \frac{1}{n_{sh}} \quad (12)$$

This equation is possibly more enjoyable to structural engineers because they can calculate  $n_s$  for one secondary member and  $n_{sh}$  for a steel sheeting part of one meter width. It is indifferent if they prefer to determine  $n$  as the ratio of  $D$  and  $W$  or like to determine it as the ratio of  $EI$  and  $EI_{cr}$ . In the latter case  $EI_{cr}$  is determined for the primary member, the secondary member and for a one meter wide sheeting element respectively. From these values  $n_p$ ,  $n_s$  and  $n_{sh}$  are computed and substitution in (12) yields  $n$  of the system.

### 2.5.1 Application

The horizontal roof exists of primary members HEA 900 of length  $l_p = 20$  m and  $EI_p = 886400$  kNm<sup>2</sup> and secondary members IPE 300 of length  $l_s = 10$  m and  $EI_s = 17600$  kNm<sup>2</sup>. The secondary members, in-between distance 5 m, continue over the primary members. Now case **a** in Figure 6 applies, and the stiffness ratio is 5. The deformation of the steel sheeting is not taken into account. In this problem is:  $d_w = 150$  mm. We restrict ourselves to the calculation of the moment due to water. A roof part between two primary members is

considered. The number of secondary member fields is  $N_s = 4$ . The deflections due to permanent loading of the primary and secondary members are 12.5 mm and 2.2 mm respectively. In the model 80% of these values is taken as initial displacement (rounded-off):  $\delta_{ip} = 10$  mm en  $\delta_{is} = 2$  mm. So we start from:  $d = 0.150 + 0.010 + 0.002 = 0.162$  m. The following calculations are done successively:

$$W = \gamma a_p l_p = 10 \cdot 10 \cdot 20 = 2000 \text{ kN/m}$$

$$D_p = \frac{96EI_p}{l_p^3} = \frac{96 \cdot 886400}{20^3} = 10640 \text{ kN/m}$$

$$D_s = 5 \cdot \frac{96N_s EI_s}{l_s^3} = 5 \cdot \frac{96 \cdot 4 \cdot 17600}{10^3} = 33790 \text{ kN/m}$$

$$\frac{1}{D} = \frac{1}{D_p} + \frac{1}{D_s} = \frac{1}{10640} + \frac{1}{33790} = \frac{1}{8090}$$

$$n = \frac{D}{W} = \frac{8090}{2000} = 4.045 \text{ m}$$

$$w = \frac{n}{n-1} d = \frac{4.045}{3.045} \cdot 0.162 = 0.215$$

$$F_{water} = Ww = 2000 \cdot 0.215 = 430 \text{ kN}$$

$$M_p = \frac{1}{8} F_{water} l_p = \frac{1}{8} \cdot 430 \cdot 20 = 1075 \text{ kNm}$$

$$M_s = -\frac{1}{12} \frac{F_{water} l_s}{N_s} = -\frac{1}{12} \cdot \frac{430 \cdot 10}{4} = -93.6 \text{ kNm (at support)}$$

For comparison, the commercial software ESA.PT yields the results 1060 kNm and -93.6 kNm, in the model 1.4% and 4.3% error respectively.

### 2.5.2 Remark

We could have calculated  $n_p$  and  $n_s$  for the separate members and deduce  $n$  for the total roof from them. One obtains the same result:

$$n_p = D_p / W = 10640 / 2000 = 5.320$$

$$n_s = D_s / W = 33790 / 2000 = 16.895$$

$$\frac{1}{n} = \frac{1}{n_p} + \frac{1}{n_s} = \frac{1}{5.320} + \frac{1}{16.895} = \frac{1}{4.045}$$

Again, the same value for  $n$  is found if we compute it from  $EI/EI_{cr}$  instead from  $D/W$ . It must only be kept in mind that we now do not multiply by the stiffness ratio  $sr$ , but have to divide  $EI_{cr}$  by it.

$$EI_{p,cr} = \frac{\gamma a_p l_p^4}{96} = \frac{10.0 \cdot 10.0 \cdot (20.0)^4}{96} = 16.667 \cdot 10^4 \text{ kNm}^2$$

$$EI_{s,cr} = \frac{1}{sr} \cdot \frac{\gamma a_s l_s^4}{96} = \frac{1}{5} \cdot \frac{10.0 \cdot 5.0 \cdot (10.0)^4}{96} = 0.1042 \cdot 10^4 \text{ kNm}^2$$

$$n_p = \frac{EI_p}{EI_{p,cr}} = \frac{88.64}{16.667} = 5.318$$

$$n_s = \frac{EI_s}{EI_{s,cr}} = \frac{1.76}{0.1042} = 16.89$$

We obtain the same value  $n$  for the roof system:

$$\frac{1}{n} = \frac{1}{n_p} + \frac{1}{n_s} = \frac{1}{5.318} + \frac{1}{16.89} = \frac{1}{4.045}$$

## 2.6 Sloping roof

The application of a slope is an efficient way to improve the insensitivity of flat roofs to water accumulation. We study the effect of a slope by means of a roof with a pitch  $\alpha$  and initial displacement  $\delta_i$  as drawn in the left-hand part of Figure 9. Practical pitch values are between 1 and 2 percent, so  $\alpha$  varies between 0.01 and 0.02. As long as the roof is fully covered with water, the mid-span water depth can be accurately chosen as  $d$ . In fact, the asymmetric moment diagram is replaced by a symmetric diagram with the same value at mid-span. The formula for  $d$  is:

$$d = d_w + \delta_i - \frac{1}{2} \alpha l \tag{13}$$

If the water surface is lower than the roof peak, a dimensionless parameter  $p$  is introduced, which is zero if no water occurs on the roof and is 1 if the water reaches the roof peak; see the mid part of Figure 9. The bending moment diagram is getting more asymmetric than for the fully covered roof, but still the diagram can be replaced by a symmetric one with the same value mid-span. It appears that the factor  $\frac{1}{2}$  in equation (20) must be replaced by a factor  $c$ :

$$d = d_w + \delta_i - c\alpha l \quad (14)$$

in which  $c$  depends on  $p$ . The relation between parameters  $c$  and  $p$  is plotted in the right-hand part of Figure 9. It was derived on basis of computational results for different  $n$ -values, reported in [2]. These results fit nicely in the present model if  $c$  is chosen as function of  $p$  as shown in Figure 9.

