On shape stability of panel paintings exposed to humidity variations - Part 1: Modelling isothermal moisture movement

S. Reijnen and A.J.M. Jorissen

Eindhoven University of Technology, the Netherlands

A substantial part of museum's collection is painted on wood, so called panel paintings. In an attempt to protect this part of our cultural heritage against degradation, museums apply strict humidity requirements mainly based upon practical experience and perception: Relative Humidity RH = 55% ±5.0%. Today scientist and conservators ask themselves: "Are these strict requirements really needed and what are the consequences of changing these strict requirements?". Finite Element Analysis could provide an answer and possibly lead to a more permanent solution.

Due to its hygroscopic behaviour, wood is sensitive to variations in humidity. As a consequence of changing environmental conditions and the hygro-expansional behaviour of wood it tends to deform. When these deformations exceed the elastic limit, it could lead to permanent deformations. Simulation of this behaviour is quite complex, due to wood being a heterogeneous, hygroscopic, cellular and anisotropic material. Without numerical simulation, it is almost impossible to predict the deformation when exposed to moisture variations. This paper discusses the development of a constitutive model for simulations and an example of the analyses for a limewood cylinder. This exercise was carried out previous to the so-called Climate4Wood project, one of the NWO projects within the Science4 Arts projects.

1 Introduction

It is important to preserve our cultural heritage against degradation. Different materials demand different conservation methodologies. Perception, based on years of experience, plays an important role in making the decision on which method of conservation is to be applied. History learns that this approach does not (always) deliver the desired results. Despite the high value of this experience and empirical approach, it is not (always) sufficient. Thorough knowledge of the more or less changing environmental factors, mainly characterized by temperature, relative humidity, light and even more the effect of these changes on the internal stresses and deformations in the wooden panel, as well as the response of the paint layers, could lead to a more permanent solution.

A panel painting is painted on wood. Not many people are aware that famous paintings, painted by great masters, are painted on wood. The Mona Lisa by Leonardo da Vinci is a good example. The Mona Lisa is painted on a wooden panel made from poplar and is almost 500 years old. Wood, and the better known canvas, varies widely in material properties.

In order not to expose the artefacts to big changes in temperature and relative humidity, museums, invest in climatic control systems. Strict requirements, completely based on empirical evidence, should protect the art against degradation. Besides the cultural responsibility, there is also a financial stimulation to review the current way of thinking and to try to provide a scientific basis on acceptable climate variations.

Wood is sensitive to fluctuations in relative humidity. Absorption from and desorption of moisture to the immediate environment is unfortunately not without conflict. Absorption and desorption of moisture give rise to swelling and shrinking in radial and tangential directions. The shrinkage and swelling values in tangential direction are roughly twice the values in radial direction, as indicated in figure 3, resulting in unwanted shape deformation, strain and stress. In the case of shrinking tension stresses perpendicular to the grain in tangential direction are decisive; in the example discussed these stresses are focussed on – see e.g. Figure 7.

The objectives of this research are:

1 to examine and experience the possibility of ABAQUS CAE (Complete Abaqus Environment) standard to model moisture flow and shape stability using the procedure of heat conduction instead of mass diffusion within a multi-physical environment. (See also Mirianon, Fortino and Toratti 2008),

2 to examine the influence of changing environmental conditions and changing material properties on shape stability and the related stress field. (See also Ormarsson, Dahlblom and Petersson 1997),

3 to examine the influence of a gesso coating layer (mixture of hide glue, gypsum or sometimes ground chalk and water for smoothing a panel surface) on shape stability.

The content of this paper is limited to the development of a constitutive model for simulations, to solve moisture diffusion and the influence of gesso on shape stability, which is discussed in the second part of this paper.

