Digital path towards Timber Reciprocal Frame Structures

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Reciprocal Frames (RFs) are structures that are feasible by means of circulating shear with compression or tension interactions between their constituent members. Beams do not meet at their ends but somewhere along their length. RFs can create planar to complex 3d surfaces. 3D shapes will increase the level of geometrical complexity drastically. When using straight elements the curvature is created by using stacked connection details. This results in a large variation of details regarding angles of intersection, profile dimensions and forces to transfer.

To date, an RF form finding tool that regards both beam depth analysis and the structural design of connections has not yet been developed. Although researchers developed computational form finding methods to create geometrical solutions and described the global structural design, computational complexity may have prevented a direct inclusion of detailing in the overall RF design. This paper presents a digital path from design to production for RF structures. A new RF form finding method is developed that includes both the structural design of beam dimensions and detailing and results automatically in production files for each element. This parametric model - named 'Reciprocal Frame Designer' (RFD) - has been developed to design RF assemblies of wood from any arbitrary shape.

Key words: Digital technologies, reciprocal frame, timber detailing, free form, form finding, computational design, design to production

1 Introduction

A Reciprocal Frame (RF) is a structure in which a combination of balanced structural members create a span that is greater than their own length. Figure 1 shows that this balance is established at intermediate nodes where the elements mutually support each other in a cyclic manner. History demonstrates that for thousands of years mankind has been able to span distances greater than the lengths of available materials by using RFs.

Where historic RF examples seemed to be induced by material shortages [Thönnissen, 2015], complex examples of today have an investigative and aesthetic function. The practical adoption still needs to be explored.

Figure 1. The structural principle of a three- and four-member single unit reciprocal frame

This paper presents a parametric model, developed within the Rhino/Grasshopper environment that covers the complete design to production process; including detailing of arbitrary shaped RF structures. The parametric model is referred to as the Reciprocal Frame Designer (RFD). The first section discusses fundamental RF (parametric) parameters, geometry, theoretical basics and design decisions. Hence, we outline the ideas and theory behind the computational RFD tool based on these principles. A practically verified method to design RFs is presented together with general theories of bringing a timber structure to production. Finally, resulting physical models are discussed.

2 Parameters and eccentricity

Reciprocal Frames are described by several parameters, including the ones described below (see figure 2). The global form of an RF with eccentric jointing depends on the unit style (see figure 8) or member orientation, the number of members per unit, and the configuration of members in the structure [Thönnissen, 2015]. The center axes of members are connected by eccentricity lines. One unit is created by 3 or more members placed in a pattern that repeats itself.

The approach how excentricity is managed is crucial for the geometry, structural behavior, detailling and production process. Eccentricity is therefore of paramount importance in this research and is structurally schematized as the shortest distance between the center axis of two members.

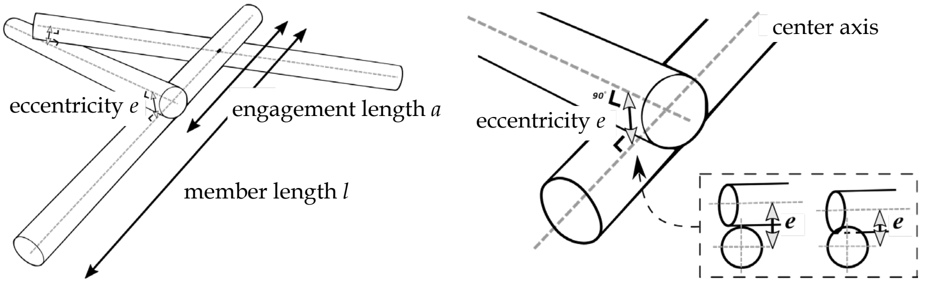

Figure 2. Parameters in a single-unit eccentric RF and eccentricity with its relation to member depth

In general, forces will naturally search for the shortest load path, which makes this schematization close to the RFs actual structural behavior. The geometrical rule of defining the eccentricity (line e) as the shortest distance between both beams center axis, ensures that line e is perpendicular to both intersecting beams.

The eccentricity length is not necessarily dependent on the member depth. However, when using an eccentricity that is smaller than the member depth, additional connection measurements, such as notching, needs to be considered. Consequently, detailing cannot be separated from the structural RF design that has started a design method based on geometrical rules.

3 Basic geometry

The design of RFs cannot be accomplished without the use of geometrical rules. In contrast to other structural systems where members often meet at their extremities, members of an RF meet each other somewhere along the length of the member, which geometrically can be achieved by modifying the geometry of polygons (see figure 3).

To create an RF by means of rotation, polygon sides are rotated around a predefined point and angle. Translation modifies the basic polygon by translating its sides towards the polygon center point. When combining the described rotation and translation method, the complexity is increased: it does not add extra freedom of form with respect to solely rotation.

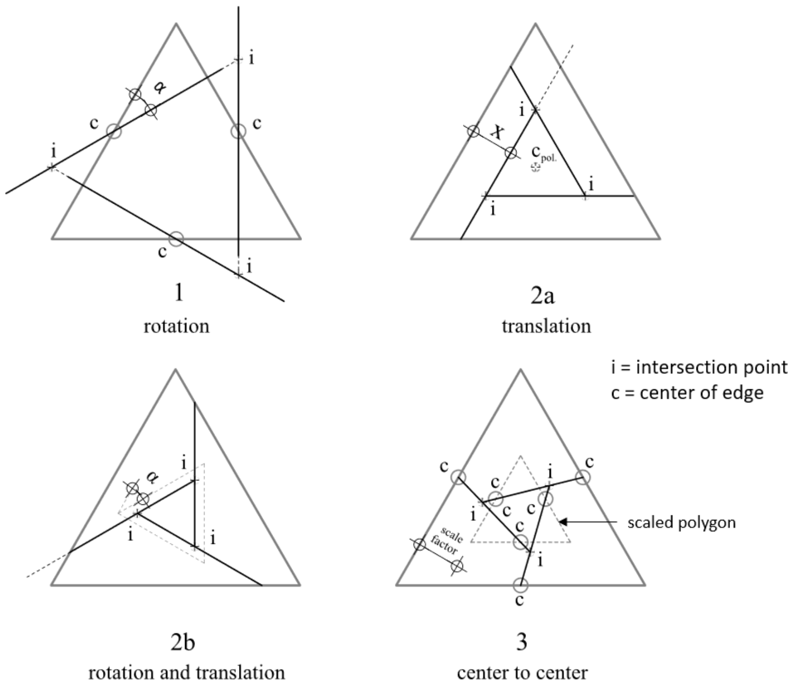

Figure 3. Four geometrical RF design methods applied to triangular polygons [Anastas, Rhode-Barbarigos, & Adriaenssens, 2016], [Rizzuto, Saïdani, & Chilton, 2001]

The first step in the center to center method is to create a new smaller duplicate polygon by scaling the polygon inwardly. Second, the midpoints at each side of the basic and scaled polygon must be determined. Third, midpoints of the basic and scaled polygon are connected by a line as shown in figure 3. This line is extended to the intersection point "i". Single-unit RFs with eccentric jointing can now be created by moving these intersection points "i" in height or z-direction (see figure 4).