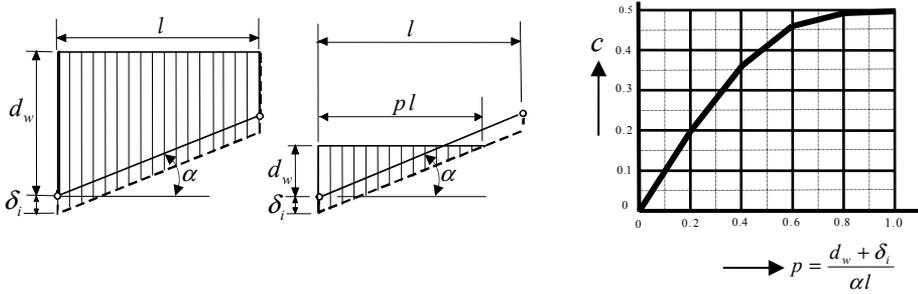


Figure 9: Sloping roof completely under water (left) and partially (middle).  
Relation between  $c$  and  $p$  (right)

Structural engineers who want to work with the relationship in the program Excel can work accurately with the formula:

$$c = 0.5 - 0.3(1 - p)^2 - 0.2(1 - p)^3 \quad (15)$$

The reader is reminded of the statement that the model is valid for values  $n > 1$ . The discussed approach is at least accurate for values  $\frac{1}{2} \leq p \leq 1$  and in this domain  $c$  keeps close to  $\frac{1}{2}$ . If smaller  $p$  values must be considered, usually other problems will arise, which will be discussed hereafter in Chapter 3. In a composed roof with pitch additional attention must be paid to the calculation of the bending moment in the profiled steel sheeting. The piston-spring model tacitly presupposes that each member in a given category is more or less equally loaded and the maximum loaded member occurs somewhere in the mid of the members of the category under consideration. However, this is not true for the steel sheeting. Most probably, the highest water column will occur on top of the steel sheeting element at the edge. Therefore, in case of sloping roofs, a second calculation must be done for the edge element with supports that are supposed to be infinitely rigid.

### 2.6.1 Application

A roof is considered existing of simply supported IPE 270 sections (here denoted as secondary members ) of length  $l_s = 10$  m en  $EI_s = 12160$  kNm<sup>2</sup> and steel sheeting of length  $l_{sh} = 4.5$  m and  $EI_{sh} = 157.5$  kNm<sup>2</sup>/m. The steel sheeting has the length of two fields and is placed in pattern. Its deformation is taken into account. The pitch of the roof is 16 mm/m, so  $\alpha = 0.016$ . The parabolic camber of the secondary members is 20 mm at mid-span, so  $\delta_c = 0.8 \cdot 0.020 = 0.016$  m. The initial water depth at the edge is  $d_w = 116$  mm. We consider a roof part between two secondary members, for which  $N_{sh} = 10$ . The dead weight of the secondary member is 0.36 kN/m and the dead weight of the steel sheeting, insulation and roof covering is 0.40 kN/m<sup>2</sup>. The calculation procedure is done in six steps.

#### Step 1. Initial moments due to permanent loading

Permanent loading and moments due to secondary members and roofing:

$$F_{i,s} = 10 \cdot 0.36 + 0.40 \cdot (10 \cdot 4.50) = 21.6 \text{ kN}$$

$$F_{i,sh} = 0.40 \cdot (10 \cdot 4.50) = 18.0 \text{ kN}$$

The reader is reminded of the fact that loads are always calculated for the same roof part area. The moments in each secondary member and 1 meter wide sheeting are respectively:

$$M_{i,s} = \frac{1}{8} F_{i,s} l_s = \frac{1}{8} \cdot 21.6 \cdot 10 = 27.0 \text{ kNm (at mid-span)}$$

$$M_{i,sh} = -\frac{1}{8} \frac{F_{i,sh} l_{sh}}{N_{sh}} = -\frac{1}{8} \cdot \frac{18.0 \cdot 4.5}{10} = -1.01 \text{ kNm/m (at support)}$$

#### Step 2. Calculation of initial deflections

Because the steel sheeting has the length of two spans and is placed in pattern the stiffness ratio is 2.5, see Figure 6.

$$D_s = \frac{96EI_s}{l_s^3} = \frac{96 \cdot 12159}{10^3} = 1167 \text{ kN/m}$$

$$D_{sh} = \frac{5}{2} \cdot \frac{96N_{sh}EI_{sh}}{l_{sh}^3} = \frac{5}{2} \cdot \frac{96 \cdot 10 \cdot 157.5}{4.5^3} = 4148 \text{ kN/m}$$

Deflection due to permanent loading:

$$\delta_{is} = \frac{F_{i,s}}{D_s} = \frac{21.6}{1167} = 0.019 \text{ m}; \quad \delta_{i,sh} = \frac{F_{i,sh}}{D_{sh}} = \frac{18.0}{4148} = 0.004 \text{ m}.$$