2 Modelling isothermal moisture movement in wood, using ABAQUS transient heat conduction

When a wooden object is exposed to varying environmental conditions, especially fluctuations in relative humidity, the wood moisture content also varies due to the hygroscopic behaviour of wood. Especially fluctuations in relative humidity below the fibre saturation point result in deformation in the form of shrinking or swelling. If these deformations do not exceed the elastic range, theoretically there is no problem. Beyond this elastic range, the so called plastic range, the material no longer returns into the initial state after removing of the load. This is called plastic deformation. Jakiela, Bratasz and Kozlowski (2008) developed a numerical model describing moisture movement due to changing environmental conditions to calculate the related stress field in the elastic range. This model has been applied to lime wood cylinders. Schellen and Schijndel (2011) verified the work done by Jakiela, Bratasz and Kozlowski (2008) with help of a numerical model developed in COMSOL, a finite element package designed to solve building physics problems.

To validate the outcome by Jakiela *et al*. and Schellen *et al*., an ABAQUS transient heat conduction analysis is performed using a transient heat conduction analysis procedure instead of mass diffusion analysis. Due to the similarities in the basic differential equations for heat transfer – equation (1) - and mass diffusion – equation (4) – the mass diffusion problem can be solved by a heat transfer analyses. One of the possible great advantages of using ABAQUS CAE (Complete ABAQUS Environment) heat conduction analysis is the ease of modelling. Within the CAE, it is possible to perform a heat conduction analysis and apply the outcome as a predefined field to a static stress/strain analysis. This is a so called sequentially coupled thermal stress analysis. A sequentially coupled thermal stress analysis can be used when stress/displacement is dependent on a temperature field and there is no inverse dependency.

ABAQUS uses Fourier's law of heat conduction to analyse heat transfer, see equation (1), (2) and equation (3) to perform a mass diffusion analysis. Equation (3) is an extension of

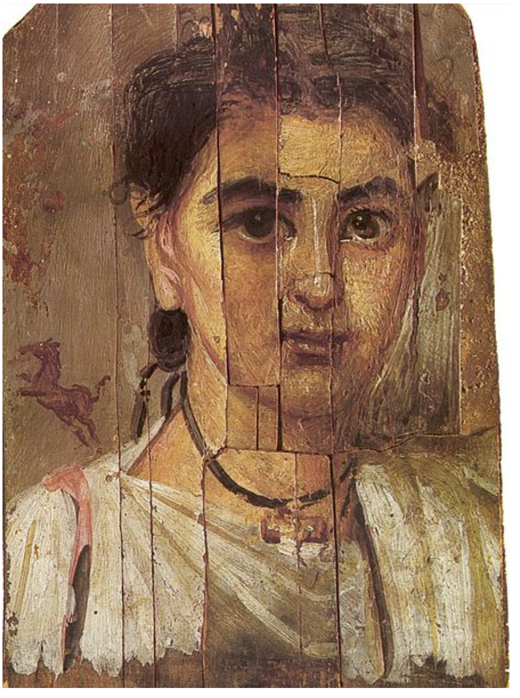

Figure 1. The boy from Al–Fayum;

2nd century; Encaustic panel painting; The figure shows local cracking in the painting

Fick's law, see equation (4) and (5). The difference can be found in the fact that the equations used by ABAQUS allow for a non-uniform solubility (the ability of a liquid (solute) to dissolve into a solid (solvent)) of the solute through the solvent and for mass diffusion driven by gradients of temperature and pressure. Fick's first law of mass diffusion is a linear equation (4). The extended law used by ABAQUS, equation (3), becomes non-linear since the diffusion coefficient, the Soret factor κ_s and the pressure stress factor κ_p depend on the concentration. Due to the analogy between Fourier's equations and Fick's equations, mass diffusion can be modelled using a heat transfer analysis and vice versa.

$$q_x = -k\frac{\partial T}{\partial x} \text{ Fourier's law (steady state)} \tag{1}$$

