Vibrations of machine foundations and surrounding soil

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Rotating or pulsing machines are often placed on concrete foundations supported by soil. The machines cause vibrations in the building and in the surrounding soil. This paper provides information, formulas and calculation examples to predict these vibrations. The formulas have been experimentally tested for both soil foundations and pile foundations. In addition, criteria are provided for evaluating the vibrations.

1 Introduction

In consulting practice, predicting vibrations due to machines is a regularly occurring task. There are several textbooks that provide a comprehensive introduction to the field of foundation dynamics, for example [1] and [2]. However, these books are incomplete, particularly with respect to pile foundations and the significant influence of the soil layer surrounding a foundation. The first author has performed research on foundation dynamics and has been a consultant in numerous foundation vibration problems for TNO in the Netherlands. This paper is intended as knowledge transfer to the next generation of engineers, providing a short but hopefully useful introduction to the field foundation dynamics.

2 Forces caused by rotating machines

A machine with a rotating component causes forces on its foundation. The amplitude of the centrifugal force F depends on the rotating mass m, the mass unbalance e and the angular frequency ω.

$$F = m e \omega^2 \qquad (1)$$

The mass unbalance e is the distance between the centre of gravity of the rotation mass and the axis of rotation. The angular frequency ω depends on the frequency f.

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Table 1. Balance quality grades for various groups of representative rigid rotors [3]

Quality grade	Rotor types
G 4000	Crankshaft/drives of rigidly mounted slow marine diesel engines with uneven number of cylinders
G 1600	Crankshaft/drives of rigidly mounted large two-cycle engines
G 630	Crankshaft/drives of rigidly mounted large four-cycle engines Crankshaft/drives of elastically mounted marine diesel engines
G 250	Crankshaft/drives of rigidly mounted fast four-cylinder diesel engines
G 100	Crankshaft/drives of fast diesel engines with six or more cylinders Complete engines (gasoline or diesel) for cars, trucks and locomotives
G 40	Car wheels, wheel rims, wheel sets, drive shafts Crankshaft/drives of elastically mounted fast four-cycle engines with six or more cylinders Crankshaft/drives of engines of cars, trucks and locomotives
G 16	Drive shafts (propeller shafts, cardan shafts) with special requirements Parts of crushing machines Parts of agricultural machinery Individual components of engines (gasoline or diesel) for cars, trucks and locomotives Crankshaft/drives of engines with six or more cylinders under special requirements
G 6.3	Parts of process plant machines Marine main turbine gears (merchant service) Centrifuge drums Paper machinery rolls; print rolls Fans Assembled aircraft gas turbine rotors Flywheels Pump impellers Machine-tool and general machinery parts Medium and large electric armatures (of electric motors having at least 80 mm shaft height) without special requirements Small electric armatures, often mass produced, in vibration insensitive applications and/or with vibration-isolating mountings Individual components of engines under special requirements
G 2.5	Gas and steam turbines, including marine main turbines (merchant service) Rigid turbo-generator rotors Computer memory drums and discs Turbo-compressors Machine-tool drives Medium and large electric armatures with special requirements Small electric armatures not qualifying for one or both of the conditions specified for small electric armatures of balance quality grade G 6.3 Turbine-driven pumps
G 1	Tape recorder and phonograph (gramophone) drives Grinding-machine drives Small electric armatures with special requirements
G 0.4	Spindles, discs and armatures of precision grinders Gyroscopes

$$\omega = 2\pi f \tag{2}$$

The centrifugal force can be replaced by two perpendicular forces with the same angular frequency ω that are also perpendicular to the rotating axis. The phase difference between these forces is 90°.

A well-balanced machine causes small forces on the foundation. The balance requirements for machines are formulated in ISO 1940/1 [3]. The purpose of this code is to prevent large stresses in engines. The code classifies machines based on the geometry of the rotating

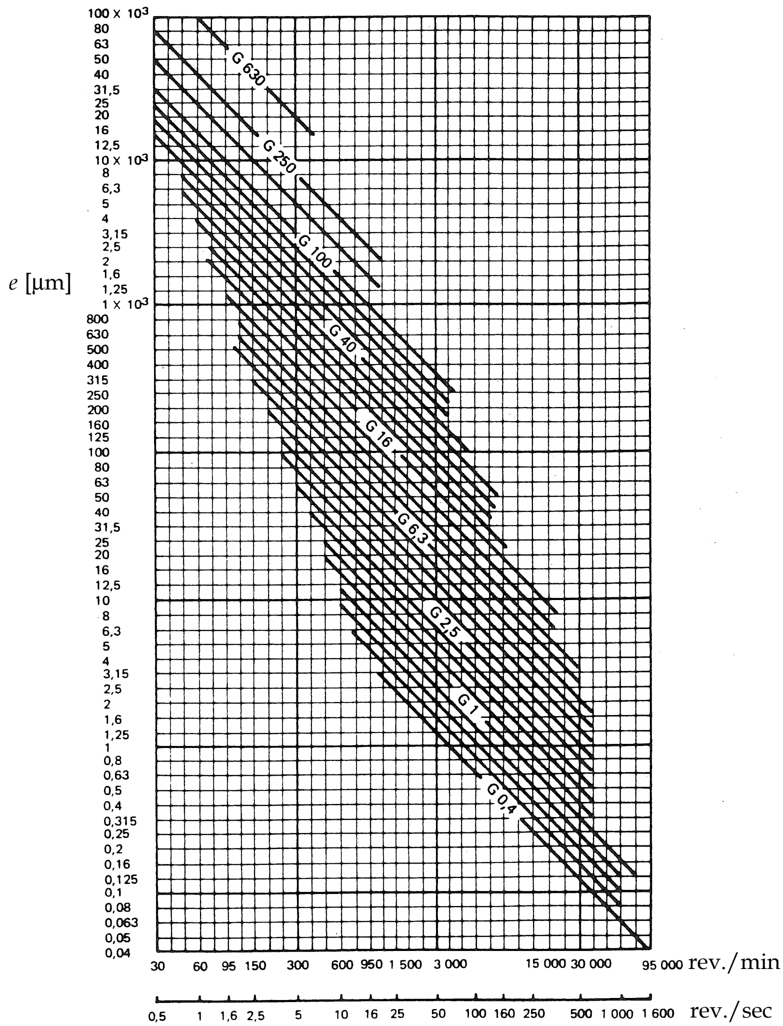

Figure 1. Accepted unbalance e [µm] *as a function of the service speed of rotation* ω [rev./min and rev./s] *for various balance quality grades [3]*

parts (Table 1, Fig. 1). This is based on the fact that geometrically similar rotors running at the same speed will have similar stresses in the rotor and its bearings. Each type of machine has a balance quality grade. For example, a steam turbine has a balance quality grade G 2.5. This means that e times ω should be smaller than 2.5 mm/s. If the engine has a maximum service speed of 600 revolutions per minute, its angular frequency ω is $600 \times 2\pi / 60 = 63$ rad/s. The maximum centre of gravity displacement is called *permissible residual unbalance*, $e_{per} = 2.5 / 63 = 40\ \mu$m. A mechanical engineer will adjust small masses on the rotating parts of this engine to obtain an unbalance smaller than 40 μm.

3 Forces caused by machines with pistons

Figure 2 shows the parts of a piston engine. This section shows that this machine causes forces with more than one frequency. The length of the rotating bar OA is r_1 . The length of piston bar AB is r_2 . The distance between point O and the piston is $r_1 + r_2 - z$. For the moving piston, two coupled kinematic equations can be formulated [4].

$$\begin{aligned} r_1 + r_2 &= r_1 \cos(\omega t) + r_2 \cos\alpha + z \\ r_1 \sin(\omega t) &= r_2 \sin\alpha \end{aligned} \tag{3}$$

From Eqs (3) the piston movement z can be solved.

