Effective section calculation of aluminium plate assemblies under uniform compression considering interactive local buckling

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In this paper, elastic interactive local buckling formulas of box and channel sections are established according to the classic plate stability theory. The restraint effects of adjacent plate elements on the bearing capacity of cross-sections are studied and the corresponding formulas of the restraint coefficient are derived. The effective thickness method is then modified to calculate the ultimate strength of box and channel sections, which is adopted by current codes of various countries. Non-linear finite element analysis is carried out and its results are compared to that of the modified method presented in this paper. It is found that the plate assembly restraints have an obvious influence on the bearing capacity of box and channel sections. The modified method of this paper can lead to safe results in most cases.

Key words: *Interactive buckling, effective width, effective thickness, plate assembly effect*

1 Introduction

One of the main advantages of aluminium profiles is the possibility of forming any shape due to the hot-extrusion molding process, in comparison to steel profiles that are made by hot rolling, welding, or cold forming processes. Due to the higher material expenses, aluminium profiles are often designed to be economic by adopting thinner and weaker plate elements. Moreover, aluminium has a small elastic modulus which is about one third of that of steel. These aspects make that local buckling is the dominant failure mode of aluminium profiles. In this paper, the effective thickness method is adopted according to Eurocode 9 [1] and the Chinese code GB50429 [2], in which the post-buckling strength of a section is considered by using an effective section instead of the true section. In calculation of the effective section the restraints between adjacent elements are not ignored. In this paper these restraint effects are investigated and a modified calculation method for effective sections is provided. A similar method has been adopted as in some design codes for cold-formed steel structures with sections of equal thickness [3], however, in this paper plate assemblies of different thicknesses are considered too.

2 Effective section calculation of compressed plate elements in EC9 and GB50429

In the codes EC9 [1] and GB50429 [3], the post-buckling strength of plates is utilized by introducing effective sections instead of whole sections in calculating the strength and stability of members. For example, the stability equation for axially loaded columns in EC9 is as follows.

$$N_{b,Rd} = \kappa \chi A_{eff} f_{0.2} / \gamma_{M1} \tag{1}$$

where κ is the heating influence factor due to welding. χ is the overall instability coefficient. A_{eff} is the effective area considering local buckling.
In GB50429 the equivalent formula is

$$N = \eta_{haz} \varphi \eta_e A f_{0.2} / \gamma_R \tag{2}$$

where η_{haz} , φ and $\eta_e \mathrm{A}$ are corresponding to κ , χ and A_{eff} of Equation 1, respectively. Due to the complexity of aluminium extruded profiles, EC9 and GB50429 adopted the effective thickness method instead of the effective width method to calculate the effective area. Figure 1 shows the effective sections of a flexural member by effective thickness method and effective width method, respectively, where the part enclosed by solid lines is the effective area. It can be seen that there is a certain difference between the cross-section bearing capacities obtained by the two methods because the sectional parameters including the neutral axis location and effective section modulus are different [4].

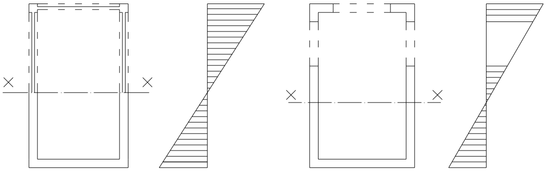


Figure 1: Effective thickness method (left) and effective width method (right)

Although the expression of the effective thickness reduction coefficient ρ_c in EC9 is different from that of the effective width reduction coefficient ρ in EC3 [5], they are derived from the same theory and thus equivalent for none-welded aluminium sections. According to the test work of Winter et al. [6 <is not referring to the paper of Winter et al.>], the effective width reduction coefficient of a plate simply supported along two longitudinal edges can be written as

$$\rho = \sqrt{\frac{\sigma_{cr}}{f_{0.2}}}\left(1 - \frac{0.415}{c}\sqrt{\frac{\sigma_{cr}}{f_{0.2}}}\right) \approx \frac{1}{\overline{\lambda}}\left(1 - 0.22\frac{1}{\overline{\lambda}}\right) \tag{3}$$

where σ_{cr} is the critical buckling stress, $f_{0.2}$ is the nominal yield strength, $c = \pi \big/ \sqrt{3\left(1-\nu^2\right)}$, and $\nu = 0.3$ is Poisson's ratio. $\overline{\lambda}$ is the non dimensional slenderness of the plate,

$$\overline{\lambda} = \sqrt{\frac{f_{0.2}}{\sigma_{cr}}} = \sqrt{\frac{12\left(1-\nu^2\right)b^2 f_{0.2}}{k\pi^2 E t^2}} = \sqrt{\frac{250}{E}} \cdot \sqrt{\frac{12\left(1-\nu^2\right)}{k\pi^2}} \cdot \frac{\beta}{\varepsilon} \tag{4}$$

where E is the material elastic modulus, k is the element buckling coefficient, $\beta = b/t$ is the width-thickness ratio of the plate, and $\varepsilon = \sqrt{250/f_{0.2}}$.