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} \text{ Fourier's law (transient)} \tag{2}$$

$$J_x = -sD\left[\frac{\partial\phi}{\partial x} + \kappa_s \frac{\partial}{\partial x}(\ln(T - T^Z)) + \kappa_P \frac{\partial p}{\partial x}\right] \tag{3}$$

$$J_x = -D\frac{\partial\phi}{\partial x} \text{ Fick's first law (steady state)} \tag{4}$$

$$\frac{\partial\phi}{\partial t} = D\frac{\partial^2\phi}{\partial x^2} \text{ Fick's second law (transient)} \tag{5}$$

q_x ……………….... heat flux [W/m]

k ………………… thermal conductivity [W/(mK)]

T ………………… temperature [K]

α ………………… thermal diffusivity [m²/s] $\alpha = \frac{\lambda}{\rho c_p}$

λ ………………… thermal conductivity (k) [W/(mK)]]

ρ ………………… density [kgm³]

c_p …………………specific heat capacity [J/(kgK)]

J_x …………………diffusion flux [mol/(ms)]

D …………………diffusion coefficient [m/s]

ϕ ………………… concentration [mol/m³]

$D(c, T, f)$ ……….. diffusivity [m²/s]

$s(T, f)$ …………… solubility [ppm]

$K_s(C, T, f)$ …….... factor, providing diffusion due to a temperature gradient [-]

T …………..……. temperature [K]

T^Z ……………..…. value of the absolute zero on the temperature scale used [-]

$K_p(C, T, f)$ ..…..…. factor, providing diffusion, due to a stress gradient [-]

C ……………….. concentration of the diffusing material [kg/m³]

f …………………. other predefined field variables (potential) [-]

Although mass diffusion can be modelled using a transient heat transfer analysis, heat transfer analysis and mass diffusion analysis are not the same. For example, heat transfer analysis based on Fourier's law can only use a temperature gradient as the driving force behind the diffusive process. Within mass diffusion analysis other driving forces (ABAQUS calls this chemical potentials) like pressure, temperature and concentration can control the diffusive process . Consequently, these has to be translated into a temperature gradient.

2.1 *Example: lime wood cylinder*

What is happening in a lime wood cylinder exposed to relative humidity fluctuations? A reproduction of the research done by Jakiela, Bratasz and Kozlowski (2008).

2.1.1 *Geometry (figure 2)*

Ø = 0.13 m lime wood

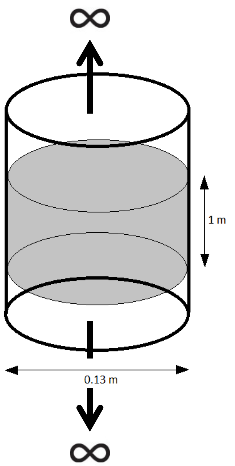

Figure 2. Geometry of the lime wood model

2.1.2 *Plane strain situation*

Strain directed perpendicular to x-y plane equals zero

normal strain $\varepsilon_z = 0$

shear strain $\gamma_{xz} = 0$

shear strain $\gamma_{yz} = 0$

The wood reaction to moisture parallel to the grain is neglected, the surface can be regarded as fully closed; consequently "one dimensional drying" is simulated.

2.1.3 *Thermal boundary conditions (isothermal)*

$t < 0$ $T_S = T_0$

$t > 0$ $q = h(T_0 - T_S)$; the cylinder gradually heats up or cools down to T_0

t = ∞ $T_\infty = T_0$

T_0 ……... 20 [°C]

T_∞ …….. 20 [°C]

T_S …….. surface temperature [°C]

q ……... heat flux at surface [W/m²]

h ……… heat transfer coefficient [W/(m²K)] $h = \dfrac{Q}{A\,\Delta T}$

Q …….. heat flow [W]

A …….. surface area [m²]

2.1.4 Hygric boundary conditions

$t < 0$ $\quad u_S = u_0 = 14\%$

$t > 0$ $\quad g = \beta(u_0 - u_S)$; the wood moisture content gradually decreases from μ_S to μ_∞