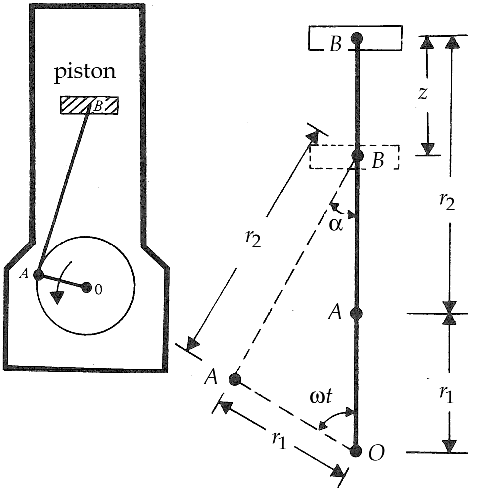

Figure 2. Kinematics of a piston engine

$$z = r_1 - r_1 \cos(\omega t) + \frac{r_1^2}{r_2} \frac{1}{4}(1 - \cos(2\omega t)) \tag{4}$$

In this is used that $\cos\alpha \approx 1 - \frac{1}{2}\sin^2\alpha$ and $\sin^2\alpha = \frac{1}{2}(1-\cos(2\alpha))$. The acceleration is

$$\frac{d^2 z}{dt^2} = r_1 \omega^2 \cos(\omega t) + \frac{r_1^2}{r_2}\omega^2 \cos(2\omega t) \tag{5}$$

The force is the product of the mass of the piston plus bar and the acceleration. The first order force is

$$F_1 = m r_1 \omega^2 . \tag{6}$$

The second order force is

$$F_2 = m \frac{r_1^2}{r_2} \omega^2 . \tag{7}$$

Note that these forces act in the direction of motion *z*. Most piston engines have several pistons which partly balance each other out. Nonetheless, perfect balancing is not possible and the foundation is loaded by forces with angular frequency ω and 2ω. Most machines are more complicated than shown in Figure 2 and therefore also 3ω, 4ω, etcetera occur.

4 Soil stiffness

The soil stiffness at low stresses can be described by a modulus of elasticity. The soil can be modelled as a linear elastic half space. A shock excitation on the half space causes three waves; a compressive wave, a shear wave and a Rayleigh wave (Fig. 3). The compressive wave contains 7% of the shock energy. The shear wave contains 26% of the energy and the Rayleigh wave contains 67% of the energy [1]. The compressive wave is fastest with a velocity of

$$v_c = \sqrt{\frac{E}{\rho} \frac{1-\nu}{(1+\nu)(1-2\nu)}} , \tag{8}$$

where E is the modulus of elasticity, ρ is the soil density and ν is Poisson's ratio. The shear wave velocity is

$$v_s = \sqrt{\frac{E}{\rho} \frac{1}{2(1+\nu)}} . \tag{9}$$

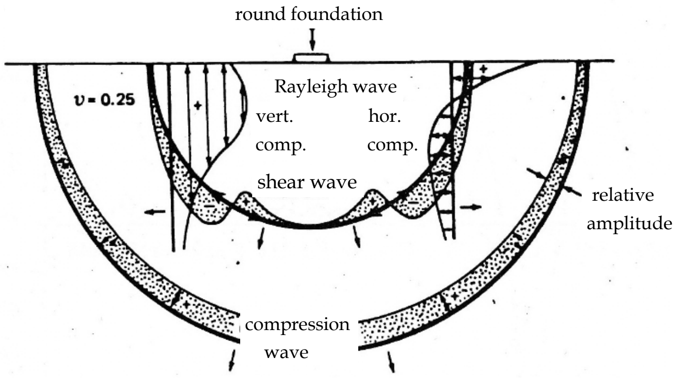

Figure 3. Waves in a halve space excited by a dynamic point load [1, p. 91]

The Rayleigh wave velocity is

$$v_R = \sqrt{\frac{E}{\rho}\frac{1}{2(1.4-0.8\nu)(1-\nu)}} \; . \qquad (10)$$

The velocities of the shear wave and the Rayleigh wave are almost the same. The Rayleigh wave has not only more energy than the other waves, it also travels at the surface only and loses less energy during travelling. Therefore, at some distance of the source the Rayleigh wave is much larger than the other waves.

In an experiment a round steel plate was put onto the soil of a clear field. A displacement receiver was positioned at 10 m distance of the plate centre. The steel plate was hit by a hammer four times. The recorded result is shown in Figure 4. After 28 milliseconds the receiver recorded the compression wave and after 56 milliseconds it recorded the shear and Rayleigh waves. Consequently, the compression wave velocity is 10/0.028 = 357 m/s. The shear and Rayleigh wave velocity are 10/0.056 = 179 m/s.

From Eq. (8) and (9) Poisson's ratio can be solved

$$\nu = \frac{\frac{1}{2}-(\frac{v_s}{v_c})^2}{1-(\frac{v_s}{v_c})^2} \; .$$

In case of the present experiment

$$\nu = \frac{\frac{1}{2}-(\frac{179}{357})^2}{1-(\frac{179}{357})^2} = 0.33 \; .$$

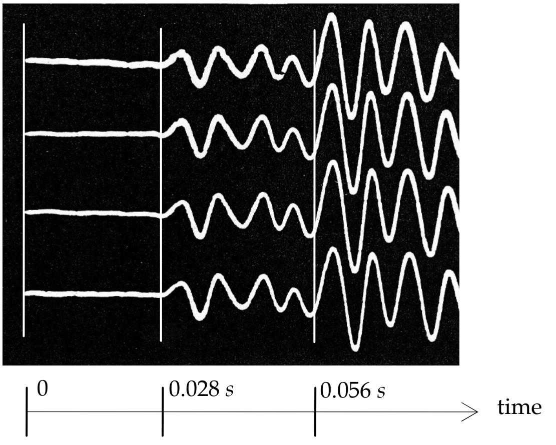

Figure 4. Four times the measured displacement at 10 m distance due to four hammer blows

The density of the soil ρ has been determined as ρ = 1600 kg/m³. The soil *E* modulus is calculated by Eq. (8).

$$E = \frac{(1+\nu)(1-2\nu)}{1-\nu}\rho v_c^2 = \frac{(1+0.33)(1-0.66)}{1-0.33} 1600 \times 357^2 = 1.4 \times 10^8 \ \text{N/m}^2.$$

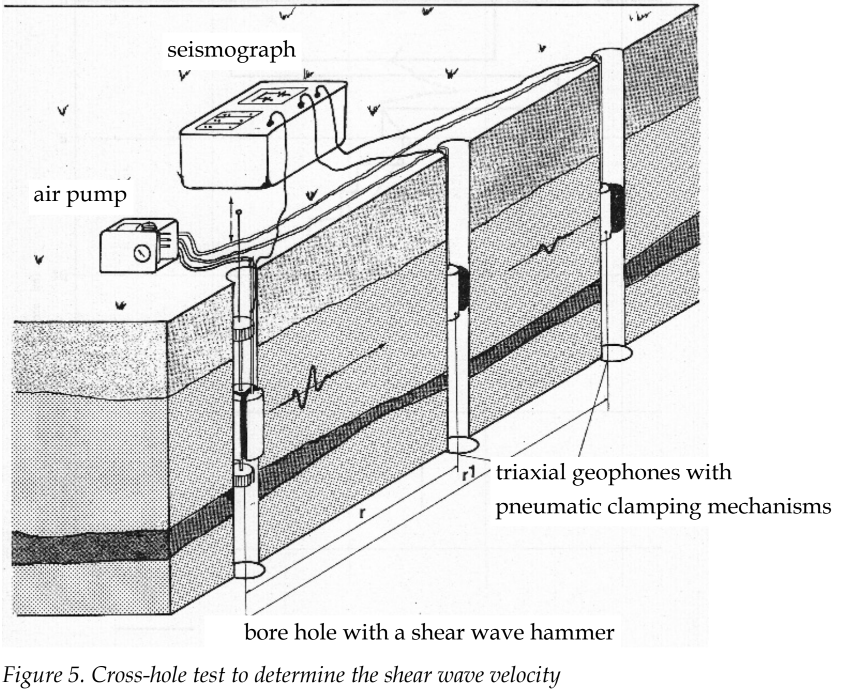

Figure 5. Cross-hole test to determine the shear wave velocity

Similar tests have been performed in vertical holes in the soil to obtain information about soil layers at a lower level (Fig. 5). A shear wave is excited by dropping a weight onto steel blocks that are clamped in a bore hole. The receiver is positioned in another hole at the same depth. These are called *cross-hole tests*. The results are shown in Figure 6 (dashed line). The continuous line shows the results of excitation at the surface and measurements in one hole at various depths. These are called *down-hole tests.* For soil supported machine foundations the surface wave tests showed to give sufficient information.