Equation 3 is the basic expression used to calculate the effective width in EC3, and the calculation formula of effective thickness in GB50429 is $\frac{t_e}{t} = \alpha_1 \frac{1}{\overline{\lambda}} - \alpha_2 \frac{0.22}{\overline{\lambda}^2}$, which is identical to Equation 3. For stiffened plates with four simply supported edges [1], is obtained $\overline{\lambda} = 0.031\frac{\beta}{\varepsilon}$ by introducing k = 4, E = 70000 N/mm², ν = 0.3 into Equation 4, and then from Equation 3 is obtained

$$\rho_c = \frac{t_e}{t} \approx \frac{32}{\beta/\varepsilon} - \frac{220}{(\beta/\varepsilon)^2} \tag{5}$$

For non-stiffened plates with three simply supported edges and one free edge [1], $k = 0.425$, is obtained similarly

$$\rho_c = \frac{t_e}{t} \approx \frac{10}{\beta/\varepsilon} - \frac{24}{(\beta/\varepsilon)^2} \tag{6}$$

Equations 5 and 6 are for calculating the effective thickness of non-welded plates in EC9.

Both in EC9 and GB50429 cross-sections of stiffened plates and non-stiffened plates are all treated as independent elements and restraint effects are ignored.

3 Calculation of restraint coefficients between adjacent plates

The Levy solution [7] for uniformly compressed rectangular plates with simply supported edges is adopted to derive the restraint coefficient of an assembly of plates with different widths and thicknesses in box and channel sections. Taking k_0 as the buckling coefficient of an independent plate element, k as the interactive buckling coefficient considering plate assembly effect,

$$\psi = k/k_0 \tag{7}$$

is defined as the restraint coefficient. Then we can write the buckling stress of plate considering the interactive effect of plate assembly as

$$\sigma_{cr} = \frac{\psi k_0 \pi^2 E}{12\left(1-\nu^2\right)\left(b/t\right)^2} \tag{8}$$

When buckling occurs under uniform compression, the flange and web in a section will have the same magnitude of buckling stress [8], i.e.

$$\sigma_{cr} = \frac{k_w \pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_w}{h}\right)^2 = \frac{k_f \pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_f}{b}\right)^2 \tag{9}$$

where t_w and t_f are the web thickness and flange thickness respectively. h and b are the web width and flange width respectively. k_w and k_f are the interactive buckling coefficients of web and flange respectively, which will have the following relationship

$$k_w = k_f \left(\frac{h}{b}\right)^2 \left(\frac{t_f}{t_w}\right)^2 \tag{10}$$

Denoting k_{w0} and k_{f0} as the buckling coefficients of the web and flange as independent plates ignoring the interactive effects of plate assembly, we can obtain the restraint coefficients of the web buckling and flange buckling as $\psi_w = k_w / k_{w0}$ and $\psi_f = k_f / k_{f0}$, respectively. Equation 11 can be written as

$$\psi_w k_{w0} = \psi_f k_{f0} \cdot \left(\frac{h}{b}\right)^2 \left(\frac{t_f}{t_w}\right)^2 \tag{11}$$

*3.1 **Buckling of box sections considering the interactive effects of the plate assembly***

Figure 2 shows the analysis model of a box-section stub under uniform compression. According to the buckling theory, local buckling of the section has the following features [9],

a) Buckling of all plates in a section happens simultaneously;
b) The connection edges between adjacent plates is straight before and after buckling;
c) The angle between adjacent plates is a right angle before and after buckling;
d) The plates in a section will have the same buckling half-wave length;
e) Any point in the connection edges between adjacent plates will have the same value of stress or angular rotation of the two plates.

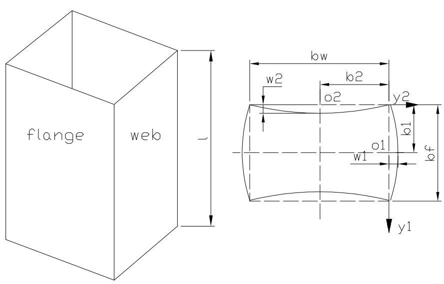


Figure 2: Analysis model of a box section stub

Taking the x-axis of the plate along the longitudinal loading direction, the y-axis as transverse sectional direction and its origin at the symmetric middle centre, shown as in Figure 2, we can express the buckling deflection equations of the flange and web as,

$$w_i = \sin\frac{m\pi x}{l}\sum_{n=1}^{N} F_n Y_{in}(y_i) \tag{12}$$

$$Y_{in}(y) = A_i \cosh \alpha_i y + C_i \cos \beta_i y \tag{13}$$

$$\alpha_i = \left[\lambda_m{}^2 + \lambda_m\sqrt{\frac{N_{cri}}{D}}\right]^{1/2}\ ,\quad \beta_i = \left[-\lambda_m{}^2 + \lambda_m\sqrt{\frac{N_{cri}}{D}}\right]^{1/2} \tag{14}$$

Here, $\lambda_m = m\pi/l$ in which m is the number of buckling half-waves along the loading direction and l is the plate length. $N_{cri} = \sigma_{cr}t_i$ in which the flange is marked as i = 1 and the web is as i = 2, where σ_{cr} is the buckling stress of the section. $D = Et^3/12(1-\nu^2)$ is the bending stiffness of plates where t is the plate thickness. A_i and C_i are undetermined coefficients, and F_n is deflection amplitude coefficient.