$t = \infty$ $\quad u_\infty = 6\%$

$u_0 = 14\% \ \rightarrow \mathrm{RH}_0 = 70\%$

$u_\infty = 6\% \ \rightarrow \mathrm{RH}_\infty = 30\%$

g ……… moisture flux at surface [kg/(m²s)]

β ……… moisture transfer coefficient [kg/(m²s)]

2.1.5 Mechanical boundary conditions

The model surface is constraint (type: coupling) to a reference point in space. The vertical movement is constraint. By coupling the surface to a reference point in space, rigid body rotation cannot take place and the model is free to move in radial direction.

2.1.6 Mechanical stress and strain (generalized Hooke's law)

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{E_x} & -\dfrac{\nu_{xy}}{E_y} & 0 \\ -\dfrac{\nu_{yx}}{E_x} & \dfrac{1}{E_y} & 0 \\ 0 & 0 & \dfrac{1}{G_{xy}} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ 0 \end{pmatrix} \Delta\theta + \begin{pmatrix} \kappa_x \\ \kappa_y \\ 0 \end{pmatrix} \Delta w \tag{6}$$

ε_x , ε_y ………….. normal strain components [-]

γ_{xy} ……………... shear strain component [-]

E_x , E_y ….……… moduli of elasticity [N/m²]

ν_{xy} , ν_{yx} ………… Poisson's ratios [-]

G_{xy} ……………..…. shear modulus [N/m²]

σ_x , σ_y ………….. normal stress components [N/m²]

α_x , α_y …………... linear thermal expansion coefficient [1/K]

$\Delta\theta$ ……………….. temperature increment [K]

Δw ………………...moisture content increment [kg/m³]

κ_x , κ_y ………….. linear relative deformation due to changing moisture content [1/(kgm³)]

2.1.7 Material properties

For a complicated material as wood, one should not expect ideal elastic behaviour, as described by Hooke's law. The stress-strain diagram is therefore not the same as for an ideal elastic body.

Table 1 shows the modulus of elasticity perpendicular to the grain of lime wood in both radial and tangential directions used to model the lime wood cylinder. Notice that the modulus of elasticity in tangential direction is approximately half the value of the modulus of elasticity in the radial direction.

It is known that there is a difference between tangential and radial shrinkage and that this explains the typical radial cracks and cracks along the annual rings when wood dries. Different wood species have different properties, and show different shrinkage and swelling behaviour. One thing is always the same for all species: wood shrinks the most in the tangential direction, about two times more than in the radial direction, see figure 3.

Table 1. Modulus of elasticity of lime wood (Jakiela, Bratasz and Kozlowski (2008))

RH [%]	Tangential direction [MPa]	Radial direction [MPa]
20	600	1120
35	490	900
50	450	820
65	420	770

Between two systems, whose concentration of moisture content differ, there is a natural tendency for moisture transfer. Due to moisture transfer, both systems seek for equilibrium, minimizing the difference in moisture concentration, this process is called diffusion .