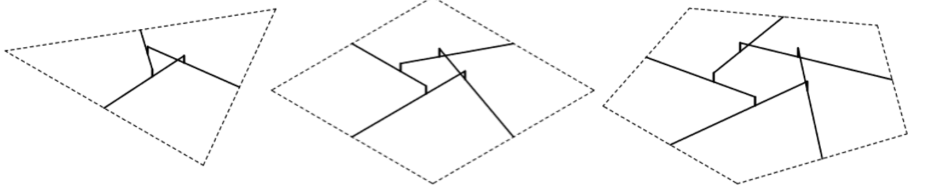

Figure 4. Three single RF units with elevated intersection points

4 Computational RF form finding

The center to center method can be used to create a single-unit RF design within one polygon. However, when multiple polygons are connected the complexity increases drastically. A precise algorithmic definition is needed to control all variables within a polygon and strong relations between the different polygons.

It seems inevitable to employ computational aid to find the optimal geometry and the input for Finite Element (FE) analysis, detail analysis and to allow for Computer Aided Manufacturing (CAM).

The past decades showed an increased interest in computational RF design among researchers, which resulted into three main approximation computational design methods [Pugnale & Sassone, 2014]. First, the analytic approach determines the RF design exactly and uses geometrical equations to create an RF that is in most research based on platonic solids [Sénéchal, Douthe, & Baverel, 2011]. Second, the bottom-up approach uses iterative processes or self-generating geometries to create an RF. These so-called iterative additions rely on choosing an elementary unit style that is then distributed over a preferred growth direction after which the transitions are optimized. These two approaches are completely different but share one similarity: the possibility to take preliminary design requirements into account is limited. The third method however, the top-down approach, is an exception to this. It relies on an optimization algorithm and is the most widely used method in RF form finding. To date, numerous strategies have been developed, but not all immediately result in a geometrical solution or take the structural feasibility of connections into account [Pugnale & Sassone, 2014]. The RFD is based on the third method and has been designed to enable direct parameter control to visualize their influence on geometry, structural behavior and detailing.

5 The Reciprocal Frame Designer (RFD)

The RFD has been developed by means of a top-down approach. It was decided that this is the best suited form finding approach for this research because in practice designers often start by creating a certain architectural shape. Based on this shape the RF design is created by using the center to center method. The RFD then uses several geometric optimization strategies to transform the shape into a triangular or rectangular subdivided NURBS (Non-Uniform Rational B-Splines) surface or mesh.

Surfaces are subdivided in UV-direction by using parametric curvature or equal point distance (see step b and c in figure 5), after which geometric rules must be applied to create panels (see step d and e in figure 5). The meshes may require a rebuild, equalization or smoothening before being further used in the RFD.

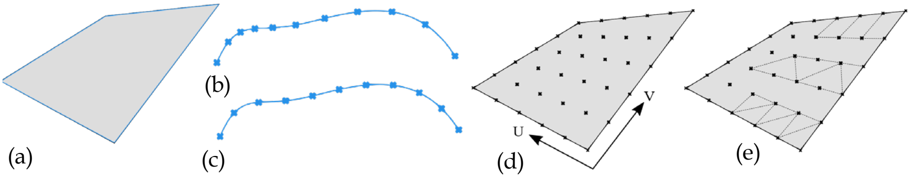

Figure 5. NURBS surface (a), by means of parametric curvature (b) or equal point distance (c) subdivided in UV direction by points (d), after which these can be panelized (e) by geometrical rules

5.1 *Creating a basic RF geometry*

By using the center to center method, the number of members in an RF unit depend on the number of mesh edges. Triangles create three-member RFs and rectangles create four-member RFs. At all common surface edges, RF members naturally meet each other because the starting point of an RF line lies at the center of a polygon edge (see figure 6).

In this situation, eccentricities between the elements are not yet created since each RF unit is still considered as planar single-unit. This is changed when two individual members, in neighboring panels, connected in a center point, are transformed into a single polyline with

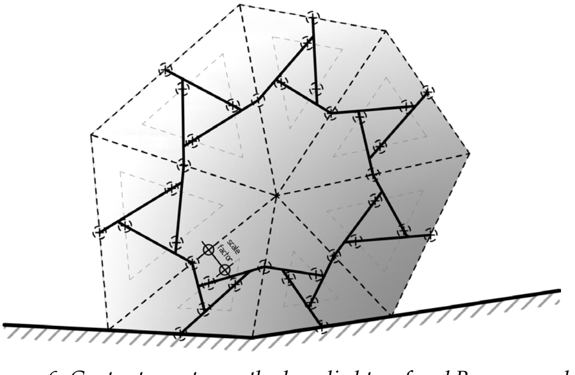

Figure 6. Center to center method applied to a faced Brep or mesh

a kink (see figure 7) [also described by Anastas et al., 2016]. The end points of this polyline can be used to create a new straight line that is defined by connecting endpoint A and B (see figure 7 line A-B). This straight line is the center line of an RF profile.

The center line A-B is moved towards the kink node - the exact distance of movement is related to the connection height of the attached profile (point C and D). The transformation distance e_g is determined by evaluating the mean length of the two-line parts L_{S1} and L_{S2} that exist between points A and C and between B and D. The e_g is determined by:

$$e_g = \frac{L_{S1} \sin\alpha + L_{S2} \sin\beta}{4} \quad (1)$$

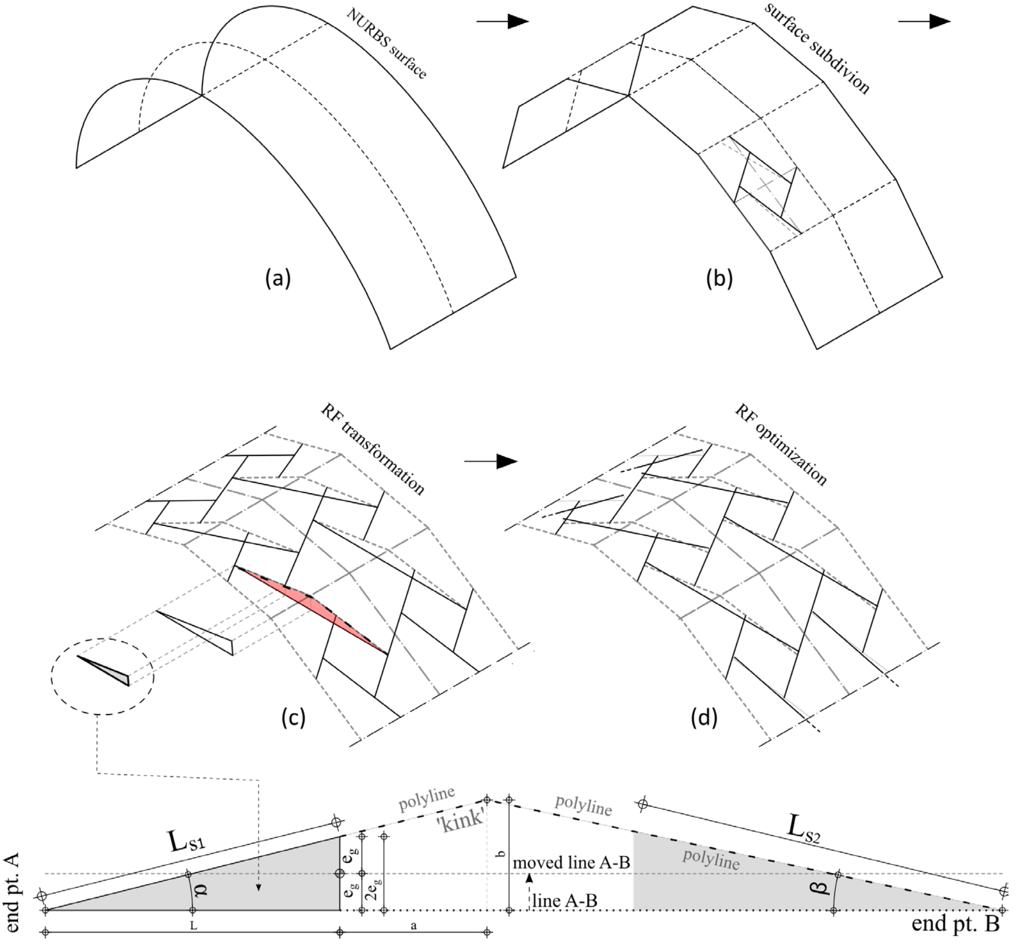

Figure 7. Basic geometrical design procedure that has been applied in the RFD

Figure 7 shows the required geometric RF transformation applied to a rectangular subdivided single curved surface. Here, a 'kinked' polyline is only created when the neighboring surfaces are noncoplanar. If it is coplanar, there is no kink and e_g is zero. These lines are moved over a distance equal to the e_g of neighboring profiles.

Consequently, member configuration and unit style depend on the geometry of the basic surface. A geodesic polyhedron, for example, results in unit-style and member configuration (a) in figure 8.

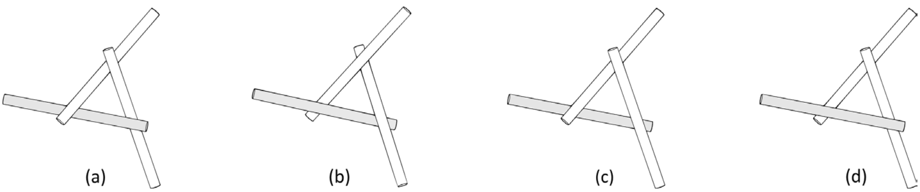

Figure 8. Four possible three-member unit styles. Unit style (a) and (b) can be structurally stable from itself by utilizing friction or using notching between the members

5.2 ***Eccentricity optimization***

Eccentricities between members depend on the curvature of the geometry and surface subdivision. This means that there can be a large variety in eccentricity lengths, i.e., the distance between individual members (see figure 2). This is not preferable in structural design since it can result in different types of details and reduces the control on the capacity of the connection details. Therefore, an optimization method is developed that enables direct eccentricity control. With this method, each individual eccentricity line is optimized towards the preferred eccentricity length *e*.

The length adjustment is done by applying initial deformations to the eccentricity lines according to:

$$\varepsilon_i = \frac{\Delta L_i}{L_i} = \frac{e - e_i}{e_i} \tag{2}$$

Here, the strain ε_i is determined by the preferable eccentricity length e and initial eccentricity length e_i . For each individual eccentricity line, an individual strain is then determined to increase, reduce, or maintain its length.

Figure 9 shows the optimization procedure applied to a simplified problem. Different eccentricity lines with length e_i are optimized towards an equal preferred length e. To make this length change possible, the beam length of members b_i also have to change. Length e_i can be changed by modeling all members (b) as 'infinitely' stiff to shear and bending and 'infinitely' flexible to axial stresses and torsion. Eccentricity lines e_i , are modeled 'infinitely' stiff in all directions avoiding torsional resistance to be able to maintain the member configuration. To limit the adjustments to the initial surface shape the value e should be chosen close to the average of the mean lengths of e_i . When a higher value of e is chosen, more curvature is created as can be seen in the dome in figure 10. This optimization procedure is applied in the RFD by using Karamba's FE solver [Preisinger, 2013] that allows geometrical nonlinear deformation analysis.

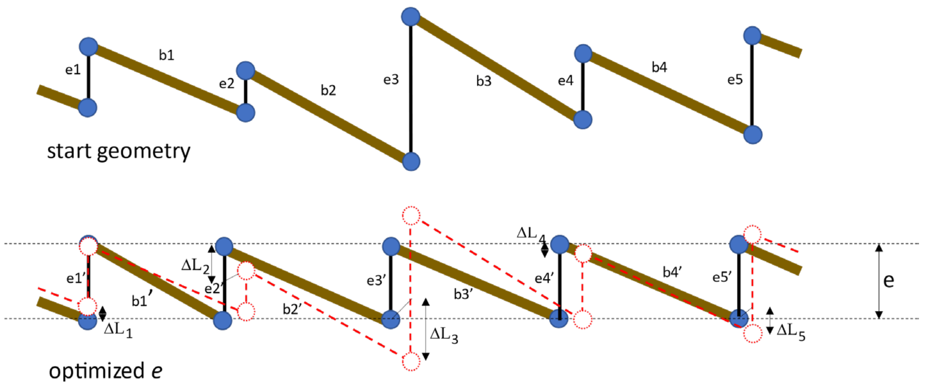

Figure 9. Eccentricity optimization method

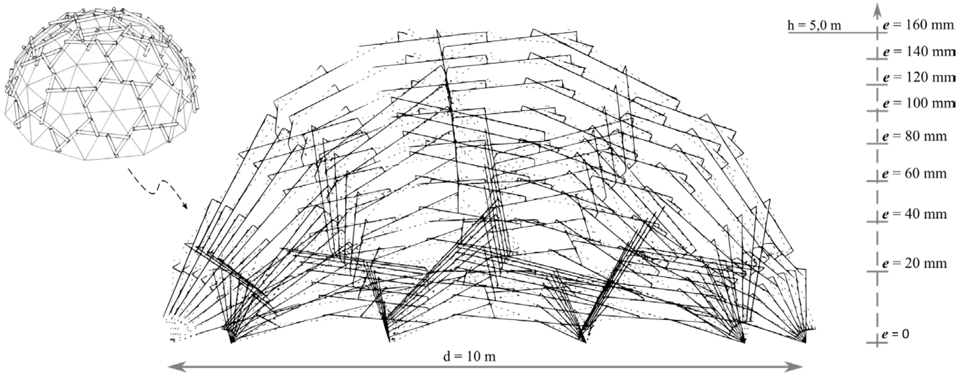

Figure 10. The influence of different chosen eccentricity values (e) on the structural height of an RF while using the same scaling factor (scaling factor see figure 3.3)