Equations 12-14 are from the Levy solution of the partial differential equilibrium equation $\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \frac{\sigma_x \cdot t}{D}\cdot\frac{\partial^2 w}{\partial x^2} = 0$ for elastic buckling plates. The unknown factors and thus the buckling load N_{cr} can be found from the following boundary conditions [7]:

$$w_1\Big|_{y_1=-b_1} = 0 \qquad A_1\cosh\alpha_1(-b_1) + C_1\cos\beta_1(-b_1) = 0 \tag{15}$$

$$w_2\Big|_{y_2=b_2} = 0 \qquad A_2\cosh\alpha_2 b_2 + C_2\cos\beta_2 b_2 = 0 \tag{16}$$

$$\begin{aligned} &\frac{\partial w_1}{\partial y_1}\Big|_{y_1=-b_1} = \frac{\partial w_2}{\partial y_2}\Big|_{y_2=b_2} \quad \Rightarrow \\ &A_1\alpha_1\sinh\alpha_1(-b_1) - C_1\beta_1\sin\beta_1(-b_1) = A_2\alpha_2\sinh\alpha_2 b_2 - C_2\beta_2\sin\beta_2 b_2 \quad \Rightarrow \\ &A_1\alpha_1\sinh\alpha_1 b_1 - C_1\beta_1\sin\beta_1 b_1 + A_2\alpha_2\sinh\alpha_2 b_2 - C_2\beta_2\sin\beta_2 b_2 = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} &M_y\Big|_{y_1=-b_1} = M_y\Big|_{y_2=b_2} \text{ and } \frac{\partial^2 w_1}{\partial x^2}\Big|_{y_1=-b_1} = \frac{\partial^2 w_2}{\partial x^2}\Big|_{y_2=b_2} = 0 \quad \Rightarrow \\ &\frac{\partial^2 w_1}{\partial y_1{}^2}\Big|_{y_1=-b_1} = \frac{\partial^2 w_2}{\partial y_2{}^2}\Big|_{y_2=b_2} \quad \Rightarrow \\ &A_1\alpha_1{}^2\cosh\alpha_1(-b_1) - C_1\beta_1{}^2\cos\beta_1(-b_1) = A_2\alpha_2{}^2\cosh\alpha_2 b_2 - C_2\beta_2{}^2\cos\beta_2 b_2 \quad \Rightarrow \\ &A_1\alpha_1{}^2\cosh\alpha_1 b_1 - C_1\beta_1{}^2\cos\beta_1 b_1 - A_2\alpha_2{}^2\cosh\alpha_2 b_2 + C_2\beta_2{}^2\cos\beta_2 b_2 = 0 \end{aligned} \tag{18}$$

A non-zero solution of w requires non-zero values of A_1, C_1, A_2, C_2. It means that Equation 19 should be satisfied from Equations 15-18.

$$\begin{vmatrix} \cosh\alpha_1 b_1 & \cos\beta_1 b_1 & 0 & 0 \\ 0 & 0 & \cosh\alpha_2 b_2 & \cos\beta_2 b_2 \\ \alpha_1 \sinh\alpha_1 b_1 & -\beta_1 \sin\beta_1 b_1 & \alpha_2 \sinh\alpha_2 b_2 & -\beta_2 \sin\beta_2 b_2 \\ \alpha_1{}^2 \cosh\alpha_1 b_1 & -\beta_1{}^2 \cos\beta_1 b_1 & -\alpha_2{}^2 \cosh\alpha_2 b_2 & \beta_2{}^2 \cos\beta_2 b_2 \end{vmatrix} = 0 \tag{19}$$

Providing some parameter values of the plate element geometry, the transcendental Equation 19 can be solved by mathematic software. The buckling stress σ_{cr} can be obtained from $\alpha_1, \beta_1, \alpha_2, \beta_2$.

3.2 *Buckling of channel sections considering the interactive effects of the plate assembly*

Figure 3 shows the analysis model of a channel section stub under uniform compression.

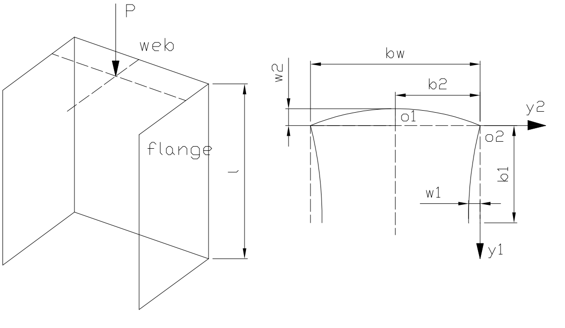


Figure 3: Analysis model of channel section stub

Taking x-axis as longitudinal compression direction, y-axis as transverse sectional direction, the origin of y-axis in the web at its symmetric center, and the origin of y-axis in the flange at its connection edge with web, we can express the buckling deflection as,

$$w_i = \sin\frac{m\pi x}{l}\sum_{m=1}^{N} F_m Y_{im}(y_i) \tag{20}$$

For flange,

$$Y_{1m}(y) = A_1 \cosh\alpha_1 y + B_1 \sinh\alpha_1 y + C_1 \cos\beta_1 y + E_1 \sin\beta_1 y \tag{21}$$

For web,

$$Y_{2m}(y) = A_2 \cosh\alpha_2 y + C_2 \cos\beta_2 y \tag{22}$$

where $A_1, B_1, C_1, E_1, A_2, C_2$ are undetermined coefficients and the parameters α_i and β_i are defined as in Equation 14.