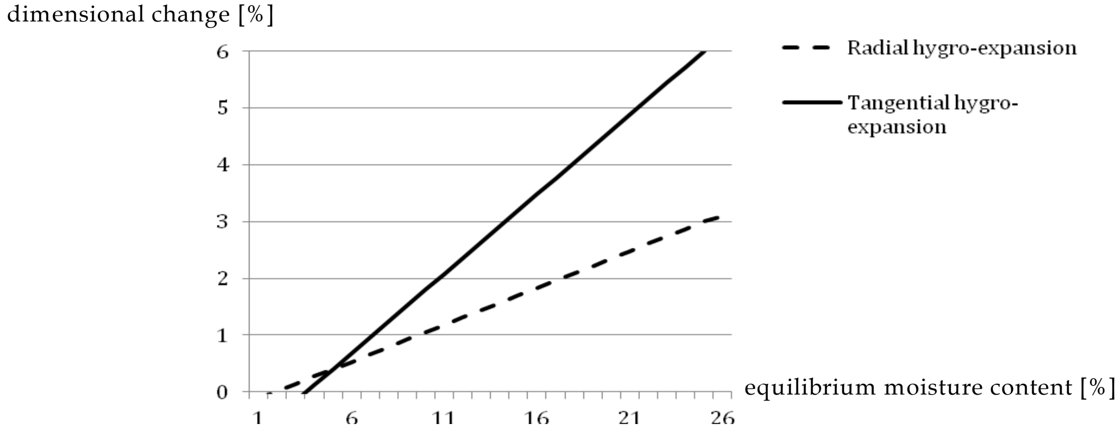

Figure 3. Relation between the Equilibrium Moisture Content (EMC) and dimensional change for shrinkage coefficients α_R *= 0.13 (radial direction) and* α_T *= 0.28 (tangential direction).*

Diffusion seems to be equal for radial and tangential directions and larger for the longitudinal direction; this is expressed in table 2. Furthermore table 2 also expresses that diffusion depends on temperature – in this example constant at 20 ^{o}C – and relative humidity – both determining the equilibrium moisture content; see also equation (7).

The Equilibrium Moisture Content (EMC) for Lime Wood as a function of Relative Humidity (RH) and temperature T is calculated according to equation (7) (Kollmann and Cote (1968)

$$\mathrm{EMC} = \frac{1800}{W}\left[\frac{K(\mathrm{RH})}{1-K(\mathrm{RH})} + \frac{K_1K(\mathrm{RH})+2K_1K_2K^2(\mathrm{RH})^2}{1+K_1K(\mathrm{RH})+K_1K_2K^2(\mathrm{RH})^2}\right] \tag{7}$$

with:

$$W = 345 + 1.29T + 0.0135\,T^2 \tag{8}$$

$$K = 0.805 + 0.000736\,T - 0.00000273\,T^2 \tag{9}$$

$$K_1 = 6.27 - 0.00938\,T - 0.000303\,T^2 \tag{10}$$

$$K_2 = 1.19 + 0.0407\,T - 0.000293\,T^2 \tag{11}$$

T in [^{o}C]

RH in 0,X ($0 \le X \le 1{,}0$) [-]

Table 2. Moisture diffusion coefficient of lime wood as a function of equilibrium moisture content (EMC) (Jakiela, Bratasz and Kozlowski (2008))

equilibrium	diffusion coefficient		
moisture content	radial	tangential	longitudinal
[%]	[m^2/h]	[m^2/h]	[m^2/h]
0	0.0003888	0.0003888	0.0009000
5	0.0004751	0.0004751	0.0050400
5.5	0.0004841	0.0004841	0.0053500
7	0.0005137	0.0005137	0.0056700
8.5	0.0005461	0.0005461	0.0058500
9	0.0005572	0.0005572	0.0056700
13.5	0.0006690	0.0006690	0.0045400
18	0.0008026	0.0008026	0.0030700
23	0.0009690	0.0009690	0.0021000
28	0.0012029	0.0012029	0.0013500

2.2 ***Results from numerical analyses***

At time t = 0, the wooden cylinder is in equilibrium with its surrounding. The relative humidity at t = 0 equals 70%, resulting in a wood equilibrium moisture content of 14% (for T = 20 °C), see figure 4. Suddenly the surrounding conditions change. The relative humidity drops from 70% to 30%, corresponding, when T = 20 °C, to 6% wood equilibrium moisture content. After this event the environment is kept constant at = 30% and the wooden cylinder is slowly drying as it releases moisture. The wooden cylinder seeks for a new equilibrium with its surrounding condition. With help of ABAQUS finite element model the time to reach complete equilibrium was calculated at ± 40 days after the event.