5.3 *Timber detailing*

The effect of increasing the eccentricity length *e* while preserving the scale factor results in a higher RF structure (see figure 10). It seems that a certain structural height is already created with a relatively small eccentricity. When notching is avoided, the diameter of the the members determine the height of the structure, meaning small height of structure needs small member depths. However, load application to the structure is likely to result in higher member depth with a larger center to center distance than the eccentricity length. Consequently, notching seems unavoidable. Furthermore, one creates geometrical freedom and enables the necessary transferring of shear and normal forces by keeping the eccentricity independent of the member depth.

To define the geometry of the notches, first the element axis orientation must be defined. The x axis is following the center line of a profile, the z-axis is defined by the mean direction of the two central eccentricity lines.

The developed detailing creates notches in both intersecting members causing equal section weakening (see figure 11). When just half of the overlap is cut out in each profile, the notched detailing works in theory but is not very efficient in terms of force transferring and impossible to construct.

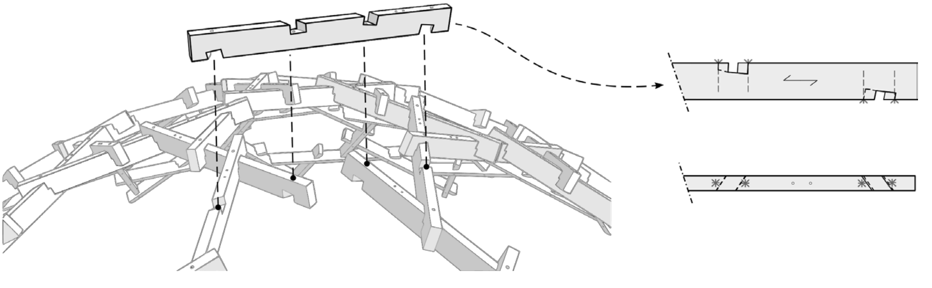

Figure 11. Detailing applied in the RFD with reinforcing screws and timber grain direction

In terms of force transferring, the weakest part of the connection is determined by stresses perpendicular to the grain. In general, timber is less capable to bear stresses perpendicular to the grain in comparison to stresses parallel to the grain/fiber. In terms of construction, each element is connected to multiple eccentricity lines which are not orientated in the same direction. Therefore, the sliding-in direction of each node is not the same and causes clashes during the building sequence.

A common method to create lightweight structures is by implementing the principle of form follows force. In lightweight timber structures the principle of **force follows fiber** may be adopted by considering the different strength properties in the different directions. In case of the RF structure, equilibrium planes between the perpendicular and parallel grain are created to minimize the stresses perpendicular to the grain. The connecting plane between two profiles (bottom plane of the each notch) is scaled inwardly so that angled edge planes are created. This new connection topology (see figure 12 and figure 13) improves the load-transfer and assures a good construction process.

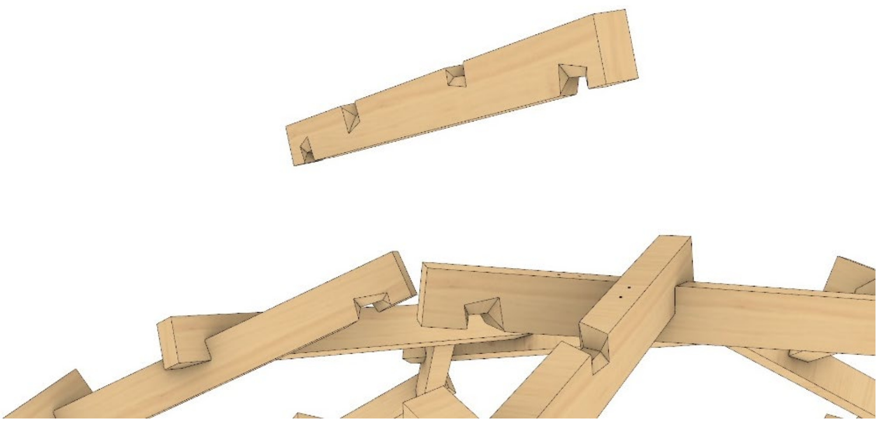

Figure 12. Improved detailing by scaling the connecting planes of the notches

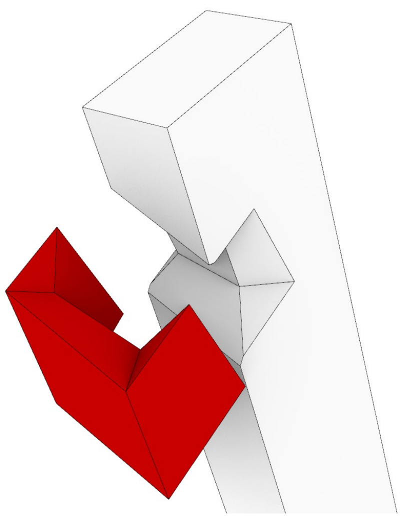

Figure 13. Boolean intersection to create detailing with cutting shape (in red) and resulting detail

The detailing is parametrically created in the Rhino/Grasshopper environment by using Boolean intersections (see figure 13 and figure 14). Cutting shapes with the preferred detailing geometry are generated automatically and used to subtract from the solid rectangular beams. Each individual plane of the beam needs to be planar to allow for easy timber milling procedures. Figure 14 briefly describes and visualizes this process.

