# Heron's fountain 20

Finds and ideas with a surprising element similar to the playful inventions of Heron of Alexandria, after whom this journal is named

## Elegant way of measuring Poisson's ratio

Poisson's ratio can be determined in an alternative, non-classical way. This requires a simple experimental setup and an equally simple analysis. The method was developed in 1970 and covered by layers of dust, which are removed in this Heron's fountain [1, 2].

The test involves a rhombic plate specimen that is subjected to so called anti-clastic bending. Hence, the two opposite corners of the short axis are supported, while the corners of the long axis are loaded (fig. 1). As a result, the plate bends downwards in the direction of the long axis and upwards in the direction of the short axis.

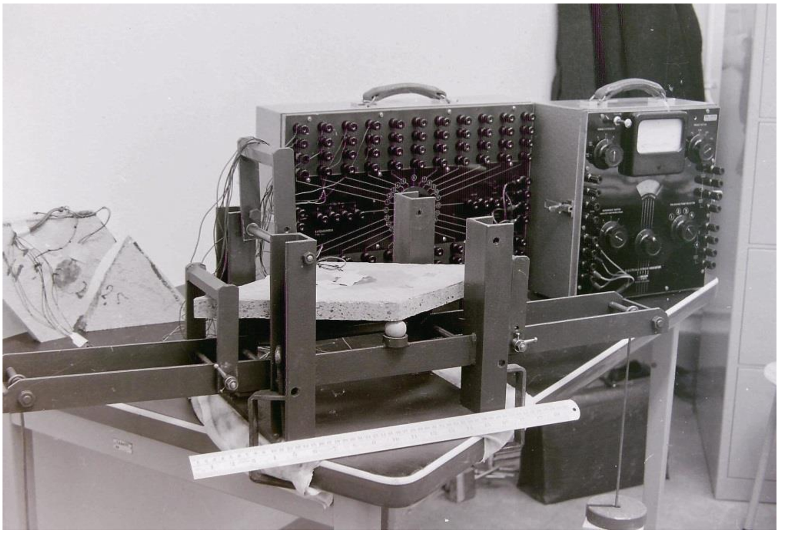

*Figure 1. Testing a rhombic plate in anticlastic bending. This setup does not include glass plate (obstruction grid), light or camera. Instead it shows strain gauges, Weatstone bride and amplifier, which were used for verification [3].*

The deformation is measured with the shadow moiré method. This involves an obstruction grid, a light source and a camera (fig. 2). The grid of parallel lines is printed on a glass plate and mounted just above and parallel to the surface of the rhombic plate. The light projects the grid onto the surface of the rhombic plate. The camera jointly records the grid and its shadow. The result is a shadow moiré photo (fig. 3).

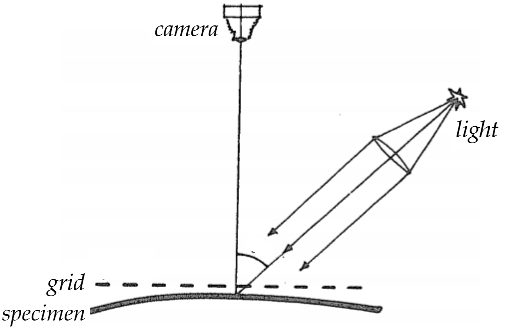


*Figure 2. Shadow moiré method*

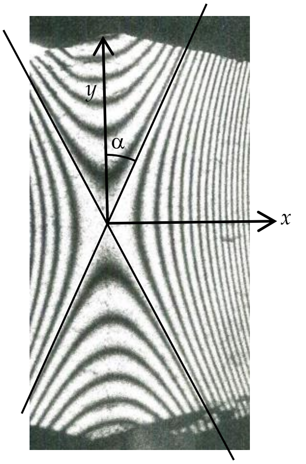


*Figure 3. Shadow moiré interference fringes showing the deformation contour lines of a mortar rhombic plate (a / b = 4) in anti-clastic bending. The total deflection in the centre is 0.1 mm.*

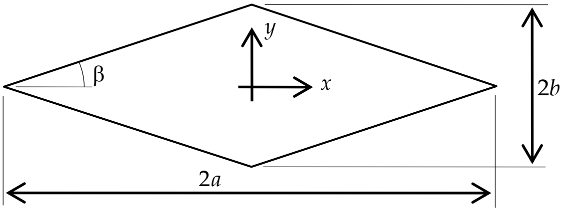


*Figure 4. Rhombic plate dimensions*

The moments in the rhombic plate are approximately uniform (fig. 4).

$$
\begin{aligned}
m_{xx} &= \frac{aP}{2b} \\
m_{yy} &= -\frac{bP}{2a} \\
m_{xy} &= 0
\end{aligned}
\qquad (1)
$$

where $P$ is the point load on each of the corners of the rhombic plate.

The constitutive equations of the thin plate theory are [4]

$$\begin{aligned} m_{xx} &= D(\kappa_{xx} + \nu\kappa_{yy}) \\ m_{yy} &= D(\kappa_{yy} + \nu\kappa_{xx}) \\ m_{xy} &= D\tfrac{1}{2}(1-\nu)\rho_{xy} \end{aligned} \tag{2}$$

where $D = \dfrac{Et^3}{12(1-\nu^2)}$ is the plate stiffness, $\kappa_{xx}$, $\kappa_{yy}$ and $\rho_{xy}$ are the plate curvatures and $\nu$ is Poisson's ratio. Substitution of equation 1 in 2 gives

$$\nu = -\frac{\dfrac{\kappa_{yy}}{\kappa_{xx}} + \dfrac{b^2}{a^2}}{1 + \dfrac{\kappa_{yy}}{\kappa_{xx}}\dfrac{b^2}{a^2}} \tag{3}$$

The deflection of the plate is described by [4]

$$w(x,y) = \tfrac{1}{2}\kappa_{xx}x^2 + \tfrac{1}{2}\kappa_{yy}y^2 \tag{4}$$

At a deflection contour, deflection $w$ has a constant value. All deflection contours have the following asymptote.

$$y = x\sqrt{-\frac{\kappa_{xx}}{\kappa_{yy}}} \tag{5}$$

We define

$$\tan\alpha = \sqrt{-\frac{\kappa_{yy}}{\kappa_{xx}}} \quad \text{and} \quad \tan\beta = \frac{b}{a} \tag{6}$$

Substitution of equations 6 in 3 and rearranging produces the following elegant solution for Poisson's ratio.

$$\nu = \tan(\alpha-\beta)\tan(\alpha+\beta) \tag{7}$$

P. Stroeven

Delft University of Technology, the Netherlands

## References


[1] Stroeven, P., Voorsluis H. (1968). Optical method for determination of very small deformations – employed to measure Poisson's ratio of concrete. *HERON*, 16(2):60-69 (in Dutch)

[2] Stroeven, P. (1970). Poisson's ratio in uniaxial tension and anticlastic bending of micro-concrete and perspex, *HERON*, 17(3):1-23:

[3] Stroeven, P. (1982). Proceedings of the *International RILEM Symposium on Ferrocement*, "Recent Developments in Ferrocement", 22-24 July 1982

[4] Blaauwendraad, J. (2010). *Plates and FEM, Surprises and Pitfalls*, Spinger