Equations 20-22 satisfy the partial differential equilibrium equation of elastic buckling plate, i.e. $\nabla^2\nabla^2 w + \frac{\sigma \cdot t}{D} \cdot \frac{\partial^2 w}{\partial x^2} = 0$. The boundary conditions are as follows [7],

$$\begin{aligned} & w_1\big|_{y_1=0} = 0 \quad \Rightarrow \\ & A_1 + C_1 = 0 \quad \Rightarrow \\ & C_1 = -A_1 \end{aligned} \tag{23}$$

$$\begin{aligned} & w_2\big|_{y_2=b_2} = 0 \quad \Rightarrow \\ & A_2 \cosh \alpha_2 b_2 + C_2 \cos \beta_2 b_2 = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} & \frac{\partial w_1}{\partial y_1}\bigg|_{y_1=0} = \frac{\partial w_2}{\partial y_2}\bigg|_{y_2=b_2} \quad \Rightarrow \\ & B_1\alpha_1 + E_1\beta_1 = A_2\alpha_2 \sinh \alpha_2 b_2 - C_2\beta_2 \sin \beta_2 b_2 \quad \Rightarrow \\ & B_1\alpha_1 + E_1\beta_1 - A_2\alpha_2 \sinh \alpha_2 b_2 + C_2\beta_2 \sin \beta_2 b_2 = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} & M_y\big|_{y_1=0} = M_y\big|_{y_2=b_2} \quad \text{and} \quad \frac{\partial^2 w_1}{\partial x^2}\bigg|_{y_1=0} = \frac{\partial^2 w_2}{\partial x^2}\bigg|_{y_2=b_2} = 0 \quad \Rightarrow \\ & \frac{\partial^2 w_1}{\partial y_1^2}\bigg|_{y_1=0} = \frac{\partial^2 w_2}{\partial y_2^2}\bigg|_{y_2=b_2} \quad \Rightarrow \\ & A_1\alpha_1^2 - C_1\beta_1^2 = A_2\alpha_2^2 \cosh\alpha_2 b_2 - C_2\beta_2^2 \cos\beta_2 b_2 \quad \Rightarrow \\ & A_1\left(\alpha_1^2 + \beta_1^2\right) - A_2\alpha_2^2 \cosh\alpha_2 b_2 + C_2\beta_2^2 \cos\beta_2 b_2 = 0 \end{aligned} \tag{26}$$

$$\begin{aligned} & M_y\big|_{y_1=b_1} = 0 \quad \Rightarrow \\ & \left(\frac{\partial^2 w_1}{\partial y_1^2} + \nu \frac{\partial^2 w_1}{\partial x^2}\right)\bigg|_{y_1=b_1} = 0 \quad \Rightarrow \\ & A_1\left(\alpha_1^2 - \nu\lambda_m^2\right)\cosh \alpha_1 b_1 + B_1\left(\alpha_1^2 - \nu\lambda_m^2\right)\sinh \alpha_1 b_1 - C_1\left(\beta_1^2 + \nu\lambda_m^2\right)\cos \beta_1 b_1 \\ & - E_1\left(\beta_1^2 + \nu\lambda_m^2\right)\sin \beta_1 b_1 = 0 \quad \Rightarrow \\ & A_1\left[\left(\alpha_1^2 - \nu\lambda_m^2\right)\cosh \alpha_1 b_1 + \left(\beta_1^2 + \nu\lambda_m^2\right)\cos \beta_1 b_1\right] + B_1\left(\alpha_1^2 - \nu\lambda_m^2\right)\sinh \alpha_1 b_1 \\ & - E_1\left(\beta_1^2 + \nu\lambda_m^2\right)\sin \beta_1 b_1 = 0 \end{aligned} \tag{27}$$

$$M_{yx}\big|_{y_1=b_1} = 0 \quad \text{and} \quad Q_y\big|_{y_1=b_1} = 0 \quad \Rightarrow$$

$$\left(Q_y - \frac{\partial M_{yx}}{\partial x}\right)\Bigg|_{y_1=b_1} = 0 \quad \Rightarrow$$
$$\left(\frac{\partial^3 w_1}{\partial y_1{}^3} + (2-\nu)\frac{\partial^3 w_1}{\partial x^2 \partial y_1}\right)\Bigg|_{y_1=b_1} = 0 \quad \Rightarrow$$
$$A_1\alpha_1\left[\alpha_1{}^2 - (2-\nu)\lambda_m{}^2\right]\sinh\alpha_1 b_1 + B_1\alpha_1\left[\alpha_1{}^2 - (2-\nu)\lambda_m{}^2\right]\cosh\alpha_1 b_1 + C_1\beta_1\left[\beta_1{}^2 + (2-\nu)\lambda_m{}^2\right]\sin\beta_1 b_1$$
$$- E_1\beta_1\left[\beta_1{}^2 + (2-\nu)\lambda_m{}^2\right]\cos\beta_1 b_1 = 0 \quad \Rightarrow$$
$$A_1\left[\alpha_1\left[\alpha_1{}^2 - (2-\nu)\lambda_m{}^2\right]\sinh\alpha_1 b_1 - \beta_1\left[\beta_1{}^2 + (2-\nu)\lambda_m{}^2\right]\sin\beta_1 b_1\right] + B_1\alpha_1\left[\alpha_1{}^2 - (2-\nu)\lambda_m{}^2\right]\cosh\alpha_1 b_1$$
$$- E_1\beta_1\left[\beta_1{}^2 + (2-\nu)\lambda_m{}^2\right]\cos\beta_1 b_1 = 0 \tag{28}$$