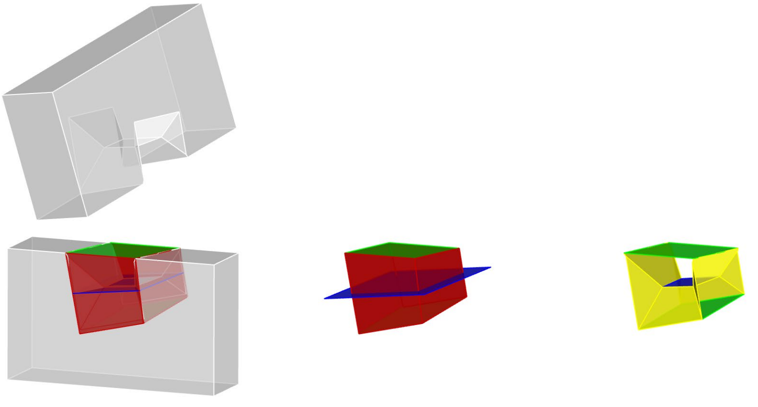

Figure 14. The Boolean intersection geometry creation method with a red/green cube as geometrical basis with resulting geometry on the right in blue and yellow

Here, the blue plane determines the planarity of each individual neighboring detailing plane, necessary for the timber milling process. The blue plane orientation is determined by developing a mean plane and by an intersection operation with its neighboring (orange) planes. Its neighboring four planes (in orange) arrive under an angular rotation with respect to the red geometry creating the polyhedral detailing shown in figure 13. The angular rotation of the orange planes can be chosen by the user of the script considering stresses perpendicular to the timber grain. The intersection lines of all computed planes create the final Boolean intersection geometry. Because all plane angles are manually implemented or computed by mathematical expressions, and all area sizes are known structural analysis on each detail based on its exact geometry can be accurately implemented. Subsequently, drill holes, beam identification and other information may be added by Boolean operations. These operations are purely geometrical, and the resulting beams are solid NURBS surfaces. All details may differ depending on the input geometry. To allow for structural analysis and design to production, geometrical data such as angles between plane normal vectors and grain directions are all subtracted from the model.

5.4 *Structural analysis*

At this point the geometry of the center lines, the eccentricity lines and the xyz orientation of the profile and grain direction is known. They are all generated in the Rhino/Grasshopper environment. Consequently, because it is possible to link forces to these data, the member sizes and the related detailing can be performed (see figure 15). In general, details in timber structures dictate in most cases the member sizes and overall structural behavior.

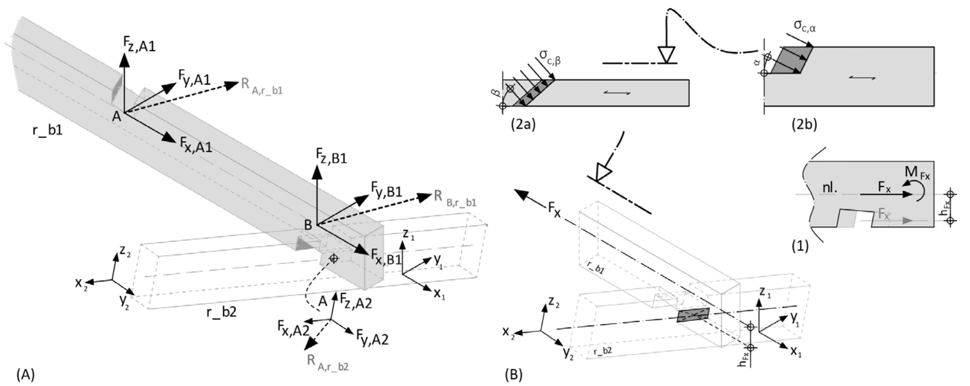

Figure 15. Force distribution/identification from one member to another, stress at angle to the grain

In the developed RFD, the structural behavior is linked to the geometry by implementing GeometryGym. GeometryGym creates a link between Grasshopper and the Finite Element Modelling (FEM) software Oasys GSA. The Finite Element Model is automatically generated in Grasshopper. Load application in the FE model is implemented by means of load panels (see figure 16). These load panels simulate possible roofing and are modeled by a triangular mesh (see figure 16).

The model can be directly exported with Geometry Gym to GSA to check for instance the maximum stresses and the deformation behavior. (see figure 16b). However, this is not mandatory since Unity Checks UC) can be directly monitored for exceedances within the RFD according to Eurocode 5 [EN, 2004]. The EC5 checks are scripted in formulas within Grasshopper and combine actual geometry with forces (see figure 15).

Consequently, timber failure can directly be visualized by colored indications for each different UCs (see figure 16c). In this case the UCs show that strengthening by means of reinforcement perpendicular to the grain is unavoidable when using Glued Laminated timber (GL28h).

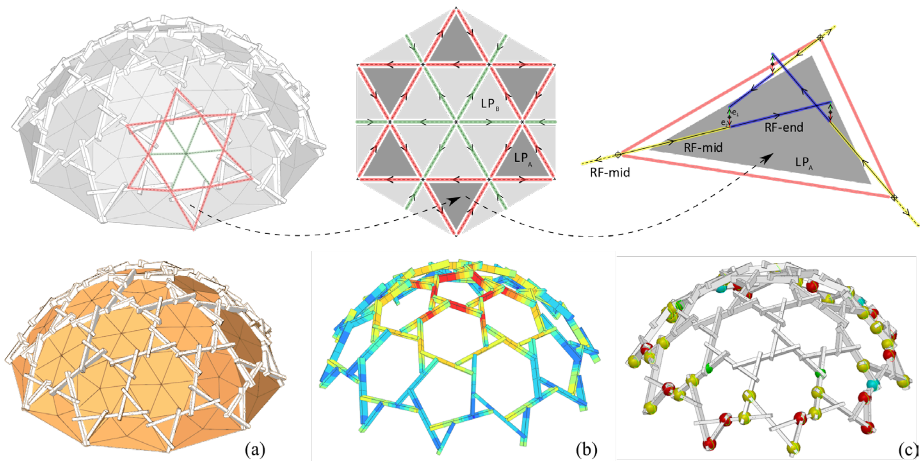

Figure 16. Structure of FE model: Load panels (a), illustrative FE model with bending moment M_{yy} (b), UCs > 1 according to EN1995 (Eurocode 5) showing timber failure in the RFD (c)

An advantage of combining parametric geometrical and structural design is that parameters can be directly changed to modify geometry or member depths to satisfy the checks. For example, the structural influence of changing the eccentricity parameter "*e*" in the RF structure visualized in figure 10 can be easily identified.

When reducing the eccentricity towards zero, this RF will start to behave as a planar grid structure. It will transfer its forces primarily through shear and bending. Increasing the eccentricity results in a reduction of shear forces and an increase of axial forces in the elements. Due to the principles of RFs these axial forces will still be transferred by bending due to engagement length and eccentricity. A final model based on these assumptions that satisfies all UCs can now be produced.

5.5 *From design to production*

The RFD enables structural designers to find a geometrical optimum by varying design parameters and section dimensions till a satisfying structural optimum has been found. Subsequently, production can start by a design conversion to CAM software.

All members and its detailing can be different. They are automatically numbered and marked in Rhino by indicating placement and member orientation in space. The third step of transferring the geometrical data from Rhino to machine language can be conducted using two methods:

1. Export geometry to timber industry CAD software that allows for CAM data generation
2. Directly export CAM data from Grasshopper to CAM software

5.5.1 CAM and the timber industry

Today, a great part of the mass timber construction companies use CAM machines to create connections and to size timber beams. Timber CAM machines from Weinmann, Technowood and Hundegger are widely used in the industry (figure 17).