Step 3. Calculation of  $n$ ,  $d$  and  $w$

$$W = \gamma a_s l_s = 10 \cdot 10 \cdot 4.5 = 450 \text{ kN/m}$$

$$n_s = \frac{D_s}{W} = \frac{1167}{450} = 2.59$$

$$n_{sh} = \frac{D_{sh}}{W} = \frac{4148}{450} = 9.32$$

$$\frac{1}{n} = \frac{1}{n_s} + \frac{1}{n_{sh}} = \frac{1}{2.59} + \frac{1}{9.352} = \frac{1}{2.02}$$

The same result is found if  $n_s$  and  $n_{sh}$  are calculated from  $EI$  and  $EI_{cr}$ .

$$\delta_i = \delta_{i,s} + \delta_{i,sh} - \delta_c = 0.018 + 0.004 - 0.016 = 0.006 \text{ m}$$

$$d_w = 0.116 \text{ m}$$

$$p = \frac{d_w + \delta_i}{\alpha l} = \frac{0.116 + 0.006}{0.016 \cdot 10} = 0.76$$

$c = 0.48$  (read from Figure 9)

$$d = d_w + \delta_i - c \alpha l = 0.116 + 0.006 - 0.48 \cdot 0.160 = 0.045 \text{ m, see formula (14)}$$

$$w = \frac{n}{n-1} d = \frac{2.02}{1.02} \cdot 0.045 = 0.089 \text{ m}$$

Step 4. Calculation of water load and moments

$$F_{water} = Ww = 450 \cdot 0.089 = 40.1 \text{ kN}$$

$$M_s = \frac{1}{8} F_{water} l_s = \frac{1}{8} \cdot 40.1 \cdot 10 = 50.1 \text{ kNm}$$

$$M_{sh} = -\frac{1}{8} \frac{F_{water} l_{sh}}{N_{sh}} = -\frac{1}{8} \cdot \frac{40.1 \cdot 4.5}{10} = -2.25 \text{ kNm/m.}$$

Step 5. Extra calculation for steel sheeting at the edge of the roof

$$d_{sh} = d_w + \delta_{i,sh} = 0.116 + 0.004 = 0.120 \text{ m}$$

$$w_{sh} = \frac{n_{sh}}{n_{sh}-1} d_{sh} = \frac{9.22}{9.22-1} \cdot 0.120 = 0.135 \text{ m}$$

$$W_{sh} = \gamma \cdot l_{sh} = 10 \cdot 1 \cdot 4.5 = 45 \text{ kN/m}$$

$$F_{water,sh} = W_{sh} w_{sh} = 45 \cdot 0.135 = 6.08 \text{ kN}$$

$$M_{sh}^{edge} = -\frac{1}{8} F_{water,sh} l_{sh} = -\frac{1}{8} \cdot 6.08 \cdot 4.5 = -3.42 \text{ kNm/m}$$

This value is larger than the earlier calculated moment  $M_{sh}$  and thus normative.

#### Step 6. Comparison

The following moments must be used in the unity check:

Secondary member:  $M_{i,s} = 27.0$  kNm ;  $M_s = 50.1$  kNm .

Steel sheeting:  $M_{i,sh} = -1.01$  kNm/m ;  $M_{sh} = -3.42$  kNm/m .

The input in the unity check will be (partial safety factor 1.2 for permanent load and 1.3 for water load):

$$M_s = 1.2 \cdot 27.0 + 1.3 \cdot 50.1 = 97.5 \text{ kNm}$$

$$M_{sh} = -1.2 \cdot 1.01 - 1.3 \cdot 3.42 = -5.63 \text{ kNm/m.}$$

The solution by the commercial software ESA.PT is: 93.9 kNm and -5.54 kNm/m respectively. The maximum deviation is only 4%.

### 3 Roofs failing by loss of stability

The method in the preceding chapter is not applicable to roofs with  $n < 1$ , however such roofs are really built. Many light-weight steel roofs fall in this category. It appears in practice that they can carry loads in a stable state if two conditions are met. The roof must be sloped and only be covered over a small part of its area ( $p \ll 1$ ), see Figure 10, where a situation for small  $p$  is sketched. It goes without saying that symmetry in displacement and moment diagram does not occur anymore, which tacitly was the starting point of the piston-spring model. Therefore it is replaced by an alternative model which is able to meet asymmetry, shown in Figure 11. The flat inclined roof is modelled by two rigid bars connected to each other by an elastic rotational spring. The position of the spring is not fixed, but keeps continuously in the middle of the expanding water surface. For raising water height the rotational spring moves in the end towards the middle of the span. In Figure 11 the position of the spring is shown for two different values of  $d_w$ . Formula's (4) and (8) for the determination of  $n$  are applicable again. In the limit case of a fully covered, sufficiently stiff roof the model yields the same results as the piston-spring model. However, for partially covered, very flexible roofs, the response is completely different. Full derivations for the model have been reported in [1]. Here an overview of the findings will be given and their consequences are discussed.