Equations 24-28 are linear equations of A_1, B_1, E_1, A_2, C_2, and Equation 29 can be obtained from its non-zero solution requirement.

$$\begin{vmatrix} 0 & 0 & 0 & \cosh\alpha_2 b_2 & \cos\beta_2 b_2 \\ 0 & \alpha_1 & \beta_1 & -\alpha_2\sinh\alpha_2 b_2 & \beta_2\sin\beta_2 b_2 \\ \alpha_1{}^2 + \beta_1{}^2 & 0 & 0 & -\alpha_2{}^2\cosh\alpha_2 b_2 & \beta_2{}^2\cos\beta_2 b_2 \\ \phi_1\cosh\alpha_1 b_1 + \phi_2\cos\beta_1 b_1 & \phi_1\sinh\alpha_1 b_1 & -\phi_2\sin\beta_1 b_1 & 0 & 0 \\ \alpha_1\varphi_1\sinh\alpha_1 b_1 - \beta_1\varphi_2\sin\beta_1 b_1 & \alpha_1\varphi_1\cosh\alpha_1 b_1 & -\beta_1\varphi_2\cos\beta_1 b_1 & 0 & 0 \end{vmatrix} = 0 \tag{29}$$

Where $\phi_1 = \alpha_1{}^2 - \nu\lambda_m{}^2$, $\phi_2 = \beta_1{}^2 + \nu\lambda_m{}^2$, $\varphi_1 = \alpha_1{}^2 - (2-\nu)\lambda_m{}^2$, $\varphi_2 = \beta_1{}^2 + (2-\nu)\lambda_m{}^2$. The buckling stress σ_{cr} can then be found from the solution of $\alpha_1, \beta_1, \alpha_2, \beta_2$ of Equation 29.

3.3 *Formula for restraint coefficient of plate assembly*

The critical stress σ_{cr} of a compressed box or channel section stub can be solved based on the descriptions before. From σ_{cr} the interactive buckling coefficient and the restraint coefficient can then be found as $k = \frac{12(1-\nu^2)(b/t)^2}{\pi^2 E}\sigma_{cr}$ and $\psi = k/k_0$, respectively. Further analysis proves that the interactive buckling coefficient does not depend on the material elastic modulus and the strength, and depends only on the width ratio b/h and the thickness ratio t_f/t_w of the sections. For uniformly compressed boxes and channel sections with the same plate thickness the web interactive buckling coefficient k_w can be derived and expressed as (see BS5950 [3]),

for box section:

$$k_w \approx 7 - \frac{2b/h}{0.11 + b/h} - 1.2(b/h)^3 \tag{30}$$

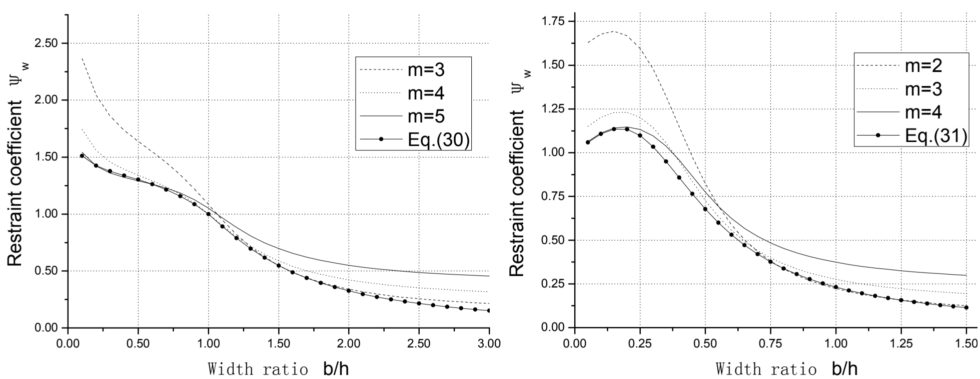
for channel section:

$$k_w \approx \frac{2}{\sqrt{1+15(b/h)^3}} + \frac{2+4.8b/h}{1+15(b/h)^3} \tag{31}$$

The flange interactive buckling coefficient k_f can be calculated from k_w by Equation 10. Taking an aluminium box section and a channel section stub with the parameters listed in Table 1 under uniform compression for example, we can obtain its relationships between the restraint coefficient ψ_w and the width ratio b/h when $t_f = t_w$, as shown in Figure 4. Here, m is the number of buckling waves. The results from Equations 30 and 31 of BS5950 and from the equations stated before are plotted and compared. In Figure 4 the horizontal line ψ_w = 1 means the buckling of independent plates ignoring interactive effects of the plate assembly. It can be seen that the results from BS5950 and from the equations stated before coincide with each other.