Figure 17. Hundegger machine (left) and timber milling in a Technowood machine (right)

Companies that use these machines need software that is able to create the machine data based on their detailing. Special BIM software for the timber industry such as Tekla Structures (timber part), HSBcad (within Revit), Sema and Cadwork (standalone) are able to derive CAM data from the designs produced by their software. The dialect used to drive the timber machines differs per machine manufacturer. A Hundegger machine for instance uses the BVX dialect that is only applicable to their machines. However, SEMA and Cadwork have been developing BTL(x) language as a universal dialect independent of machine manufacturer since 1992 (figure 18). Since then, BTL(x) has been adopted by many design software and machine sector companies worldwide.

Both languages describe detailing by means of operations on geometry. Beam ends are for instance described by saw cuts, where Rhino only describes geometry. The detailing developed in the RFD needs to be transformed into one of the described machine languages to enable automated manufacturing.

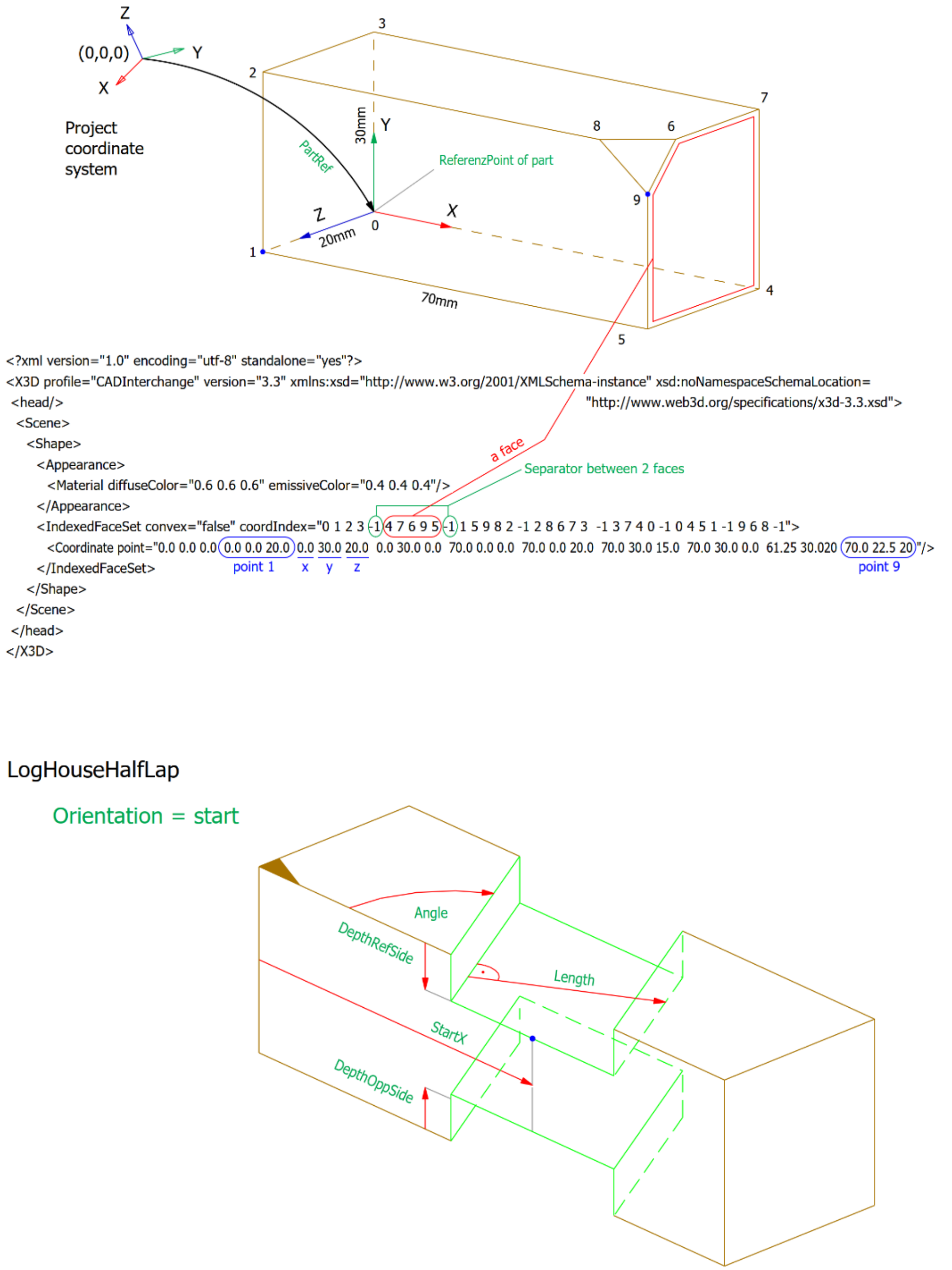

Figure 18. General Btl(x) specification to specify overall member size for timber machine control and geometrical specification to program the geometry of a half lap connection in the Btl(x) language [design2machine, 2021]

5.5.2 *Creating CAM data with timber industry CAD software*

The RF design created in the Rhino environment can directly be exported to IFC or SAT and imported into for instance Cadwork. The CAM data can now be produced using Cadwork but this requires some manual steps. Machine operations need to be assigned individually, specific geometrical data may be lost and each member needs to be individually checked. A detail that for instance requires 6-axis milling, but is adjusted to 5-axis milling in the CAD software, causes problems and non-fitting geometry. The direct specification in the Rhino/Grasshopper environment may prove to be a wonderful solution here. Computational complexity and possible errors are reduced.

5.5.3 *Directly export CAM data from Grasshopper to CAM software*

The process of directly transferring the geometric detailing to timber machine language can be conducted by rewriting in Grasshopper the geometry to machine language. The detailing specified under 5.3 needs to be rewritten into drilling and milling operations. Several plugins have been developed to allow for this transformation. The big challenge remains rewriting the detailing geometry information of complex details into operations specified by angles and distances.

Simple notches in the RF, as presented in figure 11, can be manufactured quickly and with low cost. The detail described in figure 12 and figure 13 requires more machine operations and time and is therefore more costly. But their benefits in structural capacity and the construction method are however more important and in fact will again reduce the cost. A detail that is not possible to manufacture must not be designed. Since each timber manufacturer could use different machines with different software, availability of tools and machine operation axis, final detailing always needs to be checked in close cooperation with the manufacturer.

By specifying a machine file in for instance Cambium, one can check the manufacturability of the proposed detailing. Figure 19 shows a direct export from Grasshopper into Cambium by BTL(x). Cambium is Hundegger's CAM software. A machine file covers availability of tools and operation axis.