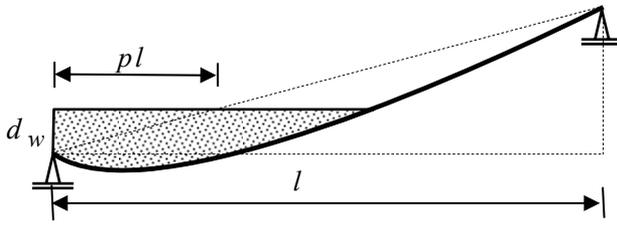


Figure 10: Example of extremely flexible roof ( $p \ll 1$ )

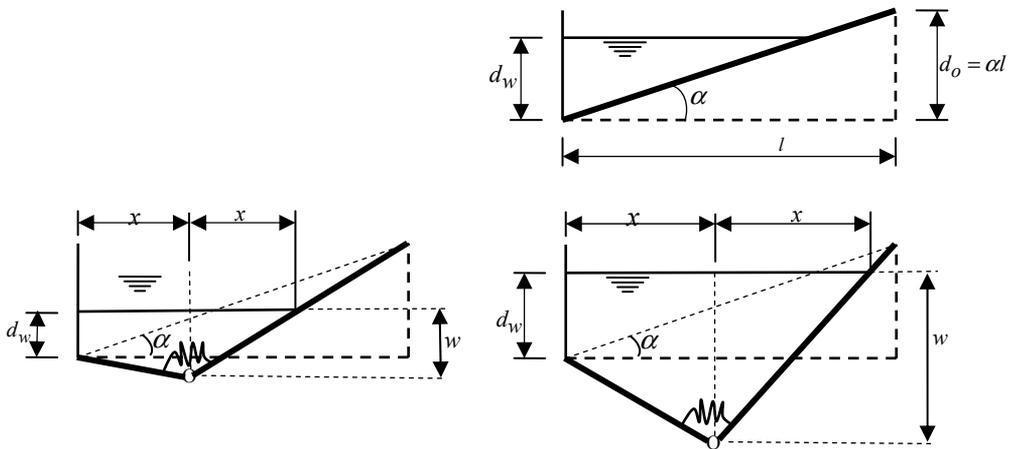


Figure 11: The bar-spring model; an alternative for the piston-spring model.

The rotational spring moves to the middle of the span at increasing water depth.

It appears that the two straight lines in Figure 4 now become curved ones. In Figure 12 is seen that the line for the water load  $F_{water}$  grows progressively with increasing water depth  $w$  (shown for different starting values  $d_1, d_2, d_3$ ). Also the line for the roof resistance  $F_{roof}$  increases, however with decreasing increments. As a result two intersections may exist (case  $d_1$ ), one point of contact may occur (case  $d_2$ ) or no contact at all (case  $d_3$ ). As a consequence, the plot of the edge water level  $d_w$  as function of the water level  $w$  on top of the rotational spring is a curve with a descending branch and a falling branch. A maximum value of  $d_w$  occurs in a full elastic state for stress levels (as it appears) far below yield strength. Apparently, stability governs the problem. The maximum value

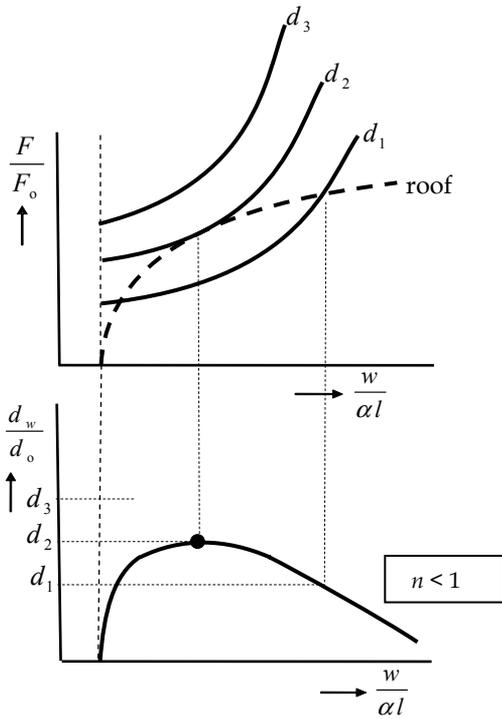


Figure 12: Nonlinear curves for (solid) and (dashed) result in a limit water level

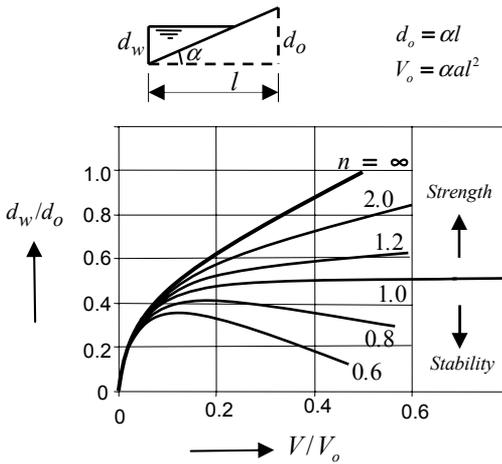


Figure 13: Result of the bar-spring model

of  $d_w$  is the *stability limit*  $d_{w,stab}$ . Wijte calls it *the water raising capacity* [8]. If we vary the stiffness ratio  $n$  and the water height  $d_w$  at the lower roof edge, Figure 13 is the result, in which the water depth  $d_w/d_0$  is plotted versus the volume  $V/V_0$ , where  $d_0 = \alpha l$  and the reference volume  $V_0 = \alpha \alpha l^2$ . As can be seen, a limit point occurs if  $n < 1$  and always increasing curves are found for  $n > 1$ . At the value  $n = 1$  a horizontal asymptote occurs at a water depth  $d_w = \frac{1}{2} \alpha l$ . For an infinite rigid roof ( $n = \infty$ ) and  $p = 1$  the volume is  $V = \frac{1}{2} V_0$ , which is the initial water volume on the roof. In case of a continuously raising curve, at last, the member will fail because of too high stresses. In case of curves with a limit point, the stresses are low and the structure is still in the elastic state. So, the roof structure fails by shortage of strength if  $n > 1$  and by loss of stability if  $n < 1$ . In the last case the water height at the edge of the roof cannot become higher than  $d_{w,stab}$ . It means that the emergency discharge must be placed sufficiently low, otherwise it will not function at all. After having reached the limit water height  $d_{w,stab}$ , increasingly more water will be accumulated at continuously down-going water surface. In the end the roof will fail because the strength limit will be as yet surpassed. Only in case not sufficient water is on the roof it will be prevented. For the extension to sloped roofs with an initial deflection or camber it is referred to [1]. After the model with rigid bars and rotational spring had unveiled what phenomenon is happening and which parameters are important, computations have been made with the water accumulation option of the commercial Finite Element package ESA.PT for various values of camber and slope. The shape of the initial deflection and camber were chosen parabolic and the measure of initial deflection or camber is expressed with the roof angle  $\alpha_e$  at the edge of the roof, because this angle is of major importance for values  $p \ll 1$ . If  $\alpha_e = \alpha$ , the roof is flat, without camber. The results are shown in Figure 14 from which a simple linear relationship between  $d_{w,stab}$ ,  $n$  and  $\alpha_e$  can be derived:

$$\frac{d_{w,stab}}{d_0} = -0.15 + 0.40 n + 0.22 \frac{\alpha_e}{\alpha} \quad (16)$$

where  $d_0 = \alpha l$ . This relationship has entered in a recently developed design directive, a code supplementary, in the Netherlands [7]. Though the relationship here has been

derived for a simply supported roof, it is valid for the other boundary conditions as well, shown before in Figure 6.

At the start of this section it was stated that the roof must be sloped in order to use values of  $n$  smaller than 1. In fact this condition is too strict if camber is applied. In that case we can yet build slope-less roofs for low  $n$ -values. Again, loss of stability determines the limit value  $d_{w,stab}$  and we can make FEM-runs for practical design purposes, see Figure 15.

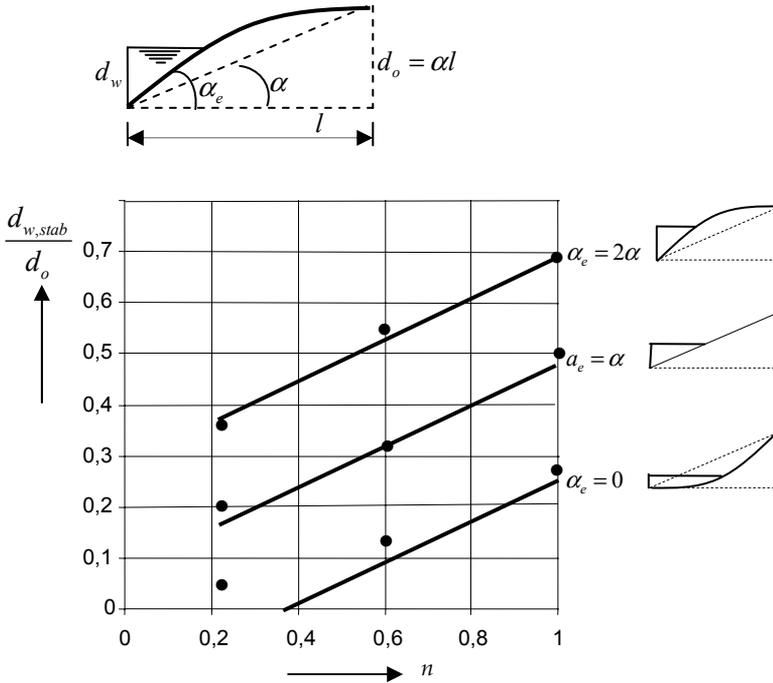


Figure 14: Limit stability values for sloping flexible members. Dots are computer results.

Solid line is the design formula.

Now only the stiffness ratio  $n$  plays a role. Again a linear relationship between the limit value  $d_{w,stab}$  and the stiffness ratio  $n$  has been derived:

$$\frac{d_{w,stab}}{\hat{\delta}_c} = 0.48 + 0.30 n \quad (17)$$

where  $\hat{\delta}_c$  is the top value of the camber. The relation should not be applied for  $n$ -values lower than 0.2. The limit value  $d_{w,stab}$  is always smaller than the camber  $\hat{\delta}_c$ . For higher values the roof will be fully covered, and the spring-piston model applies again.

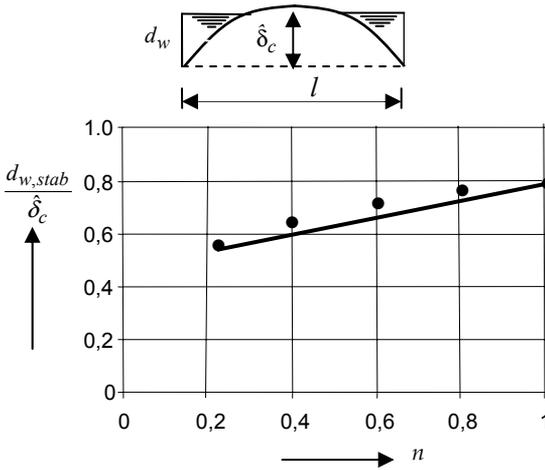


Figure 15: Limit stability values  $d_{w,stab}$  for slope-less flexible members with camber.

Dots are computer results. Solid line is the design formula.