5 Dynamic properties of a soil supported block

Experiments have been performed on a concrete block of 1×1×1.5 m (Fig. 7) [5]. The surrounding sand has been filled in layers. After applying a layer it was compacted by water and the block was left alone for a few days for the water to drain away.
Subsequently, an harmonic force has been applied straight above the centre of the block in the vertical and horizontal direction (horizontal in the direction of the 1 m width). The response has been measured. This has been repeated for every layer of filling.
The tests show that each extra layer increases the stiffness, the resonance frequency and the damping for both horizontal and vertical excitation (Fig. 8, 9). The amplitude of the

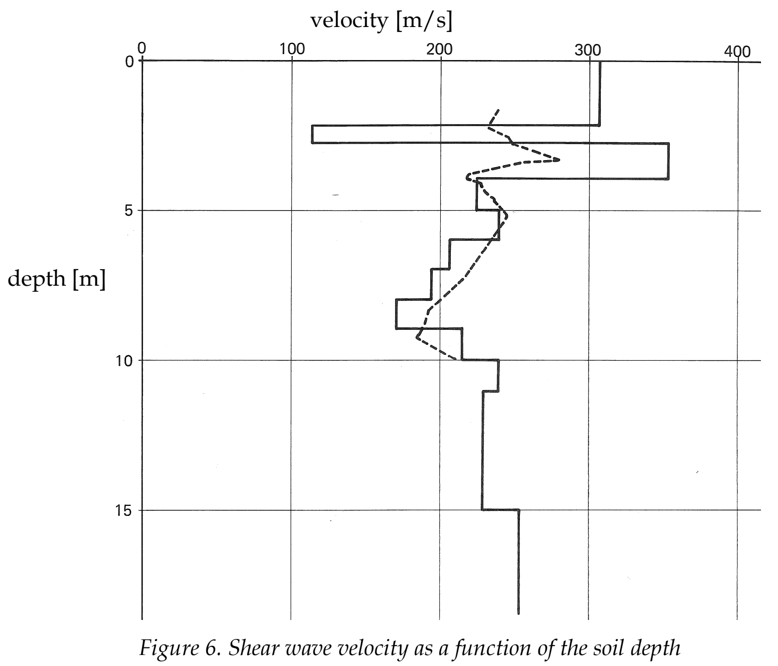

Figure 6. Shear wave velocity as a function of the soil depth

horizontal displacement is larger than the vertical amplitude. The large peaks are the resonance frequencies of the rotation mode. Smaller second peaks occurs due to horizontal shifting of the block on the soil. The peak values are based on a small number of measurements, nonetheless, they are considered to be reasonably accurate. The results have been used to determine analytical expressions for the influence of soil and filling on

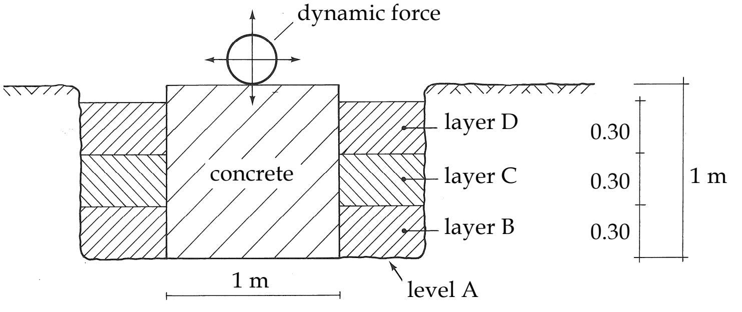

Figure 7. Cross-section of a concrete block partly enclosed by sand

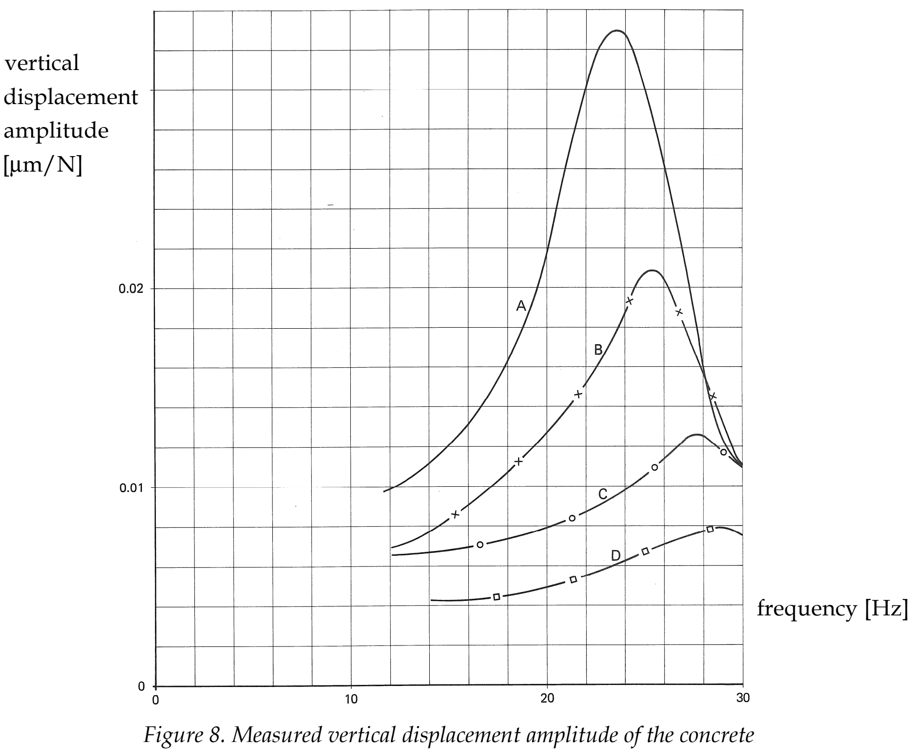

Figure 8. Measured vertical displacement amplitude of the concrete block enclosed by four levels of sand. The markers in this graph do not indicate measured data points but are to distinguish the curves.

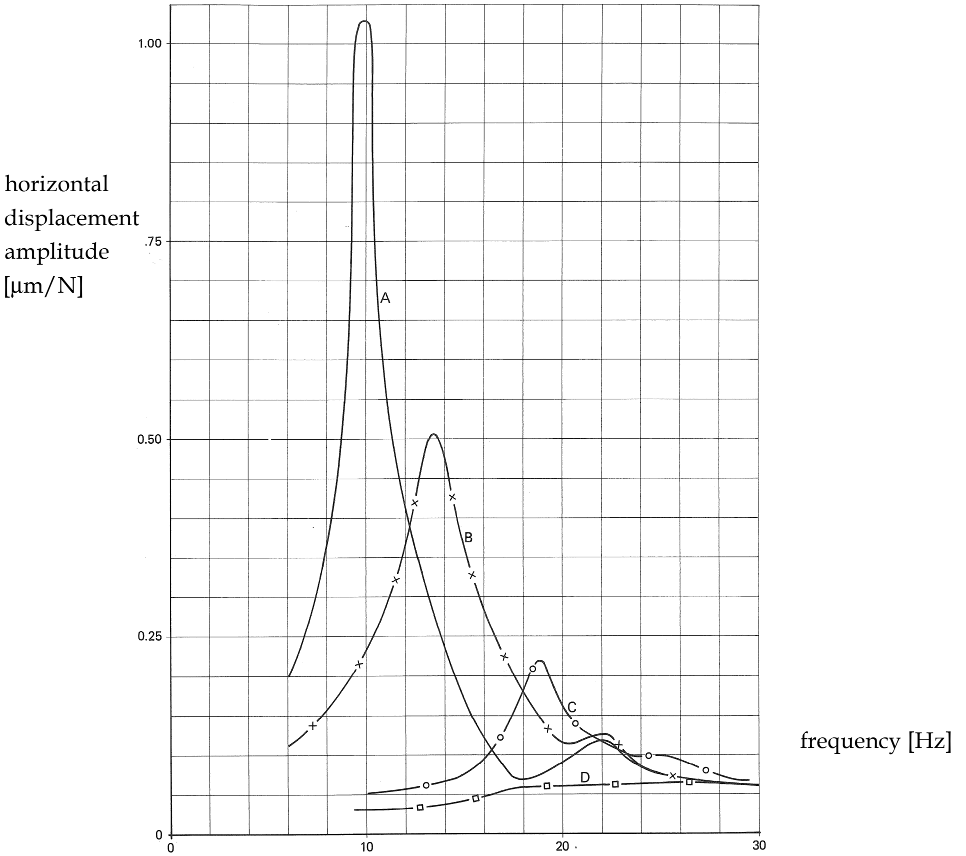

Figure 9. Measured horizontal displacement amplitude of the concrete block enclosed by four levels of sand. The markers in this graph do not indicate measured data points but are to distinguish the curves.

the vibration of the block. These expressions are presented in the following sections.