Table 1: Size and mechanical properties of aluminium section stub

Section type	Web width h (mm)	Web thickness t_w (mm)	Stub length l (mm)	Elastic modulus E (N/mm^2)	Poisson's ratio ν
Box section	180	4	720	70000	0.3
Channel section	180	4	720	70000	0.3



(a) Box section *(b) Channel section*

Figure 4: Influence of element width ratio on restraint coefficient

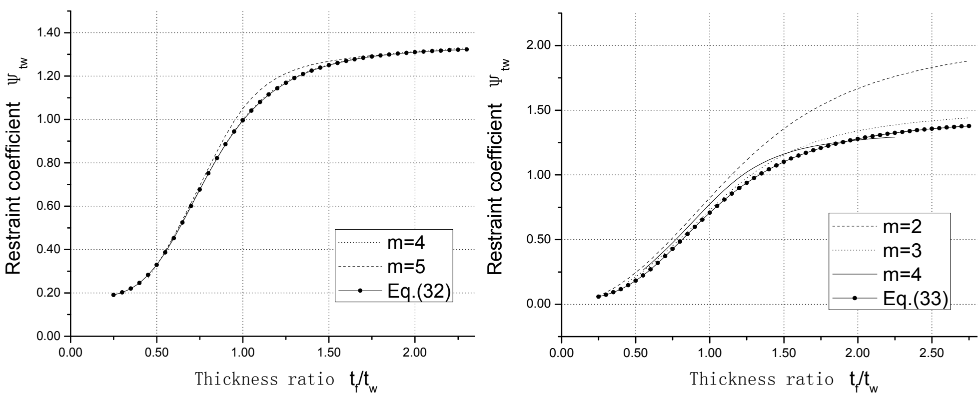
For box sections with the same width of the web and flange, the relationship between the restraint coefficient ψ_{tw} and the plate thickness ratio t_f/t_w can be expressed as

$$\psi_{tw} \approx 1.34 - \frac{1.16}{1 + 2.37\left(t_f/t_w\right)^4} \tag{32}$$

For channel section where the web width is twice of the flange, the relationship between the restraint coefficient ψ_{tw} and the plate thickness ratio t_f/t_w can be expressed as

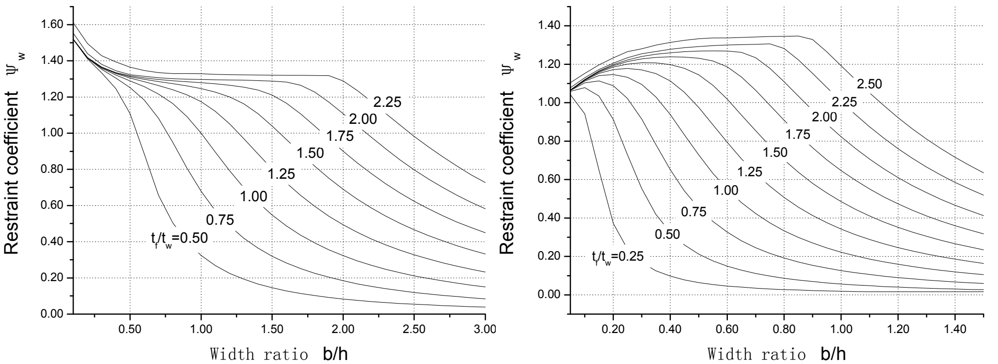
$$\psi_{tw} \approx 1.45 - \frac{1.41}{1 + 0.9\left(t_f/t_w\right)^3} \tag{33}$$

Equations 32 and 33 are approximation formulas obtained by parameter fitting from the theoretical equations stated before in Section 3.1 and 3.2. Figure 5 gives the relationship between ψ_{tw} and t_f/t_w of the two stubs with the parameters listed in Table 1. The comparison between the results from Equations 32 and 33 and theoretical equations is also made. Following ψ_{tw} we can find the other coefficients such as $k_w = k_{w0} \cdot \psi_{tw}$ and k_f from Equation 10. For general sections with different values of the width and thickness of web and flange, the restraint coefficient can be approximated get by interpolation in Figure 6 or in Table 2 and Table 3.



(a) Box section *(b) Channel section*

Figure 5: Influence of the element thickness ratio on the restraint coefficient



(a) Box section *(b) Channel section*

Figure 6: Influence of the element width ratio and the thickness ratio on the restraint coefficient