5.5.4 *Construction*

The feasibility and design to production process is verified at first by a demountable 3d-printed scale model and second by making full scale single-unit RFs to be used as table legs. The full scale model was developed using the simple notch detail (see figure 11) and

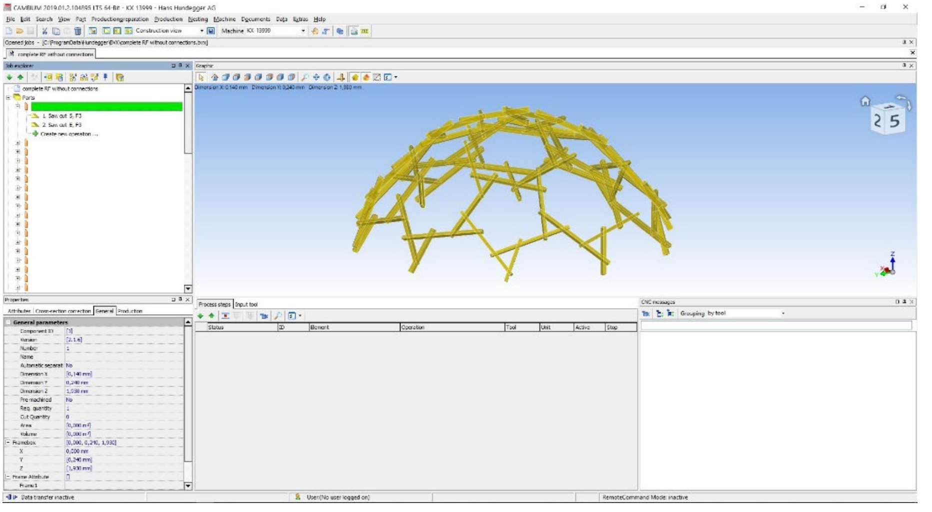

Figure 19. A direct RF import from the RFD in Grasshopper to Cambium. Cambium is the driving CAM software of Hundegger machines. Timber beams can directly be machined from this file

manufactured by a timber industry standard Hundegger machine. Results from this model revealed that a large clearance (5 mm) was needed to assemble the models. This clearance leads to a large overall sacking and was the reason to develop the details described in figure 12. However, this detail has not yet been tested at full scale.

The 3D printed scale model revealed that construction of an RF of this kind can only be constructed from top to bottom. A crane is needed to lift the structure each time a new member is added. This construction sequence has already been explored by Buckminster Fuller and Kaiser in 1957 (see figure 20) to construct aluminum domes [Cushin, 1957].

6 Practical applications

In 2019, this research has been applied in the development of an actual project for the construction of a biobased earth covered villa near Nijmegen, The Netherlands. The design evolved from the rotation of one main RF design based on a trimmed elliptical mesh. The structure is elevated on a concrete structure to be able to reach the preferred internal ceiling height and to create space of façade doors and windows. The project is under construction and is scheduled to be built by using carpenter joints and unbarked round timbers.

Figure 20. Dome construction by Fuller and Kaiser in 1957 [Cushin, 1957]

Figure 21. Full scale model made with CAM and Hundegger machine to test the feasibility of RF detailing

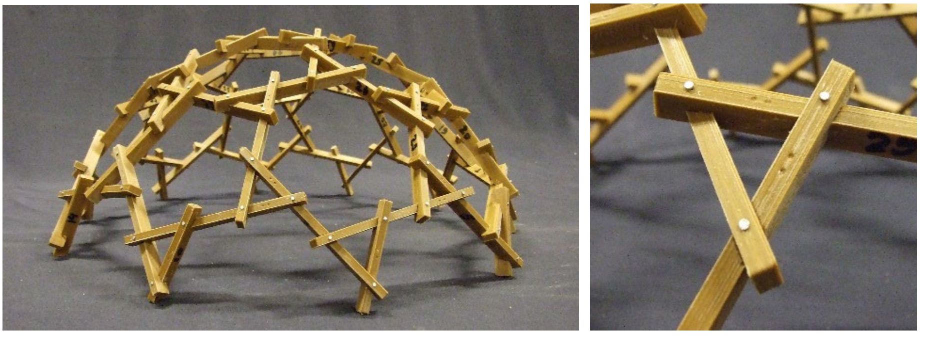

Figure 22. 3D printed 1:20 scale model using the same design as displayed in figure 10 with e = 100 mm. The two dots on each beam indicate the highest point of each beam allowing ease of construction.

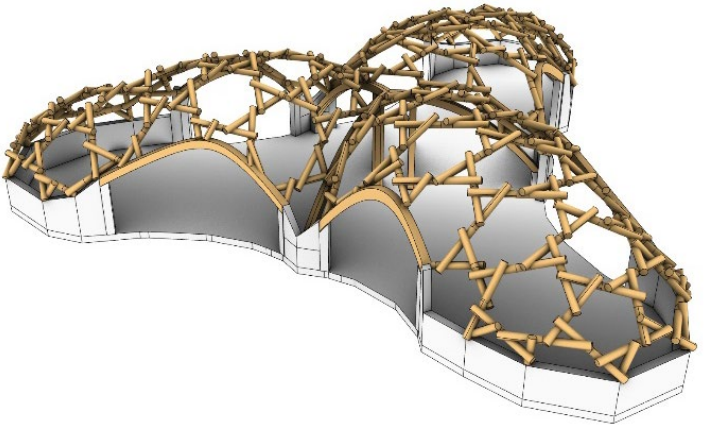

Figure 23. Rendered model of RF villa by SIDstudio

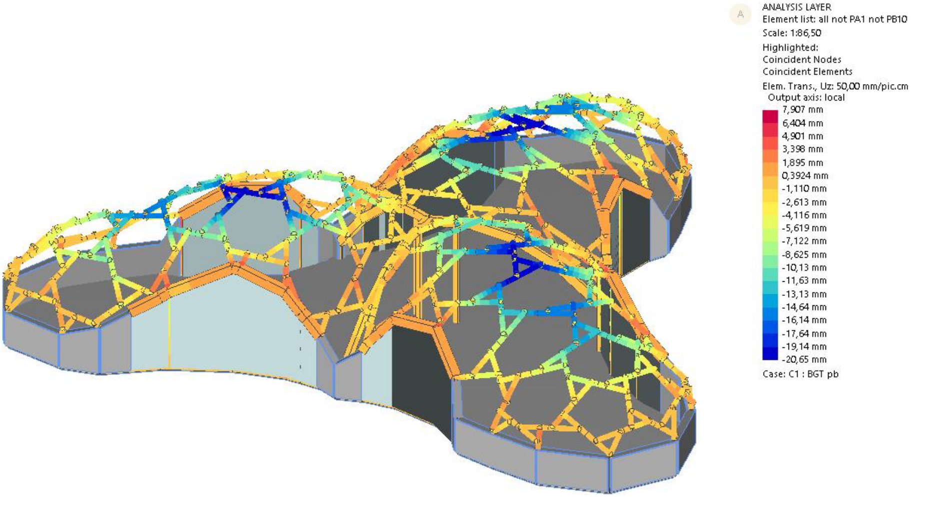

Figure 24. FE model of RF villa by SIDstudio

In 2020, the RFD research was implemented in a design competition for artwork during the 2020 Burning Man festival in Nevada (see figure 25). The project has not been built yet. When funding allows it to be build it will be a full-scale test model of the RSD from design to production including assembly.