5.1 Vertical stiffness and damping of a soil supported block

Consider a block resting on soil without side filling. The vertical stiffness of a flat rectangular plate on soil is [1 p. 350]

$$k = 1.1 E\sqrt{LB} \;, \tag{11}$$

where L and B are the horizontal dimensions of the contact area between the plate and the soil. B is smaller than L. Part of the soil mass is vibrating with the block and acts as added mass. The results of the calculations agree well with the test results if soil mass is added. This soil mass has a depth of 0.3B and is 0.3B wider than the edges of the concrete. The total mass becomes

$$m = \rho_c LHB + \rho_s 0.3B(L + 0.6B)(B + 0.6B) , \tag{12}$$

where ρ_c is the concrete density and ρ_s is the soil density. The damping of a vertically vibrating flat plate on soil is [1]

$$c = 0.32\ LB\sqrt{E\rho_s} \tag{13}$$

Subsequently, the block with side filling is considered. The stiffness is increased by the soil layers [5]

$$k = 1.1E\sqrt{LB + h_s 2(L + B)} , \tag{14}$$

where h_s is the height of the side filling. The mass is

$$m = \rho_c LHB + \rho_s \left[0.3B\left(L + 0.6B\right)(B + 0.6B) + 0.1\,Bh_s 2(L + B)\right] . \tag{15}$$

The damping is

$$c = 0.32\left[LB + h_s 2(L + B)\right]\sqrt{E\rho_s} \ . \tag{16}$$

With the above expressions for the stiffness k, the mass m and the damping c the dynamic response of the block can be calculated. The differential equation is

$$m\frac{d^2w}{dt^2} + c\frac{dw}{dt} + kw = F\sin(\omega t) , \tag{17}$$

where w is the dynamic displacement. The displacement amplitude as function of the excitation frequency ω is

$$w_{\max} = \frac{F}{\sqrt{(c\omega)^2 + (k - m\omega^2)^2}} \ . \tag{18}$$

The result of these formulas applied to the block of Figure 7 is shown in Figure 10. ($E = 8.5\times10^7$ N/m^2, ρ_c = 2400 kg/m^3, ρ_s = 1900 kg/m^3) The agreement between Figures 8 and 10 is reasonable. The calculated resonance frequency is about 90% of the measured value. The displacement amplitude without side filling is close to the measured value, however, the calculation underestimates the amplitude if side filling is applied.

5.2 *Horizontal stiffness and damping of a soil supported block*

Consider a block resting on soil with side filling (Fig. 11). The horizontal excitation of the block due to a horizontal force on top of the block causes shifting and rotation (rocking). The equation for horizontal dynamic equilibrium is

$$m\frac{d^2x}{dt^2}+k_h(x-\phi(p-q))=F\ . \tag{19}$$

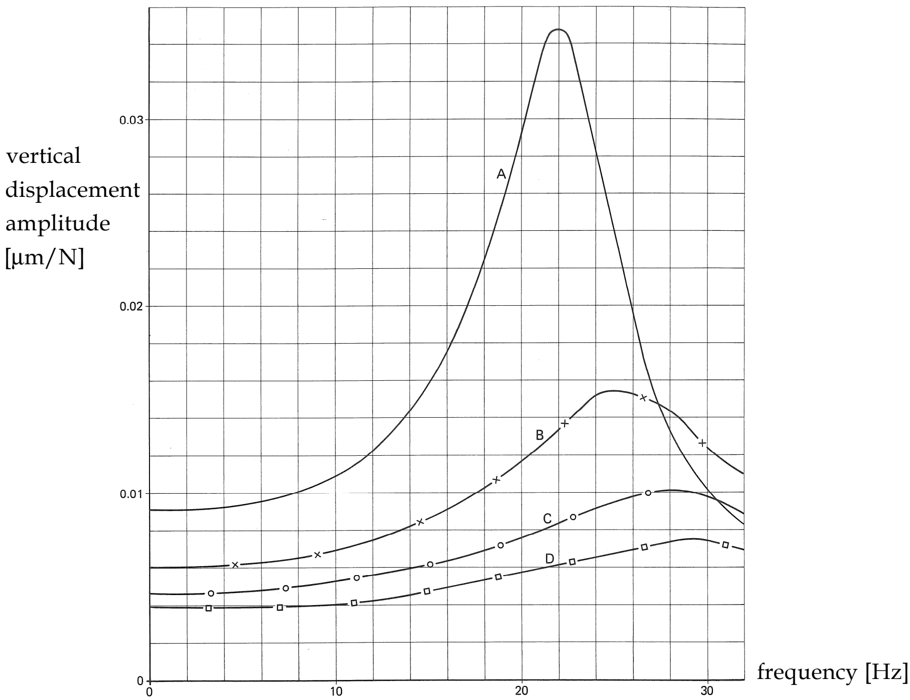

Figure 10. Calculated vertical displacement amplitude of the concrete block enclosed by four levels of sand (compare with Fig. 8)

Dynamic moment equilibrium gives

$$I\frac{d^2\phi}{dt^2}+k_v\frac{LB^3}{12}\phi-k_h\left(x-\phi(p-q)\right)(p-q)=F(H-p)\ , \tag{20}$$

where the mass m and the rotation inertia I of the concrete block are

$$m=\rho_c LBH$$
$$I=\rho_c LBH(\tfrac{1}{12}B^2+\tfrac{1}{3}H^2-p(H-p))$$

The equations are identical to the equations for a dynamically excited system with two degrees of freedom X_1 and X_2 .

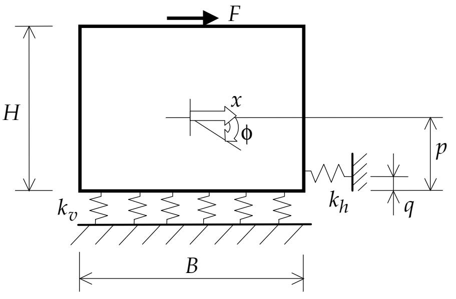

Figure 11. Idealisation of rocking and shifting of a soil supported block

$$\begin{aligned} m_1 \frac{d^2 X_1}{dt^2} + K_1(X_1 - X_2) = F_1 \\ m_2 \frac{d^2 X_2}{dt^2} + K_2 X_2 - K_1(X_1 - X_2) = F_2 \end{aligned} \quad (21)$$

This is an often solved system [6, 7]. With the following substitutions Eq. 19 and 20 are transformed into Eq. 21.

$$\begin{aligned} X_1 &= x & X_2 &= (p-q)\phi \\ m_1 &= m & m_2 &= \frac{I}{(p-q)^2} \\ K_1 &= k_h & K_2 &= \frac{k_v L B^3}{12(p-q)^2} \\ F_1 &= F & F_2 &= \frac{F(H-p)}{p-q} \end{aligned} \quad (22)$$

The mass m needs to be increased with the added mass of the soil. This is the same as for the vertical vibration.

$$m_1 = \rho_c LHB + \rho_s \left[0.3B(L+0.6B)(B+0.6B) + 0.1B h_s 2(L+B)\right] \quad (23)$$

$$K_1 = 0.25E\sqrt{LB + 4h_s 2(L+B)} \quad (24)$$

The factor K_1 has been tailored to obtain the best possible agreement with the experimental results. The positions p and q have been estimated at $p = 0.25H$, $q = 0.25h_s$. The mass m_2 needs to be increased with the added mass of the soil.

$$m_2 = \rho_c LBH \frac{(1.2B + h_s)^2 + H^2}{12(p-q)^2} \quad (25)$$

$$K_2 = 0.11E \frac{(B^2 + 4h_s{}^2)\sqrt{L(B+2h_s)}}{(p-q)^2} \quad (26)$$

The factor K_2 has been tailored to obtain the best possible agreement with the experimental results. The quotients $\frac{m_1}{m_2}$ and $\frac{K_1}{K_2}$ are used to obtain the resonance frequencies. A quick method is to use the graphs for two mass spring systems, for example in literature [6 p. 42]. Damping of vibrations in the horizontal direction can be split in damping c_1 due to horizontal sliding and the damping c_2 due to rotation. The following values give satisfactory results.