2.2.1 Distribution of moisture content after 24 hours

After 24 hours the centre of the cylinder is still at the initial 14% moisture content. The surface of the wooden cylinder shows a fast transition in moisture content, see figure 5, which shows the changing moisture content at different distances from the surface as a function of time. The selected distances from surface up to 10 mm inside the wooden cylinder show a non-linear diffusion process. The first 1 mm to 5 mm from the surface level instantaneously changes its moisture content. Figure 5 also shows that the core of the

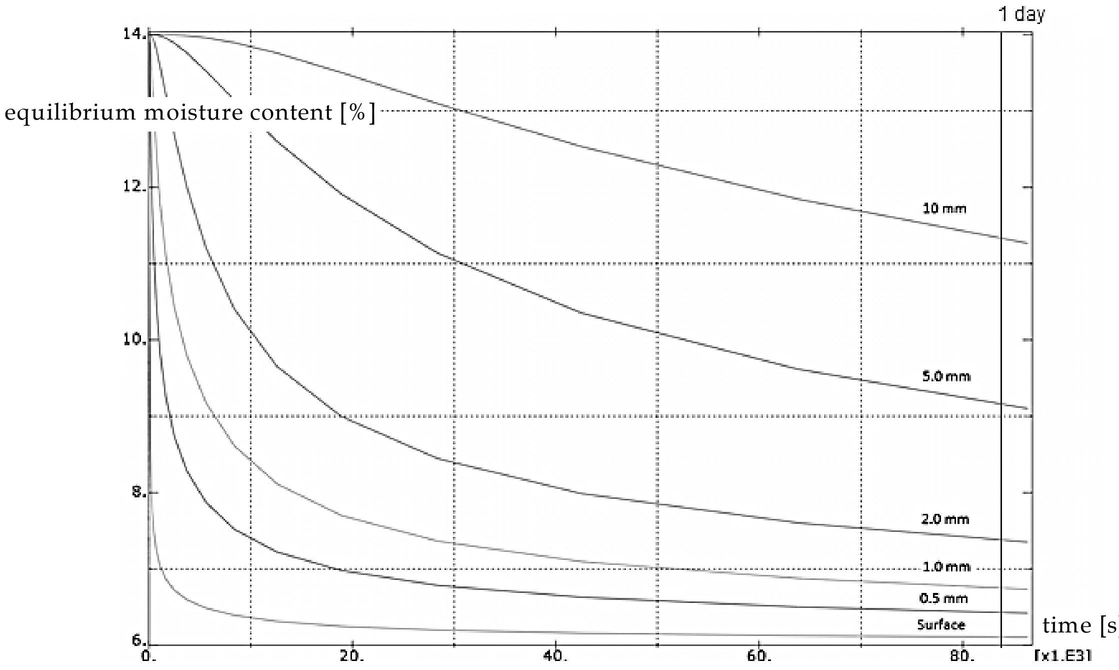

Figure 4. Distribution of moisture content at selected distances from the surface up to 10 mm into a wooden cylinder with a step change of 14% MC to 6% MC which is equal to 70% RH to 30% RH (at T = 20 °C) after 24 h

cylinder lying deeper than 1 centimetre does not experience any change in the moisture content in the first 3 hours.

2.2.2 Strain and stress development in tangential direction

Figure 5 shows the development of radial strain in compression perpendicular to the grain at different depths from the surface as a function of time. Generally, the maximum elastic strain at which wood starts to deform plastically perpendicular to the grain lies around the 0.004 (Ormarsson *et al.* (1997).