### 3.1.1 Application

Let us consider a structural engineer that has designed a two-way roof existing of primary and secondary members and profiled steel sheeting. The secondary members are continuous girders over the supporting primary members, so a stiffness ratio 5 applies. The primary members IPE 400 are long 15 m and have an in-between distance of 10 m and flexural rigidity  $EI_p = 48570 \text{ kNm}^2$ . The in-between distance of the secondary members IPE 220 is 5 m and their flexural rigidity  $EI_s = 5820 \text{ kNm}^2$ . So, three secondary fields occur between two primary members, hence  $N_s = 3$ . The steel sheeting is chosen from the SAB 100R/825 table and has the thickness 0.75 mm. The sheeting elements have the length of two fields and are placed in pattern. Their deformation is taken into account. The flexural stiffness is  $EI_{sh} = 313 \text{ kNm}^2/\text{m}$ . The slope of the roof is 1.6% and the deflection due to permanent loading is compensated by an adequate camber, resulting in  $\alpha_e = \alpha$ . This roof meets all design code requirements for load combinations other than water. The structural engineer suspects that his roof is extremely flexible and wants to judge the failure danger

due to ponding. In case of a well-functioning emergency outlet, the stationary water depth at the roof edge will become 0.12 m, which corresponds with  $p = 0.5$ . The calculation is done as follows. The structural engineer first determines  $n$ :

$$W = \gamma a_p l_p = 10 \cdot 10 \cdot 15 = 1500 \text{ kN/m}$$

$$D_p = \frac{96EI_s}{l_s^3} = \frac{96 \cdot 48570}{15^3} = 1382 \text{ kN/m}$$

$$D_s = 5 \cdot \frac{96N_s EI_s}{l_s^3} = 5 \cdot \frac{96 \cdot 3 \cdot 5820}{10^3} = 8382 \text{ kN/m}$$

$$D_{sh} = \frac{5}{2} \cdot \frac{96N_{sh} EI_{sh}}{l_{sh}^3} = \frac{5}{2} \cdot \frac{96 \cdot 45 \cdot 313}{5^3} = 27043 \text{ kN/m}$$

$$\frac{1}{D} = \frac{1}{D_p} + \frac{1}{D_s} + \frac{1}{D_{sh}} = \frac{1}{1382} + \frac{1}{8382} + \frac{1}{27043} = \frac{1}{1136}$$

$$n = \frac{D}{W} = \frac{1136}{1500} = 0.76$$

The calculated  $n$  is smaller than 1. Therefore, a limit point will occur. From formula (16) we find  $d_{w,stab} / \alpha l = -0.15 + 0.40 \cdot 0.76 + 0.22 \cdot 1 = 0.37$ . Because  $\alpha l = 0.016 \cdot 15 = 0.240$  m the final result is  $d_{w,stab} = 0.37 \cdot 0.240 = 0.089$  m. This is smaller than the required 0.120 m (apart from a safety margin in one way or another), so stiffening of the roof is a must. It is a proper place here to make a comment regarding the applicability of the model in this example. In fact, the number of secondary members ( $N_s = 3$ ) is very small, which does not really meet the assumption of a homogeneously distributed load on the primary members due to the secondary members. As a consequence, one might fear that the calculated value of  $d_{w,stab}$  (corresponding with  $p = 0.37$ ) is not very accurate. However, it is better than expected, because the roof will be almost fully covered with water at the level  $d_{w,stab}$ . For the rest, the difference between the required 0.120 m and the calculated 0.089 m is so big, that the message can not anyhow be misunderstood. Stiffening is needed.

## 4 Design hints

### 4.1 Importance of $n$

It is recommended to first make a hand calculation and determine the total  $n$  of the roof system with the formula in which the reciprocal values of the  $n$ -values are summed. Even if one intends to make use of a commercial computer program with water accumulation

functionality it is very helpful to first check if the structure belongs to the category of strength-dominated systems ( $n > 1$ ) or the stability-dominated ones ( $n < 1$ ). A computer program will not give clearly interpretable information about this question. The stiffness parameter  $n$  has an awareness function for the structural engineer; it informs him about the nature of the structure and may make him decide to take his measurements. If the structural engineer decides to check the ponding danger by hand calculation he anyhow needs information on the value of  $n$ . For a value  $n > 1$  he must apply the  $n/(n - 1)$  method because the structure will fail due to surpassing the strength limit. If  $n < 1$ , failure of the structure due to instability must be checked by the formula (16) or (17). Strength is not an issue in this case.

#### 4.2 *Effect of steel sheeting*

The formula to calculate the  $n$ -value of a roof system on basis of the reciprocal values of the individual members is an interesting help to judge the effect of the profiled steel sheeting on the ponding danger. The assessment that the steel sheeting is sufficiently stiff to neglect its deformation is not confirmed by the outcome of the formula. Instead, it was shown that this deformation can be easily taken into account.

Let us consider a roof of simply supported secondary members covered by profiled steel sheeting. The  $n$ -value of the member is about 2 and for the steel sheeting in the order of 10, so  $n_s = 2$  and  $n_{sh} = 10$ . If we neglect the deformation of the steel sheeting the system  $n$  is equal to  $n_s$ , so 2. The amplification is  $n/(n - 1) = 2/(2 - 1) = 2$ . If we take the deformation of the steel sheeting into account we must apply the formula for a two-way composed roof  $1/n = 1/n_s + 1/n_{sh} = 1/2 + 1/10 = 0.6$  and therefore  $n = 1.667$ . Now the amplification factor is  $n/(n-1) = 1.667/(1.667-1) = 2.5$ . This means that the bending moment due to water is 25 % higher if we take into account the deformation of the steel sheeting.

Those who want to use a commercial computer program need not necessarily input all steel sheeting in the program. They can account for the deformation by reducing the stiffness of the secondary member adequately. The stiffness parameter  $n$  has the value 2 for the member only and 1.667 if the steel sheeting is considered as well. The ratio is  $2/1.667 = 1.2$ . If the member flexural rigidity is divided by this factor, the correct water loading will be calculated without modelling the steel sheeting separately.