$$c_1 = 0.04 \times 2\sqrt{K_1 m_1} \quad (27)$$

$$c_2 = 0.13 \times 2\sqrt{K_2 m_2} \quad (28)$$

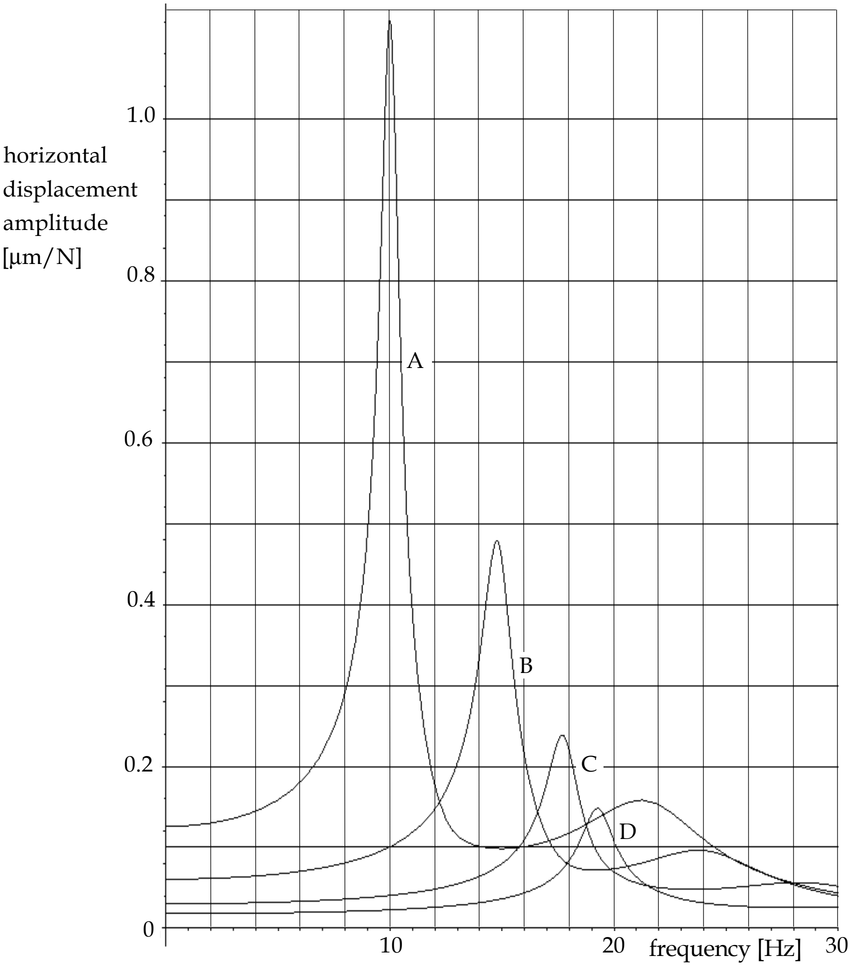

Figure 12. Calculated horizontal displacement amplitude of the concrete block enclosed by four levels of sand (Compare with figure 9)

$$X_1{}^2 = \frac{\left[F_1(-m_2\omega^2+K_1+K_2)+F_2K_1\right]^2+[F_1(c_1+c_2)\omega+F_2c_1\omega]^2}{N}$$

$$X_2{}^2 = \frac{\left[F_1K_1+F_2(-m_1\omega^2+K_1)\right]^2+[F_1c_1\omega+F_2c_1\omega]^2}{N} \quad (29)$$

where,

$$N = [(-m_1\omega^2+K_1)(-m_2\omega^2+K_1+K_2)-K_1{}^2-c_1c_2\omega^2]^2+ \\ +[c_1\omega(-m_2\omega^2-K_1+K_2)+(c_1+c_2)\omega(-m_1\omega^2+K_1)]^2 \quad (30)$$

The amplitude of the horizontal displacement at the top of a foundation block is

$$x_t = x+\phi(H-p)\,. \quad (31)$$

The results of these formulas applied to the block of Figure 7 are shown in Figure 12. (E = 8.5×10^7 N/m^2, ρ_c = 2400 kg/m^3, ρ_s = 1900 kg/m^3)

6 Individual piles

Tests have been performed to determine the horizontal and vertical dynamic stiffness of concrete foundation piles. The piles had square cross-sections of 280×280, 350×350 and 450×450 mm^2. The elasticity modulus of the soil was 10^8 N/mm^2 . De elasticity modules of the concrete was 4×10^{10} N/mm^2. The piles were harmonically excited by horizontal and vertical forces. The results of the horizontal excitation are shown in Figure 13. Clearly, the peak values in the graphs strongly depend on damping of the soil. For the purpose of determining the pile stiffness the accuracy of these peak values is not important.

The horizontal dynamic stiffness has been obtained by extrapolation of the measured motion to a zero frequency. Those values are approximated by an expression, derived from elastically supported beams. The support stiffness is [5]

$$k = 0.2\,E_s^{\frac{3}{4}}E_c^{\frac{1}{4}}D\,. \quad (32)$$

where, E_s is the elasticity modulus of the soil, E_c is the elasticity modulus of the pile, D is the diameter or width of the pile. Eq. 32 has been derived for piles with a free end. If the pile head is clamped and can translate only then the stiffness is [5]

$$k = 0.4\,E_s^{\frac{3}{4}}E_c^{\frac{1}{4}}D\,. \quad (33)$$

Only if vibrating with a frequency close to 45 Hz, the stiffness is reduced due to dynamic behaviour. This is a combined pile-soil effect.
Also the vertical dynamic stiffness of the piles has been tested, but the results are inconclusive (Fig. 14). The displacements do not show a resonance peak. Apparently, the damping is more than the critical damping. The derived vertical stiffness for dynamic excitation is $k_v = 4\times10^8$ N/m.

7 Pile foundation experiments

Four reinforced concrete foundation piles are fixed to reinforced concrete foundation plate (Fig. 15). The piles have cross-sections of 0.35×0.35 m and lengths of more than 10 m. The plate has a thickness of 1 m and a length and width of 2.5 m. Three dynamic tests have been performed in which the foundation has been exited harmonically in a horizontal

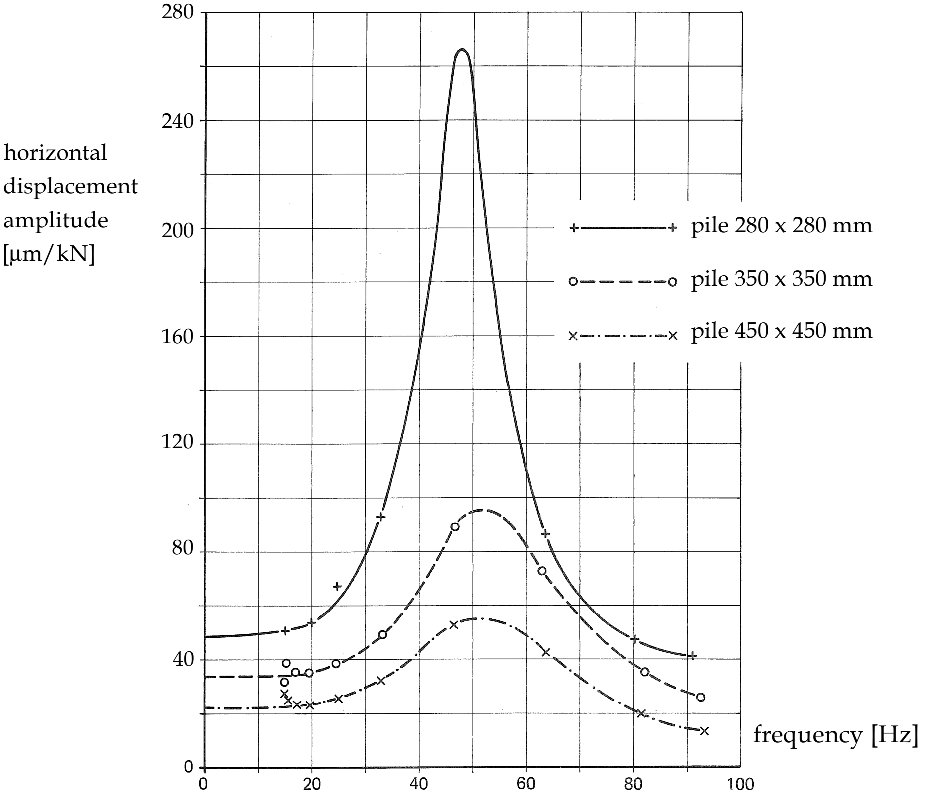

Figure 13. Horizontal displacement amplitude of pile heads as a function of the frequency