Figure 6 shows the development of tangential stress at different depths from the surface as a function of time. The elastic range up to a tangential stress of ± 2.5 MPa and maximum tangential strength of ± 5.5 MPa as denoted by Jakiela *et al.* (2008) is exceeded. With continuing drying of the interior layers, the stress slowly decreases. This slow decrease is the result of the slow vanishing of the moisture gradient as the interior layers dry, resulting in more evenly shrinkage. The content of this paper is limited to stress

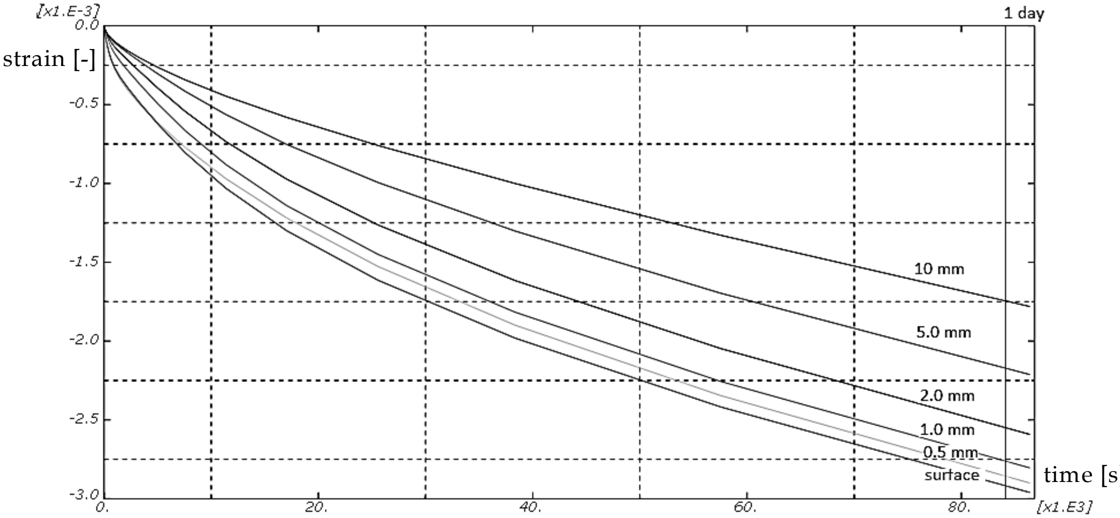

Figure 5. Radial strain at selected distances from the surface up to 10 mm into a wooden cylinder with a step change of 14% to 6% moisture content (MC) equal to 70% to 30% relative humidity (RH) after 24 h

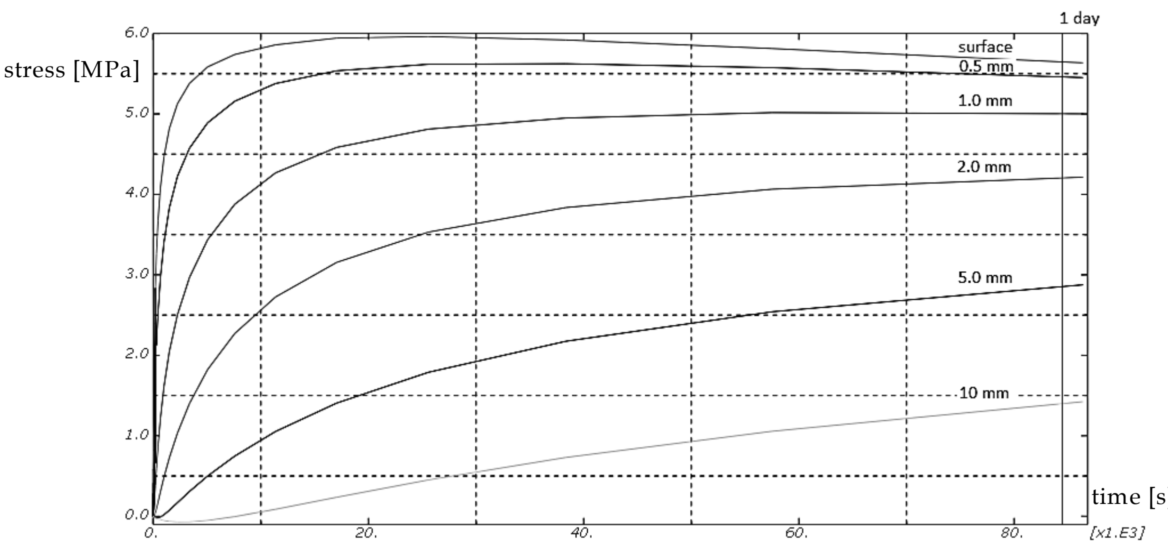

Figure 6. Tangential stress (perpendicular to the grain) at selected distances from the surface up to 10 mm into a wooden cylinder with a step change of 14% to 6% moisture content (MC) equal to 70% to 30% relative humidity (RH) after 24 h