Some applicants of commercial computer programs think that they can account for the deformation of the profiled steel sheeting by adding some additional storage to the accumulated water, say 10 mm. However, this is a misconception. Let us consider the case

of initial water depth  $d = 100$  mm. The correct amplification factor  $n/(n - 1) = 1.667/(1.667 - 1) = 2.5$  yields a final water depth  $w = 250$  mm. If we apply an additional 10 mm the initial water depth becomes  $d = 110$  mm and it is multiplied by the amplification factor  $n/(n - 1) = 2/(2 - 1) = 2$ , yielding a final water depth of 220 mm. If compared to the correct value of 250 mm it is 12% too small.

#### 4.3 *Stiffness of steel sheeting*

It is a subject apart how the stiffness of the steel sheeting must be determined. The flexural rigidity is higher at a moderate stress level than at the yield stress level. When the steel sheeting is loaded up to yield, the stiffness will be reduced because of buckling and an effective stiffness must be used. So, when strength is determining ( $n > 1$ ) the effective rigidity should be used, and when stability is judged ( $n < 1$ ) the full flexural rigidity can be used.

#### 4.4 *Suspected two-way roofs*

It is not uncommon in design codes to make a difference in the way one considers permanent load, wind and snow on the one hand and ponding on roofs on the other. Normally, the roof is designed taking permanent load, wind and snow into account and it is checked afterwards if the resulting structure is sufficiently safe from the point of view of water accumulation. To judge this, however, apparently is a hard job for structural engineers and fundamental knowledge and insight seems to be missing. What is the matter? Examination of collapsed large two-way flat roofs reveals a more or less established pattern. A conscientious design accounting for permanent load, wind and snow easily results in a roof with stiffness parameter  $n \approx 1$  for the primary members and  $n \approx 10$  for the secondary members. As a consequence, the parameter  $n$  for the total roof system is smaller than 1 and the roof collapse is not governed by strength but by stability and at stresses well below the yield stress.

#### 4.5 *Sensitivity to irregularities*

Two-way roof systems of  $n$  close to or smaller than 1 are sensitive to weak connections in the structure. In normal cases the secondary members run as continuous beams over the primary members and their deflection line and the bending moment diagram are more or less regular plots over the total length. It is not uncommon that connections between parts of the continuous secondary members are weaker than the members themselves. In the

extreme case of ‘Gerber’ girders the connections at the zero-moment positions even become hinges. Such a roof behaves sensitive to values of  $n$  close to 1 and failure occurs at a lower water level than in case of a roof with stiff connections at the zero-moment positions. The tricky thing is that this sensitivity is not noticed for the design loads dead weight, wind and snow. Then the bending moment diagrams of the secondary members are very similar with hinges and with stiff connections at the zero-moment positions of the continuous beam. However, the sensitivity comes to light in the check for ponding. The author is of the opinion that stability-dominated structures should preferably be avoided. At low costs the roof system can be shifted to the strength-dominated domain. A slightly higher profile number solves the problem at moderate additional steel costs which in fact vanish when compared to total costs over lifetime. The recommendation of Herwijnen, Snijder and Fijneman [9] to avoid  $n$ -values smaller than 1.5, is supported. A client should not accept differently, because the cheapest possible solution - penny-wise at initiation - may turn out pound-foolish at the end of the day.

**4.6 Does the structure warn at approaching failure?**

There is no difference between strength-dominated roofs and stability-dominated roofs as for the question if the structure warns in time when collapse is approached. Neither the one nor the other does. For strength-dominated structures this is clear from the left-hand part of Figure 16 where the water height is given as a function of the deflection. When a plastic hinge occurs, the loss of stiffness is such that a sudden and dramatic loss of the load carrying capacity occurs. Though the moment-curvature diagram of the member has a plastic plateau, this does not exist at the system level.

As seen in the right-hand part of the figure also stability-dominated structures do not show post-peak behaviour in which the loading is sustained on the peak level. From this point of view no difference in safety concept is needed for the two categories.

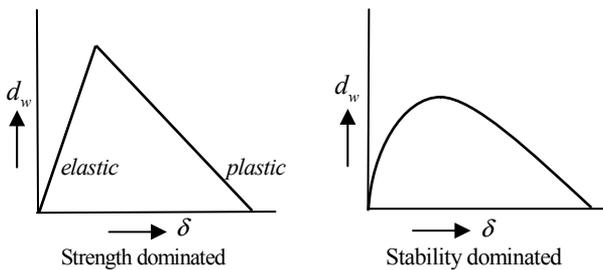


Figure 16: The structure does not warn at failure

## 5 Recommendations

### 5.1 *Applicability of method*

The  $n/(n - 1)$  method for  $n > 1$  is a generic method on the knowledge level of the  $\frac{1}{8}ql^2$  - method, familiar to structural engineers for homogeneously distributed loads. Thinking in terms of this easy-to-handle load category suffices, and the method is applicable for all possible end conditions of the roof members. Deflections due to permanent loading and camber can be accounted for, sloping of roofs is included and composed (two-way) structures can be examined.

The stability check for  $n < 1$  works out in a different way. The  $n/(n - 1)$  method is not valid anymore and formulae (16) and (17) must be used instead for sloping and slope-less roofs respectively, which formulae again are applicable for all member end conditions mentioned.

### 5.2 *Implementation of a safety margin in strength-dominated roofs*

The theory confirms the common sense that the more the roof is sloped, the safer it is. Applying sufficient camber is also profitable. A combination of slope and camber is even better and yields the safest roof. The slope at the edge of the roof is determining for the water depth that can be allowed on the roof. In that respect it is recommended to strive at an angle  $\alpha_e$  at the lower roof edge that is larger than the pitch  $\alpha$  of the roof. It is not obvious in which way a safety margin must be assured. Strength-dominated roofs can be treated as is done for permanent load in combination with snow. The bending moment due to water accumulation is determined by the  $n/(n - 1)$  method or by an iterative computer analysis and is combined with the bending moment for permanent load, each one multiplied by the respective partial safety factor. It has to be shown that the structure is strong enough under these design loads.