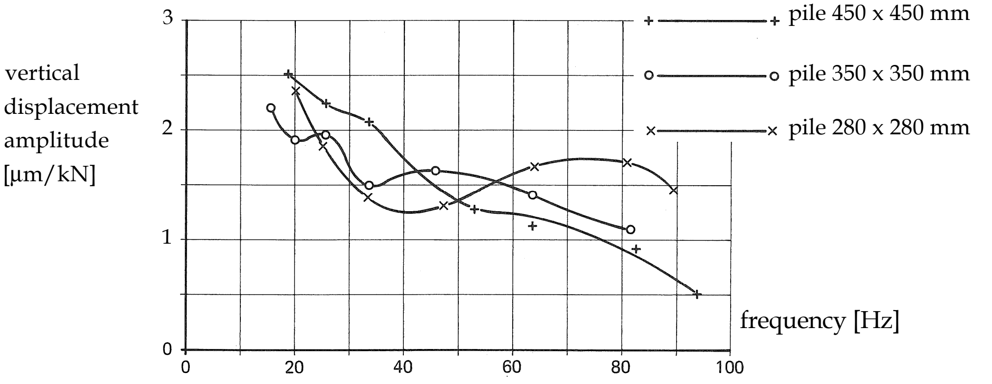

Figure 14. Vertical displacement amplitude of pile heads as a function of the frequency

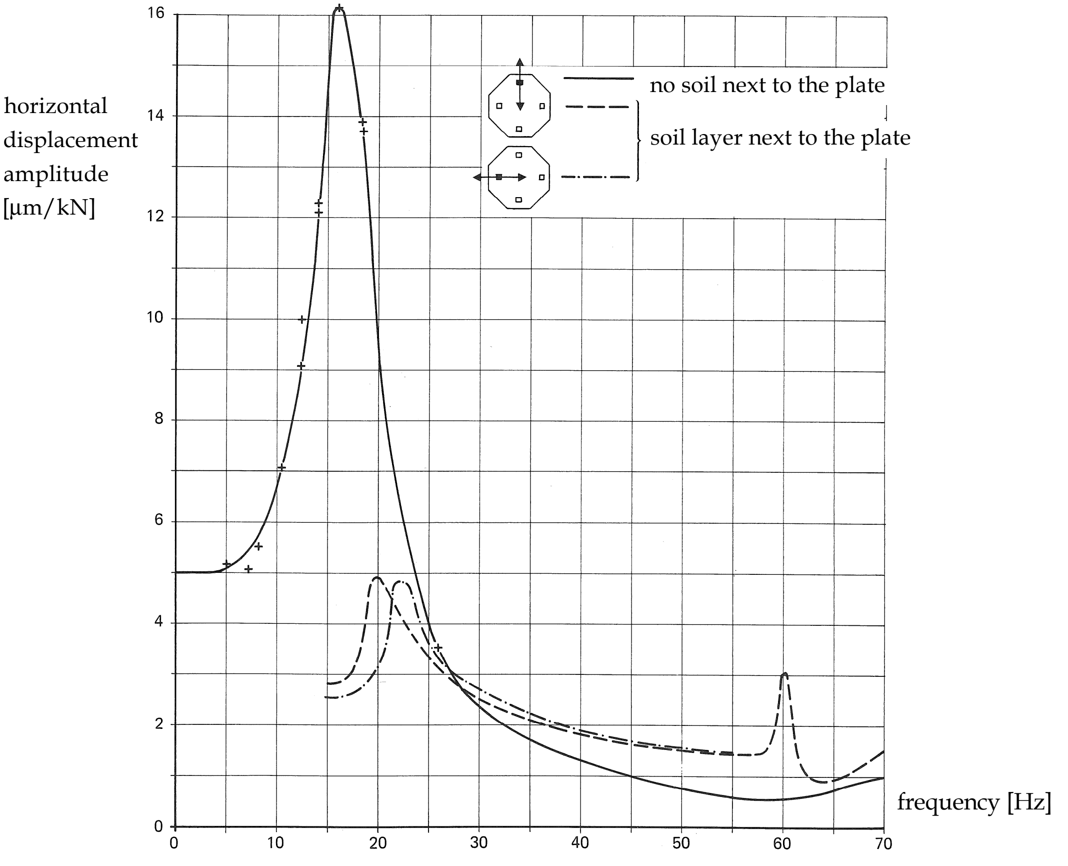

Figure 15. Measured horizontal displacement amplitude of a reinforced concrete pile foundation

direction; 1) without soil around the sides of the plate; 2) with soil around the plate; 3) with soil around the plate and loaded in a direction perpendicular to the previous tests. The mass and damping of the soil around the foundation plate have a strong influence on the vibrations (Fig. 15).

As an example the resonance frequency is calculated of this foundation without soil at the edges. Foundation mass 2.5×2.5×1×2400 = 15000 kg

Mass of the soil below the foundation $\rho_s 0.3BLB$ = 1600×0.3×2.5×2.5×2.5 = 7500 kg

Horizontal stiffness of the piles (Eq. 33) $4\times 0.4\times (10^8)^{\frac{3}{4}}(4\times 10^{10})^{\frac{1}{4}}\times 0.35 = 2.5\times 10^8\ \text{N/m}$

Resonance frequency $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2.5\times 10^8}{15000+7500}} = 16.8\ \text{Hz}$

This is close to the measured frequency (Fig. 15).

In some tests the foundation was excited vertically (Fig. 16). A resonance peak was not observed. The amplitudes are larger at higher frequencies. The soil around the reinforced concrete plate has no influence on the amplitudes.

8 Vibration of the surrounding soil

Vibrations of foundations and the surrounding soil need to be limited to a for people acceptable level. Figure 17 shows the perception of vibrations as a function of the acceleration and the frequency. In many situations – such as houses and offices – people should not feel the vibrations. This means that accelerations less than 10^{-2} m/s² are

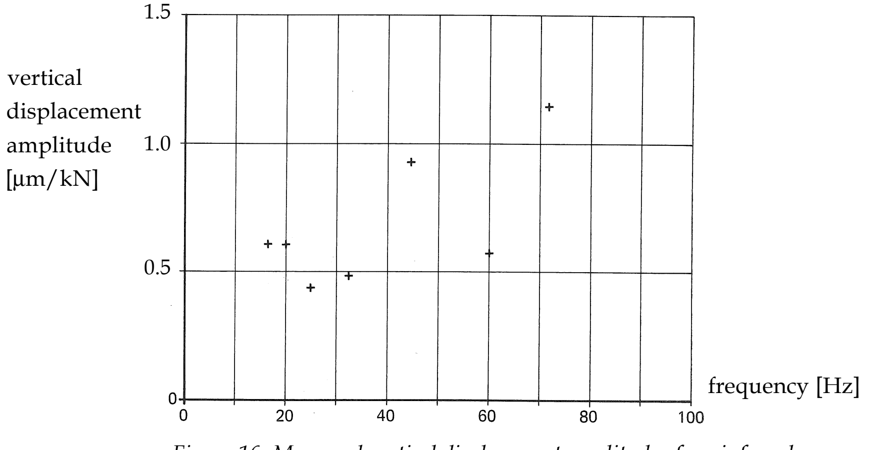

Figure 16. Measured vertical displacement amplitude of a reinforced concrete pile foundation without soil next to the plate

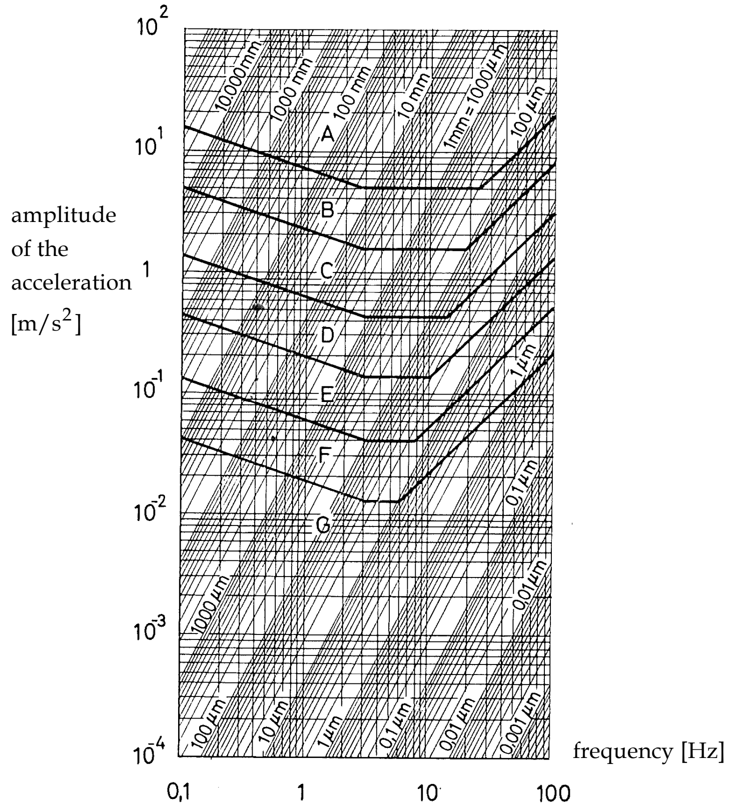

Figure 17. Perception of vibration (Table 2) [7]