Table 2: Restraint coefficient ψ_w *of box sections*

Width ratio b/h	Thickness ratio t_f/t_w							
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
0.10	1.519	1.519	1.519	1.519	1.519	1.519	1.553	1.611
0.20	1.411	1.416	1.417	1.418	1.418	1.418	1.440	1.496
0.30	1.334	1.354	1.360	1.363	1.364	1.365	1.381	1.429
0.40	1.246	1.305	1.322	1.329	1.333	1.335	1.346	1.395
0.50	1.108	1.252	1.290	1.305	1.313	1.317	1.324	1.364
0.60	0.885	1.183	1.257	1.284	1.298	1.305	1.315	1.346
0.70	0.658	1.084	1.217	1.264	1.285	1.297	1.307	1.335
0.80	0.505	0.951	1.164	1.241	1.274	1.291	1.302	1.330
0.90	0.400	0.807	1.092	1.212	1.261	1.285	1.299	1.330
1.00	0.324	0.678	1.000	1.174	1.247	1.279	1.297	1.328
1.10	0.268	0.572	0.896	1.121	1.228	1.273	1.295	1.326
1.20	0.226	0.488	0.793	1.052	1.201	1.266	1.294	1.324
1.30	0.193	0.420	0.700	0.971	1.162	1.255	1.293	1.323
1.40	0.167	0.365	0.618	0.885	1.108	1.239	1.291	1.322
1.50	0.145	0.320	0.547	0.802	1.040	1.211	1.288	1.321
1.60	0.128	0.283	0.487	0.725	0.965	1.166	1.282	1.320
1.70	0.114	0.251	0.436	0.656	0.890	1.107	1.266	1.320
1.80	0.102	0.225	0.393	0.595	0.818	1.039	1.227	1.319
1.90	0.092	0.203	0.355	0.541	0.751	0.969	1.172	1.318
2.00	0.083	0.184	0.322	0.493	0.690	0.900	1.108	1.289
2.10	0.076	0.167	0.294	0.452	0.635	0.835	1.042	1.236
2.20	0.069	0.153	0.269	0.415	0.585	0.775	0.976	1.173
2.30	0.063	0.141	0.248	0.382	0.541	0.720	0.913	1.109
2.40	0.058	0.130	0.229	0.353	0.502	0.670	0.853	1.045
2.50	0.054	0.120	0.212	0.327	0.466	0.624	0.798	0.983
2.60	0.050	0.111	0.196	0.304	0.433	0.582	0.747	0.924
2.70	0.047	0.103	0.183	0.284	0.405	0.544	0.701	0.870
2.80	0.044	0.096	0.171	0.265	0.378	0.510	0.657	0.818
2.90	0.041	0.090	0.160	0.248	0.355	0.478	0.618	0.771
3.00	0.038	0.084	0.150	0.233	0.333	0.449	0.581	0.727

Table 3: Restraint coefficient ψ_w of channel sections

Width ratio b/h	Thickness ratio t_f/t_w									
	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.05	1.041	1.060	1.063	1.064	1.065	1.065	1.065	1.066	1.077	1.104
0.10	0.942	1.079	1.104	1.112	1.116	1.119	1.120	1.121	1.129	1.152
0.15	0.652	1.035	1.114	1.140	1.152	1.158	1.162	1.165	1.172	1.196
0.20	0.372	0.913	1.088	1.147	1.172	1.185	1.193	1.198	1.205	1.233
0.25	0.239	0.731	1.023	1.132	1.178	1.201	1.214	1.222	1.234	1.265
0.30	0.169	0.552	0.915	1.095	1.171	1.208	1.228	1.240	1.255	1.281
0.35	0.125	0.419	0.782	1.030	1.151	1.207	1.236	1.252	1.267	1.300
0.40	0.098	0.326	0.652	0.939	1.113	1.198	1.239	1.260	1.278	1.314
0.45	0.078	0.260	0.541	0.833	1.055	1.179	1.237	1.266	1.286	1.325
0.50	0.064	0.212	0.452	0.728	0.975	1.145	1.231	1.269	1.293	1.333
0.55	0.054	0.177	0.382	0.632	0.885	1.091	1.217	1.270	1.297	1.337
0.60	0.045	0.149	0.326	0.550	0.794	1.019	1.187	1.269	1.300	1.337
0.65	0.040	0.128	0.281	0.481	0.708	0.937	1.134	1.262	1.303	1.339
0.70	0.035	0.111	0.245	0.423	0.632	0.854	1.064	1.234	1.304	1.342
0.75	0.031	0.097	0.216	0.374	0.565	0.775	0.987	1.179	1.305	1.344
0.80	0.027	0.086	0.191	0.334	0.507	0.703	0.909	1.109	1.283	1.346
0.85	0.025	0.077	0.171	0.299	0.457	0.638	0.835	1.035	1.223	1.347
0.90	0.023	0.069	0.154	0.269	0.413	0.581	0.766	0.960	1.153	1.329
0.95	0.020	0.062	0.139	0.244	0.375	0.530	0.703	0.889	1.080	1.264
1.00	0.019	0.056	0.126	0.222	0.343	0.485	0.647	0.823	1.008	1.193
1.05	0.017	0.051	0.115	0.203	0.314	0.446	0.596	0.762	0.939	1.121
1.10	0.016	0.047	0.105	0.186	0.288	0.410	0.551	0.706	0.874	1.051
1.15	0.016	0.043	0.097	0.172	0.266	0.379	0.510	0.656	0.815	0.985
1.20	0.016	0.040	0.090	0.159	0.246	0.351	0.473	0.610	0.761	0.922
1.25	0.016	0.037	0.083	0.147	0.228	0.326	0.440	0.569	0.711	0.864
1.30	0.016	0.034	0.077	0.137	0.213	0.304	0.411	0.532	0.665	0.810
1.35	0.016	0.032	0.072	0.127	0.198	0.284	0.384	0.497	0.623	0.761
1.40	0.016	0.030	0.067	0.119	0.185	0.266	0.360	0.466	0.585	0.716
1.45	0.016	0.028	0.063	0.112	0.174	0.249	0.338	0.438	0.551	0.674
1.50	0.016	0.026	0.059	0.105	0.163	0.234	0.317	0.412	0.518	0.635