### 5.3 *Implementation of a safety margin in stability-dominated roofs.*

For stability-dominated roofs it is an impassable road to safeguard a margin to yielding, because collapse occurs at a stress level below the yield stress. More than one way exists to implement safety in this category. One is to introduce a safety margin through a reduction of  $EI$  of the roof members by a model factor larger than 1 (therefore reduction of  $D$ ). It means that we work with a fictitious lower value of the stiffness parameter  $n$  and as a consequence find from equation (16) or (17) a lower value of  $d_{w,stab}$ . This way has been

chosen in the design code in the Netherlands. An alternative choice is to multiply the specific weight of water by the load factor prior to the start of the analysis (therefore increase  $W$ ). Because of the definition of  $n$  in equation (4) this yields the same reduction of  $n$ , hence of  $d_{w,stab}$ .

Normally it is not clear at forehand whether a structure falls in the category of strength-dominated structures or in the category of stability-dominated ones. Therefore, application of the model factor in the first approach or increase of the specific water weight in the second approach, are prescribed to every structure. It does imply an additional punishment in the  $n/(n - 1)$  method, if the partial safety for the bending moment due to water accumulation is maintained.

There is a third possible way to introduce a safety margin which avoids the disadvantages of the model factor. We can prescribe an additional water margin above the water depth at stationary flowing emergency outlets. It easily accounts for building tolerances and does not show preferential treatment of the one above the other. At the end of the day it will depend on the local design code philosophy and engineering culture which solution will be chosen.

## 6 Conclusions

The hard problem of determining the structural response to water accumulation on roofs has become more accessible to structural engineers. For roof systems of regular plan a simple approach was presented to judge the safety of the structure.

The important stiffness parameter  $n$  can be calculated at choice from  $n = D/W$  or  $n = EI / EI_{cr}$ . In case of composed (two-way or three-way) structures separate values are calculated for the primary member, secondary member and steel sheeting, after which the reciprocal value of  $n$  for the overall roof system is obtained by adding the reciprocal primary, secondary and steel sheeting values. For member end conditions other than simple supports a stiffness ratio is defined, which is used to raise  $D$  or lower  $EI_{cr}$ , depending on the method used to compute  $n$ . A corresponding moment ratio is derived to reduce or increase moments obtained for a simply supported member.

The stiffness ratio  $n$  clearly is an informative parameter for the structural engineer, providing awareness on possible risks. For  $n > 1$  the structural response is governed by strength and the structure will fail because of the occurrence of plastic hinges. For  $n < 1$  the

structural behaviour is governed by stability in the elastic state. A limit value of the water level at the edge of the sloped roof occurs at stresses far below the yield stress. Strength-dominated roofs can be ideal flat or have a slope and/or a camber. Stability-dominated roofs cannot exist without a slope or camber.

For the strength-dominated class of structures, a simple-to-use  $n/(n - 1)$  method was developed, based on a piston-spring model, and closely related to the  $\frac{1}{8}ql^2$ -type of analysis, familiar to the structural engineer in case of homogeneously distributed loads. The alternative model for stability-dominated structures roofs is based on a bar-spring model, two rigid member parts connected by an elastic rotational spring. It is more complicated than the piston-spring model, and therefore design diagrams have been derived to check in a quick way the limit capacity of the roof.

One of the revealing results is the importance of taking into account the deflection of the profiled steel sheeting. In structural engineering practice it is often presupposed that the deformation of the steel sheeting can be neglected. It is shown that this results in moments which may be 25% too low. Yet, it is not necessary to include the steel sheeting into the structural scheme when a structural engineer applies software to perform an iterative water accumulation analysis. It was shown that the stiffness of the secondary members can be judiciously reduced in order to obtain good results, without inputting the steel sheeting.

## References

- [1] Blaauwendraad J., 'Ponding on light-weight flat roofs: Strength and stability', *Engineering Structures*, Vol. 28, 2006. Available online August 2005.
- [2] Snijder H.H., Herwijnen F. van, Fijneman H.J. 'Sensitivity to ponding of roof structures under heavy rainfall', *IABSE Symposium Lisbon 2005, Structures and Extreme Events*, IABSE Report No. 90, pp. 190-191 and full 8 pages paper on CD.
- [3] Bontempi F., Colombi P., Urbano C., 'Nonlinear analysis of ponding effect on nearly flat roof', *Proceedings of fifth pacific structural steel conference*, 1998.
- [4] Colombi P., Urbano C., 'Ponding effect on nearly flat roofs of industrial or commercial single story buildings'. *Proceedings of Eurosteel '99*, 1999.
- [5] Urbano C.A., Ponding effect on flat roofs. *Structural Engineering International* 1/2000, p. 39-43.

- [6] Colombi P., The ponding problem on flat steel roof grids. *Journal of Constructional Steel Research* 62, (2006) 647-655.
- [7] *Nederlandse Praktijkrichtlijn* (Dutch Guidelines for Practice) NPR 6703, 'Wateraccumulatie - Aanvullende rekenregels en vereenvoudigingen voor het belastinggeval regenwater' in NEN 6702 (Ponding on flat roofs caused by rainwater - Supplementary to NEN 6702 with additional and simplified rules, in Dutch), NEN, Delft, The Netherlands, 2006.
- [8] Wijte N.M. 'Analysis of ponding collapse of light-weight roof structures by water raising capacity', *HERON* Vol. 51 (2006) No. 2/3, pp. 97-113.
- [9] Herwijnen F., Snijder H.H., Fijneman H.J., 'Structural design for ponding of rainwater on roof structures', *HERON* Vol. 51 (2006) No. 2/3, pp. 115-150.