Table 2. Perception levels of Figure 17 [7]

	perception	acceptability in buildings	structural effects	examples
A	very unpleasant	not acceptable	danger of collapse	- earthquake
B	unpleasant	not acceptable	local damage	- emergency braking of a car
C	strongly noticeable	hardly acceptable	cracks in masonry	- in a tram or elevator
D	well noticeable	only rough work	small cracks	- start of seasickness
E	noticeable	shortly in rooms	no influence on building	
F	hardly noticeable	acceptable	no influence	
G	not noticeable			

always acceptable. The inclined lines show the displacement amplitude of a harmonic vibration.

In theory, the amplitude of a Rayleigh wave reduces with one over the distance to the vibration source, however, due to damping in soil the reduction is considerably stronger. Collected experimental results are shown in Figure 18. The soil amplitude w at a distance r can be calculated by [1]

$$w = w_0 \sqrt{\frac{r_0}{r}} e^{-\alpha(r - r_0)} \, , \tag{34}$$

where w_0 is the amplitude at the foundation, r is the distance to the foundation, r_0 is half the foundation width and α = 0.03 m^{-1} approximately (Fig. 18).

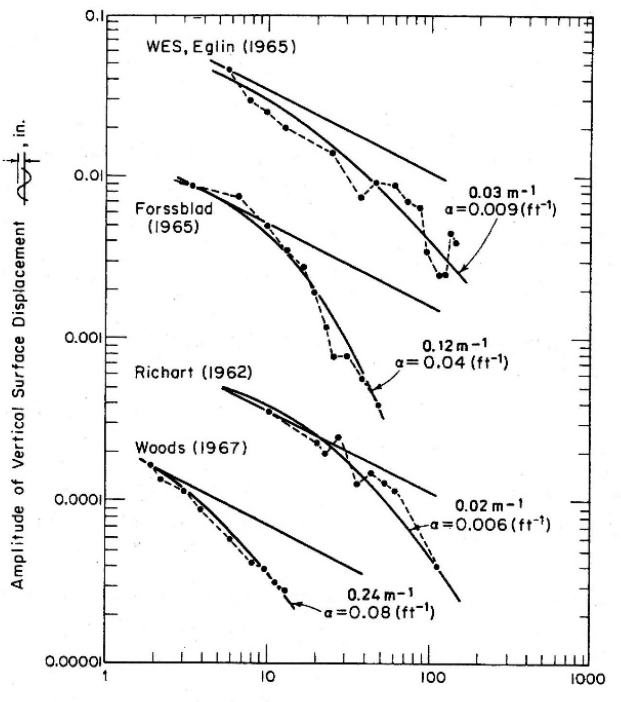

Figure 18. Amplitude of the vertical surface displacement [in] as a function of the distance to the vibration source [ft] [1, p. 246]

9 Concluding remarks

Machine foundations on soil with or without foundation piles can be analysed as one or two mass-spring systems. The harmonic loading by rotating machines and piston engines can be determined accurately. Clear formulas are available for the stiffness, equivalent mass and damping. The natural frequencies and displacement amplitudes can be obtained by graphs and a hand calculator, without complicated computer software. The amplitude can be evaluated as acceptable or not. This provides an engineer with the tools to develop a sound skill in machine foundation design.
Soil at the sides of foundations considerably increase the damping and strongly reduce the displacement amplitudes. The vertical vibrations are well damped by the soil, but the horizontal vibrations can be disturbing even at a large distance from the foundation.

Acknowledgement
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Appendix

This appendix contains a calculation example of a turbine foundation. The foundation consists of three connected parts made of reinforced concrete supported by long foundation piles (Fig. 19). The total length is 31.65 m. The soil modulus of elasticity is $E = 10^8$ N/m². This is the average of measurements at several locations in the Netherlands.

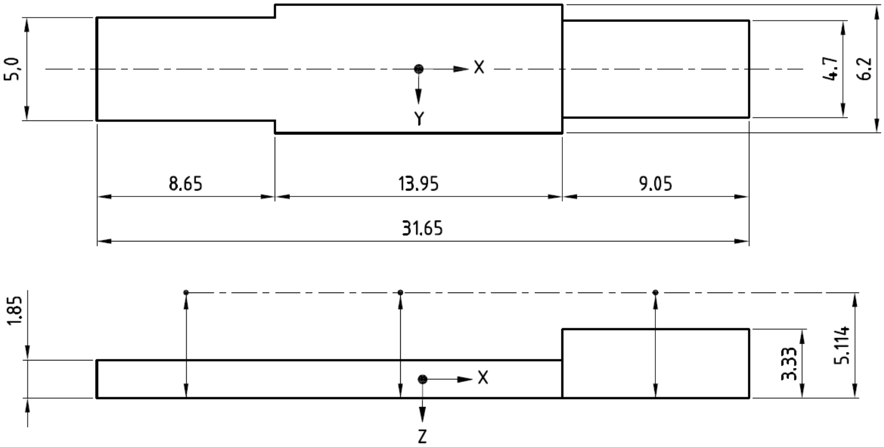

Figure 19. Turbine foundation dimensions

Part 1: Auxiliaries compartment support

Concrete mass	$1.85\times5.00\times8.65\times2400 = 1.92\times 10^5$ kg
Static load	$2\,(2.9+5.8+11.5+11.5)\,10^4 = 63.4\times 10^4$ N (64700 kg)
Supported by 8 piles	

Part 2: Turbine support

Concrete mass	$1.85\times6.20\times13.95\times2400 = 3.84\times 10^5$ kg
Static load	$2\,(4.5+44.5+57.8+7.0+7.0)\,10^4 = 2416000$ N (246000 kg)
Supported by 15 piles	

Part 3: Generator support

Mass of the concrete	$3.33\times4.70\times9.05\times2400 = 3.39\times 10^5$ kg
Static load	$2(27.5+45.8+47) = 240.6\times 10^4$ N (245000 kg)
Supported by 10 piles	

Totals

Concrete mass	9.15×10^5 kg (900×10^4 N)
Static load	545.6×10^4 N (5.56×10^5 kg)
Supported by 33 piles	

Centre of gravity

The centre of gravity is in the axis of symmetry.

From the bottom of the concrete

$1.92\times 10^5 \times 0.925 + 0.647\times 10^5 \times 5.114 = 5.08\times 10^5$ kg m

$3.84\times 10^5 \times 0.925 + 2.46\times 10^5 \times 5.114 = 16.13\times 10^5$

$3.39\times 10^5 \times 1.665 + 2.45\times 10^5 \times 5.114 = 18.16\times 10^5$

$(5.08+16.13+18.16)\,10^5 / ((9.15+5.56)\,10^5) = 2.68$ m

From left side of the concrete

$1.92\times 10^5 \times 4.325 + 0.647\times 10^5 \times 4.325 = 11.09\times 10^5$ kg m

$3.84\times 10^5 \times 15.65 + 2.46\times 10^5 \times 15.65 = 97.03\times 10^5$

$3.39\times 10^5 \times 27.12 + 2.45\times 10^5 \times 27.12 = 158.11\times 10^5$

$(11.09+97.03+158.11)\,10^5 / ((9.15+5.56)\,10^5) = 18.11$ m

Displacement in the z direction

Vertical stiffness of one pile is 4×10^8 N/m. The soil has settled around the piles and is not in contact with the foundation. Therefore, it is not included in the stiffness and not included in the mass. Damping in the vertical direction is critical damping.

Resonance frequency in the vertical direction $f = \frac{1}{2\pi}\sqrt{\frac{33\times 4\times 10^8}{(9.15+5.56)\times 10^5}} = 15.1$ Hz.