3.4 *Formula of effective thickness considering restraints of adjacent plates*

Based on the above discussion, the restraint coefficient of web can be obtained. Substituting $k = \psi k_0$ into Equation 4, we can obtain the plate slenderness considering the plate assembly effect as

$$\overline{\lambda} = \frac{\overline{\lambda}_0}{\sqrt{\psi}} \tag{34}$$

where $\overline{\lambda}_0$ is the slenderness of an independent plate. Substituting Equation 34 into Equation 3, we can write the reduction coefficient ρ_c' of the effective thickness considering the plate assembly effect as follows,

$$\rho_c' = \sqrt{\psi} \cdot \frac{\bar{\lambda}_0 - 0.22\sqrt{\psi}}{\bar{\lambda}_0 - 0.22} \cdot \rho_c \tag{35}$$

For stiffened plates, k_0 = 4, substituting Equation 4 into 35 we have

$$\rho_c{}' = \sqrt{\psi} \cdot \frac{\beta/\varepsilon - 7\sqrt{\psi}}{\beta/\varepsilon - 7} \cdot \rho_c \tag{36}$$

For non-stiffened plates, k_0 =0.425, substituting Equation 4 into 35 we have

$$\rho_c{}' = \sqrt{\psi} \cdot \frac{\beta/\varepsilon - 2.4\sqrt{\psi}}{\beta/\varepsilon - 2.4} \cdot \rho_c \tag{37}$$

where, ρ_c is the reduction coefficient of an independent plate, expressed by Equations 5 and 6.

4 Finite element analysis and comparison of bearing capacity of an aluminium stub

In order to verify the calculation method for the ultimate bearing capacity of the box and the channel section short stub, the finite element software ANSYS has been adopted, in which SHELL181 element suitable for material nonlinearity and large deformation analysis is used and initial imperfections are taken into account. The Ramberg-Osgood stress-strain constitutive law is adopted (Eq. 38), which has been accurately approximated by more than 20 stress-strain points and linear interpolations.

$$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{f_{0.2}}\right)^n, \quad n = \frac{\ln 2}{\ln(f_{0.2}/f_{0.1})} \tag{38}$$

In this, E is the initial elastic modulus, $f_{0.1}$ is the stress corresponding to residual strain $\varepsilon_0 = 0.001$. The maximum amplitude of the initial geometric imperfection is taken as 1/200 of the plate width and in the form of the first-order buckling mode.

Table 4: Mechanical properties of the box section and the channel section stub

Section type	Alloy grade	E (N/mm²)	$f_{0.2}$ (MPa)	f_u (MPa)	n
Box section	6061-T6	70000	240	285	35
Channel section	6061-T6	70000	240	285	35

Table 5: Dimensions of the box section and the channel section stub

Section type	Web width h (mm)	Web thickness t_w (mm)	Flange width b (mm)	Column length l (mm)
Box section	180	4	180	765
Channel section	180	4	90	765

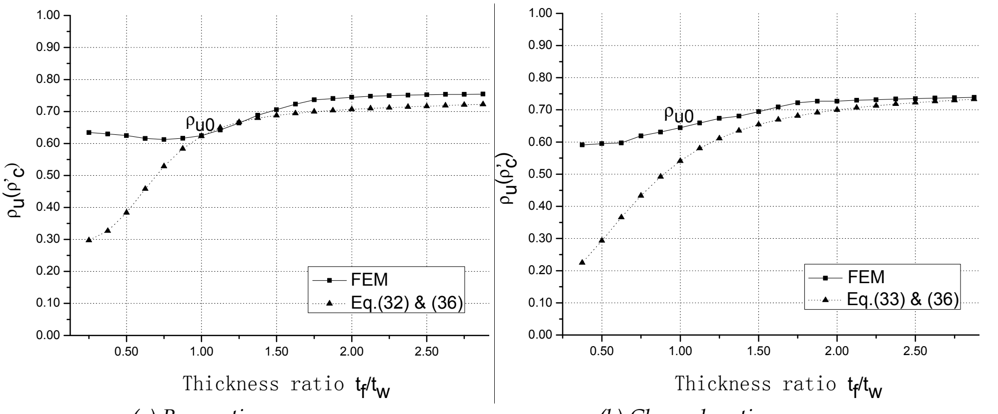
The thickness of the flange has been changed to vary the thickness ratio t_f / t_w , and the ultimate bearing capacity P_u of web is computed by the FEM software. The reduction coefficient of web can be expressed as

$$\rho_u = \frac{P_u}{A_w f_{0.2}} = \frac{P_u}{h t_w f_{0.2}} \tag{39}$$

Figure 7 shows the result of ρ_u of Equation 39 and ρ_c' of Equation 32, 33 and 36. It can be seen that, due to the plate assembly effect, ρ_u may be less than ρ_{u0} when flange thickness is less than the web thickness, where ρ_{u0} is the reduction coefficient from Equation 39 when flange thickness and web thickness are equal. This shows that Equation 5 used in EC9, which ignores the weakening effect of the flange to the web, may lead to unsafe results. Figure 7 shows that the proposed formulas in this paper considering the plate assembly effect can basically reflect the bearing capacity variation of the plate with changing of the adjacent plate thickness, and in most cases they can lead to safe results.

5 Conclusions

Aiming at the local buckling capacity, considering plate assembly effects, this paper derived the formula for calculation of the restraint coefficient of box and channel sections with different values of plate width and thickness. This restraint coefficient can be easily introduced to the effective thickness formula in the current design codes for aluminium structures so that the plate assembly effect on the local buckling of plates in sections can be taken into account.



(a) Box section *(b) Channel section*

Figure 7: Comparison of the results obtained by the finite element method and the proposed formula

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