For more research information see:
https://www.sidstudio.nl/timber-reciprocal-frame-structures/

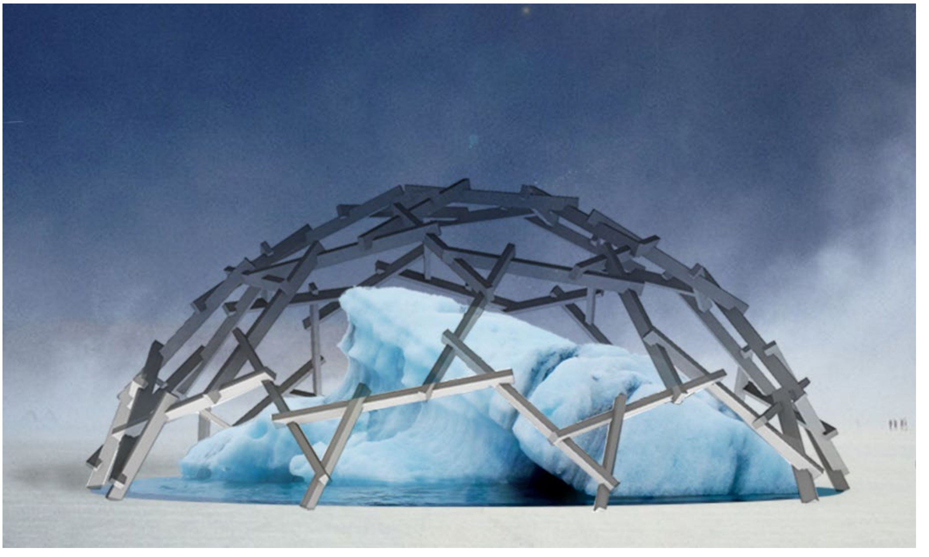

Figure 25. Concept Recip-ICE for the 2020 Burning Man proposal (Image by Dex Weel)

7 Conclusion

This paper presents the development of a design tool for RF structures (RFD), including a design to production process with timber detailing. The practical models prove that the proposed designs can be transferred to build examples able to satisfy building regulations and allowing direct manufacturing and construction. Computational design with controllable parameters allows for complex optimization processes to control RF structures. The parameters can be directly optimized through a structural designers common sense. Despite the fact that an RF transfers its forces mainly through bending, the proposed connection typologies result in relatively simple connections but may require some complex milling. No steel plate connectors are needed; the RF can be completely made in timber, reinforced with self-drilling screws. The improved detailing allows for easier construction of RFs and implements the discussed **force follows fiber** principle.

Although the results are promising, implementation and roofing have not been fully developed. Furthermore, the RFD is only applicable to three- and four-member RFs. Future work is therefore encouraged by a need for applications and should concentrate on implementation, roofing, and surface subdivisions especially suited for RF design in order to create greater design variety (see figure 26). Nevertheless, this research aims to encourage new applications by making its design accessible to structural designers.

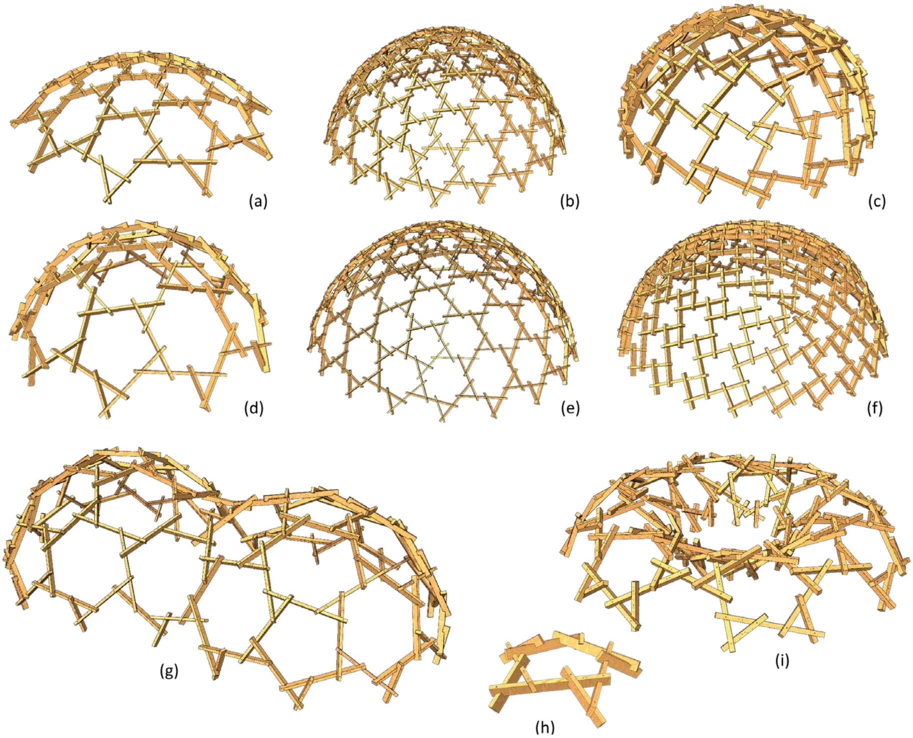

Figure 26. Nine possible RFD results covering the same span based on geodesic icosahedra (a, b, d, e), cubes (c, f)

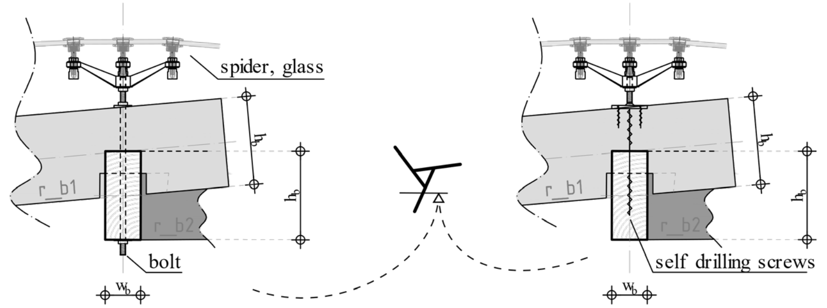

Figure 27. Concepts for roofing on an RF structure

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