Rotation around the x axis and displacement in the y direction

Moment of inertia [9]

$I = \{1/12\,(5.0^2 + 1.85^2) + (2.64 - 0.925)^2\}\; 9.15\times 10^5 +$

$\quad + \{1/12\,(3.5^2 + 3.4^2) + (5.11 - 2.64)^2\}\; 5.56\times 10^5$

$\quad = 48.54\times 10^5 + 34.03\times 10^5 = 82.57\times 10^5$ kgm^2

Horizontal stiffness of one pile is 6.2×10^7 N/m.

$k_h = 33\times 6.2\times 10^7 + 2\times 1.1\times 10^8 \sqrt{(31.61\times 1.85)} = 2.05\times 10^9 + 1.08\times 10^9 = 3.13\times 10^9$ N/m

This resistance of piles and ground together act at $1.08\times 0.6/3.13 = 0.21$ above the base.

$p = 2.68 - 0.21 = 2.47$ m

k_v = 14×4× 10^8 = 56× 10^8 N/m

a = 4 m

The mass m_1 = 9.15× 10^5 +5.56× 10^5 = 14.70× 10^5 kg

m_2 = 82.57× 10^5 / 2.47^2 = 13.53× 10^5 kg $\qquad m_1 / m_2 = 1.08$

The added mass of the soil is neglected because it has little influence on m_1 / m_2 .

K_1 = 8.36× 10^9 N/m

K_2 = 56× 10^8 $4^2 / (2\times 2.47^2)$ = 7.34 × 10^9 N/m $\qquad K_1 / K_2 = 1.14$

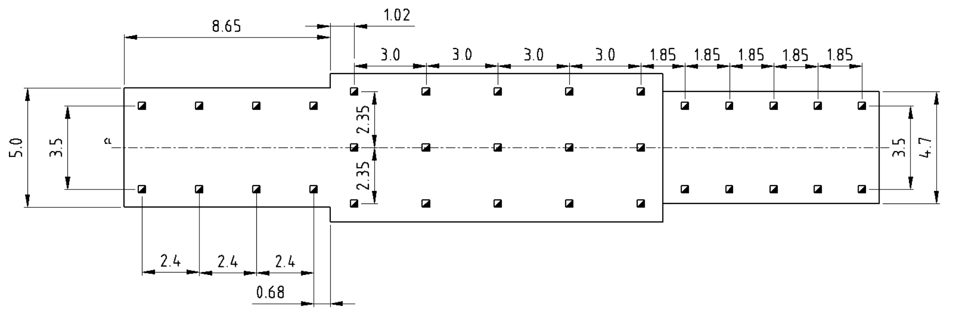

Figure 20. Pile positions [m]

The resonance frequency in the y direction is

$f_1 = 1.7/2\pi\sqrt{8.36\times 10^9 / 14.7\times 10^5} = 6.6$ Hz.

The resonance frequency around the x direction (rocking) is

$f_2 = 1.5/2\pi\sqrt{8.36\times 10^9 / 14.7\times 10^5} = 20.4$ Hz.

Rotation around the y axis and displacement in the x direction

$I = \{ 1/12(31.65^2 + 1.82^2) + (18.11 - 15.80)^2 \}\ 9.15\times 10^5 +$

$+\{ 1/12\left(31.65^2 + 3.5^2\right) + (18.11 - 15.80)^2 \}\ 2.45\times 10^5 = 9.24\times 10^7$ kgm²

$k_h = 33\times 6.2\times 10^7 + 10^8\ \sqrt{2\times 5\times 1.82} = 2.046\times 10^9 + 0.43\times 10^9 = 2.476\times 10^9$ N/m

The resistance of ground and piles act together at 0.76/2.476×1.2 = 0.36 m above the base.

p = 2.68 – 0.36 = 2.32 m

k_v = 2×4× 10^8 = 8× 10^8

a = 2/3×31.61 = 21.07 m

The mass m_1 = 14.70× 10^5 kg

m_2 = 9.24× 10^7 / 2.32^2 = 171.7× 10^5 kg $\qquad m_1 / m_2 = 0.085$

K_1 = 2.814× 10^9 N/m

$K_2 = 21.07^2 \times 8\times 10^8 / 2\times 2.32^2 = 3.30\times 10^{10}$ $\quad\quad K_1 / K_2 = 0.085$

The resonance frequency for displacements in the *x* direction is

$f_1 = 0.65/2\pi\sqrt{}2.814\times 10^9 / 14.70\times 10^5 = 4.53$ Hz.

The resonance frequency around the *y* direction (rocking) is

$f_2 = 1.5/2\pi\sqrt{}2.814\times 10^9 / 14.70\times 10^5 = 10.4$ Hz.

Rotation around the z axis

The distance between the *z* axis and the outer pile is 18.3 m.

Horizontal stiffness of one pile is 6.2×10^7 N/m.

Assume a rotation α around the *z* axis.

The moment around the *z* axis is

$2\times 6.2\times 10^7\ \alpha\ (18.3^2 + 16.0^2 + 13.5^2 + 11.3^2 + 11.0^2 + 9.2^2 + 7.3^2 + 5.5^2 + 3.9^2) +$

$+3\times 6.2\times 10^7\ \alpha\ (9.5^2 + 6.7^2 + 3.9^2 + 0.9^2 + 2.2^2) = 18.3\times 10^{10}\ \alpha$ Nm

The rotation stiffness of the piles is $18.3\times 10^{10}\ \alpha / \alpha = 18.3\times 10^{10}$ Nm/rad

The rotation stiffness of the sand is $10^8 \sqrt{1.1\times 31.65\times 2/3} = 0.48\times 10^{10}$ Nm/rad

The total rotation stiffness is 18.8×10^{10} Nm/rad

Polar moment of inertia 1/12 $(9.15\times 10^5 + 5.56\times 10^5)\ 31.65^2 = 1.23\times 10^8$ kgm²

The resonance frequency is $f = \frac{1}{2\pi}\sqrt{\frac{18.8\times 10^{10}}{1.23\times 10^8}} = 6.23$ Hz.

Summary of frequencies

The resonance frequencies are:

Displacement in the direction of the *z* axis	$f_1 = 15.1$ Hz	damping $D = 1$	(piles vert.)
Rotation around the *z* axis	$f_2 = 6.23$ Hz	$D = 0.16$	(piles hor.)
Displacement in the direction of the *x* axis	$f_3 = 4.53$ Hz	$D = 0.16$	(piles hor.)
Rotation around the *y* axis	$f_4 = 10.4$ Hz	$D = 1$	(piles vert.)
Displacement in the direction of the *y* axis	$f_5 = 6.60$ Hz	$D = 0.25$	(soil)
Rotation around the *x* axis	$f_6 = 20.4$ Hz	$D = 1$	(piles vert.)

Static displacements

The displacement caused by a static force of 1 N in the *x* direction. This load acts at the *y* axis at 9.05 m from the centre of gravity and 5.114 – 2.64 = 2.47 m above the centre of gravity.

Vertical displacement and rotation

$w = 1/(33\times 4\times 10^8) + 1/(10\times 4\times 10^8) = 1/52\times 10^8$ m/N

Horizontal displacement in the x direction and rotation

$w = 1/(2.814\times 10^{9}) + 2.47/(9.05\times 3.42\times 10^{10}) = 1/2.0\times 10^{9}$ m/N

Horizontal displacement in the y direction

$w = 1/(8.36\times 10^{9}) + 2.47/(2\times 56\times 10^{8}) = 1/2.94\times 10^{9}$ m/N

Dynamic forces and displacements

The unbalance of the generator in the y direction is 275.2 kN.

$w = 275.2\times 10^{3} / 2.94\times 10^{9} = 9.55\times 10^{-6}$ m. The dynamic displacements are smaller because of the resonance frequency of 6.6 Hz and the loading of 50 Hz.

$w = 9.55\times 10^{-6} \times (6.6/50)2 = 0.17 \times 10^{-6}$ m = 0.17 μm

The foundation will rotate around the z axis. The moment causing this rotation is $2\times 137.6\times 21 = 0.58\times 10^{7}$ Nm. The total resistance against rotation of soil and piles is 18.8×10^{10} Nm/rad. The rotation is $0.58\times 10^{7} / 18.8\times 10^{10} \times (6.23/50)\ 2 = 0.48\times 10^{-6}$ rad

The maximum motion of one side is $0.48\times 10^{-6} \times 31.45/2 = 8\times 10^{-6}$ m = 8 μm.