development in tangential direction. Information about radial stress development, tangential and radial strain development can be found in Jakiela *et al.* (2008) and Reijnen (2012).

2.2.3 Numerical verification

The results from the analyses described in this paper are compared to numerical results found by other researchers, Jakiela *et al*. (2008) and Schellen *et al*. (2011), who carried out similar calculations. The results found by Schellen *et al*. (2011) were computed with help of a numerical model within COMSOL as a comparative benchmark of the research done by Jakiela *et al*. (2008). Comparing the results regarding deformations of the limewood cylinder found by Jakiela et al. (2008) and Schellen *et al*. (2011), it can be concluded that these results are very much the same as the results calculated by ABAQUS, see Reijnen (2012).

3 Summary and conclusion

Within ABAQUS finite element software, it is shown that due to the analogy between Fourier's law of heat conduction and Fick's law of mass diffusion, it is possible to solve a moisture movement problem, which is a mass diffusion problem, with a heat transfer analysis. Careful implementation, proper material data and using an appropriate driving potential, for which water vapour pressure P and water vapour content w look suitable, deserves full attention.

Unfortunately, the results (high tension stresses perpendicular to the grain in the tangential direction) indicate that moisture transport below fibre saturation point can possibly not be regarded as being a pure Fickian process. Other models like non-Fickian or multi-Fickian model, resulting in stresses perpendicular to the grain as indicated in figure 7, can be considered.

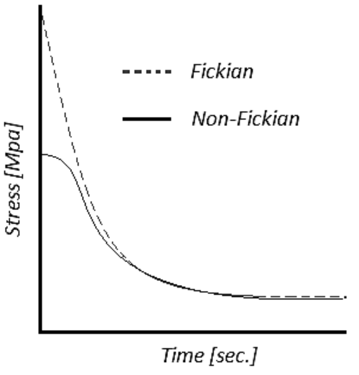

Figure 7. Stress development due to changing wood moisture content simulated with so-called Fickian and Non-Fickian model descriptions

Literature

Jakiela S., Bratasz L., Kozlowski R. (2008) Numerical modelling of moisture movement and related stress field in lime wood subjected to changing climate conditions, *Wood Science Technology*, 42,21-37

Kollmann F., Cote W. (1968) *Principles of Wood science and Technology: 1 Solid Wood*. Institute fur Holzforschung und Holztechnik , Munchen, Germany / College of Forestry, Syracuse, New York.

Mirianon F. Fortino S. and Toratti T. (2008) A method to model wood by using ABAQUS finite element software, VTT (Technical Research Centre of Finland) Publications, Laskut, Finland.

Ormarsson S., Dahlblom O., Petersson H. (1997) *A numerical study of the shape stability of sawn timber subjected to moisture variations*, Lund Institute of Technology.

Reijnen S. (2012) *Moisture transport and shape stability of wood exposed to humidity variations – A numerical study*. Eindhoven University of Technology, Department of Building and Architecture (master diploma thesis), Eindhoven, the Netherlands.

Schellen H., van Schijndel J. (2011) Numerical modelling of moisture related mechanical stress in wooden cylindrical objects using COMSOL: a comparative benchmark, Eindhoven University of Technology, Department of Building and Architecture, Eindhoven, the Netherlands.

NWO Science4Arts research program http://www.nwo.nl/en/research-and-results/programmes/science4arts